The Equilibrium Effects of Campaign Finance Deregulation on US Elections

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Abstract

The US Supreme Court's 2010 decision in *Citizens United v. Federal Election Commission* was a major deregulation of campaign finance law. A new type of political action committee emerged, coined the Super PAC, with a relatively unfettered ability to raise money. This led to an unprecedented rise in spending across the general and primary elections. I estimate the influence of Super PACs on Congressional elections with a multi-stage model of political competition. I find that Super PACs have muted equilibrium effects due to political competition but, on average, have helped Republicans. Super PACs slightly increase challenger entry and pressure opposition incumbents to moderate positions.

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1 Introduction

Campaign contributions are an integral part of U.S. elections and allow citizens to support candidates. The rules that govern these contributions, such as donor limits and corporate restrictions, were upended in the 2010 decisions Citizens United v. Federal Election Commission (FEC) and SpeechNow v. FEC. The latter case, relying on the former, created a new kind of political action committee (PAC), the "Super PAC". Unlike the existing traditional PACs, Super PACs can receive unlimited contributions per donor. Super PAC entry and subsequent spending, shown in Figure 1, reveals their potential impact; they are a major force in general elections and dominate primary election spending by non-candidate committees. Super PACs can be thought of as a new political fundraising technology that transform the preferences of select donors into electoral influence. Proponents of the Court decisions argue that election spending is akin to free speech and that "outside money" provides a counterweight to established political parties. Opponents fear that corporations and wealthy donors will have undue influence on elections. However, it is an open question whether and how this wave of money into national elections actually alters outcomes.

Cycle Cycle Super PAC General Spending Super PAC Primary Spending PAC Primary Spending PAC General Spending Party General Spending Party Primary Spending

Figure 1: House Election "Outside Committee" Ad Spending (in Millions)

The left (right) graph shows total general (primary) election ad spending by Super PACs, PACs, and party committees from 2002-2018.

¹The ruling also allowed them to accept corporate and union donations; details in Appendix A.1.

²Super PACs have supported House challengers with more than \$377 million since 2018, but Super PACs helping incumbents have spent over \$202 million. Corporate political spending has not significantly increased and Super PACs spending is largely due to donations by wealthy individuals.

In this paper, I analyze how Super PACs affect Congressional primary and general elections. I investigate how their ad spending influences voting behavior, spending by other committees, candidate platforms, and candidate entry decisions. I model a multistage game: candidates make entry and policy decisions, then committees make sequential entry and spending decisions for the primary election, then entry and spending in general elections. Voter choose their preferred candidates after each election. The model captures the dynamics within a given election cycle and the collective efforts of candidates, parties, traditional PACs, and Super PACs.³ I allow for heterogeneity along multiple dimensions, such as spending effectiveness and fundraising constraints. I first estimate the effect of candidate and committee decisions on voters and then estimate the equilibrium conditions for those decisions using backward induction to incorporate forward-looking behavior. It is vital to include the actions prior to general elections, such as primary elections and candidate entry, as any counterfactual scenario studying Super PACs should not hold these fixed. In particular, the candidates that make it to the general election are not selected at random, and the winners for non-swing districts are largely determined in the primary. Super PACs' full impact can go beyond altering general election vote shares; they operate in a strategic environment and I account for their direct and indirect influence on entry, policy, primaries, and the spending of others. Furthermore, as more districts are becoming "safe seats," meaning the general elections strongly favor one party (Kustov et al. 2021), primary elections are increasing in importance.

A key challenge is dealing with candidate unobservables. The general election winner, general election loser, primary election losers, and potential candidates who did not enter may differ in the eyes of voters in unobserved ways.⁴ To account for the unobserved heterogeneity across candidates that faced each other in an election that affects observed spending, I use exogenous variation in committee budgets from shocks to their donors that end up affecting spending.⁵ An additional important factor to account for is that forward-looking candidates' decisions at any point in the election are influenced by their expectations of what will occur in later stages. This

³It is most common to analyze one stage of the election and with one player per side (Strömberg 2008; Shachar 2009; Gordon and Hartmann 2016; Incerti 2018; Limbocker and You 2020).

⁴More generally, there is strong selection into political office (Fowler 2016; Lim and Snyder 2021).

⁵Other approaches include using lagged advertising prices as instruments (Stratmann 2009; Chung and Zhang 2020; Gordon and Hartmann 2016), discontinuities of district/media market (Strömberg and Snyder 2010; Spenkuch and Toniatti 2018; Wang 2018), repeat challengers (Levitt 1994), and lagged votes/spending (Green and Krasno 1988).

involves unobserved match-ups between candidates that must be characterized to understand each candidate's strategy. I exploit the dynamic model structure to deal with this, which is only feasible because I jointly estimate the general and primary elections. Finally, to account for the unobserved selection of candidate entry, I proxy for the difference in unobserved candidate quality by comparing the election records of State Legislature members who decide to enter versus those who do not. This provides an outside source of information to quantify the extent to which those who run for Congress have unobserved characteristics that predict electoral success.

My main results indicate that Super PACs slightly increase overall spending and help Republicans in general elections, with heterogeneity across candidates. I also find shifts in candidate entry and platforms: Super PACs promote Republican challenger entry and have moderating effects on Democratic incumbents. While Super PACs influence candidates within their own party, opposition Super PACs exert more pressure on candidate policies. Importantly, Post-Citizens United spending exhibits intense competition from both parties, largely canceling out effects. This competition reflect strategic response, not necessarily unilateral incentives. I establish this by simulating a Super PAC ban that only affects one party; this leads to lopsided effects, indicating that being outspent is a legitimate concern for candidates and donors. I simulate further restrictions to study spending incentives and free-riding. Finally, I quantify the bias from ignoring equilibrium adjustment and discuss welfare.

Concerns over Super PACs are driven by their large expenditures, but their effects in equilibrium are muted because both sides in the U.S. political duopoly utilize them. Overturning the court cases may not drastically change the electoral landscape and I find that the "independent" nature of Super PACs is not a hindrance to their effectiveness. My framework illustrates that with strategic candidates and known donors, explicit coordination is not required for Super PACs to influence candidates.

I contribute to the literature by estimating a comprehensive campaign finance model that differentiates between candidate and "outside" spending, includes within-election dynamics, and allows for entry and candidate policy choice.⁶ I provide analysis of Super PACs in national elections using a novel approach with counterfactual

⁶The dynamics extend Adams and Merrill (2008). This differs from other within-election games (Klumpp and Polborn 2006; Denter and Sisak 2015; Roos and Sarafidis 2018; Acharya, Grillo, and Sugaya 2018) or between-election dynamics (Polborn and Snyder 2017). Kawai and Sunada (2015) has between-election war-chest building and some within-election facets (abstracting away from policy, donors, outside spending, and primary-contested incumbents).

simulations on their effects. Primaries and competition between committees are often overlooked, and I highlight their importance. This paper relates to the work on spending in elections, primaries and candidate entry, "outside" influence and donors, and the new literature on *Citizens United* and Super PACs.⁷

There is little work on Super PACs in national elections,⁸ and state election evidence suggests they helped Republicans win more state legislature seats (Klumpp, Mialon, and Williams 2016). The large effects found in the state-level literature are not necessarily predictive of what will happen on the national stage. There are differences in spending, policy issues, and other variables that affect each environment. For example, state-level candidates raise substantially less money than federal candidates, which allows outside groups like Super PACs to more easily affect the outcomes of the former. For the state-level analysis, identification stems from variation prior to 2010 in state campaign finance laws (Werner and Coleman 2014).⁹ That strategy is not feasible with national elections, and my integrated approach controls for unobservables and equilibrium adjustment across multiple dimensions.

My methodology contributes to and builds on the literature on estimation of election and contest models (Coate and Conlin 2004; Diermeier, Keane, and Merlo 2005; Strömberg 2008; Bombardini and Trebbi 2011; Kawai and Sunada 2015; Gordon and Hartmann 2016; Kang 2016; Sieg and Yoon 2017; Iaryczower, Moctezuma, and Meirowitz 2017; Garcia-Jimeno and Yildirim 2017; Huang and He 2021). This paper also contributes to the literature on modeling political markets (Mulligan and Tsui 2008; Dyck, Moss and Zingales 2013). A model-based approach is important for empirically evaluating how elections would unfold without Super PACs. Simply comparing outcomes pre and post 2010 is not ideal as each election has different candidates, committees, donors, and voters, all of which respond to the policy change. Furthermore, entry, policy, and spending are functions of election specific unobservables. Finally, estimating a single stage in isolation or not controlling for strategic responses ignores equilibrium effects and biases counterfactuals.

The paper continues as follows: I start with the model in section 2, describing

⁷This includes the drivers of candidate ideology (Boleslavsky and Cotton 2015; Baker 2016b).

⁸There is a growing body of descriptive work (Hansen, Rocca, and Ortiz 2015; Baker 2016a; Barutt and Schofield 2016; Boatright, Malbin, and Glavin 2016; Miller 2017).

⁹Many use that same variation (Hamm, Malbin, Kettler, and Glavin 2014; Spencer and Wood 2014; Abdul-Razza, Prato, and Wolton 2020; Harvey and Mattia 2019; Petrova, Simonov, and Snyder 2019; Gilens, Patterson, and Haines 2021).

each stage of the game. I follow with detailing the empirical environment and data in section 3. I discuss the identification and estimation in section 4. The model has parameters that I estimate stage by stage, including voter preferences in the general and primary elections, committee preferences in the general and primary elections, and parameters that govern candidate entry and policy decisions. Section 5 discusses the parameter estimates. Section 6 studies various counterfactuals on how the elections would change if Super PACs never existed. I conclude in section 7.

2 Model

I propose a theoretical framework for studying multi-stage two-party elections. A model is useful to quantify the effects of Super PACs on electoral competition and analyze how the elections outcomes could change without Super PACs. This model captures the direct and indirect channels through which Super PACs influence the election, from the initial entry and policy-platform decisions by candidates to the general election voters' choices. I estimate the model parameters, so that the endogenous decisions can be re-solved for in the counterfactual, holding the parameters fixed.

The two principal groups in this environment are candidates and voters: candidates choose policy platforms and voters choose their preferred candidate. The two broad groups in the background are election committees and donors: committees spend money to help candidates win and donors supply these committees with campaign contributions. The main committees are the campaign committees, political party committees, traditional PACs, and Super PACs, each with spending and fundraising limitations. I describe additional institutional details in the data section.

2.1 Model Setup

The game environment is as follows: There are two sides, Republican and Democrat, competing to win a Congressional seat. Candidates make policy and entry decisions prior to the election and committees raise and spend money to help candidates win. For exposition, let there be a Republican incumbent. Bold notation denotes vectors.

There are three main classes of players: First candidates: $c \in \{R_1, R_2, D_1, D_2\}$, where R_1 is the Republican Incumbent, R_2 is the Republican Primary Challenger, D_1 is the first Democratic Primary Challenger, and D_2 is the second Democratic

Primary Challenger. Second there are committees (campaign, parties, PACs, and Super PACs) aligned to each candidate: let $i_c \in N_c$ refer to a committee aligned with candidate c; N_c is the set of committees aligned to candidate c. Third, there are many voters v for each side in the primary and the general.

The actions take place over four stages. Actions from previous stages are observed by players. First, the incumbent chooses a (policy) position in a discrete finite space with $d_I \in \Theta \subset \mathbb{Z}$ and $d_I = 0$ indicates they will not seek re-election. The nonzero positions can be interpreted as a political scale of left-to-right or extremism within party and capture how voters and donors perceive candidates. Second, the challengers decide whether to enter the election or not and choose a position d_c . Third, committees (other than the candidate's committee) make primary entry decisions $a_{i_c}^P \in \{0,1\}$. Afterwards, the committees decide how much to spend $S_{i_c}^P \geq 0$. Then, the primary voters (on each side) vote and a winner is decided $w_c^P \in \{0,1\}$ for both Republican and Democratic primaries. Fourth, the committees (including those who may not have entered the primary) make general entry decisions $a_{i_c}^G$. They then choose how much to spend $S_{i_c}^G$. Finally, voters vote to determine a general election winner w_c^G .

I distinguish between committees and donors. Donors are not absent, but engage indirectly through committees. I show how committee spending can be derived from a stage with committees choosing fundraising effort targeting donors (Appendix A.2). Since only the spending is payoff relevant to voters, I focus on that for the exposition.

2.2 Model Parameterization

I describe the payoffs in this section, going through each stage, starting at the end. The various distributional and functional forms chosen throughout are common in the discrete choice literature, and make the model tractable for estimation.

2.2.1 Voter Choice and Election Outcomes

Consider the final stage; a general election voter v chooses candidate R, D, or not to vote. Their utility from voting for candidate c, U_{vc} , is given in equation (2.1) and inspired by Gordon and Hartmann (2016). It is a function of campaign spending, exogenous observables, and private information. The spending $S_{i_c}^G \geq 0$ is by committees $i_c \in N_c$ supporting candidate c and has corresponding effectiveness pa-

rameters $\beta_{ic} \geq 0$ and $\phi \in (0,1)^{10}$ The utility per candidate is also affected by k observed exogenous district-level characteristics $\mathbf{X}_c^G \in \mathbb{R}^k$ and the policy/position choice d_c , which interact in the function $h^G(\mathbf{X}_c^G, d_c)$. The voter's policy bliss point is $\operatorname{argmax}_{d_c} h^G(\mathbf{X}_c^G, d_c)$, holding all else fixed. Thus spending can shift the voter to support a candidate beyond what policy can achieve on its own.

The unobservables include unobserved candidate-election characteristics $\xi_c^G \in \mathbb{R}$, called candidate valence, and voter private information $\varepsilon_{vc} \in \mathbb{R}$. The utility of abstention is $U_{v0} = u_0^G + \varepsilon_{v0}$. It is standard to set u_0^G to zero, but other normalizations may be appropriate, provided they do not affect the identification of ξ_c^G or the equilibrium.

$$U_{vc} = \underbrace{\sum_{i_c \in N_c} \beta_{i_c} (S_{i_c}^G)^{\phi} + h^G(\mathbf{X}_c^G, d_c)}_{u^G} + \xi_c^G + \varepsilon_{vc}$$
(2.1)

Voters observe everything except other voters' idiosyncratic shocks. Committees do not observe $\{\xi_c^G, \varepsilon_{vc}\}_{\forall v,c}$, but know their distributions. Voters observe ξ_c^G because it includes how voters perceive candidates and shocks that occur during the election up to election day that affect the voter's decision. While a voter does not know what their neighbor thinks, captured in ε_{vc} , it is reasonable to let them know the district-candidate level factors. Committees and candidates make their spending and policy decisions early enough in the election such that ξ_c^G is not exactly known at the time.

The voter has information on each candidate $\{\mathbf{X}_c^G, \xi_c^G, \varepsilon_{vc}\}$; policy and spending further affect their decision. To pivot from the voter's perspective to the committee's, construct the share of votes and the probability of winning. Let the voter's private idiosyncrasies ε_{vc} be independently and identically distributed (iid) Type 1 Extreme Value with location zero and scale one, T1-EV(0,1).¹¹ Then the share of votes s_c^G is the following for \aleph number of candidates (see Appendix Lemma 1 for details):

$$s_c^G = \frac{\exp(u_c^G + \xi_c^G)}{\exp(u_0^G) + \sum_{\iota=1...\aleph} \exp(u_\iota^G + \xi_\iota^G)}.$$
 (2.2)

Then, under a plurality rule, candidate c wins if $s_c^G > s_n^G \ \forall n \neq c$. For two candidates, the win indicator for candidate R is $\mathbb{1}[s_R^G > s_D^G]$, which is equivalent to

¹⁰The $\phi = 1$ case leads to perfect substitutability; only one player per side ever spends.

¹¹The standard Type 1 Extreme Value distribution is a continuous distribution with pdf $f(x) = \exp(x) \exp(-\exp(x))$. The difference in two T1-EV(0,1) follows a logistic distribution.

 $\mathbbm{1}[u_R^G + \xi_R^G > u_D^G + \xi_D^G]$. Committees may not perfectly know how voters will perceive candidates and thus have beliefs over the unobserved candidate shocks. I assume their beliefs are that $\xi_c^G \stackrel{iid}{\sim} \text{T1-EV}(\psi_c^G, \sigma_{\xi})$. Then the expected value of winning is a probability $P(w_R^G = 1 | \mathbf{w}^P)$ from the committee's perspective: 12,13

$$P(w_R^G = 1 | \mathbf{w}^P) = \frac{\exp((u_R^G + \psi_R^G)/\sigma_{\xi})}{\sum_{c \in \{D, R\}} \exp((u_c^G + \psi_c^G)/\sigma_{\xi})}.$$
 (2.3)

2.2.2 Committee Strategy

The general election committee spending program is given in (2.4). A committee's value associated with winning is V_{i_c} . Their spending constraint is g_{i_c} : spending on ads has a marginal cost associated with raising the sufficient funds. ¹⁴ This definition of g_{ic} naturally arises from embedding a model of donors giving to committees into this stage; I derive this in Appendix A.2. I let valuations and costs be functions of exogenous covariates $\mathbf{X}_{i_c}^V$ and $\mathbf{X}_{i_c}^g$, and allow the cost to vary with candidate positions d_c and unobserved cost shocks $\gamma_{i_c}^G$: $V_{i_c} = \exp(\mathbf{X}_{i_c}^V \varrho_c)$ and $g_{i_c} = \exp([\mathbf{X}_{i_c}^g, d_c]^\top \varphi_c^G + \gamma_{i_c}^G)$.

$$\max_{S_{i_c}^G \in \mathbb{R}_+} V_{i_c} \cdot P(w_c^G = 1 | w_c^P = 1, \mathbf{w}_{-c}^P) - g_{i_c} \cdot S_{i_c}^G$$
(2.4)

Before the general election, the committees make entry decisions. I model this as a standard entry game with private information in entry costs shocks (Seim 2006). Committees choose entry a_{ic}^G to maximize the expected payoff with a belief about the probability of other committees entering $p_i(\mathbf{a}_{-i_c}^G)$. I allow for private information in payoffs, $\lambda_{ic}^G \stackrel{iid}{\sim} \text{Logistic}(0,1)$. Committees have equilibrium beliefs over the entry decisions of others. Let $\pi_{i_c}^G = V_{i_c} \cdot P(w_c^G = 1|\cdot) - g_{i_c} \cdot S_{i_c}^G$. The expected payoff for a given entry decision $u_{i_c}^G(a_{i_c}^G|\cdot) - \lambda_{i_c}^G a_{i_c}^G$, integrates over these beliefs. $N = \dim\{N_c\}$. The summation is across all 2^{N-1} combinations of decisions $\mathbf{a}_{-i_c}^G$; denote the belief by committee i_c in the probability of committee j choosing a_j^G from the decision profile $\mathbf{a}_{-i_c}^G$ with $p_j(\mathbf{a}_{-i_c}^G)$, where $-i_c$ refers to committees except i_c . The entry program is

¹²Rewrite ξ_c^G in terms of a T1-EV(0,1) random variable $\xi_c^* = (\xi_c^G - \psi_c^G)/\sigma_\xi$, so $\xi_c^G = \xi_c^* \sigma_\xi + \psi_c^G$. Rewrite that as: $\mathbbm{1}[u_R^G + \xi_R^* \sigma_\xi + \psi_R^G > u_D^G + \xi_D^* \sigma_\xi + \psi_D^G] \implies \mathbbm{1}[(u_R^G + \psi_R^G)/\sigma_\xi - (u_D^G + \psi_D^G)/\sigma_\xi > \xi_D^* - \xi_R^*]$.

¹³This is only for a plurality voting rule. A majority rule could use $P = \exp(-\exp(s_c - 0.5))$ with a runoff. 2 states have majority rules for the general; 11 have it for the primary. 3 states currently use open primaries. I exclude unique designs (Louisiana) and use the run-off as the "main" election.

¹⁴Using implicit costs to capture contribution limits is an alternative to from explicitly modeling constraints (Avis, Ferraz, Finan, and Varjao 2022; Maloney and Pickering 2018).

given in (2.5), where S^* is the vector of optimal spending for a given entry profile.

$$\max_{a_{i_c}^G \in \{0,1\}} u_{i_c}^G(a_{i_c}^G | \mathbf{p}_{-i_c}) - \lambda_{i_c}^G a_{i_c}^G \ni u_{i_c}^G = \sum_{\mathbf{a}_{-i_c}^G \in \{0,1\}^{2N-1}} \pi_{i_c}^G(\mathbf{S}^* | a_{i_c}^G, \mathbf{a}_{-i_c}^G) \prod_{j \neq i_c} p_j(\mathbf{a}_{-i_c}^G) \quad (2.5)$$

The committee's primary private information is $\lambda_{ic}^P \stackrel{iid}{\sim} \operatorname{Logistic}(F_{i_c}^P, 1)$, where $F_{i_c}^P \geq 0$ is a common knowledge entry cost mean.¹⁵ Let $\pi_{i_c}^P = V_{i_c} \cdot E[P(w_c^G = 1|\cdot)] - g_{i_c}^P S_{i_c}^P$ and $u_{i_c}^P - \lambda_{i_c}^P a_{i_c}^P$ be the expected payoff. The entry program is $\max_{a_{i_c}^P \in \{0,1\}} u_{i_c}^P (a_{i_c}^P | a_{-i_c}^P) - \lambda_{i_c}^P a_{i_c}^P$. Note that spending has a public good aspect; a committee can have a non-entry positive payoff because the win probability is not necessarily zero with non-entry.

The previous stages are repeated in the primary election, but the committees now use the expected outcome of the general election: $E[P_c^G|\mathbf{w}^P] = \sum P_c^G(\mathbf{a}^G) \prod_j p^*(a_j^G)$, where $P_c^G(\mathbf{a})$ is the win probability from equation (2.3) evaluated at the equilibrium spending levels \mathbf{S}^* for a given entry profile, $p^*(a_j^G)$ is the equilibrium probability of that entry profile, the summation is over all combinations of \mathbf{a}^G . For the Republican side, the program is given in (2.6), where $c \in \{R_1, R_2\}$. Note that they take into account the primary in which they are spending, but also the opposition party's primary, and the possible general elections for each opponent they might face.

$$\max_{S_{i_c}^P \in \mathbb{R}_+} V_{i_c} \cdot \left[P(w_c^P = 1) E[P(w_c^G = 1 | w_c^P = 1 \cap w_{D_2}^P = 1)] \cdot P(w_{D_2}^P = 1) + P(w_c^P = 1) E[P(w_c^G = 1 | w_c^P = 1 \cap w_{D_1}^P = 1)] \cdot P(w_{D_1}^P = 1) \right] - g_{i_c}^P S_{i_c}^P$$
(2.6)

Before the primary election, the committees make entry decisions in an analogous manner as in the general election. A key aspect to the behavior in the primary is that committees are forward-looking and predict what may happen in hypothetical general elections at the time of making their primary spending decisions.

