Climbing the Energy Ladder: How Energy Resources Hinder, Facilitate, and Fuel Economic Growth*

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I show that the nature of the energy resources available to an economy qualitatively determines long-run growth outcomes. A harvested resource such as biomass drags on growth, a mined resource such as coal enables output per capita to hold constant, and both a tapped resource such as oil and a manufactured resource such as solar panels risk degrowth if energy return on energy invested (EROI) cannot stay above a threshold. The only energy resource that can fuel long-run growth is a manufactured resource such as solar panels. Either that resource must rely on substitutable energy inputs that have a sufficiently large EROI, or it must be produced by robots that are themselves produced from robots and energy. Even in these cases, coal and oil economies may have been necessary stages on the way from a biomass economy to a solar economy.

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1 Introduction

The ignition of sustained economic growth around the Industrial Revolution roughly coincided with the first sustained exploitation of fossil energy resources.[1] Over the ensuing centuries, both economic output and fossil resource use have continually grown, generating global wealth and global warmth without precedent in human history. Now the global economy is in the midst of a transition from fossil resources to renewable resources, in particular solar photovoltaics. Some fear that this transition will impede further economic growth by removing the fossil energy dividend that underpins it (e.g., King and van den Bergh, 2018; Jackson and Jackson, 2021).

I study the energetic basis of economic growth. In particular, I consider how fossil energy resources may have fostered growth and the implications of solar resources for growth. To this end, I extend a neoclassical Solow-Swan growth model to include energy as a factor of production. I show that the origin of that energy determines the types of growth outcomes that are possible.

Until the nineteenth century, all human societies were run on "harvested resources" in the form of biomass. These resources are extracted from an exogenous and renewable resource base via application of labor and capital. I show that the limited availability of land constituted a drag on growth in output per capita. I call such an outcome "energy-scarce degrowth". Output per capita could tread water or grow only if technological change was sufficiently rapid.

In the nineteenth century, several European economies began large-scale exploitation of "mined resources" in the form of coal. These resources require inputs of labor, capital, and energy, both to initially open a mine in a coal seam and to subsequently extract coal from that seam over time. I show that mined resources cannot drive growth in output per capita but do enable output per capita to tread water even in the absence of technological change. As a result, positive growth arises from even the slightest rate of technological change. I call such an outcome "energy-enabled growth". The essential element of growth around the Industrial Revolution may well have been some factor other than energy, but the increasing use of coal energy may have enabled that other factor to drive growth.

In the twentieth century, developed economies came to rely on "tapped resources" in the form of oil and gas. Initially tapping these resources requires an upfront input of labor, capital, and energy, but once tapped, energy production does not scale with further inputs. In particular, oil and gas wells need inputs at the point of drilling, but once a well has been drilled, the pressure in the well drives production, without needing to send workers or machines into the well to coax the oil or gas out. I show that such resources can sustain output per capita, as mined resources do. But I also show that another outcome is possible

[1] Some attribute economic growth to that shift in the energetic basis of the economy (e.g., Wrigley, 1988; Pomeranz, 2000; Allen, 2009; Wrigley, 2010), whereas others argue that energy was not critical to growth (e.g., Mokyr, 1990; Clark and Jacks, 2007; Clark, 2014; Mokyr, 2016).

when energy is complementary to capital and labor in deposit-tapping: if each deposit does not produce enough energy, then energy cannot be reinvested at a high enough rate to ensure the continued supply of energy. The limited availability of energy prevents the economy from reaching a balanced growth path. I call such an outcome "energy-scarce degrowth".[2]

In the twenty-first century, economies are increasingly reliant on solar energy. Solar panels have consistently outpaced expectations for installed capacity (Economist, 2024) and cost (Way et al., 2022), and solar power is on track to become the largest single source of electricity generation around 2030 (IEA, 2024). Solar panels are similar to oil and gas resources in that an initial investment produces energy for years on end. Solar panels' distinguishing feature is that they are "manufactured resources", with machines and energy combining to convert silicon and other minerals into a standardized product.[3] I show that a solar economy with panels produced from capital, labor, and energy contains the long-run outcomes described above for the economy with a tapped resource. In addition, I show that solar panels can drive long-run growth if energy inputs to their manufacture are substitutable with other inputs to their manufacture and the productivity of those energy inputs is sufficiently large. In that case, solar energy becomes the dominant input to solar panel production, with solar energy producing ever more panels. Energy becomes abundant and can cause output per capita to grow over time. I call such an outcome "energy-fueled growth".

Many companies are working to automate the production and installation of solar panels via use of robots and artificial intelligence (Liu, 2024; Plumer, 2024).[4] One day, these robots may themselves be produced in an automated fashion, from other robots and energy. In this case, I show that the economy can enter a regime of "energy-fueled growth" even when production functions are Cobb-Douglas. Again the critical condition is that the productivity of energy in making energy be sufficiently large.

I tie each of these outcomes to energy return on energy investment (EROI). This metric is much analyzed and discussed among scholars of energy systems.[5] EROI captures how much energy is produced per unit of energy invested in making energy. Energy analysts have been especially interested in the possibility that the economy is approaching a "net energy cliff": society might eventually have to invest so much energy into producing energy

[2]A mined resource avoids this outcome because production from existing mines scales with labor inputs to mining.