2.2.3 Candidate Strategy

Prior to the primary, the potential challengers make entry decisions alongside policy positions. I write the program for all challengers in (2.7), based on the probability of winning the overall election minus their costs. Let V_c be the value to candidate c of winning, V_c^0 be the outside option, and $\bar{\theta}_c$ be the ideal position point. The two

¹⁵Committees in the primary do not observe the private shock for the general election, and a committee does not observe its own private shock in the general until reaching it.

valuations are functions of exogenous covariates \mathbf{W} and \mathbf{W}^0 , respectively. Let η_{d_c} be private variation in payoffs per choice, where $\eta_{d_c} \stackrel{iid}{\sim} \text{T1-EV}(0,1)$. The probability of winning the general election from the challenger's perspective, $E[P_c^G|\mathbf{d}]$, is an expectation over both the general and primary election committee equilibrium entry. This is key to understanding the main tension a candidate faces: they must balance the effects of a policy on voters and committees in the primary and general elections.

$$\max_{d_c \in \Theta} V_c \cdot E[P_c^G | \mathbf{d}] + V_c^0 \cdot (1 - E[P_c^G | \mathbf{d}]) - (d_c - \bar{\theta}_c)^2 \cdot \mathbb{1}[d_c \neq 0] + \eta_{d_c} \quad \forall c \in \{R_2, D_1, D_2\}$$
(2.7)

Finally there is the first stage in which the incumbent I chooses a position. The expected win probability is now defined as $E[P_I^G|d_I] = \sum E[P_I^G(d_I|\mathbf{d}_C')] \prod p(\mathbf{d}_C')$, taking an expectation over the equilibrium distribution p of challenger decisions \mathbf{d}_C , and where the summation is over $\mathbf{d}_C' \in \dim\{\Theta\}^{|\mathbf{d}_C|}$.

$$\max_{d_I \in \Theta} V_I \cdot E[P_I^G | d_I] + V_I^0 \cdot (1 - E[P_I^G | d_I]) - (d_I - \bar{\theta}_I)^2 \cdot \mathbb{1}[d_I \neq 0] + \eta_{d_I}$$
 (2.8)

The extent to which committees, like Super PACs, affect policies as opposed to simply electing candidates of an unchanged policy (Lee, Moretti, and Butler 2004) can be separated by observing various aspects of the model. First, one can map out the equilibrium response of candidate policy with respect to committee influence parameters, such as the spending effectiveness. Second, the gap between the incumbent's ideal and their chosen policy tells us how far they deviated; counterfactual analysis can parse out whether the voters, challengers, or committees drove that policy gap.

2.3 Model Discussion

I solve the game with backward induction. Proposition 1 addresses equilibrium existence (proof in Appendix A.12). I discuss uniqueness in Appendices A.3 and A.12.

Proposition 1. There exists a pure strategy Bayesian Nash equilibrium in which all agents condition on payoff relevant actions.

The *Citizens United* case affected this environment in multiple ways, and I focus on how Super PACs entered the game with possibly different valuations, costs, and effectiveness. Their presence may affect spending, candidate decisions, and election outcomes. The campaign finance laws that each committee is subjected to show up

$${}^{16}E[P_c^G|\mathbf{d}] = \sum_{\mathbf{a}^P \in \{0,1\}^{4N}} \left[\sum_{\mathbf{a}^G \in \{0,1\}^{2N}} P_c^G(\mathbf{a}^P|\mathbf{a}^G,\mathbf{d}) \prod_j p_j^*(a_j^G) \right] \prod_j p_j^*(a_j^P).$$

PACs to have lower costs since they are unrestricted, but that need not always be the case: fundraising efficacy is a function of factors beyond the donation limit. Some candidates outraise Super PACs despite having strict limits per donor. Super PACs are simply a new entity that have the potential to raise and spend well beyond what was previously possible. One concern is that Super PACs may be playing a "long game" across elections. While they may have long-term goals, their decisions per election cycle are still aimed at affecting the immediate election.¹⁷ My framework captures these aspects. Next, I describe the data to match the elements in the model.

3 Data

3.1 Voting and Candidate Data

The main election outcome I study is the share of votes a candidate receives. The primary, runoff, general, and general runoff election data are from the FEC, and I use data from the 2002-2018 cycles for House elections. In primaries, incumbents win re-election at a high rate and uncontested primaries were the norm prior to 2010. The number of contested primaries increased during 2010 and stayed high afterwards, as shown in Figure 2. The 2010 surge was largely driven by the "Tea-Party" movement in which establishment Republicans faced a higher rate of contested primaries.

My measure for candidate policy/position/platform/ideology comes from Bonica (2014). This commonly used measure is based on a spatial model of donors where they contribute to candidates to whom they are ideologically aligned. Bonica uses correspondence analysis to construct the "CF-scores" based on the network of donors and recipients. See Appendix A.4 for more details on the voting and policy data. The scores are available for candidates that received donations. Practically all candidates fit between -4 and 4, where -4 is most liberal, 0 is in the middle, and 4 is most conservative. Figure 3 shows the distribution of scores for pre and post (including) 2010. The distribution is slightly wider post 2010, indicating higher polarization. The peaks around -1 and 1 are due to most candidates maintaining a moderate position.

 $^{^{17}\}mathrm{Others}$ also study Super PAC strategies (Dwyre and Braz 2015; Herrnson et al. 2018).

 $^{^{18}\}mathrm{An}$ alternative measure is based on Congressional voting records (DW-NOMINATE scores), and is insufficient for this analysis as it is only observed for incumbents with a voting record. I find that the correlation between DW-NOMINATE and CF-scores is 93.46% among House incumbents.

Figure 2: Primary Entry

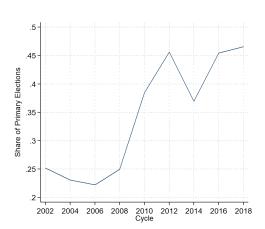


Figure 3: Candidate Positions

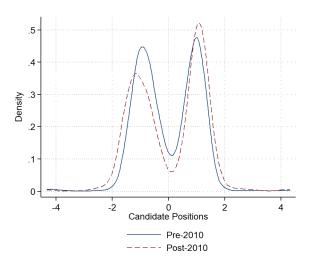


Figure 2 shows the share of contested elections from 2002-2018: at least one primary opponent in a primary election divided by all of the races in that election cycle. Figure 3 shows the distribution of candidate positions for elections prior to 2010 and post (including) 2010, based on Bonica's score. -4 is most "left-wing" (liberal) and 4 is most "right-wing" (conservative).

There is a local trough at 0 as most candidates are slightly positioned on one side.

Republican incumbents with a primary challenger are slightly more extreme than the unopposed. For all Republican candidates, less extreme candidates are generally more likely to win the primary. The mean position for Republican incumbent primary winners is more extreme than for incumbent losers, but there are few incumbent losers. Candidates that are outspent are more likely to lose and the variance increases with position; for more extreme candidates, large spending gaps may be necessary to win.

3.2 Committee Data

Political action committees are formal entities, regulated by the FEC, that can raise and spend money in elections. PACs support candidates through multiple channels: they donate money to the candidate's campaign committee, rally supporters, and spend on "communications" in support or opposition of a candidate. Direct contributions to a given candidate have strict limits that prevent a single PAC from "buying" too much influence. Individuals can give only a few thousand dollars to a PAC per election cycle; party limits are slightly higher.

Prior to 2010, non-PAC groups such as corporations, nonprofits, unions, and trade associations were limited in their ability to spend in elections. They could form

their own PAC, but they could not donate money directly nor make ads targeting candidates. Ads targeting candidates but not coordinated with the candidate or party are called "independent expenditures" (IEs). The 2010 case Citizens United v. FEC allowed these non-PAC groups to make independent expenditures. A following 2010 case SpeechNOW v. FEC allowed individuals and corporations to donate unlimited amounts to IE-only PACs (coined Super-PACs). Super PACs cannot give money directly to candidates and they are only allowed to receive unlimited contributions for the purpose of financing IEs and some other independent political activity. Super PACs spend the vast majority of their money on advertising with the remaining funds on transfers and administrative/fundraising costs. Other kinds of PACs, such as traditional or leadership PACs (e.g. Save America PAC) can be directly affiliated with a candidate; their expenses are often not primarily advertising based. See Appendix A.1 for additional information and related campaign finance issues.

I link each "outside" committee (PAC, Super PAC, party) to the candidates they support, combine that with donor data (discussed below) per district in which the committee is active. I distinguish between spending targeted in the primary and general. The FEC requires that IEs designate which candidate the ad is targeting and whether it supports or opposes the candidate. I combine ads supporting the candidate and attacking the opponent. See Appendix A.5 for data details.

Table 1 displays total general election ad spending in House election pre and post (including) 2010 for four committee types based on the party and incumbency status of the candidate they support. Presidential election cycles often have more Congressional spending as there are donor spillovers, and there are two sets of Presidential and non-Presidential cycles in both pre/post periods. Candidates consistently spend the most, and this is because there is candidate spending in every single race, whereas parties and Super PACs spend sporadically. Total spending increased since 2010 across all committee types, with the new \$497 million in Super PAC spending neary matching the total increase of \$681 million by candidates, parties, and PACs. Super PACs spend more on challengers than on incumbents and Republican incumbents have seen the smallest increase in spending since 2010.

While there are more Republican incumbents after 2010, the 2010 Congressional re-districting may have favored Republicans (Eguia 2021), leading to less competitive districts and less spending by incumbents to defend their seat. The large increase in Democratic incumbent spending is mirrored by the increase in Republican challenger

Table 1: Total General (Ad) Spending (in Millions)

	Democrat		Repu		
	Challenger Incumbent		Challenger Incumbent		Total
	Pre, Post	Pre, Post	Pre, Post	Pre, Post	Pre, Post
Candidate	238, 400	148, 261	152, 225	284, 345	823, 1230
Party	135, 175	26, 122	80, 179	98,113	339, 589
PAC	24, 23	6, 21	6, 13	14, 18	50, 75
Super PAC	0, 202	0, 48	0, 107	0,139	0, 497
Total	397, 800	178, 451	238, 524	397, 615	

This table show pre and post (including) 2010 total general election ad spending by candidate election committees and general election independent expenditures by parties, PACs, and Super PACs, separated by whether the committee is aligned with a Democrat or Republican candidate and whether the candidate is an incumbent or challenger.

spending as that is a common match-up for competitive races. In these, candidates, parties, and Super PACs have large spending expenditures. The substantial increase in total spending for Democratic challengers is largely driven by the 2018 elections, which saw unprecedented levels of fundraising for Democratic House challengers.

Traditional PACs (called PACs) are distinct from parties and Super PACs as they spend relatively little on independent expenditures and their main method is through giving money directly to candidates, especially incumbents. Despite their limitations in fundraising, their role has not necessarily diminished with the rise of Super PACs (Baker 2018), and thus I include their ad spending in the analysis. A major concern for parties, beyond retaining majorities, is re-electing incumbents. Their spending patterns align with these goals and they often focus on competitive races, such as districts with weak opposition incumbents and open seats in swing states. Super PACs are similar in that they spend large amounts in few but highly competitive races. Both will also occasionally spend in a safe race, often to challenge an important incumbent. Parties and Super PACs differ most in primary elections.

Table 2 shows total primary election ad spending for races with an incumbent. Prior to (and including) 2010, candidate committees dominated spending. This changed after 2010, when Super PACs started to spend; while their average is low, they can outspend candidates when they participate. Party and PAC spending have seen a downward trend in primaries. One explanation of this behavior is that parties are relatively ineffective spenders or have high primary costs. There may be some substitution from party to Super PAC spending as the decrease in spending to support Democratic incumbents by parties is closely matched with an increase by Democratic Super PACs. Republican Super PACs spend more in primaries than their Democratic

Table 2: Total Non-Open Race Primary (Ad) Spending (in Millions)

	Democrat		Repu		
	Challenger Incumbent		Challenger Incumbent		Total
	Pre, Post	Pre, Post	Pre, Post	Pre, Post	Pre, Post
Candidate	9, 12	45, 79	5, 22	52, 108	112, 222
Party	0, 2	8, 3	0, 1	4, 3	12, 10
PAC	2, 1	2, 5	1, 1	0, 3	5, 10
Super PAC	0, 3	0, 6	0, 11	0, 9	0, 29
Total	11, 19	55, 94	8, 34	56, 124	

This table show pre and post (including) 2010 total non-open race primary election ad spending by candidate election committees and primary election independent expenditures by parties, PACs, and Super PACs, separated by whether the committee is aligned with a Democrat or Republican candidate and whether the candidate is an incumbent or challenger. The terminology "Open Race Primary" is used to not confuse races without incumbents to "Open Primaries", a term commonly used for primaries in which party affiliation is not required.

counterparts, and there has been an increase in spending across all candidate types. The changes for primaries without an incumbent (open races) are even larger; candidate spending increased from \$167 to \$245 million, party spending decreased from \$22 to \$3 million, and Super PACs spent \$54 million since 2010.

The total spending statistics do not tell us about the strategic responses between committees, such as whether or not they mirror each other in which races they enter. When a committee helps a candidate, the opposing committees often match their spending. For example, if at least one Super PAC spends during the general, then in 94% of those races, at least one party committee or PAC would also spend. Also, Super PACs outspend parties in 66% of the races in which they spend. In primaries, Super PACs are the lone non-candidate spenders 43% of the time. Prior to 2010, parties were alone 73% of the time, which decreased to 37% after 2010. The primaries are becoming more crowded, but this could be due to either increased levels of participation or simply lower number of primaries spent in. Parties spent in about 7% of primaries before and after, and Super PACs spent in 13% after 2010.

3.3 Donor Data

Donors supply committees with campaign contributions. A committee's ability to spend is determined by how much they raise, which is influenced by their donors' financial well-being. Committees are thus vulnerable to shocks in their donors' in-

come/wealth.¹⁹ Super PACs are particularly sensitive as they can receive large contributions from a single individual. Contribution limits for candidates/PACs (\approx \$5,000) and parties (\approx \$35,000) force them to have a broader set of donors.

Donors are known because all political committees (those regulated under the FEC) are required to disclose the identities of their individual donors, including the donation amount, date, name, address, and employment information. I do not observe financial information of donors directly and instead consider IRS zip code level incomes (Gimpel, Lee, and Kaminski 2006). To gage donor ideology, I use donor Bonica CF scores from their historical donation record.²⁰ The donor data are primarily used to get variation in a committee's budget. How these data factor into estimating the effects of Super PACs can be illustrated by combining the data with the model. Next, I estimate the model to understand the magnitude and direction of the effects.

4 Identification and Estimation

I estimate the parameters that govern preferences for voters (parameters from equation (2.1) for the general and primary elections), committees (parameters from programs (2.4) and (2.6)), and candidates (parameters from programs (2.7) and (2.8)). The main estimation steps are: 1. estimate voter preferences for general and primary elections; 2. estimate valuations and general election costs with general election optimality conditions; 3. estimate primary election costs using primary optimality conditions; 4. estimate challenger valuations and costs using entry and position variation; 5. estimate incumbent valuations, costs, and ideal points using position variation.

I assume that the observed data are in equilibrium and are selected from the same equilibrium across observations. Each estimation step is robust to multiple equilibria and the uniqueness conditions for calculating counterfactual outcomes can be checked ex-ante. Due to the across-stage dependencies, I estimate confidence intervals for counterfactuals and committee/candidate parameters with non-parametric bootstrap.

¹⁹The strength of this variation is based on the elasticity of campaign contributions, and the wealth elasticity of contributions by billionaires is significant (Bonica and Rosenthal 2015).

²⁰In Appendix A.10, I discuss other sources of donor variation, including address level housing characteristics of individual donors and billionaire donors' wealth.

4.1 Estimation Of General Election Voter Preferences

Voter preferences are captured by the spending effectiveness parameters β , observed candidate and district characteristic parameters in h, and unobserved candidate factors ξ ; each of these varies across the general and primary elections. Recall that the last term captures election day common value shocks and unobserved heterogeneity, collectively called candidate valence, and which committees and candidates know in expectation ψ_c^G for the general and ψ_c^P for the primary. The main threat to identifying voter preferences is ξ , which influences voters directly and affects committees and candidates through their choices. I address identification using instrumental variables for the endogenous spending and policy present in the vote share equation.

An ideal instrument for spending would be a shock to a committee's budget unrelated to the district; I use shocks to their donor base from outside the state. The intuition for how this works is illustrated in the model. Donors are influenced by committee fundraising effort, and the donations affect voters indirectly through election spending. A shock to a committee changes their ability to raise funds, which exogenously varies how much they spend.²¹ I differentiate between donors in and out of the state in which the committee is spending. Within-state donor shocks may correlate with a given district's electoral outcome, and factors affecting out-of-state donors are less related to a given district (Gimpel, Lee, and Pearson-Merkowitz 2008; Rhodes, Schaffner, and La Raja 2018).²² Variation based on outside donors is only conditionally exogenous as variables that affect the overall economy or political climate will affect all donors. The key is that conditional on the pre-spending controls, the variation explained from the outside is only related to the election through spending.²³

The excluded donor instruments that correlate with spending but not unobserved candidate quality differences in a given election include the change in out-of-state donor income and the variance in out-of-state donor ideology scores. The latter

²¹Shocks to donors may affect all of the committees to which they give, and that can be correlated across donors who are in similar areas or professions. These overlapping donors do not pose a problem as long as the shocks only influence voters via spending.

²²We do not observe a committee that was interested in spending but did not. Endorsement data could reveal this, but many committees do not report this. To define the IV in such cases, I use the within-state average for committees that are aligned with the same party-incumbency status.

²³A concern is that some committees do not rely on out-of-state donors; thus the IV relevance may vary across non-excludable dimensions like committee size or scope. The median number of states in which they get donations is 23 and 3.16% have donors from only one state. The average (dollar) share from each state is 10%, and the maximum share over all states is on average 52%.

affects spending ability as a high variance in donor ideology provides a fundraising challenge; a homogeneous donor base is easier to corral. I control for the within-state versions of these to justify the exclusion restriction; some of the within-state donors are the voters in that district. Since there is concern about behavior by large donors, I also use an inverse-donation weighted version of the IVs, which captures the variation in smaller donors, who may act differently (Bouton, Cagé, Dewitte, and Pons 2022).

To instrument for candidate position, I use lagged mean Senate incumbent positions from the state. The inclusion of lagged incumbent success, district characteristics, expected competitiveness, and state fixed effects controls for pertinent variables within the district such that out of district variation in recent state partisanship of other incumbents correlates with the candidate's policy choice, but does not otherwise affect the election odds in their race.²⁴ The exogenous variation in positions and spending across differential vote shares (accounting for turnout) identify β and h.

The spending effectiveness parameters β_{i_c} are pooled across committee types (candidate, Super PAC, and party/PAC), meaning there are three distinct spending coefficients for the general election. I specify $h^G(\mathbf{X}_c^G, d_c) = \mathbf{X}_c^G \delta_0 + \mathbf{X}_c^{G'} [d_c > 0] \delta_1 + \delta_2 |d_c|$ to capture heterogeneous preferences over party/partisanship and within-party policy, where $\mathbf{X}_c^{G'} \subset \mathbf{X}_c^G$. A policy $d_c < 0$ ($d_c > 0$) is equivalent to being Democrat (Republican), conditional on entry $d_c \neq 0$; this is reasonable as there are very few candidates whose ideology scores overlap across party. The candidate's policy choice is within-party extremism. To construct the estimating equation, I transform the vote share equation (2.2).²⁵ The log vote share is $\ln(s_c^G) = u_c^G - u_0^G + \xi_c^G + \ln(s_0^G)$, where s_0^G is the absenteeism share.²⁶ I estimate this with 2SLS using excluded instruments s_0^G is the residual.

$$\ln(s_c^G/s_0^G) = \sum_{i_c \in N_c} \beta_{i_c} (S_{i_c}^G)^{1/2} + \mathbf{X}_c^G \delta_0 + \mathbf{X}_c^{G'} [d_c > 0] \delta_1 + \delta_2 |d_c| - u_0^G + \xi_c^G$$
 (4.1)

and Endersby (2004) make a similar point on the usefulness of multinomial logit in voting research.
$$^{26}s_0^G = 1 - \sum_{c=1}^C s_c^G = 1 - \frac{\sum_{c=1}^C \exp(u_c^G - u_0^G + \xi_c^G)}{1 + \sum_{c=1}^C \exp(u_c^G - u_0^G + \xi_c^G)} \implies \ln(s_0^G) = -\ln(1 + \sum_{c=1}^C \exp(u_c^G - u_0^G + \xi_c^G)).$$

²⁴This approach is similar to Iaryczower, Moctezuma, and Meirowitz (2017). I choose Senate as that is less sensitive to local district variation; a downside is that it does not vary between districts or candidates within the state. Results are not sensitive to using average outside-of-district by-party lagged position of House candidates within the same state.

²⁵Some alternative specifications include interacting spending with covariates (which is difficult to instrument for) or random coefficients to allow for more flexible substitution patterns; Gordon and Hartmann (2016) note that the latter specification does not significantly change results. Dow and Endersby (2004) make a similar point on the usefulness of multinomial logit in voting research.

Since I use the ratio of candidate vote share to turnout in the dependent variable, differential turnout can have large effects on ξ_c^G . In an election with an expectation of a lopsided outcome, one may posit that voter turnout would be low. To formally incorporate such an aspect, one would have to specify a model where pivotality affects the voter's turnout decision, which has been shown to not match the data well (Coate, Conlin, and Moro 2008). I include a variety of controls that are predictive of lopsided outcomes. It is important to include control variables \mathbf{X}_c^G related to a candidate's expected election performance. The results are not sensitive to some of the controls, but their inclusion is meant to alleviate concerns about the exclusion restrictions.

I include numerous control variables, interacting them with incumbency status and party whenever there is heterogeneity across that dimension. I include district unemployment rate, income, and total unemployed (all interacted with incumbency), district high school graduation rate, mean age, racial and gender demographics, election day city precipitation (all interacted with party), incumbency status, party, Republican vote share from the last presidential race (interacted with party), the vote share of the district's last incumbent (interacted with incumbency), the number of Senate candidates running in the state, an open-race indicator, whether the governor has the same party as candidate, within-state donor (zip) income changes, within-state donor ideology variance, and safe-seat ratings (interacted with incumbency and party).²⁷ Finally, I include state and election cycle fixed effects, with the latter interacted with party and incumbency.²⁸ To account for relative advertising costs across media markets, I divide expenditures by local ad prices.²⁹ Some election structures, such as nonpartisan blanket primaries, are not well approximated with the model, and so I drop Louisiana, California after 2012, and Washington after 2008.

Super PAC ads are predominately negative in tone and the data suggests their spending may depress turnout in certain races. In general, attack ads may affect turnout (Malloy and Pearson-Merkowitz 2016) and some ads in the primary may be divisive. Thus to best fit the data, I allow the mean utility of abstaining u_0^G to

²⁷These are from assessments of incumbency weakness and the "safety" of the seat for the general election. Some years scraped from Cook's website and others generously shared by Jim Campbell.