[3]Solar panels do require mined minerals, but there is a salient difference with respect to a "mined resource" such as coal: the minerals embodied in solar panels subsequently produce a flow of energy that does not scale with ongoing inputs, whereas coal produces energy only at the moment of combustion.

[4]Nemet (2019) describes the key role that automation played in the declining cost of solar panels. The scope for automating production of solar panels potentially exceeds that of other sectors of the economy. The manager of solar panel manufacturing plant in Florida boasted, "City and state officials who have seen our plant contend this is likely the most automated factory in Florida" (Mendenhall, 2022).

[5]See Murphy and Hall (2010) and Brandt and Dale (2011) for histories of the concept. A small prior literature formally relates EROI and growth in numerical simulations (Court et al., 2018; Fagnart et al., 2020) and Leontief production structures (Fagnart and Germain, 2016).

that it becomes hard (or even impossible) to generate the energy required for the rest of civilization.[6] EROI looms especially large in current discussions for two reasons. First, the EROI of fossil fuels has been declining over time.[7] Second, many analysts worry that a shift to renewable energy would reduce EROI and risk approaching the net energy cliff.[8]

I show that EROI is endogenous to an economy, in the sense that it depends on input choices. EROI can even decrease in the physical productivity of an energy resource, depending on how energy is substituted for other inputs to energy production. As a result of this endogeneity, EROI in an economy with a mined resource always exceeds unity plus the rate of population growth ($1+g_L$). In an economy with a tapped resource having complementary inputs in its tapping or a solar resource having complementary inputs in its manufacture, energy-scarce degrowth occurs when EROI cannot exceed $1+g_L$ along a balanced growth path. This possibility substantiates fears of a net energy cliff.[9] In economies with energy-fueled growth, EROI exceeds $1+g_L$ by a margin that increases with the rate of economic growth. When solar panels are manufactured from substitutable inputs or by robots, a high-EROI solar resource accumulates energy indefinitely, with each panel making enough energy to both accumulate more energy and devote energy to final good production. Such a solar resource promises a "net energy ramp", which is the optimistic counterpart of the dreaded net energy cliff.

Even if a solar-fueled net energy ramp materializes, it will occur in a world full of carbon dioxide from prior fossil resource use. I show that a climate cost may have been an unavoidable byproduct of the path to a solar economy. Accessing a new type of resource and retooling the economy's capital stocks to work with it requires diverting a nontrivial portion of economic output from consumption. The upfront cost of learning to drill for oil is not trivial, and the upfront cost of learning to harness and commercialize the photoelectric effect at the heart of solar panels is especially large. I show that when these fixed costs are

[6]Analysts disagree about the minimum EROI required to sustain society: Hall et al. (2009) suggest that the minimum EROI is 5; Hall and Klitgaard (2012) say the minimum is probably around 10; Brandt (2017) argues for a minimum between 1.1 and 15; and Fizaine and Court (2016) suggest 11.

[7]See, among others, Cleveland (1992), Gagnon et al. (2009), Guilford et al. (2011), Murphy (2014), Court and Fizaine (2017), and Brockway et al. (2019).

[8]There is much debate about the EROI of particular renewable technologies. Two analyses of EROI for a renewable energy system suggest that it would be around 5, engendering concern about reaching the net energy cliff (Trainer, 2018; Capellán-Pérez et al., 2019). Others argue that the EROI for renewables will fall as the energy system changes (Trainer, 2018; Fabre, 2019) and that a transition to a renewable energy system would require a period in which so much energy is devoted to renewable energy development that energy becomes less abundant for the rest of society and energy sector emissions may even increase (King and van den Bergh, 2018; Sers and Victor, 2018; Slameršak et al., 2022). On the other hand, some argue that the EROI of renewable energy is now relatively large and will increase further over time as technology improves (e.g., Steffen et al., 2018; Diesendorf and Wiedmann, 2020).

[9]But the cliff is not diagnostic. Numerical simulations show that EROI can exceed $1+g_L$ even in the midst of energy-scarce degrowth and that EROI can decline towards $1+g_L$ even in the midst of energy-fueled growth.

high, an economy with a harvested resource may not generate enough output to be able to access oil or solar resources, which would leave the economy trapped in a biomass world in the absence of an alternative. However, an alternative did exist, in the form of coal resources that were visible from the surface and readily usable for production. I show that when the coal resource's productivity is sufficiently large, the biomass economy optimally transitions to using the coal resource. Moreover, a productive coal resource can raise output per capita high enough to be able to pay the fixed cost of the oil resource, so that the economy may subsequently transition to using the oil resource. Finally, that oil resource may be sufficiently productive to enable society to afford the fixed cost of the solar resource, and if that solar resource is sufficiently productive, the economy does subsequently transition to the solar resource. The economy climbs an energy ladder towards the solar resource and the potential for emission-free, energy-fueled growth. Emission-free solar panels thereby stand on the shoulders of dirty fossil resources.[10]

Energy resources are of potential importance to long-run economic growth because the second law of thermodynamics means that energy cannot be completely recycled. The ability to sustain energy consumption therefore ultimately depends on stocks of fossil resources and flows of renewable resources. Economists have long studied how the finitude of fossil resource stocks affects the economy's ability to sustain long-run consumption growth (e.g., Dasgupta and Heal, 1974; Solow, 1974; Stiglitz, 1974; Krautkraemer, 1998; Hassler et al., 2021) and can induce transitions between resources (e.g., Nordhaus, 1973; Chakravorty and Krulce, 1994; Tahvonen and Salo, 2001).[11] Motivated by evidence that fossil reserves are actually quite abundant (e.g., Rogner et al., 2012), I here focus on how each energy resource is accessed, albeit permitting extraction costs to depend on the availability of resource deposits (see Heal, 1976).[12] My analysis can thus describe nearer-term growth outcomes rather than the much longer-run outcomes for which resource exhaustion may become a concern.

The present paper analyzes the implications for growth of some aspects of energy production that have been studied in the applied microeconomics literature. First, some prior papers study the role of fixed costs in accessing deposits of energy resources (e.g., Hartwick et al., 1986; Holland, 2003; Venables, 2014). I integrate more general forms of fixed costs

[10] Hansen and Prescott (2002) model a transition from a "Malthusian" land-based economy to a "Solow" economy that lacks a fixed factor. The decreasing returns to scale of the present paper's harvested resource are similar to the Malthusian economy of Hansen and Prescott (2002), and the constant returns to scale of the present paper's mined resource are similar to their Solow economy. In Hansen and Prescott (2002), the transition occurs because exogenous (and, implicitly, energy-free) technical progress raises productivity to the point where it is profitable to operate the Solow technology. I instead emphasize how the Solow technology can raise output per capita to the point where other resources become accessible.

[11] Other literature endogenizes the role of innovation in determining the types of energy resources used (e.g., Acemoglu et al., 2012; Hart, 2019; Lemoine, 2024).

[12] I assume that new deposits of the tapped and mined resources are discovered at a constant, exogenous rate. Prior work has analyzed the equilibrium implications of endogenous exploration for new resource deposits (e.g., Pindyck, 1978; Gilbert, 1979; Arrow and Chang, 1982; Ekeland et al., 2022). Future work might synthesize the literatures on growth and endogenous exploration.

into a growth model. Second, recent work emphasizes that the flow of oil from a well is, for practical purposes, exogenous (Thompson, 2001; Mason and van't Veld, 2013; Anderson et al., 2018; Kellogg, 2024), based on a physical relationship known as Darcy's Law. The present paper's "tapped resource" is consistent with these papers' models.[13]

The "linearity critique" holds that sustained economic growth has fragile preconditions within macroeconomic models. To generate a trajectory with constant growth in output, economic models require a linear relationship somewhere under the hood: AK models place the linearity in physical capital accumulation and production, Lucas-type models place it in human capital accumulation, ideas-based endogenous growth models place it in productivity growth, and semi-endogenous growth models place it in population growth (Jones, 2005). These linear relationships are all knife-edge conditions, with small deviations ruling out sustained exponential growth (Growiec, 2010). The question is which type of linearity is well-founded, both in principle and empirically. Jones (2005) observes that there is no intuitive reason to prefer a model with linear growth in ideas to a model with either fishing-out or spillover effects, and Jones (1995) reports that linearity in idea accumulation is inconsistent with empirical evidence. Instead, Jones (2005) argues that population growth is sensibly linear since "people reproduce in proportion to their numbers", but others find such linearity contrary to both intuition and evidence (e.g., Solow, 2003; Growiec, 2010).

I show that energy can sustain growth in two ways. First, if energy is a substitutable input to solar panel manufacturing, then it can crowd out other inputs and thereby relieve that manufacturing of other constraints.[14] Energy production becomes linear in energy. This mechanism resembles the potential for capital to crowd out labor at high elasticities of substitution in aggregate production functions (Solow, 1956; Pitchford, 1960; de La Grandville, 1989).[15] In reality, capital requires energy, so the present analysis clarifies the preconditions for substitution from labor to drive growth. Second, if solar panels are produced by robots, then the stocks of robots and solar panels grow in tandem and the transition equation for solar panels becomes, in equilibrium, effectively linear. The mechanism has parallels in the recent literature on artificial intelligence and robots (see Trammell and Korinek, 2023). In particular, Mookherjee and Ray (2017) analyze when it is optimal for robots to crowd out labor in the production of robots and output. They show that this possibility depends on an additional robot producing sufficiently more than one extra robot even as robot production becomes entirely reliant on capital, which is an EROI-like condition (they refer to it as their "von Neumann singularity condition"). In that case, sustained growth is achievable, driven by capital accumulation. However, this literature generally ignores that operating robots

[13]Moreno-Cruz and Taylor (2017) model how a resource's energetic density per unit of land affects the spatial distribution of economic activity. I abstract from space and study how a resource's energy produced per unit energy input affects the evolution of energy and output over time.

[14]In particular, energy would crowd out labor along the solar panel manufacturing chain.

[15]And the potential for energy-scarce degrowth resembles the potential for degrowth at low elasticities of substitution in aggregate production functions.

requires energy. Incorporating a scarce energy input would tend to prevent sustained growth in economic output. I show that solar energy resources can restore the link between robots and growth if their production function permits self-sustaining feedback between energy and robots.[16]

The next section outlines the broader economic environment. Subsequent sections analyze harvested, mined, tapped, and manufactured resources. Section 7 analyzes resources manufactured by robots. Section 8 discusses actual energy transitions within the context of the model. Section 9 describes how the model generates an energy ladder. The final section concludes. The appendix contains additional analysis, numerical details, and proofs.