²⁸Summary statistics for these variables are reported in Appendix Table A1.

²⁹Measured with cost-per-point using off-election year lagged prices. Generously shared by Gregory Martin for 2000-2008 (Martin 2019). I use SRDS for 2010 onward and impute missing. There is price variation across committees (Moshary 2020) and heterogeneous coefficients absorb the mean. I also include a cost estimate per committee type for ads per market (from the Wesleyan Media Project) to further control for heterogeneity in prices faced by committees.

be affected by Super PAC spending in the general election and Democratic primary (and party/PAC spending in Republican primaries). Since the turnout effect must be normalized to identify valences, I let Super PAC spending depress turnout and help the candidate equally. This normalization does not affect the equilibrium properties of the game, as the probability of winning remains unchanged. This is equivalent to using the negative of the opponent Super PAC's spending in the regression.

4.2 Estimation Of General Election for Committees

The parameters from equation (4.1) tell us the elements that influence voters directly. Next I estimate the remaining objects relevant to committees, namely the their valuation for winning the election and a cost function that varies across the general and primary elections. Recall the general election post-entry committee payoff: $\pi_{ic}(S_{ic}^G, \mathbf{S}_{-ic}^G) = V_{ic}P_c^G - g_{ic} \cdot S_{ic}^G$. This is a function of V_{ic} : the value to committee i of candidate c winning, P_c^G : the probability of candidate c winning the general election defined in equation (2.3) and a function of voter utility u_c^G and expected valence ψ_c^G for all general election candidates, and g_{ic} : the marginal cost of spending (fundraising constraints and donor preferences). I let the committee's expectation of a candidate's valence equal the (estimated) realized valence, $\psi_c^G = \hat{\xi}_c^G$. Without more assumptions, I cannot separately identify a committee's expectation of a given candidate's valence (Gordon and Hartmann 2016).³⁰ This is not restrictive as committees observe the full set of controls. The probability of winning P_c^G can then be calculated for the observed pair of candidates in the general election with one normalization on the variance of uncertainty of candidate valence: $\sigma_{\xi} = 1$ (see Appendix A.9).

Valuations and costs are not separately identified off post-entry spending variation alone: low committee spending could signal either low valuations or high costs. Separate identification is achieved by exploiting both spending and entry variation. Recall that $V_{ic} = \exp(\mathbf{X}_{ic}^V \varrho_c)$ and $g_{ic} = \exp([\mathbf{X}_{ic}^g, d_c]^\top \varphi_c^G + \gamma_{ic}^G)$, where γ_{ic}^G is unobserved cost heterogeneity. The vector \mathbf{X}_{ic}^V includes a constant, incumbency status of the candidate, year, lagged presidential votes, and the incumbent's tenure length relative to the state average, with all variables interacted with committee type, incumbency, and party. Allowing the coefficients to vary across party is important as

³⁰We only observe their single spending decision and a single election outcome. For separate identification, track how spending changes with new polls that allow committees to update expectations. This is difficult given House race polling data quality (see Appendix A.11).

there is asymmetry in motivations and behavior (Grossmann and Hopkins 2016).

While the valuation is exogenous, cost is a function of policy and unobserved heterogeneity. Also, one may argue that the value of winning is affected by outcomes of other races, particularly if a given race will swing the majority control of Congress. This concern is reduced by Incerti (2018), who studies party spending in House races with majority versus total-seat seeking models, finding evidence for the latter. Also, I control for aspects of seat importance like its safety and incumbent's tenure.

The vector $\mathbf{X}_{i_c}^g$ includes a constant, the number of senate candidates in the state (to measure competition for resources and state political activity), district voting age population, and average ad prices in the state that year, all interacted with committee type, incumbency status, and party. The estimator is based on the derivative of the committee's post-entry payoff for a given set of entrants: $V_{i_c}\partial P_c^G/\partial S_{i_c}^G - g_{i_c} = 0$. I rearrange this to set marginal benefit to marginal cost, and then isolate the marginal probability of winning. The observed candidate decision is a function of the unobservable $\gamma_{i_c}^G$ and I control for it using the policy IV. I estimate equation (4.2) and recover the unobserved marginal cost shock $\gamma_{i_c}^G$ for entrants.³¹

$$\log \left(\frac{\partial P_c^G}{\partial S_{i_c}^G} \right) = -\mathbf{X}_{i_c}^V \varrho_c + [\mathbf{X}_{i_c}^g, d_c]^\top \varphi_c^G + \gamma_{i_c}^G$$
(4.2)

The term $\partial P_c^G/\partial S_{i_c}^G = \beta_{i_c}^G \phi(S_{i_c}^G)^{\phi-1} P_c^G (1-P_c^G)$. The moment $E[\mathbf{Z_c}^\top \gamma_{i_c}^G | S^G > 0]$ identifies the ratio of valuations to costs with variation in the marginal effect of spending on the probability of winning for different levels of the instruments. This equation can only identify valuation coefficients that are excluded from costs, meaning it cannot separately identify variables in both. The costs are identified using the variation in estimated entry probabilities p_{i_c} and expected win probability from entry for a given value to cost ratio and expenditure. I rewrite the entry condition to form a moment $E[\mathbf{Z_c}^\top \gamma_{i_c}^G]$. See Appendix A.6 for the derivation from the entry stage.

³¹For non-entrants, I impute it by averaging across party and committee type. Relying on spending requires assuming that $\gamma_{i_c}^G$ do not systematically differ in unobserved ways across entrants and non-entrants; this is because the first order conditions do not hold with equality for non-entrants. For an alternative approaches, see Erikson and Palfrey (1998) or Box-Steffensmeier and Lin (1996).

4.3 Primary Election Estimation

I estimate the primary election analogs to general election parameters, except the valuation for winning the whole election. The key challenge for the primary is the presence of an additional unobservable, namely unobserved general election valences for primary losers. For the primary voter preferences, I mirror the general election approach to estimate spending effectiveness, policy effects, and primary valences ξ^P (letting $\psi_c^P = \hat{\xi}_c^P$). I estimate the Republican and Democratic primaries vote share equations separately without exclusions: $\ln(s_c^P/s_0^P) = \sum_{i_c \in N_c} \beta_{i_c} (S_{i_c}^P)^{1/2} + h^P(\mathbf{X}_c^P, d_c) - u_0^P + \xi_c^P$. The results are largely robust with 2SLS, but the spending instruments are weaker in the primary elections because there are fewer donors and more zeros.

I discuss the setup and intuition for estimating committee costs in the primary, leaving the details to Appendix A.7. To utilize the committee's first order condition, one must deal with the unobserved (counterfactual) general election outcomes. For example, the R_1 candidate aligned committee considers both general election outcomes of R_1 facing either D_1 or D_2 when they choose their primary spending. To see this, rewrite a Republican committee's payoff with two candidates per side, where the expected probability of winning the general election for a given set of primary winners is $E[P_c^G|\mathbf{w}^P] = \sum_{\mathbf{a}^G \in \{0,1\}^{2N}} P_c^G(\mathbf{a}^G|\mathbf{w}^P) \prod_j p_j^*(a_j^G|\mathbf{w}^P)$, the expected probability of winning the general election against D_1 is $E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_1}^P = 1)]$, and the probability of D_1 beating D_2 in the primary is $P_{D_1}^P = P(w_{D_1}^P = 1)$:

$$V_{i_c} P_c^P \cdot \Omega_c - g_{i_c}^P S_{i_c}^P \quad s.t. \quad \Omega_c = E[P(w_c^G = 1 | w_c^P = 1 \cap w_{D_2}^P = 1)] \cdot P_{D_2}^P$$

$$+ E[P(w_c^G = 1 | w_c^P = 1 \cap w_{D_1}^P = 1)] \cdot P_{D_1}^P \quad .$$

$$(4.3)$$

In the Ω_c expression, only one object is unobserved for candidates that won their primary, namely the general election probability against the candidate on the other side that lost their primary. For candidates that lost their primary, both general election probabilities are unobserved. I already backed out the general election expected valence ψ_c^G for candidates that made it to the general election in the data, but one does not observe it for the primary election losers. This valence term affects the decisions of committees in the primary (and candidates decisions before that), and thus identification of the remaining parameters hinges on recovering it.

I recover ψ_c^G for primary losers using variation in the general and primary that exploits beliefs revealed by equilibrium spending. A committee takes the probability

of their preferred candidate winning the general election into account when spending in the primary; their behavior reveals information about their underlying expectations of ξ_c^G . This approach relies on inverting the equilibrium win probability to solve for the primary loser ψ_c^G as a function of observed objects and primary costs.

The logic of how to recover this counterfactual ψ_c^G can be seen through the available variation. Since I estimated the primary voter preferences, the effects of primary spending on election outcomes are known, allowing one to isolate how costs affect spending. The primary cost function $g_{i_c}^P$, conditional on a known valuation V_{i_c} , shifts a committee's willingness to spend. Thus variation in primary spending and expected outcomes in the realized match-ups for a given cost implies a single expected probability of winning the general election for the counterfactual match-up. Then, given the probability functional form and exogenous inputs, it implies a single counterfactual expected valence. The moment I target is $E[\mathbf{X}^{\top}\psi_c^G|S^P>0]$ for primary losers.

For robustness to $\gamma_{i_c}^P \neq 0$, the primary moment for the unobservable ψ_c^G is augmented with an estimate of $\gamma_{i_c}^P$, which is identified from leveraging variation in the number of primary opponents (see Appendix A.7). I back out mean fixed entry costs from entry variation. Now we have recovered valences and committee valuations and costs across both the primary and general elections for candidates that entered. For candidates who did not enter, we must exploit a different source of variation.

4.4 Estimation Of Candidate Stages

Now that the general and primary elections are characterized, I can calculate a candidate's probability of winning for any combination of opponents and positions, conditional on valence. Using this, I estimate the candidate stages. Recall the candidate's objective $\Pi_c = V_c \cdot E[P_c^G|\mathbf{d}] + V_c^0 \cdot (1 - E[P_c^G|\mathbf{d}]) - (d_c - \bar{\theta}_c)^2 \cdot \mathbb{1}[d_c > 0] + \eta_c(d_c)$, where V_c : value to candidate c of winning, V_c^0 : outside option, $\bar{\theta}_c$: ideal position point, and η_c : private variation in payoffs. The probability of winning is now the expected probability pre-entry, where the candidate positions \mathbf{d} are now written as explicit arguments: $E[P_c^G|\mathbf{d}] = \sum_{a_i^P \in \{0,1\}^{4N}} E[P_c^G|\mathbf{d}, \mathbf{a}^P] \prod_j p_j^*(a_j^P|\mathbf{d})$.

The unknowns $\{V_c, V_c^0, \bar{\theta}_c\}$ must be restricted as candidate decisions can be rationalized by a variety of combinations (Diermeier et al. 2005, Tillmann 2014). I allow the value from office and the outside option value to vary only at the district-party level. Specifically, $V_c = \exp(\mathbf{W}_c \lambda)$, where \mathbf{W}_c is a data vector including the incumbent's tenure length to date, election cycle, and district income (all interacted

with party). I specify $V_c^0 = \exp(\mathbf{W}_c^0 \lambda^0)$, where \mathbf{W}_c^0 includes party and election cycle fixed effects. I restrict the ideal points to vary at the election cycle-party level for incumbents and set them for challengers to be their observed choices.³²

For a candidate that entered, I observe their entry decision and their policy position, and thus there are two sources of variation to compare across candidates. To estimate the candidate entry stage, one needs to know the identity of each potential entrant in the event that they do not enter.³³ I construct potential entrants, with as many potential entrants as there are "empty" spots with two candidates per party per primary. This approach is feasible because \mathbf{W} and \mathbf{W}^0 do not rely on individual characteristics. However, there may be selection on unobservables.

The general and primary election valences of candidates that never ran, i.e. the potential entrants that chose $d_c = 0$, are not recoverable from Congressional elections data.³⁴ Identifying candidate preferences requires an estimate of these terms as one needs them to calculate expected win probabilities. I let the expected valences ψ_c for non-entrants follow a distribution: $\psi_c^{NE} \stackrel{iid}{\sim} N(\mu_{NE}, \sigma_{NE})$. The mean expected valence for non-entrants, μ_{NE} , is likely different from that of entrants. To allow for this selection, I use a proxy to estimate the difference in means of the valences for entrants and non-entrants. State legislature members are a significant source of the candidate pool for Congressional elections (over 40% of current members of Congress since 2010). I compare the state legislature election valences for state legislature incumbents who ran for Congress and those who did not. This reveals how different entrants are from similar non-entrants. I estimate a state legislature vote share regression and recover the mean valence difference for potential entrants to calculate μ_{NE} .³⁵

For a given vector of valences for all candidates $(\psi_c^G, \psi_c^P) \, \forall c$, either estimated or drawn from the proxy distribution, I calculate $E[P_c^G|\mathbf{d}]$ for every combination of candidate choices. I allow for two positions per party beyond non-entry, namely

³²Non-entrant ideal points cannot be separately identified from valuations.

³³Tillmann (2014) estimates a Congressional candidate entry model and generates a list of potential entrants; because he has their identities, he uses their characteristics to predict entry.

 $^{^{34}\}mathrm{In}$ addition, any valence for a candidate in an uncontested race in which the total number of votes is zero (or party convention where turnout cannot be measured like CT and UT) is also underidentified; this occurs for 20% of primary incumbents and 12% of primary non-incumbents. Since 67% of uncontested primaries still have ballots, I draw valences for those unidentified uncontested primary winners from the estimated primary winner distribution from balloted uncontested primaries.

 $^{^{35}}$ I get the election results for state legislatures from ICPSR, campaign spending from the National Institute on Money in State Politics, and candidate/donor information from state-level DIME. The s.d. σ_{NE} is estimated with maximum likelihood using the variation in estimated entrant valences.

moderate and extreme for both Republicans and Democrats.³⁶ As in the committee stages, I define the system of equilibrium challenger choice probabilities $p_c(d_c = \theta) = \frac{\exp(E[\pi_c(d_c = \theta|\mathbf{p}_{-c})])}{\sum_{w=0}^{\Theta} \exp(E[\pi_c(d_c = w|\mathbf{p}_{-c})])}$, where $\pi_c = \Pi_c - \eta_{d_c}$. I construct estimating equations based on this equilibrium probability. Variation in the estimated probability, controlling for differential expected win probabilities, identifies the valuations. The incumbent's estimating equation is similar. For both stages, I target two moments. I estimate a flexible conditional choice probability with $E[\frac{dp}{d\mathbf{x}}^{\top}(d_c - p_c(\mathbf{x}))]$ for a sufficient set of inputs \mathbf{x} and estimate the payoff parameters with $E[\log(p_d/p_{d'}) - \log(E[\pi_d]/E[\pi_{d'}])]$ for candidate choices $d \neq d'$. See Appendix A.8 for details.

5 Parameter Estimates

To recap, I estimate voter preferences using vote share regressions and deal with unobserved candidate quality with out-of-state donor income and ideology shocks. I estimate committee preferences using spending first order and entry conditions. I leverage the dynamic structure to recover the general election valence of primary election losers. I estimate candidate preferences using entry and policy conditions. I address the selection bias of non-entrant quality using state legislature variation. Each stage feeds into the next, capturing how each influences the rest of the election.

Table 3 reports the main model parameters. In particular it reports the committee spending and candidate position coefficients from the general and primary election voter preferences, candidate position coefficients from the committee cost functions, and candidate absolute ideal points.³⁷ The voter preferences estimation uses robust errors. The committee and candidate preference estimation use non-parametric biascorrected percentile bootstrap confidence intervals with 600 draws.

I find that candidates are statistically the most effective per dollar in converting spending into votes. Super PACs are weaker but precise, whereas parties and PACs have noisy effects. This is intuitive as candidate ads are most likely to be on message, less likely to be negative, and receive subsidized ad rates (Moshary 2020). The candidate position coefficient reflects how voters respond to more extreme positions, measured here as 1 for a moderate position and 2 for extreme (binning CF scores at the

 $^{^{36}}$ I scale Bonica CF-scores by dividing by the max of all absolute-positions and then set cutoff points at the ≤ 60 th percentile across the position distribution for moderate and > 60th for extreme.

³⁷Table A2 reports the 1st stage estimates for the general election and Table A3 and A4 show the controls for the general and primary elections, respectively.

60th percentile). The coefficient is negative, implying general election voters prefer moderate candidates, but the effect is noisy.³⁸ A reduced form interpretation would be that for a candidate, a one standard deviation increase in spending (\$437,846) at the average ad price leads to a 21% increase in vote share relative to absenteeism. Due to the likely anticipated response by others, the best way to interpret the effects is in the context of the whole model, which I do in the counterfactual analysis.

In the Republican primary, candidates still have the largest spending effectiveness but Super PACs are similar. Parties and PACs again have small and imprecise effects. In the Democratic primary, candidates dominate and Super PACs have a smaller noisy effect. Thus Super PACs play an outsized role in Republican primaries; this is in part due to concerted efforts by Super PAC funders like the Koch Brothers. Also, primary voters reward extreme candidates with a precise positive coefficient on position in both primaries.³⁹ Recall that I allow the candidate's position to affect the committee costs. The coefficient is slightly positive (negative) for Republican (Democratic) committees in the general, and negative in the primaries. A positive coefficient implies that as the candidate becomes more extreme, the implicit cost of spending increases. Thus, primary donors seem to prefer more partisan candidates. Finally, recall the policy choice is either moderate (1) or extreme (2) per party, and I find that the mean ideal policy for incumbents is 1.18; many incumbents choose policies that deviate from their ideal, driven by donor and voter pressure.

Table 4 reports the estimated valuations and costs for committees and candidates, averaged for different types, elections, and parties. Valuations are insufficient to determine how much a committee will spend as their spending effectiveness and costs also influence their decision. For example, PACs have high valuations, large marginal costs, moderate fixed costs, and low effectiveness. This aligns with their behavior of spending small amounts in many races. The estimates and confidence intervals on party spending indicate noisy effects, which is a byproduct of their limited spending variation. Challenger valuations are quite high as there is often entry despite a large incumbency advantage that results in frequent challenger losses. Costs for challengers are typically higher, which may indicate weaker fundraising abilities. Valuations for

 $^{^{38}}$ Due to concerns about large donors affecting the IV, I also consider an inverse-donation weighted version which captures variation of many small donors. IV strength decreases, but results are similar: candidate spending has a coefficient estimate and standard error of 0.0381 & 0.0251; Super PACs have 0.0137 (0.0081), party & PAC are 0.0166 (0.0277), and the candidate position is -0.2871 (0.1891).

³⁹Estimates using donor real estate shock IVs (Appendix A.10) are similar.

Table 3: Main Parameter Estimates

Parameter	Estimate	95% Confidence Interval

General Election Voter preferences (marginal effects on vote shares)

Candidate Spending β_C^G	0.0445	[0.0117, 0.0773]
Super PAC Spending β_S^G	0.0153	[0.0048, 0.0259]
PAC/Party Spending β_P^G	0.0183	[-0.0121, 0.0487]
Candidate Policy δ_2^G	-0.2772	[-0.5978, 0.0435]

Control variables: within-state donor income variation for candidates, super pacs, parties, and pacs, district unemployment rate, income, and unemployed number [interacted with incumbency], lagged Republican presidential votes [interacted with party], incumbency status, party, lagged incumbent votes, number of senate candidates, contested primary, governor same party as candidate, district high school graduation rate, median age, white percent, male percent [all interacted with party and election day precipitation, average ad costs, Cook's competitiveness rating and cycle time trends [interacted with party and incumbency], state and year fixed effects. The R^2 is 0.619.

Primary Election Voter preferences (marginal effects on vote shares)

V	1	(0	
R Candidate	Spending $\beta_{C_R}^P$		0.0272	[0.0177, 0.0368]
R Super PAC	Spending $\beta_{S_R}^{P}$		0.0208	[0.0069, 0.0347]
R PAC/Party	y Spending $\beta_{P_R}^{P^*}$		0.0039	[-0.0088, 0.0165]
R Candidate			0.1935	[0.1208, 0.2662]
D Candidate	Spending $\beta_{C_D}^P$		0.0415	[0.0288, 0.0542]
D Super PAC	Spending $\beta_{S_D}^{P}$		0.0043	[-0.0057, 0.0143]
D PAC/Part	y Spending $\beta_{P_D}^{P}$		0.0233	[-0.0093, 0.0559]
D Candidate			0.2475	[0.1572, 0.3378]

Control variables: Same as for general election, separately for each party

Committee Mean Valuations: See Table 4

Control variables [per committee type, incumbency status, and party]: constant, incumbency, year, lagged presidential votes, incumbent tenure length relative to state average

Committee Mean Costs: See Table 4

Marginal Effect of Policy on Costs

Marghar Effect of Folicy on Costs		
General Election R Cans φ_{Pol,C_R}^G	0.0575	[-0.0733, 0.1601]
General Election R S-PACs $\varphi_{Pol,S_R}^{\widehat{G}^{R}}$	0.4051	[0.0987, 0.7582]
General Election D Cans φ_{Pol,C_D}^G	-0.3402	[-0.4788, -0.1857]
General Election D S-PACs φ_{Pol,S_D}^{G}	-0.3122	[-0.6656, -0.0024]
Primary Election Winners $\varphi_{Pol,W}^{P}$	-0.2082	[-0.5124, 0.1111]
Primary Election Loseres $\varphi_{Pol,L}^{P}$	-0.3203	[-0.4951, 0.0040]

Control variables [per committee type, incumbency status, and party]: constant, number of senate candidates, voting age population, average ad prices

Absolute Ideal points mean $\bar{\theta}$ 1.17964 [0.97648, 1.34468]

Candidate Entry Valuation V_c : See Table 4

Control variables: incumbent's tenure length to date, election cycle, and district income (all interacted with party)

Candidate Outside Valuation V_c^0 : See Table 4

Control variables: party and election cycle fixed effects

Republican challengers are on average larger than Democratic challengers; this mirrors Gordon and Hartmann (2016) who find a similar result for Presidential candidates. Republicans are willing to spend more in races in which they are more likely to lose, implying Democrats may be more risk averse, with Democratic PACs as the exception.

How do estimates in one stage affect the others? Suppose the marginal effects of spending β are biased. This would cause an additional bias in the valuation/cost estimates as those attempt to rationalize observed spending conditional on an estimated β . Thus if the β coefficient is upwardly biased, the valuation to cost ratio would be downwardly biased. Here, one bias negates the other, leading to a smaller bias in the counterfactual prediction relative to a method that ignores one of the stages.⁴⁰

I also find primary election losers have a lower average and higher variance of general election valences than winners. This indicates that the pool of candidates that successfully make it to the general are not always the highest "quality" in unobserved dimensions (corroborating Tillmann (2014)). Finally, I find that state legislature incumbents who did not run for Congress have on average 14% lower quality, conditional on controls, than the state legislature incumbent Congressional entrants.