2 Setting

I begin by introducing the elements of the model that are common across types of resources.

At each instant t, the economy is populated by $L(t)$ households who discount future utility at rate ρ. The population grows at rate $g_L \geq 0$, with $g_L < \rho$. Households supply a fixed unit of labor to the economy. They save a fixed share $s \in (0, 1)$ of the final good $Y(t)$ and consume the rest. They have utility $u(\cdot)$ over time t per-capita consumption, with $u(\cdot)$ strictly increasing and concave.

The aggregate capital stock is increased by savings:

$$\dot{K}(t) = sY(t) - \delta K(t),$$

with $\delta > 0$ the depreciation rate. A representative firm produces the final good $Y(t)$ by combining labor $L_Y(t)$, capital $K_Y(t)$, and energy $E_Y(t)$ in a Cobb-Douglas production function:

$$Y(t) = A(t)L_Y(t)^{\alpha_L} K_Y(t)^{\alpha_K} E_Y(t)^{\alpha_E},$$

where $\alpha_L, \alpha_K, \alpha_E > 0$ and $\alpha_L + \alpha_K + \alpha_E = 1$. $A(t) > 0$ is productivity, which grows at rate $g_A \geq 0$. After the first part of the paper, I fix $g_A = 0$ in order to focus on the role of energy in growth.[17]

Available energy $E(t)$ depends on the time t resource base $R(t)$. Subsequent sections will specify how energy is produced and how the resource base evolves. Depending on the section, the resource base will represent land, coal mines, oil deposits, or solar panels.

I analyze the economy's optimal path, ranked according to the following welfare criterion:

$$\int_0^\infty e^{-\rho t} L(t)\, u\Big((1 - s)Y(t)/L(t)\Big)\, \mathrm{d}t. \tag{1}$$

[16] Nordhaus (2021) shows that artificial intelligence can increase the rate of economic growth if it makes capital substitutable for labor. This possibility is similar to my discussion of solar panels produced from substitutable energy inputs.

[17] Ignoring productivity growth also means that I do not have to take a stand on how productivity growth depends on the availability of energy (see Suzuki, 1976).

I am interested in the long-run evolution of output per capita. In particular, I am interested in the existence of a balanced growth path along which output is strictly positive and output, capital, and energy each grow at constant rates while maximizing (1) subject to resource constraints. Throughout, lower-case symbols indicate variables in per-capita form.

3 Harvested Resources: Biomass

Begin by considering a pre-industrial world in which energy derives from harvesting the products of land and sun, such as crops or trees. Because land and solar radiation flows are fixed from year to year, the aggregated resource base R is fixed (so I here drop its time argument). Energy produced from the land depends on labor $L_E(t)$ and capital $K_E(t)$ devoted to harvesting the resource:

$$E(t) = Q_H L_E(t)^{\phi_{HL}} K_E(t)^{\phi_{HK}} R^{\phi_{HR}},$$

where $\phi_{HL}, \phi_{HK}, \phi_{HR} > 0$, $\phi_{HL} + \phi_{HK} + \phi_{HR} = 1$, and $Q_H > 0$. There is only one use for energy, so $E(t) = E_Y(t)$. The labor market clears when $L(t) = L_Y(t) + L_E(t)$, and the capital market clears when $K(t) = K_Y(t) + K_E(t)$.

Substituting for $E(t)$ and converting to per-capita form, the final good production function becomes

$$y(t) = A(t)\ell_Y(t)^{\alpha_L} k_Y(t)^{\alpha_K} \left[Q_H \ell_E(t)^{\phi_{HL}} k_E(t)^{\phi_{HK}}\right]^{\alpha_E} r(t)^{\alpha_E \phi_{HR}},$$

where $r(t) \triangleq R/L(t)$ decreases over time. From the capital transition equation, capital and output must grow at the same rate on a balanced growth path. However, growth in capital is not sufficient for continual growth in output. First, the labor supply is constrained by population growth. Second, the land resource is fixed. The greater the value share of the land resource (i.e., the larger is $\alpha_E \phi_{HR}$), the greater the drag on growth in output per capita.

The following proposition establishes the growth rate of output per capita:

Proposition 1. *Along a balanced growth path, the growth rate of output per capita is*

$$\frac{g_A - \alpha_E \phi_{HR} g_L}{\alpha_L + \alpha_E(\phi_{HL} + \phi_{HR})}.$$

Proof. See Appendix C. □

This economy exhibits two types of long-run behavior, depending on the pace of technical change.[18] First, output per capita eventually collapses to zero if $g_A < \alpha_E \phi_{HR} g_L$. If technical change is not sufficiently fast to overcome the diminishing returns in energy production, then

[18] There is also a knife-edge case with constant output per capita.

output per capita collapses to zero. This is *energy-scarce degrowth* because the collapse is driven by lack of energy. Importantly, this case arises when technical change is nonexistent ($g_A = 0$). Second, output per capita grows forever if $g_A > \alpha_E \phi_{HR} g_L$. If technical change is sufficiently fast, then output per capita can grow forever, but this growth occurs in spite of, not because of, the availability of the energy resource.

4 Mined Resources: Coal

Now consider an economy fueled by coal mines. There are two stages to obtaining coal from a mine. First, the mine must be opened: the hole must be bored and the shaft set up.[19] Second, coal must be extracted from the mine. The resource base $R(t)$ represents mines already opened and available for production. Opening a mine at time t uses labor $L_R(t)$, capital $K_R(t)$, and energy $E_R(t)$ inputs, and mining coal uses labor $L_E(t)$ and capital $K_E(t)$ inputs.[20] The energy market clears when $E(t) = E_Y(t) + E_R(t)$, the labor market clears when $L(t) = L_Y(t) + L_R(t) + L_E(t)$, and the capital market clears when $K(t) = K_Y(t) + K_R(t) + K_E(t)$.

Energy production from existing mines is

$$E(t) = Q_M L_E(t)^{\phi_{ML}} K_E(t)^{\phi_{MK}} R(t)^{\phi_{MR}}, \tag{2}$$

where $\phi_{ML}, \phi_{MK}, \phi_{MR} > 0$ and $\phi_{ML} + \phi_{MK} + \phi_{MR} = 1$. The multiplier $Q_M > 0$ accounts for both the productivity of mining and the energy contained in each unit of coal.[21] Through appropriate rescaling, this production function is consistent with requiring energy to extract coal from a mine and transport it to market.[22]

This energy production function is superficially similar to the one in Section 3, but there is an important difference: rather than being fixed by the availability of land and sun, the

[19]See Ashton and Sykes (1929, Chapter II) for a description of this process. Underground mines still constituted most of U.S. coal production as recently as 1970 (EIA, 2024).

[20]Fouquet (2008, 222) observes, "At the pit face, the principal method of extraction was the miner's pick and shovel." Capital in the form of steam engines kept mines clear of water: see Ashton and Sykes (1929, Chapter III), among others. And see footnote 22 below regarding the fuel for those steam engines.

[21]When describing the expansion of the coal industry, Wrigley (2010, 46) argues that labor productivity was static over 1700–1900, and Fouquet (2008, 57) writes, "…the main reason the industry expanded was simply because it used existing practices and multiplied the number of men and seams being exploited."

[22]First, imagine that time t energy production were instead $\tilde{Q}_M L_E(t)^{\tilde{\phi}_{ML}} K_E(t)^{\tilde{\phi}_{MK}} E_E(t)^{\tilde{\phi}_{ME}} R(t)^{\tilde{\phi}_{MR}}$, with $\tilde{\phi}_{ML} + \tilde{\phi}_{MK} + \tilde{\phi}_{ME} + \tilde{\phi}_{MR} = 1$. The representative energy producer's first-order condition for $E_E(t)$ would require that $E_E(t) = [\tilde{\phi}_{ME} \tilde{Q}_M L_E(t)^{\tilde{\phi}_{ML}} K_E(t)^{\tilde{\phi}_{MK}} R(t)^{\tilde{\phi}_{MR}}]^{1/(1-\tilde{\phi}_{ME})}$. Substituting, time t energy production becomes $\left[\tilde{Q}_M (\tilde{\phi}_{ME})^{\tilde{\phi}_{ME}} L_E(t)^{\tilde{\phi}_{ML}} K_E(t)^{\tilde{\phi}_{MK}} R(t)^{\tilde{\phi}_{MR}}\right]^{1/(\tilde{\phi}_{ML}+\tilde{\phi}_{MK}+\tilde{\phi}_{MR})}$. This is equivalent to our case with ϕ_{ML}, ϕ_{MK}, ϕ_{MR}, and Q_M appropriately defined. Second, imagine that production of energy does not require energy inputs but bringing energy to market imposes a fixed cost of c units of energy. If this production function has productivity $\tilde{Q}_M$, it is equivalent to our case if we define $Q_M \triangleq \tilde{Q}_M - c$. Moreno-Cruz and Taylor (2017) consider the implications of c varying with distance.

resource base is here dynamic and endogenous. The transition equation for the resource base is:

$$\dot{R}(t) = Z(t)^{\omega} F(L_R(t), K_R(t), E_R(t))^{1-\omega} - \lambda R(t), \tag{3}$$

with $\omega \in [0, 1)$. $\lambda > 0$ is the depreciation rate of mines. The quantity of new mines opened depends on inputs of labor, capital, and energy. $Z(t)$ represents the stock of available sites. When $\omega > 0$, mines are easier to open when more mine sites are available to choose from, in the spirit of models of resource extraction following Heal (1976), whereas when $\omega = 0$, mine sites are not depletable, as when the highest quality coal resources are abundant. The stock of available sites evolves as

$$\dot{Z}(t) = -Z(t)^{\omega} F(L_R(t), K_R(t), E_R(t))^{1-\omega} + \Omega Z(t). \tag{4}$$

Opening a mine reduces the stock of mine sites. New deposits are discovered at rate $\Omega \in [0, \rho)$.[23] This discovery rate can also be interpreted as technical change that increases the economically recoverable resources from known deposits. I assume that $\omega(\Omega - g_L) > 0$, so that either resources are not depletable or the discovery rate keeps up with population growth. This assumption is a good fit for coal resources, which remain physically abundant after hundreds of years of exploitation (Rogner et al., 2012, Section 7.4).