I calculate the means and correlations for the observed and estimated model outcomes (Appendix Table A5). The candidate positions, entry choices, and election outcomes fit well, indicating that the model's first and final stages reach outcomes similar to the data; this provides reassurance that the model dynamics mirror the data generating process. Entry totals differ for some committees despite similar spending means; this is because the model occasionally predicts more entry with less spending. The model cannot fully replicate some of the data's asymmetries and outliers.

6 Counterfactuals

I consider multiple counterfactual policies to study the impact of Super PACs on all stages within a given election cycle. The main counterfactual I consider reverses the *SpeechNow v. FEC* decisions. Comparing the model predictions in this hypothetical scenario to the status quo allows one to determine whether and in what way Super PACs affected Congressional elections. I consider additional counterfactual scenarios

 $^{^{40}}$ Similarly, voter and committee parameters affect candidates through the win probability. A downwardly biased estimate on δ (influence of policy on vote share) causes the policy choice to be rationalized with an upwardly biased valuation of winning relative to the outside option. This affects the counterfactual by downplaying how much candidates react, attenuating policy change effects.

Table 4: Committee/Candidate Valuations and Costs Estimates

Committee Valuations (in Thousands)			Committee General Election Costs (in Tens)			
Inc R Candidate	0.5259	[0.1126, 1.5227]	Inc R Candidate	0.0135	[0.0207, 0.3972]	
Inc R Super PAC	0.5185	[0.2629, 9.4834]	Inc R Super PAC	0.0097	[0.0137, 0.9496]	
Inc R Party	0.3663	[0.1876, 1.3556]	Inc R Party	0.0014	[0.0038, 0.0547]	
Inc R PAC	1.4161	[0.3967, 23.3659]	Inc R PAC	0.0325	[0.2236, 17.4309]	
Cha R Candidate	0.4877	[0.1052, 1.3812]	Cha R Candidate	0.0208	[0.0149, 0.2917]	
Cha R Super PAC	0.5850	[0.3000, 12.7342]	Cha R Super PAC	0.0104	[0.0112, 0.6100]	
Cha R Party	0.4829	[0.2726, 1.9711]	Cha R Party	0.0598	[0.0038, 0.0520]	
Cha R PAC	1.9914	[0.6138, 34.2989]	Cha R PAC	0.0322	[0.1516, 10.4116]	
Inc D Candidate	0.2794	[0.1388, 1.6633]	Inc D Candidate	0.0122	[0.0208, 0.2075]	
Inc D Super PAC	0.4314	[0.1470, 2.0259]	Inc D Super PAC	0.0013	[0.0113, 0.2583]	
Inc D Party	0.2865	[0.1389, 2.3298]	Inc D Party	0.0077	[0.0079, 0.2203]	
Inc D PAC	3.3444	[1.4334, 47.1684]	Inc D PAC	0.7761	[0.5153, 40.3875]	
Cha D Candidate	0.1877	[0.1009, 1.1518]	Cha D Candidate	0.0161	[0.0219, 0.1979]	
Cha D Super PAC	0.5721	[0.4725, 7.5908]	Cha D Super PAC	0.0019	[0.0143, 0.3184]	
Cha D Party	0.2796	[0.2310, 3.9818]	Cha D Party	0.0073	[0.0055, 0.1453]	
Cha D PAC	3.8027	[3.0020, 132.2389]	Cha D PAC	0.6784	[0.5874, 38.0509]	
	e Entry Va		Committee Primary Election Costs			
Inc R	21.3663	[0.0048, 39.3725]	Inc R Candidate	0.0918	[0.0563, 1.1269]	
Cha R	79.3051	[72.7964,83.9186]	Inc R Super PAC	0.0663	[0.0467, 2.5689]	
Inc D	38.9213	[0.4686, 89.1583]	Inc R Party	0.0100	[0.0077, 0.0872]	
Cha D	81.7723	[79.4525, 87.8268]	Inc R PAC	0.2212	[0.1553, 9.6182]	
			Cha R Candidate	0.1156	[0.0552, 1.0158]	
Candidate I			Cha R Super PAC	0.0628	[0.0340, 1.7726]	
Inc R	28.0700	[0.0000, 57.6772]	Cha R Party	0.1510	[0.0096, 0.1291]	
Cha R	72.7875	[70.7003, 73.8260]	Cha R PAC	0.2015	[0.1133, 6.2827]	
Inc D	0.010	[0.0000, 47.1628]	Inc D Candidate	0.0759	[0.0464, 0.4991]	
Cha D	76.9479	[75.0249, 78.0273]	Inc D Super PAC	0.0079	[0.0059, 0.1388]	
			Inc D Party	0.0467	[0.0330, 0.7367]	
Committee Primary Election Fixed Costs		Inc D PAC	4.7774	[2.7755, 154.9467]		
R Super PAC	1.7536	[1.6901, 1.8225]	Cha D Candidate	0.0989	[0.0577, 0.6661]	
R Party	2.4530	[2.3059, 2.6158]	Cha D Super PAC	0.0121	[0.0087, 0.1930]	
R PAC	1.6913	[1.6368, 1.7464]	Cha D Party	0.0450	[0.0303, 0.8642]	
D Super PAC	1.9491	[1.8719, 2.0499]	Cha D PAC	4.0673	[2.5704, 120.1414]	
D Party	2.6726	[2.4481, 2.9395]				
D PAC	1.9310	[1.8370, 2.0460]	Incumber			
			Inc R	1.1544	[0.7649, 1.4991]	
			Inc D	1.1528	[0.5684, 1.2981]	

The 95% confidence intervals are bias corrected percentile bootstrap. This shows valuations and costs for committees and candidates. 'Inc' refers to incumbent. 'Cha' refers to challenger. R and D refer to Republican and Democrat aligned groups.

to better understand the strategic incentives of committees: by banning all Super PACs, we may miss out on whether their influence is simply due to their equilibrium effects on the opposition or if there is influence even without the need for strategic spending. Disentangling these effects also allows us to understand the extent to which free-riding drives spending decisions by candidates, parties, and traditional PACs.

6.1 Equilibrium Effects of Banning Super PACs

I first consider the counterfactual scenario of Super PACs never existing (or a law banning them). To evaluate the 2010-2018 elections in this setting, I first use the parameter estimates to fully solve the model under the observed data with Super PACs, and then solve the model with the same parameter estimates but now excluding Super PACs. There are two sources for changes in a given stage: first the change in behavior conditional on the same outcomes from the previous stage, and then the change in behavior given a different outcome from a previous stage. Comparing the differences in equilibrium outcomes between the observed and counterfactual scenarios incorporates both. I am holding un-modeled variables, like exogenous covariates, constant in the counterfactual, which is not innocuous. For example, ad prices could readjust without Super PACs as spending decreases. In this case there would be a second order equilibrium adjustment, which could attenuate the overall effect.

I separate the analysis for elections with an incumbent and without; the main discussion is on the former, and I highlight notable differences with the latter. Only 11% of races in the sample are open (meaning no incumbent in the primary or general), and the existence of an incumbent can create differences in equilibrium outcomes. Finally, across the counterfactual distributions, there is a pile-up near zero, which is driven by the fact that Super PACs did not spend anything or very little in many races.⁴² I report the mean and median effects for each distribution in the table notes.

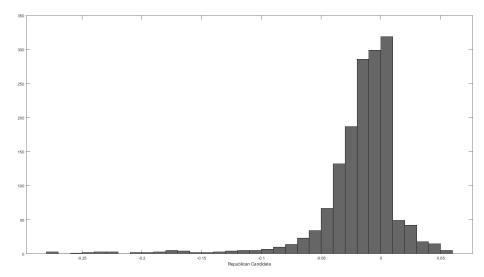
⁴¹The fixed point algorithm needs to use the same equilibrium across the two scenarios, and sufficient for that is a unique equilibrium; uniqueness conditions can be checked ex-ante (see Lemmas 3, 4, 6, and 7); results are not sensitive to starting values. I consider one simulation and private info. draw to study choices instead of probability distributions (and due to computational constraints).

⁴²Super PACs spent in 48% of general elections and 13% of primaries (20% of contested primaries). For histograms, I trim the bottom and top 1.5% of observations as a few outliers skew the graphs. There are also large tails on some of the distributions, and these outliers are partially a natural consequence of the data; for example, the distribution of observed spending has a large right tail.

6.1.1 Effects on General Election Voting and Spending

In Figure 4, I report the percent change in Republican general election vote share (excluding abstention) without Super PACs. The average Republicans vote share changes by -2.2% [-3.03, 0.21] with substantial variation. Republican incumbent shares increase by 1.3 percentage points without Super PACs and Republican challengers see their chances decrease by 1.2 points. This is intuitive as Super PACs help challengers more than incumbents. The change is slightly larger in Democratically leaning states, so Super PACs may provide higher benefits in difficult environments.

Figure 4: Percent Change in Republican General Election Vote Share without Super PACs



This plots the histogram of percent changes in Republican general election vote share (excluding abstention) with and without Super PACs. I compare the simulated equilibrium and counterfactual shares if Super PACs can't enter. The mean is -2.17% and the median is -1.09%.

The large left tail indicates that Super PACs provided a lifeline for some Republican candidates that otherwise would have performed poorly. Overall, Republicans may lose on average 2.1% of House seats in the counterfactual analysis sample (6 to 7 seats of the 325 districts on average per cycle studied) without Super PACs, which also represents an average 3.5% decline of their currently held seats in the sample; this is significant in close Congresses. The result is similar for open races, where Republicans see a -2.6% change in vote share. However, Democratic Super PACs are gaining on Republicans each new election cycle, so the trend could change. Further study on the U.S. Senate may be illuminating as Super PAC spending represents a larger share of total spending in Senate races and there are fewer seats.

Suppose one held the spending, policy, and entry constant and simply evaluated the change to general election vote shares without Super PACs. In this case, the counterfactual prediction would be biased by -92% on average with a 40% sign reversal rate across districts. Thus the counterfactual would be biased towards zero and predict that Super PACs on average slightly helped Democrats. This illustrates the importance of allowing agents to optimally respond to regime changes in predictions.

Super PACs also increased general election spending, as that would change (in races with incumbents) by -3.0% [-14.76 -0.05] if Super PACs did not exist. The remaining committees see a total 8.5% [2.75, 12.55] spending increase. Many races have increased spending as candidates cannot rely on Super PAC support. The lack of large Super PAC expenditures, not sufficiently compensated for with spending elsewhere due to contribution limits and ineffectiveness, depresses total spending. ⁴³ In open races, general election spending decreases by 7.5%; Super PACs are likely more influential in open races as challengers cannot rely on war-chests like incumbents. The changes in vote share and spending in the general election are due not only to the absence of Super PACs within this stage, but also to changes in previous stages of the election. The set of candidates that enter the general election from the primary, and their policies, are affected by the absence of Super PACs.

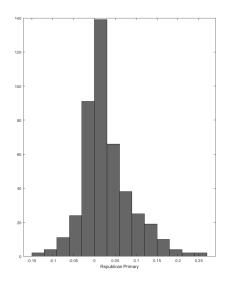
6.1.2 Effects on Primary Election Voting and Spending

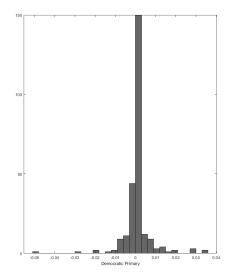
Figure 5 shows the percent change in incumbent primary election vote share without Super PACs for contested primaries; incumbents are generally helped as the distribution skews to the right. The effect for Democrats is relatively small, with most of the distribution falling between \pm 1 percentage point changes, whereas Republican incumbents see slightly larger increases. Super PACs mainly help challengers, with a 3.2% [-0.22, 4.23] change in vote share for Republican incumbents without them.

Total primary spending (in races with incumbents) changes without Super PAC by -11.1% [-38.0, -4.69] in the absence of Super PACs. Total candidate spending changes by -1.2% [-1.90, 3.47] without Super PACs whereas party spending increases

 $^{^{43}}$ Appendix Figure A1 shows the counterfactual distribution of the percent change in general election spending without Super PACs for different committee types; the median is a 1% increase with large right tails for some committees. I also compare the simulated equilibrium and counterfactual committee entry probabilities if Super PACs cannot enter for parties and PACs. For Republicans, the mean is 0.43% and the median is 0.00% for parties, and 0.69% & 0.01% for PACs. For Democrats, the mean is 1.74% and the median is 0.07% for parties, and 1.86% & 0.03% for PACs.

Figure 5: Percent Change in Incumbent Primary Vote Share Without Super PACs





This plots the histogram of percent changes in incumbent primary election vote share (excluding abstention) with and without Super PACs. I compare the simulated equilibrium and counterfactual shares if Super PACs cannot enter. For Republicans, the mean is 3.16% and the median is 1.63%. For Democrats, the mean is 0.30% and the median is 0.02%.

3.7% [-5.20, 19.83], and PAC spending changes by -5.9% [-12.05, 29.42]. Super PACs play heterogeneous roles in the primaries with complementing some and crowing out others.⁴⁴ The effects on open races are larger with a total spending decrease of 23%. This again suggests that Super PACs play a role in creating competition in open races. Super PACs have out-sized effects on primaries relative to general elections, which reiterates the importance of accounting for the primary election. Not only does ignoring the primary miss out on these direct changes, but this spills over into general election predictions; the set of candidates that make it there are affected by Super PACs. For this same reason, it is important to study changes to candidate choices.

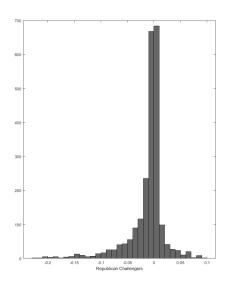
6.1.3 Effects on Candidate Entry and Policy

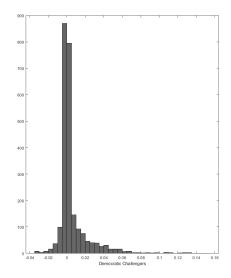
Figure 6 reports the percent change in challenger entry probability without Super PACs. There is a concentration near zero for all challengers, but with a left tail

 $^{^{44}}$ Appendix Figure A2 displays the counterfactual distribution of the percent change in primary election spending without Super PACs. The small number of bins for Democratic party committees is a function of their selective spending and many uncontested primaries. I also compare the equilibrium and counterfactual committee entry probabilities if Super PACs cannot enter. For Republicans, the mean is -1.98% and the median is -0.17% for parties, and -2.18% & -0.20% for PACs. For Democrats, the mean is 0.56% and the median is 0.22% for parties, and 0.50% & 0.30% for PACs.

for Republicans and right tail for Democrats. The tail indicates that Republican challenger entry decreases without Super PACs. The average change in Republican challenger entry without Super PACs is -1.5% [-1.64, -0.64]; the average change for Democrats is 0.9% [0.27, 0.63]. The effect is also larger in states that are dominated by the candidate's party compared to opposition states. Overall, we see that Super PACs encouraged Republican challenger entry and slightly depressed Democratic challenger entry. The median effects are much smaller, -0.15% and 0.02% for Republican and Democratic challengers respectively. This reflects the data: most districts have noncompetitive primaries, and only a few capture the interest of Super PACs.

Figure 6: Percent Change in Challenger Entry Without Super PACs





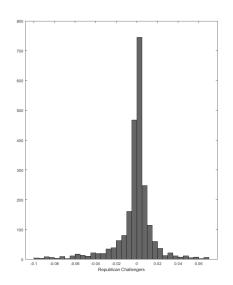
This plots the histogram of percent changes in challenger entry with and without Super PACs. I compare the simulated equilibrium and counterfactual challenger entry probabilities if Super PACs cannot enter, for Republican and Democrats. For Republicans, the mean is -1.48% and the median is -0.15%. For Democrats, the mean is 0.90% and the median is 0.02%.

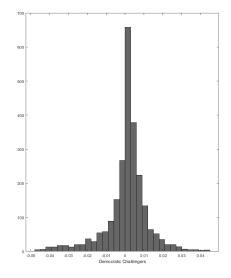
Figure 7 displays the percent change in challenger extreme position probability without Super PACs. Both types of challengers barely change on average without Super PACs. Average Republican challenger change in extreme position is -0.2% [-0.01, 0.81]. The average extreme change for Democrats is 0.13% [-0.06, 0.03]. Super PACs are more likely to support challengers and increase their chances of winning the primary; challengers now have higher expected probabilities of winning the general,

⁴⁵The model may over-assign credit to Super PACs in 2010 given the large Tea-Party induced entry with only fledgling Super PAC primary activity; the election cycle fixed effects and interactions soak up some of this. Also, if Super PACs never existed, the set of possible entrants may change beyond what the observables and estimated valence ranges considered here can capture.

and since general election voters have a preference for moderation, challengers could increase their general election chances through a more moderate position. However, Republican Super PACs also have a slight preference for extremism in the general election. These countervailing forces combine into null effects.

Figure 7: Percent Change in Challenger Extreme Position Without Super PACs





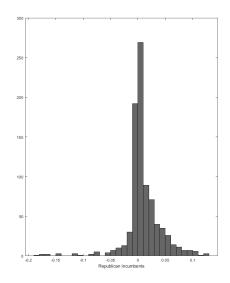
This plots the histogram of percent changes in challenger extreme position with and without Super PACs. I compare the simulated equilibrium and counterfactual challenger extreme position probabilities if Super PACs cannot enter, for both Republican and Democratic candidates. For Republicans, the mean is -0.19% and the median is 0.05%. For Democrats, the mean is 0.13% and the median is 0.17%.

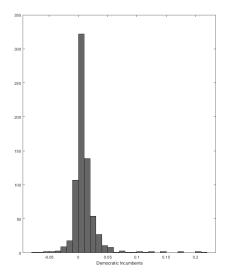
Figure 8 reports the percent change in incumbent extreme position probability without Super PACs; the average change is 1.4% [1.21, 7.29] for Democratic incumbents and 0.5% [0.01, 2.71] for Republicans. Thus Super PACs seem to be a slight moderating force for Democratic incumbents. One explanation is that since Super PACs helped Republicans in the general, their absence relieves general election pressure on Democrats, and so the incumbent focuses on the primary. This is backed up by the fact that the moderating effect for Democrats is stronger in districts where the Republican candidate fairs well in the general election with Super PAC support. How should one reconcile this slight moderating effect with the trend towards polarization since 2010 as seen in Figure 3? Super PACs may not be part of the cause,

⁴⁶Without Super PACs, challengers are more vulnerable; an incumbent can deter entry by going extreme in the primary. This is less effective when Super PACs support the challenger as they are less deterred. Without Super PACs, Democratic challenger entry decreases a few percentage points.

and their subtle influence is likely not affecting the overall trend.⁴⁷

Figure 8: Percent Change in Incumbent Extreme Position Without Super PACs





This plots the histogram of percent changes in incumbents extreme position with and without Super PACs. I compare the simulated equilibrium and counterfactual incumbent extreme position probabilities if Super PACs cannot enter, for both Republican and Democratic candidates. For Republicans, the mean is -0.52% and the median is 0.26%. For Democrats, the mean is 1.41% and the median is 0.56%.

6.1.4 Voter Welfare

Next I consider the welfare effects from the hypothetical Super PAC ban. Given the small to moderate effects on policy, the zero-sum election spending may be seen as "social waste". Formally, the model has voters receiving utility from this spending; without Super PACs, general election voter utility decreases by 3.4%, largely because there is less spending. However, if we ignore this component and only consider the utility from policy and candidate characteristics, utility increases by 0.4% without Super PACs; thus Super PACs may slightly "distort" the set of candidates who win.

⁴⁷I measure policy in one dimension, which may miss out on heterogeneity: for example, Gilens et al. (2021) find state-level evidence that *Citizens United* changed corporate tax policy but not on other issues. The extent to which a change in one dimension in a multidimensional framework would show up in changes to a one dimensional measure is unclear, but it may suggest that a null finding using a one dimensional scale could conceal nuanced effects that a richer analysis could capture.

6.2 Equilibrium Effects of a One-Sided Super PAC Ban

The cannibalization of spending efforts is evident in both the data and the model; spending by one side is often matched by the other. The counterfactual considered so far removes Super PACs from both sides, resulting in often small changes since the Super PACs were canceling each other out in the first place. So why do donors contribute at all if it has little effect? One explanation is the fear of being swamped in opponent spending with no response, which could result in large election outcome changes. I study this with a one-sided counterfactual: suppose the environment changed such that only one party was affected and their Super PACs could not spend. While an unlikely policy, this counterfactual is a useful way determine whether unregulated donations lead to competitive incentives like those in patent contests (Jensen 2016).

With a ban on Republican Super PACs, Republican general election vote share decreases by an additional 1.03 percentage points beyond the symmetric ban, indicating that Super PAC spending is helpful on its own and counteracts opponent spending. Challengers suffer slightly as well as Super PACs were a nontrivial source of support. The party without Super PACs has their incumbents perform better in primaries; this helps in the general election due to incumbency advantage but is simultaneously counteracted by opponent general election Super PAC spending. Overall, the asymmetric ban on Republican Super PACs alters outcomes (compared to the symmetric ban) more than a ban on Democratic Super PACs. This is intuitive as, so far, Republican Super PACs have been relatively more influential.

6.3 Separating Strategic Response and Unilateral Incentives

Relatedly, how much of Super PAC spending is due to the unilateral incentive to spend as opposed to strategic response? I study this by considering how much a committee may spend without opposition. In this case, the incentive to spend is due to differential candidate characteristics that affect the election outcome. I estimate two additional counterfactuals to study this. First, I only allow spending by candidate committees, banning all outside groups. Second, I only permit spending by a single committee. Note that in both of these scenarios, the candidates are still strategically responding to each other with their policy and entry choices.

⁴⁸Democratic (Republican) challengers are less likely to enter under a Democratic (Republican) Super PAC ban as not only do they have less support, but opposing Super PACs spend unfettered.

When banning all outside committees, candidate spending on both sides increases as they no longer can free-ride. Relative to the status-quo with outside committees and spending on all sides, Republican general election share decreases by 5.2%, indicating that outside support is vital for many candidates. Candidate spending increases by 15.8% for Republicans and 13.3% for Democrats. When banning all but one committee (the Republican candidate committee), Republican win percentages increase by 14.4% and their spending increases by 0.98%; overall spending decreases by 16.7% in primaries and 80.7% in the general elections, compared to the status quo. Compared to the Republican Super PAC ban, banning all but the Republican candidate leads to higher vote shares for Republicans and lower spending increases. This is intuitive as when only banning Super PACs from one side, other committees, who were previously free-riding, can now compensate by increasing spending.

7 Conclusion

In this paper I tackle the role of unregulated money in national campaigns. I focus on Super PACs and their effect on House elections. I solve a novel model and estimate it using spending, policy, and donor data. I find that while Super PACs have nuanced equilibrium effects, their presence has changed the campaign finance environment. The slight changes in spending, entry, and policy may have unforeseen consequences and affect eventual legislation. I corroborate the result in the literature that Republicans are helped in the general election by Super PACs. They appear to help challengers in primaries, and the effects on candidate positions are varied. Incumbents rarely lose primaries, and Super PACs have only slightly changed that.

Figure 1 alludes to possibly large Super PAC effects, but the counterfactuals indicate a muted influence. The simultaneous rise of the "Tea-Party" movement, spending cannibalization, and candidate selection demonstrate the need to account for confounders and equilibrium responses. The small effects of special interest spending on policy is not an uncommon result (Besley and Coate 2001; Kang 2016), and I provide an affirmation of this with Super PACs. Furthermore, I track their influence throughout the entire dynamic process within an election, including direct and indirect effects. This rich environment provides insights into which channels are most affected and how influence dissipates into only moderate changes to policy outcomes.

I do not study direct effects on legislation, but Super PAC funded Republican state

legislature electoral gains (Klumpp, Mialon, and Williams 2016) have not necessarily resulted into major policy changes (Grossmann 2019). Furthermore, the literature on campaign contributions has largely found mixed effects, indicating that donors may primarily target their contributions to simply help their preferred candidates (Ansolabehere, De Figueiredo, and Snyder 2003; Fowler, Garro, and Spenkuch 2020).

Also, I do not incorporate 501(c)(4) nonprofits, known as "dark money" groups. They do not report their spending or donors to the FEC as they do not engage in "express advocacy" for a candidate. Their ads can be tracked with advertising data, but their influence can be indirect; a growing number of Super PAC donations come from these nonprofits, providing a discreet alternative for donors (see Appendix A.1).

Finally, election spending is gradually shifting towards digital ads. For example, the Super PAC "The Lincoln Project" creates "viral" content and "America First Action" operates a news website spreading their content on social media. Modeling these distinct strategies is important for understanding 21st century campaign finance.

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Appendix (For Online Publication)

A.1 Background

Independent expenditures (IEs) were created by the 1976 Buckley v. Valeo case that allowed unlimited spending on political messaging (by individuals or PACs). Peech-NOW v. FEC (not a Supreme Court ruling, but a DC court of appeals), ruled that individuals could contribute unlimited funds to committees that make IEs. The SpeechNOW committee wanted to raise funds for IEs without forming a PAC (to avoid limiting itself to receiving at most \$5,000 per person). The court ruled that if the organization is IE only (not a PAC that can make both direct contributions and IEs), then it has no restrictions on fundraising (still no foreign funding however). This allowed individuals to basically pool IEs through Super PACs, making large sums more coordinated. Before SpeechNow individuals could either donate to a PAC (subject to contribution limits) or act on their own (not with a PAC).

Prior to Citizens United v. FEC, corporations had to form their own PACs. The case allowed corporations and unions to use their general treasury funds to make IEs.² This was partially a response to the 2002 "Bipartisan Campaign Reform Act", which banned "electioneering communications" (EC) [TV ads mentioning candidates 60 days prior to general or 30 days prior to primary] by non-PACs. See Prato and Wolton (2017) for a discussion. Spending on ads that do not support/oppose a candidate (issue advocacy) are less regulated. If the issue ad mentions a candidate and is within 60(30) days of a general (primary) election, then the ad must be disclosed (called an electioneering communication). Furthermore, prior to 2010, corporations could not make ECs; Citizens United overturned that.³

While most committees are either independent expenditure-only (Super) or traditional PACs, the district court case Carey v. FEC allowed for the "hybrid PAC" (Carey Committee), which is a single PAC that operates as both a normal and Super PAC, with the requirement that the funding for each activity stems from two separate bank accounts. Unlimited donations aimed at IEs originate from one and none of that money can be used in coordination expenses, and vice-versa. "Hard money" is money donated with a donation limit. "Soft" money has none and has been limited to parties ever since the Federal Election Campaign Act (FECA) of 1971 and were subsequently upheld in the 1976 Buckley v. Valeo case and were further limited in the 2002 "Bipartisan Campaign Reform Act". There is substantial legal scholarship on

¹ "Independent" communication "expressly advocating the election or defeat of a clearly identified candidate that is not made in cooperation, consultation, or concert with, or at the request or suggestion of, a candidate, a candidate's authorized committee, or their agents, or a political party or its agents" [11 CFR 100.16(a)].

²The ruling also allowed them to accept corporate and union donations; the exceptions are foreign nationals, federal contractors, national banks, and federally chartered corporations.

 $^{^3}$ The 2007 case Wisconsin Right to Life v. FEC loosened the restrictions on what classified ads to be EC, allowing more politically charged non-EC ads.

IEs and soft money. Some consider IEs to be the new form of soft money (Tokaji and Strause 2014). The 2014 *McCutcheon v. FEC* case overruled some of the 2002 "Bipartisan Campaign Reform Act" (The BCRA was upheld in the 2003 case *McConnell v. FEC*), removing "aggregate contribution limits" made to national parties and federal candidate committees (the total amount one can give across all contributions in a cycle). This made it possible for individuals to give to many more candidates.⁴

Social welfare nonprofits (501(c)(4)s), known as "dark money" groups, used to be limited in that they could not directly do IEs. They could still lobby and make non-EC issue ads. Citizens United allowed them to make political expenditures; they still cannot spend the majority (>50%) of their operating budget on these funds. They do not disclose their donors and they can raise unlimited amounts (see Oklobdzija (2018)). Their spending totaled 257 million in 2012 ($\approx 20\%$ of outside spending), but declined to 106 mil. in 2018.⁵ As the Center for Responsive Politics reports, they often spend earlier in the cycle (well before Super PAC spending) and they often do not target individual candidates. Finally, when a 501(c)(4) donates to Super PAC, the original donor is undisclosed. This is allowed as long as the donor does not instruct the 501(c)(4) to give to the Super PAC; otherwise they risk being a "straw donor".

A.2 Committee Effort and Donor Decision Details

Let primary election committees exert fundraising effort $e_{i_c}^P \in \mathbb{R}_+$. Then, donors m make their primary election donations $y_{mi_c}^P \in \mathbb{R}_+$, which then gets converted into spending $S_{i_c}^P$. In the general election, committees choose fundraising efforts $e_{i_c}^G$. Then donors donate $y_{mi_c}^G$, which gets converted into spending $S_{i_c}^G$. To construct the committee payoff, I model donors to map committee fundraising efforts $e_{i_c}^G$ into spending $S_{i_c}^G$ (similar for primary). The general election donor m maximizes the utility from giving to the causes they support. Their program is in (A.1) and they choose how much to give to committee i_c with $y_{mi_c}^G$. They give based on their political alignment with the committee to which they are donating, α_{mi_c} , which is function of the candidate's policy. The benefit is also a function of how much they give and the committee fundraising effort $e_{i_c}^G$. This setup is similar to the "naive" donors from Bouton, Castanheira, and Drazen (2020). Their costs are a function of their

 $^{^4}$ The 2011-2012 limits: \$46,200 for federal candidates + \$70,800 for national parties.

 $^{^5}$ See CRP. This spending is predominantly issue ads. Any IEs or ECs must be reported to the FEC. Few 501(c)(4)s file reports so either these groups stick to non-EC issue ads or do not properly disclose. 501(c)(5) unions and 501(c)(6) trade associations have similar rules but spend much less.

⁶In their main model, donors internalize their influence over the election outcome, which in my model is done by the committees. My approach also differs from Schnakenberg and Turner (2020), who model the donor's decision between two kinds of candidates based on policy preference.

donation, weighted by their wealth α_m^0 and the committee i_c 's fundraising ability $\alpha_{i_c}^F$.

$$\max_{y_{mi_c}^G \in \mathbb{R}_+} \alpha_{mi_c} y_{mi_c}^G e_{i_c}^G - \frac{(y_{mi_c}^G)^2}{2\alpha_m^0 \alpha_{i_c}^F}$$
(A.1)

Solving program (A.1) gives us the interpretation: the donor supplies campaign contributions $y_{mi_c}^G$ to political committees i_c by choosing their contribution level based on their preference/ability $\gamma_{mi_c} = \alpha_{mi_c} \alpha_m^0 \alpha_{i_c}^F$ and the political committee's fundraising efforts $e_{i_c}^G$. Their optimal donation function can be thought of as the fundraising production technology (from that donor) for the committee $y_{mi_c}^G = \gamma_{mi_c} e_{i_c}^G$.

One may want to model donors who directly take into account the effects of their donation on the election. In my model, equilibrium donor behavior results in outcomes basically equivalent to those with strategic donors. This is because the committee's fundraising effort is strategic. For example, a Super PAC communicates to donors the importance of a race, convincing them to give. If donors are influenced by fundraising effort, then their objective in (A.1) is appropriate. If they are not, then the donors effectively act like committees, and treating committees as separate is superfluous. The committees are vessels for donor money, but the two are distinct agents. I capture how committees strategically raise money from impressionable donors, which then becomes election spending that influences voters. The donations translate into spending: $S_{i_c} = \sum_m y_{mi_c}^G$. Thus $S_{i_c}^G = \sum_m \gamma_{mi_c} e_{i_c}^G$, where γ_{mi_c} is the fundraising yield (inverse cost) from donor m for committee i_c . Then the general election committee effort program is $\max_{e_{i_c}^G \in \mathbb{R}_+} V_{i_c} \cdot P(w_c^G = 1 | w_c^P = 1, \mathbf{w}_{-c}^P) - e_{i_c}^G$ s.t. $S_{i_c}^G = \sum_m \gamma_{mi_c} e_{i_c}^G$, and written in (2.4) in terms of spending. Let $g_{i_c} = (\sum_m \gamma_{mi_c})^{-1} \geq 0$. Thus this formulation is equivalent to the main model, with spending as the choice variable and an implicit cost g that is a function of committee constraints and donor preferences.

A.3 Model Characterizations

As noted in the proposition, I focus on equilibria where agents condition on payoff relevant actions: recall that I allow players to observe all actions from previous stages (hidden actions complicate defining the equilibrium). Thus for example, voters observe fundraising effort and an equilibrium might exist with voters conditioning on effort. Committee effort is not payoff-relevant to voters conditional on spending.

The general election spending stage has a solution, but uniqueness is not guar-

⁷The weighting by fundraising limits is an alternative to a strict limit per donor. Consider the donation production function and think of these donors as classes of donors. It is easy for a Super PAC to raise a lot of money with little effort: they can get \$1 million from one wealthy donor. For a candidate to raise that much, they would have to raise the maximum of \$5,000 from 200 people.

⁸To make this model of spatial donors that are also influenced by fundraising efforts consistent with Bonica (2014), an interpretation is that the individual donor is not influenced by fundraising (only policy), and rather just the number of donors is affected by fundraising efforts.

⁹This lets donors give to specific races per committee and ignores dynamic fundraising.

anteed given the convexity of the exp function in P_c^G , but the equilibria can be characterized. Existence and uniqueness of the entry stage is easier to demonstrate. Existence of the primary spending stage is straightforward, but uniqueness is not. However simulation evidence suggests that a sufficient expression can be empirically validated in the voter preferences estimation before needing to solve the model, thus we can check uniqueness before relying on it. Existence and uniqueness for the primary entry stage is similar to the general election argument. Challenger decisions are generalizations of the entry stages and the incumbent's decision is straightforward. I discuss the equilibrium properties of each stage in Lemmas 2-7 in Appendix A.12.

A.4 Voting and Candidate Data Details

The FEC has votes and parties for all balloted candidates in federal elections which had general elections occur on election day. Non-election day special elections are added from the FEC's reports and the CQ election database. To measure turnout, I use population data from Census. For closed primaries, the population to use for turnout is different than the district total VAP as the voting population is split based on political affiliation. I want the relevant population for that party's primary, I adjust the population using party affiliation percentages at the state level from Gallup.

Bonica (2014) constructs a contingency table of all donor-recipient committee matches with the dollar values, converts the dollars into counts using contribution limits, then performs a singular value decomposition on the normalized matrix. The final positions are based on the eigenvalues of square of that decomposition. The measure has limitations with capturing within-party dynamics (Tausanovitch and Warshaw 2017). There is a concern with using a contribution based measure. It is primarily based on the existence of a contribution, and many small donors (who were less influenced by the court decision) provide the bulk of the variation; CF's correlation with DW-Nominate scores is high, which do not suffer from these concerns.¹⁰

A.5 FEC Committee Data Details

For political party committees, I include federal, state, and "Leadership PAC" type committees. I group hybrid/Carey PACs with Super PACs. The FEC provides committee expenditures at the transaction level for everything over \$200. The groups engaged in IEs must disclose to which candidate that expenditure was targeted and whether it was for or against the candidate. The date is for when the "communication is publicly distributed or otherwise publicly disseminated" (FEC), and committees often note whether a given expenditure is aimed at the primary or general election. Campaign committee advertising spending is calculated from itemized expenditure reports. I use the self-reported transaction codes and augment that with stringmatching in the description field to determine which transactions are ad spending.

¹⁰Other methods (Ramey 2016) cannot be calculated for every challenger per district.

Summing over all transactions for candidates is inappropriate as the IEs are adspending and the expenditure files include other spending.¹¹ Incorporating other kinds of spending like transfers/contributions to other PACs will result in double-counting as a transfer from one Super PAC to another will eventually be an ad by the receiving Super PAC. Dollars are inflation adjusted to 2015. I do not consider Senate races as, given the limited time window, there are insufficient observations (to estimate Senate separately). The main downside of omitting Senate races is that the majority of Super PAC spending in Congressional races is targeted at Senate races, so the overall effect of Super PACs on Congressional outcomes cannot be fully captured in this analysis.

A.6 General Election Estimation Details

The committee's expected payoff for a given entry decision conditional on their private information $\lambda_{i_c}^G$ is denoted with $\mathcal{U}_{i_c}^G$, and p_j is i_c 's belief about committee j's choice.

$$\mathcal{U}_{i_c}^G(a_{i_c}^G|\mathbf{p}_{-i_c}) = \sum_{\mathbf{a}_{-i}^G \in \{0,1\}^{2N-1}} \pi_{i_c}^G(\mathbf{S}^*|a_{i_c}^G, \mathbf{a}_{-i_c}^G) \prod_{j \neq i} p_j(\mathbf{a}_{-i}^G) + \lambda_{i_c}^G a_{i_c}^G$$
(A.2)

The probability of entry is $p_{ic}(a_{ic} = 1) = Prob[u_{ic}^G(1|\mathbf{p}_{-i_c}) + \lambda_{ic}^G > u_{ic}^G(0|\mathbf{p}_{-i_c})],$ where $u_{ic}^G = \mathcal{U}_{ic}^G - \lambda_{ic}^G a_{ic}^G$, and with the Logistic distribution leads to probabilities:

$$p_{i_c} = \frac{\exp(u_{i_c}^G(1|\mathbf{p}_{-i_c})/\sigma)}{\exp(u_{i_c}^G(1|\mathbf{p}_{-i_c})/\sigma) + \exp(u_{i_c}^G(0|\mathbf{p}_{-i_c})/\sigma)} = f(p_{-i_c}).$$
(A.3)

This system defines a fixed point $\mathbf{p}=f(\mathbf{p})$; note that calculating $u_{i_c}^G \forall i_c$ requires solving the general spending stage for all entry combinations. Rather than solving for \mathbf{p} , I flexibly estimate P_c^G and \mathbf{p} based on the sufficient set of inputs (see below), letting $\sigma=1$. Next, one could estimate costs comparing observed entry to the model prediction. We can also construct a regression that illustrates the variation that is identifying the parameters. Consider the log-odds ratio: $\log(\frac{p_{i_c}}{1-p_{i_c}}) = \log[\frac{\exp(u_{i_c}^G(1|\mathbf{p}_{-i_c}))}{\exp(u_{i_c}^G(0|\mathbf{p}_{-i_c}))}]$. This can be rewritten as $V_{i_c}\left(E[P_{i_c}^G|a_i=1]-E[P_{i_c}^G|a_i=0]\right)-g_{i_c}E[S_{i_c}^G|a_i=1]$, where $E[P_{i_c}^G|a_i]=\sum_{a_{-i}\in\{0,1\}^{2N-1}}P_{i_c}^G\prod_{j\neq i}p_j^*(a_{-i})$. Isolating g yields the linear regression that identifies the parameters common to valuations and costs.

$$g_{i_c} = \frac{\log\left[p_{i_c}/(1 - p_{i_c})\right]}{(V_{i_c}/g_{i_c})\left(E[P_{i_c}^G|a_i = 1] - E[P_{i_c}^G|a_i = 0]\right) - E[S_{i_c}^G|a_i = 1]}$$
(A.4)

I estimate P_c^G as a function of entry and use the first order condition to get spending; spending is the implicit function of equilibrium probability of winning for that

¹¹About 30% of candidates running are not listed in either the FEC committee IE or candidate expenditure datasets; most candidates without spending receive trivial votes.

given entry profile: $S_{i_c}^G = ([V_{i_c}/g_{i_c}]\beta_{i_c}\phi P_c^G(1-P_c^G))^{1/(\phi-1)}$. The general election probability of winning as a function of entry P_c^G : regression of the log-odds of the probability of winning on the sufficient inputs to characterize post-entry decision-making: the ratio of effectiveness times valuations divided by costs for all committees of the two general election candidates, the sum of observed and unobserved district/candidate characteristics (including candidate's policy) times their coefficients from the voter regression, and indicators for whether the committee entered. The adjusted R^2 is 0.98 with just linear terms and there are 1656 observations. An indicator of the fit: if no outside committee enters and all inputs are identical, then one should get a 50% win probability. I find it to be 0.505 for the Republican in the general election.

I approximate the committee entry probability p given the observed d_c (Bajari, Hong, Krainer, and Nekipelov 2010). The general election equilibrium conditional entry probabilities for the 6 different kinds of committees with entry decisions p^G : polynomial logit of the entry decision on the near-sufficient inputs to predict entry. These include the same inputs from predicting P^G above (except entry) but now fully interacted with coefficients for each of the committees (polynomial combinations).¹²

A.7 Primary Election Estimation Details

Consider a committee's spending first order condition in terms of the ex-ante expected probability of winning the general election $\Omega_c = \frac{\omega_{i_c}^P (S_{i_c}^P)^{1-\phi}}{\phi P_c^P (1-P_c^P)} \equiv K_c$, where $\omega_{i_c}^P = g_{i_c}^P / (\beta_{i_c}^P V_{i_c})$. For committees whose candidate won their primary, rewrite this in terms of the main unobservable: the general election probability of beating the other candidate that lost their primary (let D_1 be the opponent who won their primary):

$$E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_2}^P = 1)] = \frac{K_c - E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_1}^P = 1)]P_{D_1}^P}{1 - P_{D_1}^P}.$$
 (A.5)

This left hand side, denoted as EP_{CF} , takes into account general election equilibrium committee entry for the hypothetical match-up between candidates R_1 and D_2 , and is thus just a function of the exogenously given objects at the start of the general election, including the unknown $\psi_{D_2}^G$. I invert this probability with respect to $\psi_{D_2}^G$:¹³

$$\psi_{D_2}^G = EP_{CF}^{-1} \left(\frac{K_c - E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_1}^P = 1)]P_{D_1}^P}{1 - P_{D_1}^P} \right). \tag{A.6}$$

¹²The fit for R-SPAC is 0.81, R-party 0.81, R-PAC 0.90, D-SPAC 0.80, D-party 0.84, D-PAC 0.71.

¹³The non-closed form nature of $E[P_c^G|\theta,\mathbf{w}^P]$ makes a proof of invertibility difficult. Since I estimate the general election parameters first (and $E[P_c^G|\theta,\mathbf{w}^P]$ only depends on those), I check the inversion condition per observation beforehand. Graphing the function across the range of estimated general election valences at the values for the estimated general election parameters shows that it has a sigmoid shape across observations. I approximate the inverse EP_{CF}^{-1} function. It is a polynomial (non-collinear interactions and squares) of the EP, all inputs from predicting the conditional choice probabilities, with the slight alteration of separating out the opponent valence as a separate input from the candidate characteristics term. The adjusted R^2 is 0.97 with 1656 observations.

This approach works when a candidate considers only two potential general election opponents; this is not restrictive as many races have only two candidates that receive many votes and the vast majority of races only have two that spend nontrivially.¹⁴ Then a fully contested primary has four candidates: two of them move on to the general and I only have to recover the general valences for the two primary losers. I recover $\psi_{D_2}^G$ from an R_1 aligned committee's spending first order condition (FOC) and $\psi_{R_2}^G$ from a D_1 committee's FOC. We could now estimate the following moment $E[\mathbf{X}^{\mathsf{T}}\psi_{D_2}^G|S_{i_{R_1}}^P>0]=\mathbf{0}$, but that would require normalizing the unobserved heterogeneity cost shock $\gamma_{i_c}^P$ from the primary cost function $g_{i_c}^P=\exp(X_{i_c}^P\varphi_c^P+\gamma_{i_c}^P)$. Since there is significant variation in primary spending across committee types, allowing for heterogeneous $\gamma_{i_c}^P$ is important. I exploit the structure of single-contested (only one party) and double-contested primaries to partially recover both.

When a single primary is contested, the primary committee spending first order condition system has only one counterfactual general election matchup probability (EP_{CF}) and it appears only in the primary loser's FOC.¹⁵ We can rearrange the primary winner's FOC to isolate the costs, where $EP_c^G = E[P_c^G|\mathbf{w}^P]$ for the observed primary winners: $g_{i_c}^P = (EP_c^G\beta_{i_c}^PV_{i_c}\phi P_c^P \cdot (1-P_c^P))/((S_{i_c}^P)^{1-\phi})$. I estimate the cost function parameters φ_c^P and recover the unobserved cost shocks $\gamma_{i_c}^P$ for primary winners in single-contested races. Then I use the estimated φ_c^P as known for primary winners in dual-contested primaries to partially identify their cost shocks. By plugging in φ_c^P into the FOC of a dual-contested winner, we are left with two unobservables: the unobserved general election valence of their hypothetical general election opponent who lost the primary (ψ_{CF}) and their own cost shock $(\gamma_{i_c}^P)$. Rewrite their FOC in terms of the counterfactual match-up probability (as in (A.5)):

$$EP_{CF}(\psi_{CF}, \psi_c|\cdot) = K_c(g_{i_c}^P(\gamma_{i_c}^P, \varphi_c^P|\cdot)|\cdot)/(1 - P_c^P) - (EP_c^G P_c^P)/(1 - P_c^P). \tag{A.7}$$

The left hand side of equation (A.7) is bounded between 0 and 1. I refine its

¹⁴Of primaries 2010-2016 from my sample (House elections ignoring third party), 74% have fewer than three candidates, but this is because 42% are not even contested. 55% of contested primaries have only two candidates. Among contested races with at least three, 66% have only two dominants candidates, defined as where the sum of the non-top two candidates by vote share is less than 25% of the total vote, and in 90% of races three plus, the top two receive 60% of or more of the vote. Furthermore, among primary races with three plus, in 96% of races, 90% or more of the ad-spending by candidate committees is done by the top two candidates and in 99.7% of races, 75% or more of ad-spending is done by the top two. 98% of races have the top two receiving 75% or more of outside spending. Thus in most of these elections, the smaller candidates are not in the same strategic environment as major candidates and can be added to the absenteeism count.