The representative firm's production function for opening mines has the constant elasticity of substitution (CES) form:

$$F(L_R(t), K_R(t), E_R(t)) \triangleq \begin{cases} \left((1-\kappa_E)(A_{LK} L_R(t)^{\kappa_L} K_R(t)^{\kappa_K})^{\frac{\sigma-1}{\sigma}} + \kappa_E (A_E E_R(t))^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} & \text{for } \sigma \neq 1 \\ (A_{LK} L_R(t)^{\kappa_L} K_R(t)^{\kappa_K})^{1-\kappa_E} (A_E E_R(t))^{\kappa_E} & \text{for } \sigma = 1 \end{cases}, \tag{5}$$

where $\kappa_L, \kappa_K > 0$, $\kappa_L + \kappa_K = 1$, and $\kappa_E \in (0, 1)$. $\sigma > 0$ is the elasticity of substitution between energy and labor inputs, and $A_{LK}, A_E > 0$ control the productivity of inputs in mine-opening.

Murphy and Hall (2010) define energy return on energy invested (EROI) as measuring energy gained over energy required to get it. But there are many possible metrics compatible with that definition—and the energy analysis literature has indeed used many types of metrics. My metric is in the spirit of the "net external energy ratio" (NEER), which is the ratio of the net energy produced in a system to the energy inputs from external sources (Brandt and Dale, 2011). I treat the system as the economy at a given instant t. The external input is

[23] For tractability, here deposits are easier to find when they are relatively abundant, but the opposite story is also plausible if economic incentives are stronger than geologic constraints on deposit discovery. Exhaustibility is not focus of my analysis, and I leave the study of endogenous discovery in a growth model to future work.

the energy devoted to mine opening (i.e., diverted from uses that assist in contemporaneous final good production) in the previous instant. Formally,

$$EROI(t) \triangleq \frac{E_R(t) + \dot{E}(t)}{E_R(t)}. \tag{6}$$

When $EROI(t) \geq 1$, the energy committed to opening mines is technically sustainable for some finite interval of time, without reducing energy available for final good production.

The following proposition establishes a lower bound on EROI:

Proposition 2 (EROI Lower Bound)**.** *If $Y(t) > 0$, then*

$$EROI(t) > 1 + g_L + \frac{\dot{e}(t)}{e(t)}.$$

Proof. From definition (6) and the identity $\dot{E}(t)/L(t) = \dot{e}(t) + g_L e(t)$,

$$EROI(t) = 1 + \left(\frac{\dot{e}(t)}{e(t)} + g_L\right)\frac{e(t)}{e_R(t)}. \tag{7}$$

From $\alpha_E > 0$ and $e_Y(t) + e_R(t) = e(t)$, final good production $Y(t)$ is strictly positive only if $e(t) > e_R(t)$. □

Economy-wide EROI must be large enough to generate surplus energy that can be dedicated to final good production, not just reinvested in opening mines. If energy grows at the rate of population along a balanced growth path (i.e., if $\dot{e}(t) = 0$), then EROI must exceed $1 + g_L$ so that maintaining that rate of growth does not require substituting energy from final-good production. If energy is growing faster than population along a balanced growth path (i.e., if $\dot{e}(t) > 0$), then EROI must be even larger so as to generate the additional energy that can maintain investment in opening and extracting from ever more mines.

From here on, I eliminate exogenous productivity growth so as to highlight whether output growth is possible in the absence of technical change:

Assumption 1. $g_A = 0$.

The following propositions describe balanced growth paths. Begin with the Cobb-Douglas case:

Proposition 3 (Cobb-Douglas Mine-Opening)**.** *Fix $\sigma = 1$ and let Assumption 1 hold. An interior balanced growth path has all real variables growing at the same rate as population and the prices of unmined deposits and operating mines growing at rate $\rho - g_L$. Such a path exists. Along that path,*

$$EROI(t) = 1 + \frac{1}{\phi_{MR}\kappa_E}\frac{\lambda + \rho}{\lambda + g_L}\frac{(1-\omega)(\rho - \Omega) + \omega(\rho - g_L)}{(1-\omega)(\rho - \Omega)}g_L.$$

The elasticity of output per capita on the balanced growth with respect to Q_M is strictly positive and constant in Q_M.

Proof. See Appendix E. □

Proposition 3 describes a case of *energy-enabled growth*. Whereas Proposition 1 showed that an economy with a harvested resource avoids a collapse in output per capita only if technical progress is fast enough, Proposition 3 shows that an economy with a mined resource can have output grow as fast as population even in the absence of any technical progress, as long as either resources are not depletable or deposit discoveries keep up with population growth. Because resource production has constant returns to scale, resources per capita are constant as long as inputs to mine opening increase at the same rate as population. And if the shares of labor and capital allocated to energy production are constant, then energy production will increase at the same rate as population. The foregoing implies constant per-capita inputs to final good production, so final good production grows at the rate of population growth. In this world, energy resources act like a second capital stock that must be produced from labor and capital inputs.

Although energy enables output growth to match population growth, it cannot drive growth. Technical progress is still key for growth. Energy per capita can grow no faster than capital per capita. Through savings, capital per capita grows as fast as output per capita. However, labor constraints mean that output per capita cannot grow as fast as the growth rate of its inputs. As a result, output per capita must not be growing, and neither must capital per capita nor energy per capita.[24]

In equilibrium, inputs adjust to ensure that EROI remains above $1 + g_L$, as required by Proposition 2. The precise EROI observed depends on economic forces. One might expect that increasing the productivity of energy in energy production (i.e., increasing A_E or Q_M) would increase EROI, but increasing either term also leads the economy to substitute towards energy inputs to mine opening: increasing A_E makes energy more productive relative to other inputs to mine opening, and increasing Q_M makes energy more abundant relative to other inputs to mine opening. This substitution works to reduce EROI through the declining returns to scale of increasing only one input in a CES production function.

In a Cobb-Douglas case ($\sigma = 1$), the two effects exactly offset. Based on engineering considerations, we may have intuitively expected EROI to depend on A_E and Q_M, but EROI is in fact independent of these technical parameters. EROI increases in ρ because less patient agents require a greater return on their investments in mine opening, and it decreases in ϕ_{MR} and κ_E because agents will invest more of their energy supplies in mine opening when the marginal energetic return of mines is larger (ϕ_{MR} is large) or energy inputs carry a larger weight in mine opening (κ_E is large).

The next proposition describes economic growth and EROI under more general production functions for mine opening, in the special case that capital and labor inputs have the same relative weights in mine opening, energy production, and final good production:

[24]Formally, we have $g_e \leq g_k$, $g_k = g_y$, and $g_y = \alpha_K g_k + \alpha_E g_e$, which implies $g_k \leq (1 - \alpha_L) g_k$. The only solution has $g_k = 0$, which implies $g_e, g_y = 0$.

Assumption 2. $\kappa_K/\kappa_L = \phi_{MK}/\phi_{ML} = \alpha_K/\alpha_L$.

Proposition 4 (CES Mine Opening)**.** *Let Assumptions 1 and 2 hold. Any interior balanced growth path has all real variables growing at the same rate as population and the prices of unmined deposits and operating mines growing at* $\rho - g_L$*. Such a path exists. Along that path,*

i $EROI(t)$ *is constant and is strictly greater than* $1 + \frac{1}{\phi_{MR}} \frac{\lambda+\rho}{\lambda+g_L} \frac{(1-\omega)(\rho-\Omega)+\omega(\rho-g_L)}{(1-\omega)(\rho-\Omega)} g_L$.

ii $EROI(t)$ *increases in* A_E *and* Q_M *if* $\sigma < 1$.

iii $EROI(t)$ *decreases in* A_E *and* Q_M *if* $\sigma > 1$.

Proof. See Appendix F. □

We again have energy-enabled growth. As before, EROI must be large enough to compensate for population growth (and indeed is along a balanced growth path), but now EROI does depend on the technical parameters A_E and Q_M. When $\sigma < 1$, substitution towards energy inputs in mine opening is weak, leading the direct effects of A_E and Q_M on productivity to dominate the EROI calculation. EROI thus increases in A_E and Q_M, as engineering intuition would suggest. But when $\sigma > 1$, substitution towards energy inputs in mine opening is strong, so that the effects of diminishing returns to energy investment dominate the direct effects of A_E and Q_M. Contrary to engineering intuition, EROI decreases in A_E and Q_M.

Among much else, the Industrial Revolution is known for a shift towards coal use. Many have debated whether this shift towards coal use spurred the takeoff in economic growth (see footnote 1 above). We have obtained a more nuanced result. In the present setting, discovering the ability to use a coal resource saves the economy from the possibility that population growth erodes standards of living over time and does so irrespective of the productivity of that coal resource. Sustained growth in standards of living still requires technical change, but now any degree of technical change suffices, however small.

5 Tapped Resources: Oil

Now consider an economy with oil resources. These require inputs when they are first tapped but do not require ongoing inputs to produce once they have been tapped: an oil well must be drilled and connected, but pressure within the well then forces oil to flow without requiring labor to haul it out of the well.[25]

[25] Anderson et al. (2018, 987) write, "…oil extraction is more akin to a "keg-tapping" problem than a cake-eating problem: extractors choose when to drill their wells (or tap their kegs), but the flow from these wells is (like the libation from a keg) constrained because of pressure and decays toward zero as more oil is extracted." Anderson et al. (2018) assume zero marginal costs of extracting from a tapped well, up to the physically determined constraint.

The resource base $R(t)$ now represents deposits already tapped and producing, evolving as in (3). Tapping a resource deposit at time t uses labor inputs $L_R(t)$, capital inputs $K_R(t)$, and energy inputs $E_R(t)$, following (5). The energy market clears when $E(t) = E_Y(t) + E_R(t)$, the labor market clears when $L(t) = L_Y(t) + L_R(t)$, and the capital market clears when $K(t) = K_Y(t) + K_R(t)$. I again apply Assumption 1 (fixing $g_A = 0$) in order to highlight the potential for growth in the absence of technical change.

Because oil resources are depletable, I fix $\omega > 0$. In keeping with Section 4, I also fix $\Omega > g_L$.[26] Each tapped deposit produces $Q_D > 0$ units of energy:

$$E(t) = Q_D R(t). \tag{8}$$

Through appropriate rescaling, this production function is consistent with requiring energy to transport oil to market (see footnote 22 above). The economy's EROI is as in (6), and Proposition 2 still holds.