¹⁵For example, suppose the Republican primary has two candidates, R_1, R_2 , but the Democratic primary is uncontested with just one candidate, D_1 . Then the Republicans know who they will face in the general and they know D_1 's expected general election valence (since $\psi_{D_1}^G$ was already estimated). Thus R_1 can formulate their expected chances against D_1 with the only unobservable being primary cost shocks. For R_2 however, since they lost the primary, their chance against D_1 was never observed, and hence is a counterfactual, EP_{CF} that is a function of the unobserved $\psi_{R_2}^G$.

bounds by exploiting valences estimated from the vote share regressions. Consider the following valence bounds: $\psi_{obs}^{LB} = \min(\psi_{obs}) - \operatorname{std}(\psi_{obs})$ and $\psi_{obs}^{UB} = \max(\psi_{obs}) + \operatorname{std}(\psi_{obs})$. I combine these realistic bounds with the data and estimated parameters to calculate significantly tighter bounds on EP_{CF} . I solve the general election (equation (2.5)) for the pair of candidates in question, substituting in the unobserved valence ψ_{CF} for each bound to calculate bounds $EP_{CF} \in [EP_{CF}(\psi_{obs}^{LB}, \psi_c|\cdot), EP_{CF}(\psi_{obs}^{UB}, \psi_c|\cdot)]$. I plug these bounds into equation (A.7) and back out cost shock bounds. I generate an estimated cost function by drawing cost shocks from a uniform distribution based on the estimated bounds per committee $\gamma_{r,i_c}^P \in U[\gamma_{i_c}^{P,LB}, \gamma_{i_c}^{P,UB}], g_{i_c}^P = \sum_r \exp(X_{i_c}^P \varphi_c^P + \gamma_{r,i_c}^P)$. This provides substantially more information than normalizing the cost shock. ¹⁶

Next, I plug the estimated $g_{i_c}^P$ for the primary winners into equation (A.7) to recover ψ_{CF} for the losers in the double-contested primary. For this we need the inverse function EP_{CF}^{-1} from equation (A.6). We can then estimate the cost function parameters and unobserved cost shocks for the double-contested primary losers. This follows the exact same method as for the single-contested primary winners, but now we estimate the cost function parameters φ_c^P and back out the unobserved cost shocks $\gamma_{i_c}^P$ for primary losers in double-contested races. Then, I estimate the costs and ψ_{CF}^G for losers in single-contested primaries. They have one unknown EP_{CF} , and so we can mirror the approach used for primary winners in double-contested primaries: plug in the estimated φ_c^P from double-contested losers, construct bounds on ψ_{CF}^G , solve for the bounds on EP_{CF} , back out bounds on the unobserved cost shocks, average across draws from a distribution to recover the cost function $g_{i_c}^P$, and finally use the function EP_{CF}^{-1} to recover the unobserved valence for single-contested primary losers.

Next I estimate a primary fixed entry cost:¹⁷ construct the log-odds ratio for equilibrium entry and isolate the cost term. This is possible because all other terms are known from previous estimation steps.¹⁸ I estimate fixed costs for non-candidate committees with heterogeneity across party, committee type, and incumbency status: $F_{i_c}^P \cdot a_{i_c}^P = V_{i_c} \left(E[P_c^G | a_i^P = 1] - E[P_c^G | a_i^P = 0] \right) - g_{i_c}^P E[S_{i_c}^P | a_i^P = 1] - \log \left(\frac{p_{i_c}^P}{1 - p_{i_c}^P} \right).$ Similarly to the general election, I estimate the equilibrium win probability in the

Similarly to the general election, I estimate the equilibrium win probability in the primary as a function of entry P^P . Its inputs are: the primary versions of the same set of inputs for predicting P^G alongside the EPGs for each match-up combination for all entrant candidates. For Republican primaries, the adjusted R^2 is 0.78 with linear terms and there are 958 observations. For Democratic primaries, the adjusted R^2 is 0.83 with linear terms and there are 758 observations (both only contested primaries).

The primary election equilibrium conditional entry probabilities for the 12 different kinds of committees with entry decisions (Incumbent and challenger - Republican

¹⁶The large set of parameters and number of estimation steps remaining make set inference infeasible. For non-entrant committees/candidates, I draw from $N(\text{mean}(\gamma^P), \text{std}(\gamma^P))$.

¹⁷This is not possible for the general as I use spending and entry to identify valuations and costs.

¹⁸To utilize entry variation, we need to calculate all possible primary elections for different entry profiles; I flexibly estimate post-entry win probability per primary P_c^P and the committee equilibrium entry probabilities \mathbf{p}^p based on the (nearly) sufficient set of inputs to predict them (see below).

and Democratic Super PACs, parties, and PACs): p^p . Inputs: the same (non-entry) used for predicting P^P but separating out valuations from costs as distinct inputs. Given the limited variation, I group the estimation and use limited polynomials.¹⁹

A.8 Candidate Stages Estimation Details

Consider the expected payoff for a given candidate choice, where $P_c^G = E[P_c^G | \mathbf{d}]$, with the equilibrium probability $p_c(d_c = \theta) = \frac{\exp(E[\pi_c(d_c = \theta | \mathbf{p}_{-c})])}{\sum_{w=0}^{\Theta} \exp(E[\pi_c(d_c = w | \mathbf{p}_{-c})])}$, shown for a given draw (results use 100 non-entrant draws), using semi-parametric estimates for \mathbf{p} .²⁰

$$E[\pi_c(d_c|\mathbf{p}_{-c})] = \sum_{\mathbf{d}_{-c} \in |\theta|^{2N}} \left(V_c P_c^G + V_c^0 (1 - P_c^G) - (d_c - \bar{\theta}_c)^2 \cdot \mathbb{1}[d_c > 0] \right) \prod_j p_j^*(\mathbf{d}_{-c}) \quad (A.8)$$

The log-odds ratio is then based on the difference, $\Delta_{\theta,\theta'}(\cdot)$, in "benefits" and costs: $\log(\frac{\hat{p}(d_c=\theta)}{\hat{p}(d_c=\theta')}) = (V_c - V_c^0) \cdot \Delta_{\theta,\theta'}(\sum_{\mathbf{d}_{-c} \in |\theta|^{2N}} (P_c^G) \prod_j p_j^*(\mathbf{d}_{-c})) - \Delta_{\theta,\theta'}((d_c - \bar{\theta}_c)^2).$

For challengers, the cost term is known. Also, we can calculate $E[P_c^G|\mathbf{d}]$ for any position. I estimate equation (A.9), where $\Delta \mathcal{R} \equiv \Delta_{\theta,\theta'}(\sum_{\mathbf{d}_{-c} \in |\theta|^{2N}} (P_c^G) \prod_j p_j^*(\mathbf{d}_{-c}))$. Since the left hand side is the difference in valuations, W and W^0 cannot overlap.

$$V_c - V_c^0 = \left(\log\left(\frac{\hat{p}(d_c = \theta)}{\hat{p}(d_c = \theta')}\right) + \Delta_{\theta, \theta'}((d_c - \bar{\theta}_c)^2)\right) / (\Delta \mathcal{R})$$
(A.9)

Finally, I estimate the incumbent's valuations and ideal point. I use MLE and average across challenger draws to construct the incumbent's expected win probability per choice. Their summation is over policies excluding non-entry.²¹

$$\sum_{w=1}^{\Theta} (d_I - 1) \log \left(\frac{\exp(\pi_I(d_I = \theta))}{\sum_{w=1}^{\Theta} \exp(\pi_I(d_I = w))} \right)$$
(A.10)

The challenger choice probabilities for Republicans and Democrats, with three choices within each party (do not enter, moderate position, and extreme position): inputs are EPGs for candidate position combinations and valuation predictors.²²

¹⁹The fit between prediction and data are: 0.66, 0.26, 0.86, 0.51, 0.30, 0.63, 0.92, 0.29, 0.85, 0.73, 0.32, 0.50 for Inc R SPAC, Inc R Party, Inc R PAC, Cha R SPAC, Cha R Party, Cha R PAC, Inc D SPAC, Inc D PAC, Cha D SPAC, Cha D PAC, respectively.

 $^{^{20}}$ I use the cost function parameters φ_c^P from the primary loser regressions for non-entrants.

²¹There are very few instances in the data of an incumbent un-expectantly deciding not to rerun (as opposed to a previously announced retirement); from 2011-2020, of the 243 non-rerunning incumbents, 70% announced it before election-year and of the remaining 30% who mentioned it during, 65% mentioned it January-February, well before their primaries (Ballotpedia 2020). Thus I omit their re-entry choice and consider the race open when in the data they do not re-run.

²²For Rep. chal., the fit is 0.76, 0.49, and 0.45 for the 3 positions, and for Dem: 0.75, 0.48, 0.53.

A.9 Normalization of σ_{ξ}

With a value for ψ_c^G and σ_{ξ} , one can plug in observed spending and candidate characteristics to calculate the probability of winning. The probability from the committee's perspective will be biased towards 0.5 (meaning closer to the observed vote share excluding abstention) from above and below if the specified uncertainty is too high, and biased towards the corners if the specified uncertainty is too low. While I find that $\operatorname{Var}(\hat{\xi}_c^G) = 0.52$ and $\operatorname{Var}(\hat{\xi}_c^P) = 1.04$, it is likely not equal to σ_{ξ}^2 (the committee's uncertainty about valence).²³ To study the sensitive, I re-run the estimation and look at the general election vote share counterfactuals, but changing uncertainty normalizations of σ_{ξ} . Average results are not significantly different across 10%, 25%, and 50% reductions. For a 50% change, the center and tails of the distribution become more prominent, due to the higher degree of certainty.

A.10 Wealthy Donor Variation

Donors can give unlimited amounts to Super PACs, so if there are donors who want to spend a lot in a given race, their most efficient option is to go to Super PACs. Thus a Super PAC's incentive to invest in a race is largely influenced by whether there are such donors (who care about that race). Since Super PACs raise significant funds from these donors, they are especially vulnerable to a shock affecting the donor. While Super PACs are arguably more sensitive to large swings in donor incomes, donor variation may be weakened by the fact that reported incomes are right censored and the wealthy are less sensitive to local economic shocks; their contributions respond to a variety of factors (Broockman, Ferenstein, and Malhotra 2019).²⁴

I consider variation in individual large donor housing values, real-estate prices, taxes, zip code level mortgage information, and other financial indicators that are proxies for their financial well-being.²⁵ The address level real-estate transaction data are from Corelogic's nationwide database on deeds and taxes.²⁶ The zip code data on mortgage performance and origination are also from Corelogic. I track the financial well-being changes for that individual and zip code over time, which may affect how much the donors give. I weight each shock by the amount that the citizens in that zip code gave to same-party candidates in the previous election. Results for these IVs with the available 2010-2016 data are similar.²⁷

²³Gordon and Hartmann 2016 estimate σ_{ξ} from the FOC by using pre-spending race competitiveness ratings. I include those as covariates in the vote share regression.

²⁴It is difficult to define large donors pre-2010 without defining them via multiple candidates (due to contribution limits), and even that is limited before the 2014 case *McCutcheon v. FEC*.

 $^{^{25}}$ I also collected the Forbes list of U.S. billionaires since 2010. Many committees do not receive any money from a billionaire, so there are too many zeros to utilize this variation.

²⁶Access via Princeton's Data and Statistical Services. Zhao (2023) also studies donors with this.

²⁷IVs are change in: house sale price, house value, house tax, and zip code level: income, mortgage balance, mortgage interest rate, foreclosure rate, days delinquent, and max days delinquent.

A.11 Polling Data

Polling data has issues, such as more polling for competitive races, some races only having polling late into the election, and others having early in the election. Polling data variation includes between and within elections; there is variation across time within an election for some races, but the intervals are not uniform. I do not include polling data recorded throughout the election. One could track the spending effects on each poll up until election day, turning the cross-section into a panel, but this is complicated as the presence of a poll is endogenous. Many Congressional races will only have a couple polls throughout the entire election cycle, and the most competitive races have the most polling done. For example, in 2010 only 8 races had 7 polls on different days; with the vast majority having less than 2. Presidential and Senate races have significantly more polling coverage.

A.12 Proofs

Lemma 1. When voter i's indirect utility from choosing j is expressed as: $U_{ij} = u_j + \xi_j + \epsilon_{ij} \epsilon \sim iid$ Type 1 EV with $\psi = 0, \sigma = 1$, the share of votes is $s_j = \frac{\exp(u_j + \xi_j)}{\sum_{k=0...J} \exp(u_k + \xi_k)}$, with utility of abstention $U_{i0} = u_0 + \epsilon_{i0}$ and number of choices J.

Proof of Lemma 1.

Consider the voter i with preferences over alternatives j=1...J with an outside option j=0: $U_{ij}=u_j+\xi_j+\epsilon_{ij}, \quad \epsilon \sim \text{iid}$ Type 1 EV with $\psi=0, \sigma=1$. Then the probability that voter i, drawn at random from the population, votes for candidate j is: $P_{ij}=(u_j+\xi_j+\epsilon_{ij}>u_k+\xi_k+\epsilon_{ik}\ \forall k\neq j)$. Train (2009) shows that, for this distribution, $P_{ij}=\frac{\exp(u_j+\xi_j)}{\sum_{k=0...J}\exp(u_k+\xi_k)}$. Note that this term is the same $\forall i$, meaning $P_{ij}=P_j$. Since choice probabilities are not observed, we can construct the share of votes for a given candidate based on an average of choices from a sample of the voters: $s_j=\frac{\sum \mathbb{I}[choice=j]}{n}$. For the market share to be consistent for the probability, we need $s_j\to_p P_j$ as the number of votes $n\to\infty$. Assume sufficient number of votes to utilize the equivalence between shares and aggregate probability. \square

Lemma 2. The program in equation (2.4) has a strictly (finite) positive solution for $V_{i_c} > 0 \ \forall i_c \forall c, \ \beta_{i_c} > 0 \ \forall i_c \forall c, \ \phi \in (0,1), \ and \ \sum_{j \in J} \gamma_{ji_c} > 0 \ \forall i_c \forall c.$

Proof of Lemma 2.

Rewrite the effort game as the spending game with the following grouping of variables: the cost of spending $g_{ic} = \left(\sum_{j \in J} \gamma_{jic}\right)^{-1}$ and candidate characteristics $\Delta_c = h_c^G + \psi_c$.

$$\max_{S_{i_c}^G \in \mathbb{R}_+} V_{i_c} \left(\frac{\exp\left(\sum_{j_c \in N_c} \beta_{j_c} (S_{j_c})^{\phi} + \Delta_c\right)}{\sum_{c \in \{D,R\}} \exp\left(\sum_{j_c \in N_c} \beta_{j_c} (S_{j_c})^{\phi} + \Delta_c\right)} \right) - g_{i_c} S_{i_c}^G$$

First we must check whether a solution exists at all.²⁸ It is clear that the payoff is continuous in all arguments. The unrestricted strategy space is non-compact but without loss of generality we can consider a top-bounded space, despite the payoff not being globally concave. Intuitively this is clear as the payoff is a positive constant times a probability (bounded between 0 and 1) plus a linear strictly decreasing cost function. Thus at some point, the costs will overpower the benefits and any solution will be finite. The first order condition for player i_c of this program is:

$$V_{i_c}\beta_{i_c}\phi(S_{i_c})^{\phi-1}\left(\frac{\prod_{c\in\{D,R\}}\exp\left(\sum_{j_c\in N_c}\beta_{j_c}(S_{j_c})^{\phi}+\Delta_c\right)}{\left(\sum_{c\in\{D,R\}}\exp\left(\sum_{j_c\in N_c}\beta_{j_c}(S_{j_c})^{\phi}+\Delta_c\right)\right)^2}\right)-g_{i_c}=0.$$

Note that the derivative of the probability of winning function $P_c = P(w_c^G = 1 | w_c^P = 1, \mathbf{w_{-c}^P}) = ((\frac{\exp(\sum_{j_c \in N_c} \beta_{j_c}(S_{j_c})^{\phi} + \Delta_c)}{\sum_{c \in \{D,R\}} \exp(\sum_{j_c \in N_c} \beta_{j_c}(S_{j_c})^{\phi} + \Delta_c)}))$ is strictly positive and is increasing in $S_{i_c}^G$. Also note that we can write this first order condition more compactly: $[V_{i_c}\beta_{i_c}\phi(S_{i_c})^{\phi-1}P_c(1-P_c) - g_{i_c} = 0$. The second order condition is the following:

$$V_{i_c}\beta_{i_c}\phi(S_{i_c})^{\phi-1}\left((\phi-1)(S_{i_c})^{-1}P_c\cdot(1-P_c) + \frac{\partial P_c}{\partial S_{i_c}}\cdot(1-P_c) + P_c\cdot(-\frac{\partial P_c}{\partial S_{i_c}})\right).$$

To determine the sign of this expression, the following version is easier to work with, using the fact that $\frac{\partial P_c}{\partial S_{i_c}} = \beta_{i_c} \phi(S_{i_c})^{\phi-1} P_c \cdot (1-P_c)$ and combining terms: $V_{i_c} \beta_{i_c} \phi(S_{i_c})^{\phi-1} [P_c \cdot (1-P_c)] ((\phi-1)(S_{i_c})^{-1} + [\beta_{i_c} \phi(S_{i_c})^{\phi-1} \cdot (1-2P_c)])$. The expression $V_{i_c} \beta_{i_c} \phi(S_{i_c})^{\phi-1}$ is strictly positive, and thus the sign is determined by the sum in the parentheses. Since we assumed $\phi \in (0,1)$, the first term $(\phi-1)(S_{i_c})^{-1}$ is strictly negative for any $S_{i_c} > 0$. Note that if $P_c > 1/2$ then the entire expression will be negative and thus the objective function will be concave. However, if $P_c < 1/2$, then it is unclear. Thus: $\operatorname{sign} \left[\frac{\partial \pi_{i_c}^2}{\partial S^2} \right] = \operatorname{sign}[(1-2P_c)\beta_{i_c} \sqrt{S_{i_c}} - 1]$.

Since P_c is strictly increasing in S_{i_c} , as S_{i_c} increases, the term $(1-2P_c)\beta_{i_c}\sqrt{S_{i_c}}$ will become larger and eventually negative. Thus the convexity of π_{i_c} , if any, is confined to some interval [0,B] for B>0. Whether or not any optimal $S_{i_c}^G$ is strictly positive can easily be seen by comparing the payoff from positive spending and zero spending, denoting the sum of others' spending on the same side, $\sum_{j_c \in N_c \setminus \{i_c\}} \beta_{i_c}(S_{i_c})^{\phi}$, with S_{-i_c} : $V_{i_c}\left(\frac{\exp(\beta_{i_c}(S_{i_c})^{\phi} + S_{-i_c} + \Delta_c)}{\sum_{c \in \{D,R\}} \exp(\beta_{i_c}(S_{i_c})^{\phi} + S_{-i_c} + \Delta_c)}\right) - g_{i_c}S_{i_c}^G - V_{i_c}\left(\frac{\exp(S_{-i_c} + \Delta_c)}{\sum_{c \in \{D,R\}} \exp(S_{-i_c} + \Delta_c)}\right)$. Note that the other side does not have an excluded player. At $S_{i_c} = 0$, this term is zero. Thus a positive solution will always dominate a zero if this expression is ever positive for all values of the other variables. To see whether this term is strictly positive for any $S_{i_c} > 0$, we can check its derivative at zero: $V_{i_c}\beta_{i_c}\phi(S_{i_c})^{\phi-1}\left(\frac{\prod_{c \in \{D,R\}} \exp(\sum_{j_c \in N_c} \beta_{j_c}(S_{j_c})^{\phi} + \Delta_c)}{\left(\sum_{c \in \{D,R\}} \exp(\sum_{j_c \in N_c} \beta_{j_c}(S_{j_c})^{\phi} + \Delta_c)\right)^2}\right) - g_{i_c}$.

²⁸Note that we cannot rely on the Debreu, Glicksberg, and Fan Theorem: in an infinite strategic form game, if the strategy space is compact and convex, if the payoffs are continuous in other players' strategies, and if the payoff is continuous and concave in own strategies, then there exists a pure strategy Nash equilibrium. I cannot use this as the payoff is not globally concave. While a quasi-concave version of this theorem exists, I just directly show an equilibrium exists.

Since $\phi \in (0,1)$ and the expression in parentheses is strictly positive, the limit from the right is positive infinity. Thus this function initially increases, starting from zero, and hence is somewhere positive. Now we need to check for the existence of a positive solution. First take the first order condition and rearrange it: $P_c(1-P_c) = \frac{g_{i_c}}{V_{i_c}\beta_{i_c}\phi(S_{i_c})^{\phi-1}}$. Since the right hand side is the same for all players in the game, the best responses are linear functions, letting $\omega_{i_c} = g_{i_c}/(V_{i_c}\beta_{i_c})$: $S_{i_c} = S_{j_c} \left(\frac{\omega_{j_c}}{\omega_{i_c}}\right)^{1/\phi} \quad \forall j_c \ \forall c$. Thus we can rewrite the first order condition in terms of one player, say player 1_R :

$$V_{1_R} \beta_{1_R} \phi(S_{1_R})^{\phi - 1} \left(\frac{\prod_{c \in \{D, R\}} \exp\left((S_{1_R})^{\phi} \sum_{j_c \in N_c} \beta_{j_c} \left(\frac{\omega_{1_R}}{\omega_{j_c}} \right) + \Delta_c \right)}{\left(\sum_{c \in \{D, R\}} \exp\left((S_{1_R})^{\phi} \sum_{j_c \in N_c} \beta_{j_c} \left(\frac{\omega_{1_R}}{\omega_{j_c}} \right) + \Delta_c \right) \right)^2} \right) - g_{i_c} = 0.$$

We can show that this has a real and unique solution. From the preceding discussion, we know that any solution is nonzero and finite, so since the payoff function starts off positive, increases, and eventually becomes negative, we know a positive solution exists. \Box

Lemma 3. The equations that define whether there is a unique solution for the program (2.4) can be expressed as a single equation with two parameters and one variable. Sufficient for a unique solution are magnitude restrictions on the relative sizes of the two parameters.

Proof of Lemma 3.

Continuing from the proof of Lemma 2, the question now is multiplicity. It will be useful to denote terms with simpler notation: $A_c = \sum_{j_c \in N_c} \beta_{j_c} / (\omega_{j_c})$, and express the solution in terms of $X = \omega_{1_R}(S_{1_R})^{\phi}$, with shorthand $e_c = \exp(XA_c + \Delta_c)$. Then we can rewrite:

$$(1/\phi)X = \frac{e_R e_D}{(e_R + e_D)^2}.$$

The goal is to show that these two functions intersect once. First note that the term on the left is strictly increasing linear function starting at 0. The term on the right starts above zero and eventually decreases (which can be seen because the denominator is strictly larger than the numerator and increases at a faster rate). As shown below, this function may initially increase or decrease, but a single intersection with the left hand size function is guaranteed. Consider the derivative of the second term after some combining of terms:

$$\frac{e_R e_D (e_D - e_R) (A_R - A_D)}{(e_R + e_D)^3}.$$

The equation that determines the sign: $\operatorname{sign}[(\exp(XA_D + \Delta_D) - \exp(XA_R + \Delta_R))(A_R - A_D)]$. If $(A_R - A_D)$, then eventually this will be negative. However for low values of X, if $\Delta_D > \Delta_R$, this can be positive. Thus it either starts off positive then goes strictly negative, or is negative throughout. Since the left hand side function starts below the right hand side function, the only possibility of more than one intersection is when the right hand side function increases at a slow enough rate to cross the left hand side and subsequently cross

two more times: the bell shape curve can lead to either 1 crossing or three. This can occur when there are extreme differences on opposite ends: if the effective influence of one side $\sum_{i_c \in N_c} \beta_{ic}^2 V_{ic}/g_{ic}$ is much higher than the other side while simultaneously the other side has an extreme effective valence $h_d + \psi_d$ relative to the initial side (however if too extreme then again a single crossing), then 3 equilibria arise. The only possibility of 2 equilibria are when the increasing part of the bell curve function intersects the left hand side straight line with a tangent before coming back down with another intersection.

Note that we can fully characterize the right hand side in terms of just two parameters (fixing ϕ), where we define $\varpi = A_R - A_D$ and $\varrho = \Delta_R - \Delta_D$:

$$\frac{e_R e_D}{(e_R + e_D)^2} = (\exp(\varpi X + \varrho) + \exp(-(\varpi X + \varrho)) + 2)^{-1}.$$

Then uniqueness can be characterized from the relative magnitude of those two parameters, namely $(\sum_{i_c \in N_c} \beta_{ic}^2 V_{ic}/g_{ic} - \sum_{i_d \in N_d} \beta_{id}^2 V_{id}/g_{id})$ and $(h_d + \xi_d - h_c - \xi_c)$ for candidates c and d. The derivative of this expression is as follows: $\frac{\varpi(\exp(-(\varpi X + \varrho)) - \exp(\varpi X + \varrho))}{(\exp(\varpi X + \varrho) + \exp(-(\varpi X + \varrho)) + 2)^2}$. Thus the function increases when $\varpi(\exp(-(\varpi X + \varrho)) - \exp(\varpi X + \varrho)) > 0$, but if that increasing rate is small enough, it will cross $(1/\phi)X$ while it is increasing: meaning when $\varpi(\exp(-(\varpi X + \varrho)) - \exp(\varpi X + \varrho)) < (1/\phi)$. We can find when the slopes are equal: $\log\left(\frac{-(1/\phi)(1/\varpi) \pm \sqrt{((1/\phi)(1/\varpi))^2 - 4}}{2}\right)/\varpi - \varrho/\varpi = x$.

Lemma 4. The program in equation (2.5) has a pure strategy solution for strictly positive $\{V_{i_c}, \beta_{i_c}, \sum_{j \in J} \gamma_{ji_c}\}\ \forall i_c \forall c \ and \ \phi \in (0,1)$. This solution is unique for sufficiently large σ (conditional on a unique program (2.4)).

Proof of Lemma 4.

This proof follows the approach from $\operatorname{Cox}(2022)$. Denote any second stage Nash equilibrium vector of spending given an entry profile $(a_1, ..., a_N)$ as $(S_1^*, ..., S_N^*)$. The committee's interim expected payoff for a given entry decision conditional on their private information is denoted with U_i and given in equation (A.11). The summation is across all 2^{N-1} combinations of opponent decisions a_{-i} ; the term $p_j(a_{-i})$ is the belief by player i in the probability of player j choosing a_j from the decision profile a_{-i} . The term $p_{i,j}(e_{-i})$ is the belief by player i of the probability of player j choosing the a_j from the decision profile a_{-i} ; the term p_{-i} is the vector of opponent probabilities of a = 1; the term ε_i is private information:

$$U_{i}(S_{1}^{*},...,S_{N}^{*},a_{1},...,a_{N},p_{-i}) = \sum_{a_{-i} \in \{0,1\}^{2N-1}} \pi_{i}^{*}(S_{1}^{*},...,S_{N}^{*}|a_{1},...,a_{N}) \prod_{j \neq i} p_{j}(a_{-i}) + \varepsilon_{i} \cdot a_{i}.$$
(A.11)

First I show that there exists a pure strategy (Perfect Bayesian equilibrium for this stage) in cutoff strategies. Let the first part of the payoff be denoted with u_i so that $U_i = u_i + \varepsilon_i$. Given the iid distribution of ε , the beliefs are symmetric, meaning player i's belief about player j equals player k's belief about player j: $p_{i,j} = p_{k,j} = p_j$. Thus one can write out

any player's belief about player i choosing $a_i = 1$ as $p_i(a_i = 1) = Prob[u_i(1, p_{-i}) + \varepsilon_i > u_i(0, p_{-i})]$. Which, given the scaled Logistic distribution of ε , yields the functional form $p_i = \frac{\exp(u_i(1, p_{-i})/\sigma)}{\exp(u_i(1, p_{-i})/\sigma) + \exp(u_i(0, p_{-i})/\sigma)} = f(p_{-i})$. This is a continuous system of choice probabilities \mathbf{p} that defines an equilibrium if one

This is a continuous system of choice probabilities \mathbf{p} that defines an equilibrium if one exists: $\mathbf{p} = f(\mathbf{p})$. Note that $\mathbf{p} \in [0,1]^N$ and $f(\mathbf{p}) : [0,1]^N \to [0,1]^N$. Thus f is a continuous function over a compact convex set. As noted in Bajari et al. (2010), applying Brouwer's fixed point theorem to this system yields a pure strategy equilibrium for finite values of π .

The system $\Phi(\mathbf{p}) = \mathbf{p} - f(\mathbf{p}) = 0$ will have one zero if the matrix of partial derivatives of Φ with respect to p is a positive dominant diagonal matrix, meaning: $\left|\frac{\partial \Phi_i}{\partial p_i}\right| > 0 \ \forall i$ and $\left|\frac{\partial \Phi_i}{\partial p_i}\right| \geq \sum_{j \neq i} \left|\frac{\partial \Phi_i}{\partial p_j}\right| \ \forall i$.

Given the functional form, the first is satisfied with value of unity. The second can be satisfied for a sufficiently large σ . To see this, first write out the expression for a given i:

$$\sum_{j\neq i} \left| \frac{\partial \Phi_i}{\partial p_j} \right| = \frac{\exp(u_i(1)/\sigma - u_i(0)/\sigma)}{(1 + \exp(u_i(1)/\sigma - u_i(0)/\sigma))^2} \sum_{j\neq i} \left| \frac{\partial u_i(1)}{\partial p_j} - \frac{\partial u_i(0)}{\partial p_j} \right| \frac{1}{\sigma}.$$

$$\frac{\partial u_i(1)}{\partial p_j} = \sum_{a_{-\{i,j\}}} \left[\pi_i(a_i = 1, a_j = 1, a_{-\{i,j\}}) - \pi_i(a_i = 1, a_j = 0, a_{-\{i,j\}}) \right] \prod_{k \neq \{i,j\}} p_k(a_{-\{i,j\}})$$

with a complementary expression for $\frac{\partial u_i(0)}{\partial p_j}$. Note that $\frac{\partial u_i(1)}{\partial p_j}$ is less than the maximum difference in payoffs for entering M, with an analogous bounding for $\frac{\partial u_i(0)}{\partial p_j}$, equal to m. Both M and m are well-defined given the interior solution to the second stage game.

$$\frac{\partial u_i(1)}{\partial p_j} \le \max_{a_{-\{i,j\}}} \left[\pi_i(a_i = 1, a_j = 1, a_{-\{i,j\}}) - \pi_i(a_i = 1, a_j = 0, a_{-\{i,j\}}) \right] = M_{ij}$$

$$\frac{\partial u_i(0)}{\partial p_i} \ge \min_{a_{-\{i,j\}}} \left[\pi_i(a_i = 0, a_j = 1, a_{-\{i,j\}}) - \pi_i(a_i = 0, a_j = 0, a_{-\{i,j\}}) \right] = m_{ij}$$

The expression $\frac{\exp(u_i(1)/\sigma - u_i(0)/\sigma)}{(1+\exp(u_i(1)/\sigma - u_i(0)/\sigma))^2}$ can also be bounded above by noting that the function $\frac{\exp(x/\sigma)}{(1+\exp(x/\sigma))^2}$ achieves its maximum at x=0 for any positive σ with a function value of 1/4 at that point. Thus one can bound the sum of the absolute cross-partials:

$$\sum_{j \neq i} \left| \frac{\partial \Phi_i}{\partial p_j} \right| \le \frac{1}{4\sigma} \sum_{j \neq i} \left| \frac{\partial u_i(1)}{\partial p_j} - \frac{\partial u_i(0)}{\partial p_j} \right| \le \frac{1}{4\sigma} \sum_{j \neq i} |M_{ij} - m_{ij}|.$$

Thus sufficient for uniqueness (conditional on also satisfying uniqueness from the spending stage in Lemma 2) is $\sigma \ge \max_{i \in \mathcal{I}} \{ \sum_{j \neq i} |M_{ij} - m_{ij}|/4 \}$.

Lemma 5. The program in equation (2.6) has a strictly (finite) positive solution for $V_{i_c} > 0 \ \forall i_c \forall c, \ \beta_{i_c} > 0 \ \forall i_c \forall c, \ \phi \in (0,1), \ and \ \sum_{j \in J} \gamma_{ji_c} > 0 \ \forall i_c \forall c.$

Proof of Lemma 5.

$$\max_{e_{i_c}^P \in \mathbb{R}_+} V_{i_c} P(w_c^P = 1) P(w_c^G = 1 | w_c^P = 1 \cap w_{D_2}^P = 1) \cdot P(w_{D_2}^P = 1) + V_{i_c} P(w_c^P = 1) P(w_c^G = 1 | w_c^P = 1 \cap w_{D_1}^P = 1) \cdot P(w_{D_1}^P = 1) - e_{i_c}^P$$

Which can be rewritten as below, where $\Omega_c = P(w_c^G = 1 | w_c^P = 1 \cap w_{D_2}^P = 1) \cdot P(w_{D_2}^P = 1) + P(w_c^G = 1 | w_c^P = 1 \cap w_{D_1}^P = 1) \cdot P(w_{D_1}^P = 1)$: $\max_{e_{i_c}^P \in \mathbb{R}_+} V_{i_c} P(w_c^P = 1) (\Omega_c) - e_{i_c}^P$.

The arguments for the existence of a solution follow from the proof for the general election contest, as the payoffs have the same shape in own arguments, but are just scaled by the probabilities from the other primary election.

Lemma 6. The solution to equation (2.6) is determined by just two variables in two equations. Sufficient conditions for uniqueness can be expressed in terms of 4 exogenous terms.

Evidence for Lemma 6.

Continuing from the proof of Lemma 5.

$$\Omega_c V_{i_c} \beta_{i_c} \phi(S_{i_c})^{\phi - 1} \left(\frac{\prod_{d \in \{R_1, R_2\}} \exp\left(\sum_{j_d \in N_d} \beta_{j_d} (S_{j_d})^{\phi} + \Delta_d\right)}{\left(\sum_{d \in \{R_1, R_2\}} \exp\left(\sum_{j_d \in N_d} \beta_{j_d} (S_{j_d})^{\phi} + \Delta_d\right)\right)^2} \right) - g_{i_c} = 0$$

Define the term $\omega_{i_c}^P = g_{i_c}/(V_{i_c}\beta_{i_c})$. Note that the best response functions are linear with respect to the other players from your direct primary (not with respect to players from the other primary, whose actions are contained in Ω_c).

$$S_{i_c} = S_{j_d} \left(\frac{\omega_{j_d}}{\omega_{i_c}} \cdot \frac{\Omega_c}{\Omega_d} \right)^{1/\phi} \quad \forall j_d \ \forall c \in \{R_1, R_2\}$$

We have two sets of these for both sides of the primary. This mirrors the general election just now with two sets with the exception of the Ω terms which capture the forward-looking nature of committees during the primary. Thus we can write out the primary election first order condition for the Republican side as just a function of spending of a single Republican committee (from either side) and the spending from the Democratic primary (with the analogous case for the Democratic spending). Thus the solution is characterized by two sets of equations:

$$(1/\phi)\omega_{1_R}S_{1_{R_1}}^{\phi} = \Omega_{R_1} \cdot P_{R_1}([\Omega_{R_2}/\Omega_{R_1}]\omega_{1_R}S_{1_{R_1}}^{\phi}) \cdot (1 - P_{R_1}(\cdot))$$

$$(1/\phi)\omega_{1_D}S_{1_{D_1}}^{\phi} = \Omega_{D_1} \cdot P_{D_1}([\Omega_{D_2}/\Omega_{D_1}]\omega_{1_D}S_{1_{D_1}}^{\phi}) \cdot (1 - P_{D_1}(\cdot)).$$

Recall from the proof for the general election, that each equation can have a unique solution (assumed here) so that we can write out the best responses as functions (not correspondences): $S_{1_{R_1}} = BR_R(S_{1_{D_1}})$, and $S_{1_{D_1}} = BR_D(S_{1_{R_1}})$. We can write out the two

equations with simpler notation, letting $X = \omega_{1_R} S_{1_{R_1}}^{\phi}$ and $Y = \omega_{1_D} S_{1_{D_1}}^{\phi}$. Let G_{11}^R be the equilibrium expected general election probability of candidate R_1 beating candidate D_1 , with similar notation for the other terms. Note that $G_{11}^D = 1 - G_{11}^R$, $G_{12}^R = 1 - G_{21}^D$, etc.

$$(1/\phi)X = [G_{11}^R P_{D_1}(Y) + G_{12}^R (1 - P_{D_1}(Y))] \cdot P_{R_1} \left(X \cdot \left[\frac{G_{21}^R P_{D_1}(Y) + G_{22}^R (1 - P_{D_1}(Y))}{G_{11}^R P_{D_1}(Y) + G_{12}^R (1 - P_{D_1}(Y))} \right] \right) \cdot (1 - P_{R_1}(\cdot))$$

$$(1/\phi)Y = [G_{11}^D P_{R_1}(X) + G_{12}^D (1 - P_{R_1}(X))] \cdot P_{D_1} \left(Y \cdot \left[\frac{G_{21}^D P_{R_1}(X) + G_{22}^D (1 - P_{R_1}(X))}{G_{11}^D P_{R_1}(X) + G_{12}^D (1 - P_{R_1}(X))} \right] \right) \cdot (1 - P_{D_1}(\cdot))$$

We must establish the curvature of the best responses. First take the derivative of the best response for X in terms of Y by differentiating the first equation by Y and re-arranging, where it will be useful to define the a new term which is derived from to the derivative of the ratio $\Omega_{R_2}/\Omega_{R_1}$ with respect to Y: $\Omega_{\delta}^R = \frac{(G_{21}^R - G_{22}^R)\Omega_{R_1} - (G_{11}^R - G_{12}^R)\Omega_{R_2}}{(\Omega_{R_1})^2}$.

$$\frac{\partial BR_{X}(Y)}{Y} = \frac{\frac{\partial P_{D_{1}}}{\partial Y} \left(G_{11}^{R} - G_{12}^{R}\right) P_{R_{1}}(1 - P_{R_{1}}) + \Omega_{R_{1}} \frac{\partial P_{R_{1}}}{\partial X \cdot [\Omega_{R_{2}}/\Omega_{R_{1}}]} BR_{X} \frac{\partial P_{D_{1}}}{\partial Y} \Omega_{\delta}^{R}(1 - 2P_{R_{1}})}{1/\phi - [\Omega_{R_{1}}] \frac{\partial P_{R_{1}}}{\partial X \cdot [\Omega_{R_{2}}/\Omega_{R_{1}}]} [\Omega_{R_{2}}/\Omega_{R_{1}}] (1 - 2P_{R_{1}})}$$

To determine the curvature of the best responses, consider the G terms.²⁹ If $G_{11}^R = G_{12}^R$, then the best response curve is flat because player 1_{R_1} is indifferent to which Democratic candidate wins. In this case the solution from the general election contest suffices to show a unique solution. Similarly, if either of the probabilities for the opposing side are equal to 1, meaning the other candidate did not enter, then we again reach the degenerate best response. To consider the other cases, we must establish the curvature of the best responses. First take the derivative of the best response for X in terms of Y by differentiating the first equation, which is an implicit function of the best response function, by Y and re-arranging:

$$\frac{\partial BR(Y)}{Y} = \frac{\frac{\partial P_{D_1}}{\partial Y} \left(G_{11}^R - G_{12}^R \right) P_{R_1} (1 - P_{R_1})}{1/\phi - \left[G_{11}^R P_{D_1} + G_{12}^R (1 - P_{D_1}) \right] \frac{\partial P_{R_1}}{\partial X} (1 - 2P_{R_1})}.$$

The sign of the numerator is based on the following, with $A_{D_c} = \sum_{j_{D_c} \in N_{D_c}} \beta_{j_{D_c}} / \omega_{j_{D_c}}$.

$$\operatorname{sign}\left[\frac{\partial P_{D_1}}{\partial Y}\left(G_{11}^R - G_{12}^R\right)\right] = \operatorname{sign}\left[\left(A_{D_1} - A_{D_2}\right)\left(G_{11}^R - G_{12}^R\right)\right]$$

The A terms are the aggregate effective spending influence of the democratic committees for the Democratic primary. The G terms are the equilibrium expected probability of the Republican winning against either Democrat. Thus the sign is positive if Democrat 1 candidates are more effective at spending and the Republican 1 has a better chance against

 $^{^{29}}$ If we assume $\Omega_{R_2} = \Omega_{R_1}$, then it is straightforward to establish curvature. The case of $\Omega_{R_2} \neq \Omega_{R_1}$ revolves around similar terms but involves significantly more algebra.

Democrat 1 than Democrat 2 in the general. The sign of the denominator is determined by the following condition, where for shorthand $\theta = G_{11}^R P_{D_1} + G_{12}^R (1 - P_{D_1})$, and $\exp_{1_{R_c}} = \exp(A_{1_{R_c}}X + \Delta_{R_c})$:

$$sign[denom] = sign \left[1/\phi - \theta \cdot \frac{(\exp_{R_1} \exp_{R_2})(A_{R_1} - A_{R_2})(\exp_{R_2} - \exp_{R_1})}{(\exp_{R_1} + \exp_{R_2})^3} \right].$$

Note that $1/\phi$ is strictly greater than one and Ξ is strictly between zero and one. Also note that if the sign of this term ever changed, then it necessarily crosses 0 (as it is continuous) and the derivative would be undefined at that point. If $A_{R_1} - A_{R_2}$ is sufficiently large and $\Delta_{R_2} - \Delta_{R_1}$ is sufficiently large, then the sign can be positive for small X'; thus the question remains of whether there exists a Y' such that X' = BR(Y'). The best response is a Sigmoid function (with the convex-concave turning point being based on the difference in candidate characteristics for the opposite primary), either increasing if the product $(A_{D_1} - A_{D_2}) \cdot (G_{11}^R - G_{12}^R)$ is positive, decreasing if strictly negative, or flat if zero.

Lemma 7. The program in equation (2.7) has a pure strategy solution for strictly positive $V_c, V_c^0 \, \forall c$. Furthermore, the solution to program in equation (2.7) is unique for sufficiently large σ_C .

Proof of Lemma 7.

Using the same logic as from the Proof of Lemma 4, Brouwer's fixed point theorem for the multinomial logit case guarantees existence for finite payoff values. The sufficient conditions for uniqueness in the Proof of Lemma 4 have multinomial Logit analogs. However now there are additional equations, namely three per player (one for each decision). Thus player i has probability p_{i_d} : specifically p_{i_0} , p_{i_1} , and p_{i_2} such that $p_{i_0} + p_{i_1} + p_{i_2} = 1$; for example i_0 refers to the $d_c = 0$ decision for candidate i.

$$p_{i_d} = \frac{\exp(u_{i_d}(d, p_{j_d} \forall j \forall d) / \sigma_C)}{\sum_{f = \{0, 1, 2\}} \exp(u_{i_f}(f, p_{j_d} \forall j \forall d) / \sigma_C)} = f(p_{-i_d}).$$

The system $\Phi(\mathbf{p}) = \mathbf{p} - f(\mathbf{p}) = 0$ will have one zero if the matrix of partial derivatives of Φ with respect to \mathbf{p} is a positive dominant diagonal matrix, meaning:

$$|\frac{\partial \Phi_{i_d}}{\partial p_{i_d}}| > 0 \ \forall i \ \forall d \quad \& \quad |\frac{\partial \Phi_{i_d}}{\partial p_{i_d}}| \geq \sum_{(j_d \forall i \forall d) \backslash (i_d)} |\frac{\partial \Phi_{i_d}}{\partial p_{j_d}}| \ \forall i_d.$$

The summation in the second inequality, namely $(j_d \forall j \forall d) \setminus (i_d)$, includes all of i's probabilities other than their choice for d and each other player j's full set of choice probabilities.

The own-derivative condition is satisfied with value of one. The second is satisfied with own cross-choice probability with a value of zero. The second for cross-player derivatives

can be satisfied for a sufficiently large σ_C . To see this, first write out the expression for i_0 :

$$\sum_{j_d \forall j \neq i \forall d} \left| \frac{\partial \Phi_{i_0}}{\partial p_{j_d}} \right| = \sum_{e = \{1,2\}} \left(\frac{\exp((u_{i_e} - u_{i_0})/\sigma_C)}{(1 + \sum_{f = \{1,2\}} \exp([(u_{i_f} - u_{i_0})/\sigma_C])^2} \sum_{j_d \forall j \neq i \forall d} \left| \frac{\partial u_{i_e}}{\partial p_{j_d}} - \frac{\partial u_{i_0}}{\partial p_{j_d}} \right| \frac{1}{\sigma_C} \right).$$

Following the logic from the Proof of Lemma 4, each cross partial of u_{i_d} with respect to p_{j_d} can be bounded; let that maximum be denoted with M_{i_d,j_d} . Then similarly, we can rewrite that first term on the right hand side:

$$\frac{\exp((u_{i_e} - u_{i_0})/\sigma_C)}{(1 + \sum_{f=\{1,2\}} \exp([(u_{i_f} - u_{i_0})/\sigma_C])^2} = p_{i_1}p_{i_0}.$$

This product is strictly bounded between 0 and 1. Thus one can bound the sum of the absolute cross-partials for i_0 and by extension every other choice and player:

$$\sum_{j_d \forall j \neq i \forall d} \left| \frac{\partial \Phi_{i_0}}{\partial p_{j_d}} \right| < \frac{1}{\sigma_C} \sum_{e = \{1, 2\}} \left(1 \cdot \sum_{j_d \forall j \neq i \forall d} \left| M_{i_e, j_d} - M_{i_0, j_d} \right| \right)$$

Thus sufficient for uniqueness (conditional on uniqueness of the spending stage from Lemma 6) is $\sigma_C \ge \max_{i_D \forall i} \{ \sum_{e=\{1,2\}} \sum_{j_d \forall j \neq i \forall d} |M_{i_e,j_d} - M_{i_D,j_d}| \}$.

Proposition 1. There exists a pure strategy Bayesian Nash equilibrium in which all agents condition on payoff relevant actions.

Proof of Proposition 1.

The proof is by backward induction, and all steps are based on conditioning on payoff relevant only actions. By Lemma 2, the general election spending stage has a pure strategy Nash equilibrium. By Lemma 4, the general election entry stage has a pure strategy Bayesian Nash equilibrium. By Lemma 5, the primary spending stage has a pure strategy Nash equilibrium. By re-applying Lemma 4 to the primary stage, the primary entry stage has a pure strategy Bayesian Nash equilibrium. Then by Lemma 7, the challenger entry stage has a unique pure strategy Bayesian Nash equilibrium. The incumbent's discrete choice single-agent environment will have a unique pure decision rule given the discrete set of actions. Thus the game has a Bayesian Nash equilibrium in pure strategies.

A19

A.13 Additional Tables

Table A1: General Election Voter Regression Summary Statistics

 Variable	Mean	Std. Dev.	Min.	Max.
Log Share Of Votes Minus Log Abstention Share	-0.757	0.773	-7.435	1.099
Candidate Spending	4.520	5.94	0	50.221
Super PAC Spending	1.421	4.116	0	45.695
Party Spending	1.758	4.852	0	36.874
PAC Spending	0.553	1.664	0	20.813
Candidate Position	1.458	0.498	1	2
Out Of District Lagged Position	0.505	0.232	0	2.198
Out-Of-State Candidate Donor Zip Income Variation	0.48	0.543	-1.456	5.704
Out-Of-State Party Donor Ideology Variance	0.133	0.091	0	0.556
Out-Of-State Candidate Donor Ideology Variance	0.258	0.186	0	1.605
Out-Of-State Super PAC Donor Ideology Variance	0.136	0.091	0	0.815
Out-Of-State PAC Donor Zip Income Variation	0.201	0.231	-0.768	3.503
Out-Of-State Opponent Donor Zip Income Variation	0.678	0.861	-2.432	7.049
Out-Of-State Opponent S-PAC Donor Ideo. Var.	0.144	0.316	-2.452	2.872
Within-State Candidate Donor Zip Income Variation	0.301	0.41	-1.806	5.517
Within-State Party Donor Zip Income Variation	0.161	0.197	-0.373	3.783
Within-State Super PAC Donor Zip Income Variation	0.254	0.498	-3.26	5.05
Within-State Candidate Donor Ideology Variance	0.331	0.167	0	1.531
District Unemployed Rate	6.428	2.479	2.142	16.869
District Income	8.049	1.435	5.267	15.369
District Unemployed Number	9.034	6.58	1.271	29.548
Lagged Republican Presidential Votes	0.484	0.152	0.03	0.825
Incumbent	0.46	0.498	0	1
Party=Republican	0.501	0.5	0	1
Lagged Incumbent Votes	0.581	0.249	0	1
Number Of Senate Candidates	7.95	7.385	0	27
Contested Primary	0.885	0.32	0	1
Governor Same Party	0.485	0.5	0	1
District High-School Rate	29.17	6.143	11.2	46.757
District Median Age	40.237	3.454	29.306	51.269
District Election Day Precipitation	0.088	0.141	0	1.052
Average Ad Cost Per Committee	6.648	0.701	4.659	9.000
District White Percentage	0.754	0.174	0.16	0.968
District Male Percentage	0.491	0.01	0.457	0.537
Cook's Competitiveness	0.446	2.787	-3	3
N	3536			

This table shows the summary statistics for the variables used in the estimation of general election voter preferences. Spending by each committee, district income, district unemployment number, and precipitation is scaled as followed: $(X/1e3)^{0.5}$.

Table A2: General Election Voter Preference 1st stage Excluded IVs

	Candidate	-Opp SPAC	Party/PAC	Candidate
	Spending	Spending	Spending	Position
Inc=0 × Out Of District	0.5043	1.0552	0.8647	-0.0146
Lagged Position	(0.4257)	(1.1839)	(0.4749)	(0.0435)
$Inc=1 \times Out Of District$	-0.7804	0.1419	1.4956*	0.4922***
Lagged Position	(0.5918)	(1.3974)	(0.7419)	(0.0580)
Out-Of-State Candidate	0.4036**	0.1159	0.1444	0.0377*
Change in Donor Zip Income	(0.1539)	(0.2571)	(0.1567)	(0.0178)
1	,	,	,	,
Out-Of-State Party	13.5133***	-29.7319***	20.9792***	0.1144
Donor Ideology Variance	(1.0704)	(2.7011)	(1.3650)	(0.1065)
	0.0070***	0.4015	0.1100***	0.1000**
Out-Of-State Candidate	3.9278***	0.4817	2.1193***	-0.1800**
Donor Ideology Variance	(0.6500)	(1.1375)	(0.6167)	(0.0688)
Out-Of-State Super PAC	5.1858***	-14.6325***	3.4582**	0.2379*
Donor Ideology Variance	(1.0324)	(2.4457)	(1.0779)	(0.0995)
	,	,	,	,
Out-Of-State Party	3.0916***	-4.7728***	3.7194^{***}	0.0355
Change in Donor Zip Income	(0.6652)	(1.1107)	(0.7759)	(0.0435)
	0.0070**	0.1014	0.0100	0.0000**
Out-Of-State Opp. Candidate	0.3278**	-0.1814	0.0190	0.0288**
Change in Donor Zip Income	(0.1025)	(0.2895)	(0.1051)	(0.0102)
Out-Of-State Opp. Spac	3.4386***	-11.5922***	4.0074***	-0.0356
Donor Ideology Variance	(0.3429)	(0.8775)	(0.4438)	(0.0259)
Observations	3514	3514	3514	3514
R^2	0.528	0.340	0.421	0.330
F-statistic of excluded IVs	65.25	48.41	50.25	11.79
F-statistic	30.5905	8.4761	11.6747	35.7321

Robust standard errors in parentheses. Controls in Appendix Table A3. The KP under-identification test rejects the null with an LM-statistic of 51.390 and p-value of 0.000. The Hansen J-statistic for over-identification fails to reject with 4.93 and a p-value of 0.49. The first stage F-tests of the excluded instruments all have p-value<0.000 and their F-values are 65.25, 48.41, 50.25, and 11.79; the SW F-values are 10.16, 15.19, 10.19, 10.19, 10.19, 10.19, 10.19, 10.19, and 10.19, 10.

Table A3: General Election Voter Parameters: Controls

Table A3: General Ele	ction Voter P	arameters: Co	ontrols		
	1st Stage 1	1st Stage 2	1st Stage 3	1st Stage 4	2nd Stage
Within-State Candidate Donor Zip Income Variation	0.2803 (0.1822)	-0.0127 (0.4531)	-0.1524 (0.2202)	0.1074*** (0.0226)	-0.0381 (0.0357)
Within-State Party Donor Zip Income Variation	-0.3212	1.4126	1.0325	-0.1147	-0.2416*
	(0.4860)	(1.0111)	(0.5982)	(0.0725)	(0.1051)
Within-State Super PAC Donor Zip Income Variation	-0.0251	-0.6206	0.1295	-0.0117	-0.0329
Within-State Candidate Donor Ideology Variance	(0.2113) 0.3643	(0.5285) -0.4461	(0.2945) -0.8091	(0.0202) -0.0526	(0.0235) 0.1596*
Within-State Candidate Donor Ideology Variance	(0.4828)	(0.9210)	(0.5097)	(0.0639)	(0.0673)
District Unemployed Rate	0.1507	-0.2538	0.0459	0.0058	0.0073
D: I	(0.1000)	(0.1921)	(0.1199)	(0.0104)	(0.0123)
District Income	0.0405 (0.0718)	-0.0711 (0.2151)	-0.0658 (0.0766)	0.0075 (0.0101)	0.0990*** (0.0136)
District Unemployed Number	-0.1882***	0.0329	-0.1232***	-0.0035	-0.0292***
	(0.0249)	(0.0530)	(0.0318)	(0.0027)	(0.0041)
Lagged Republican Presidential Votes	1.9836	7.5329*	-1.4218	-0.3375**	-1.4231***
Incumbent	(1.3873) 177.4192	(3.1590) -8.3e+02*	(1.6470) 3.2859	(0.1252) 25.7424	(0.1745) 18.2572
	(210.2473)	(373.3865)	(206.8778)	(18.7635)	(22.9349)
Party=Republican	71.1369	158.5742	-51.9656	56.7934***	95.1111***
I d Inbt W-t	(111.2091)	(270.9655)	(115.7772)	(10.3234)	(16.3418)
Lagged Incumbent Votes	-1.7525*** (0.4336)	3.4291*** (0.9164)	-1.9051*** (0.5061)	0.0170 (0.0445)	-0.1654** (0.0515)
Number Of Senate Candidates	-0.0201	0.0246	-0.0012	0.0001	-0.0008
G 170	(0.0109)	(0.0246)	(0.0132)	(0.0012)	(0.0013)
Contested Primary	(0.0709	(0.9754)	(0.4501)	0.1239***	-0.0182
Governor Same Party	(0.3782) -0.0562	(0.9754) 0.4028	(0.4501) -0.2985	(0.0345) 0.0625***	(0.0441) 0.0179
3	(0.1637)	(0.3779)	(0.1866)	(0.0157)	(0.0229)
District High-School Rate	-0.0358	0.1437^{*}	0.0369	-0.0106***	-0.0175***
District Median Age	(0.0293) 0.0034	(0.0621)	(0.0311)	(0.0029)	(0.0040)
District Median Age	(0.0434)	-0.0873 (0.0943)	0.0030 (0.0500)	0.0289*** (0.0042)	0.0394*** (0.0067)
District Election Day Precipitation	0.7545	0.5074	-0.3822	0.0101	-0.0394
110.7.	(0.5947)	(1.4060)	(0.5781)	(0.0592)	(0.0768)
Average Ad Cost Per Committee	-0.7612*** (0.1840)	-0.4803 (0.3760)	-0.3520 (0.2350)	-0.0710*** (0.0184)	0.0478 (0.0255)
District White Percentage	0.5022	-4.4618**	0.4368	0.0902	0.3666**
ŭ	(0.7982)	(1.6983)	(0.8387)	(0.0834)	(0.1153)
District Male Percentage	27.8203*	6.9216	25.2859	0.4257	-3.6735*
R x District High-School Rate	(12.4005) 0.0271	(26.5611) -0.0079	(13.4319) 0.0266	(1.2855) 0.0062*	(1.6944) 0.0040
	(0.0278)	(0.0615)	(0.0311)	(0.0028)	(0.0036)
R x District Median Age	0.0476	-0.1460	0.0821	-0.0361***	-0.0099
R x Lagged Republican Presidential Votes	(0.0526) -3.3104	(0.1079) 3.3585	(0.0569) -0.1849	(0.0050) 1.1498***	(0.0092) 3.1559***
it x hagged hepublican i residentiai votes	(1.8061)	(4.0597)	(2.1593)	(0.1618)	(0.3057)
R x District White Percentage	-0.6410	2.1613	-0.4032	-0.1437	0.3243*
D. Division II D.	(1.0394)	(2.1356)	(1.1015)	(0.1166)	(0.1558)
R x District Male Percentage	-12.3622 (15.2209)	-23.6431 (34.3866)	-2.4519 (17.9240)	2.1511 (1.7503)	2.4989 (2.1762)
Incumbent x District Unemployed Number	-0.0739***	0.0325	0.0161	0.0032	0.0160***
	(0.0220)	(0.0571)	(0.0236)	(0.0025)	(0.0041)
Incumbent x District Unemployed Rate	0.0426	0.0260	0.0733	-0.0360**	-0.0131
Incumbent x Lagged Incumbent Votes	(0.1203) 0.7437	(0.1903) -1.7591	(0.1249) 0.4937	(0.0110) -0.2239***	(0.0142) 0.1128
	(0.5849)	(1.1819)	(0.6999)	(0.0596)	(0.0811)
Incumbent x District Income	-0.1833*	-0.0451	0.0877	-0.0032	-0.0308*
Inc=0 x Party=D x Cook's Competitiveness	(0.0862) -0.4828***	(0.2566) 0.1140	(0.0917) -0.4302*	(0.0112) 0.0291*	(0.0155) 0.0303
Inc—o x 1 ary—D x Cook s Competitiveness	(0.1386)	(0.4330)	(0.1762)	(0.0120)	(0.0194)
Inc=0 x Party=R x Cook's Competitiveness	0.5159***	-0.7325**	0.1405	-0.0009	0.0437**
I I D C D C D C	(0.0983)	(0.2351)	(0.1244)	(0.0101)	(0.0137)
Inc=1 x Party=D x Cook's Competitiveness	2.4223*** (0.3392)	-0.9685 (0.5462)	2.5524*** (0.4856)	0.0236 (0.0225)	-0.0512 (0.0418)
Inc=1 x Party=R x Cook's Competitiveness	-1.7402**	-0.4174	-1.4279	0.0008	0.0692
	(0.5427)	(0.9390)	(0.8953)	(0.0296)	(0.0575)
Inc=0 x Party=D x Cycle Time Trend	0.1171	-0.3421	-0.0236	0.0412***	0.0576***
Inc=1 x Party=D x Cycle Time Trend	(0.1277) 0.0330	(0.2619) 0.0720	(0.1230) -0.0229	(0.0105) 0.0283***	(0.0140) 0.0485***
In I are by -D & Oyde Time frend	(0.0552)	(0.1339)	(0.0574)	(0.0051)	(0.0081)
Inc=0 x Party=R x Cycle Time Trend	0.0844	-0.4130*	0.0011	0.0129	0.0090
State & Cycle Fixed Effect	(0.1041)	(0.1848)	(0.1026)	(0.0093) Voc	(0.0113)
State & Cycle Fixed Effects	Yes	Yes	Yes	Yes	Yes

Robust standard errors in parentheses. These are the controls for the regressions in Table A2.

Table A4: Primary Election Voter Parameters: Controls

	Republican	Democratic		Republican	Democratic
Within-State Candidate	0.0528	0.0730	Average Ad Cost	-0.0202	0.1131*
Donor Zip Income Variation	(0.0651)	(0.0503)	Per Committee	(0.0512)	(0.0454)
Within-State Party	-0.2804*	-0.5207**	District White %	0.9120***	0.2789
Donor Zip Income Variation	(0.1267)	(0.1901)		(0.2341)	(0.1874)
Within-State Super PAC	-0.0581	-0.0319	District Male %	-3.5479	-4.3066
Donor Zip Income Variation	(0.0590)	(0.0535)		(3.0807)	(2.8374)
Within-State Candidate	0.8698***	0.8377***	Incumbent x District	0.0237^{**}	-0.0061
Donor Ideology Variance	(0.1208)	(0.1058)	Unemployed Number	(0.0087)	(0.0073)
District Unemployed Rate	0.0631**	0.0163	Incumbent x District	0.0245	0.0331
	(0.0213)	(0.0251)	Unemployed Rate	(0.0231)	(0.0301)
District Income	0.0394	0.0318	Incumbent x Lagged	0.4336**	0.0628
	(0.0373)	(0.0233)	Incumbent Votes	(0.1383)	(0.1218)
District Unemployed	-0.0262**	-0.0173*	Incumbent x	0.0040	0.0295
Number	(0.0082)	(0.0078)	District Income	(0.0414)	(0.0276)
Lagged Republican	1.6195***	-1.9798***	Inc=0 x Party=R x	-0.0060	
Presidential Votes	(0.2820)	(0.2573)	Cook's Competitiveness	(0.0131)	
Incumbency Status	40.7314	-1.3e + 02**	Inc=1 x Party=R x	0.1373	
	(37.1421)	(46.7678)	Cook's Competitiveness	(0.0714)	
Lagged	-0.3604***	0.0125	Inc=0 x Party=R x	0.0202	
Incumbent Votes	(0.1021)	(0.0790)	Cycle Time Trend	(0.0184)	
Number Of Senate	-0.0013	-0.0048	Inc=0 x Party=D x		0.0703***
Candidates	(0.0025)	(0.0033)	Cook's Competitiveness		(0.0119)
Contested Primary	-0.1614*	-0.1475**	Inc=1 x Party=D x		-0.0668
	(0.0626)	(0.0569)	Cook's Competitiveness		(0.0462)
Governor Same Party	-0.0487	-0.1691**	Inc=0 x Party=D x		-0.0669**
	(0.0576)	(0.0605)	Cycle Time Trend		(0.0231)
District High-School Rate	0.0064	0.0078	Constant	-45.3193	130.8263**
	(0.0058)	(0.0057)		(37.0748)	(46.5863)
District Median Age	0.0279**	0.0429***	State & Cycle FE	Yes	Yes
	(0.0086)	(0.0098)	Observations	2385	2190
District Election	0.2763^{*}	0.0713	R^2	0.578	0.492
Day Precipitation	(0.1346)	(0.1362)	F	50.3706	36.9291

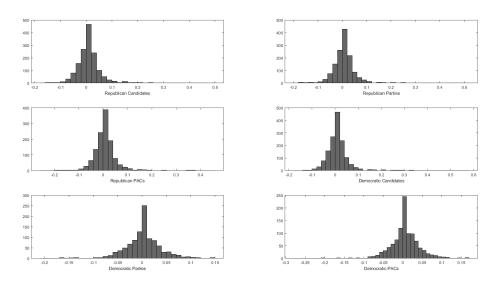
Robust standard errors in parentheses.

Table A5: Model Fit Statistics

Candidate Position				Primary Election Spending			
Variable	Data	Model	Correlation	Variable	Data	Model	Correlation
Inc R Candidate	1.377	1.2743	0.2815	Inc R Candidate	58.4044	57.9023	0.4863
Cha R Candidate	0.7128	0.6371	0.4270	Inc R Super PAC	6.4622	25.8916	0.3032
Inc D Candidate	1.3475	1.2786	0.3861	Inc R Party	1.0393	1.5921	0.5948
Cha D Candidate	0.5590	0.4945	0.4852	Inc R PAC	1.6691	1.3926	0.5953
				Cha R Candidate	20.0280	41.4971	0.4814
(Candidate	Entry Totals		Cha R Super PAC	6.8172	41.4624	0.1906
Variable	Data	Model		Cha R Party	0.9899	0.3564	0.0322
Inc R Candidate	1545	1583		Cha R PAC	0.6338	1.1480	0.2315
Cha R Candidate	787	801		Inc D Candidate	35.2046	26.0485	0.6443
Inc D Candidate	1539	1523		Inc D Super PAC	3.8535	6.1009	0.3883
Cha D Candidate	632	625		Inc D Party	0.6590	5.4525	0.0000
				Inc D PAC	1.0470	0.8783	0.7884
G	eneral Elec	tion Spending		Cha D Candidate	21.0192	14.2689	0.3625
Variable	Data	Model	Correlation	Cha D Super PAC	3.3354	5.3948	0.6476
R candidate	295.2791	169.2999	0.7444	Cha D Party	1.4934	8.5719	0.0000
R Super PAC	115.9741	67.2293	0.7462	Cha D PAC	0.6270	0.2872	0.1041
R Party	144.2056	142.2421	0.7516				
R PAC	16.0309	8.2118	0.7770	Primary	Election I	Entry Tota	als
D Candidate	335.3436	216.9161	0.6606	Variable	Data	Model	
D Super PAC	124.8620	17.3993	0.2201	Inc R Candidate	787	780	
D Party	142.8750	89.3063	0.5127	Inc R Super PAC	64	509	
D PAC	24.3584	10.2651	0.6937	Inc R Party	31	95	
				Inc R PAC	179	226	
Ger	neral Electi	on Entry Totals		Cha R Candidate	787	780	
Variable	Data	Model		Cha R Super PAC	87	505	
R candidate	1458	1543		Cha R Party	39	52	
R Super PAC	491	1289		Cha R PAC	74	167	
R Party	326	1435		Inc D Candidate	632	583	
R PAC	816	1290		Inc D Super PAC	57	124	
D Candidate	1458	1543		Inc D Party	26	195	
D Super PAC	435	403		Inc D PAC	176	247	
D Party	473	1083		Cha D Candidate	632	583	
D PAC	718	1069		Cha D Super PAC	44	82	
				Cha D Party	11	152	
Ge	neral Elect	ion Vote Share		Cha D PAC	30	218	
Variable	Data	Model	Correlation				
R average	0.5060	0.5522	0.6452	Primary	Election	Vote Shar	e
R Inc	0.6703	0.6879	0.2687	Variable	Data	Model	Correlation
R Cha	0.3118	0.3918	0.2650	Inc R Candidate	0.7605	0.7289	0.1212
				Inc D Candidate	0.7865	0.7514	0.2711

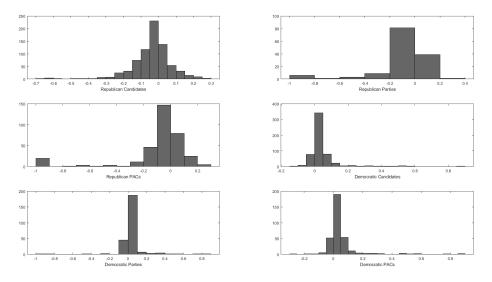
This table shows the data and model averages and correlations for the main choice variables. It also shows the total sum of binary entry decisions in the data and model (for one private draw). "Inc" refers to incumbent. "Cha" is shorthand for challenger. R and D are shorthand for Republican and Democrat aligned groups. The vote share is defined excluding abstention.

Figure A1: Percent Change in General Election Spending Without Super PACs



This plots the histogram of percent changes in general election spending with and without Super PACs. I compare the simulated equilibrium spending and counterfactual spending if Super PACs cannot enter for candidates, parties, and PACs for both Republicans and Democrats. For Republicans, the mean is 3.69% and the median is 0.03% for candidates, 3.98% & 0.00% for parties, and 14.37% & 0.00% for PACs. For Democrats, the mean is 16.68% and the median is 0.04% for candidates, 0.72% & 0.00% for parties, and 2.22% & 0.03% for PACs.

Figure A2: Percent Change in Primary Election Spending Without Super PACs



This plots the histogram of percent changes in primary election spending with and without Super PACs. I compare the simulated equilibrium spending and counterfactual spending if Super PACs cannot enter for candidates, parties, and PACs for both Republicans and Democrats. For Republicans, the mean is -4.39% and the median is -2.50% for candidates, -8.64% & -2.65% for parties, and -6.73% & -3.02% for PACs. For Democrats, the mean is 8.68% and the median is 2.15% for candidates, 7.78% & 1.18% for parties, and 8.86% & 2.22% for PACs.