This production function does not require that tapped deposits maintain constant productivity over time. Darcy's Law implies that the flow of oil from a tapped deposit decays over time, which is captured here by $\lambda > 0$ in (3).[27] Additional capital or labor inputs may be used to offset the decline in productivity by tapping new deposits. Importantly, though, energy production from already-tapped deposits does not scale with any such inputs.

The following proposition describes outcomes with this energy resource:

Proposition 5. *Let Assumption 1 hold and define*

$$\chi \triangleq (\lambda + g_L)\left(\frac{\lambda+\rho}{\kappa_E(\lambda+g_L)}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{(1-\omega)(\rho-\Omega)+\omega(\rho-g_L)}{(1-\omega)(\rho-\Omega)}\right)^{\frac{\sigma}{\sigma-1}}(\Omega-g_L)^{\frac{\omega}{1-\omega}}.$$

Any balanced growth path has all real variables growing at the rate of population and the prices of untapped deposits and operating wells growing at rate $\rho - g_L$.

i If $\sigma = 1$, such a balanced growth path exists and the elasticity of output per capita on the balanced growth with respect to Q_D is strictly positive and constant in Q_D.

ii If $\sigma < 1$, such a balanced growth path exists if and only if $A_E Q_D > \chi$.

iii If $\sigma > 1$, such a balanced growth path exists if and only if $A_E Q_D < \chi$.

iv Along such a balanced growth path,

$$EROI(t) = 1 + \left(\frac{Q_D A_E}{\chi}\right)^{1-\sigma} g_L.$$

[26] Oil resources should be fixed over some sufficiently long timescale, but even now the oil resource is rather large (Rogner et al., 2012, Section 7.2) and reserves continue to grow (Sorrell et al., 2012). The present model of continued discoveries is an adequate approximation over timescales of decades or more.

[27] The exponential decline is consistent with prior work (Mason and van't Veld, 2013; Anderson et al., 2018; Kellogg, 2024).

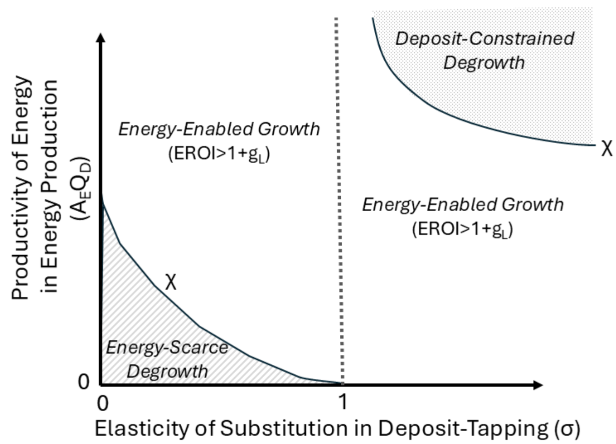


Figure 1: Schematic of long-run outcomes for tapped resources.

Proof. See Appendix G. □

The proposition describes several possibilities, which depend on σ and $A_E Q_D$. Using its results, Figure 1 divides $(\sigma, A_E Q_D)$ space into three regions, with χ from the proposition forming the borders between them.

The middle region (in white) is a case of *energy-enabled growth*, in which the energy resource enables output to keep up with population but cannot drive output to grow faster than population. When $\sigma = 1$, this region encompasses all permissible values of $A_E Q_D$, as it did for the mined resource.[28] However, whereas the middle region would have encompassed the entire plot for the mined resource, we here see that the middle region requires that $A_E Q_D$ not be too small when $\sigma < 1$ and not be too large when $\sigma > 1$. From part iv of the proposition, this middle region ensures that EROI is greater than $1 + g_L$. As we approach either boundary from the middle region, EROI approaches $1 + g_L$ and, from the proof of Proposition 5, the share of energy devoted to deposit-tapping approaches 1.

It is intuitively plausible that labor, capital, and energy are complementary inputs to tapping oil wells, so that $\sigma < 1$.[29] In that case, we obtain a different outcome when $A_E Q_D < \chi$ (lower-left shaded region in Figure 1): the economy cannot maintain constant output per capita because energy resources are not sufficiently productive in making energy. If a balanced growth path cannot have EROI greater than $1 + g_L$, then the inability to generate enough energy for deposit-tapping eventually binds the entire economy. With $\sigma < 1$, energy is an essential input, and with $A_E Q_D$ small, energy is scarce. The proof of Proposition 5 shows

[28] And taking the limit as σ goes to 1 in part iv of Proposition 5 yields the EROI from Proposition 3.

[29] Indeed, empirical work suggests that the elasticity of substitution between energy and value-added is strictly less than 1 in the energy supply sector: Koesler and Schymura (2015) estimate the elasticity of substitution between energy and value-added in resource mining as a (noisy) 0.42.

that holding output per capita constant would require more energy to be allocated to deposit-tapping than is produced in the entire economy. Because such an allocation is infeasible, the economy instead experiences *energy-scarce degrowth*, as seen with the harvested resource. This is the type of outcome that analysts have in mind when they express concerns about a "net energy cliff" in EROI (e.g., Murphy and Hall, 2010). We here see that a net energy cliff requires that the resource be a tapped deposit and not a mined resource (because labor and capital can be used to produce more energy from already-opened mines) and that energy be an essential input in deposit-tapping (because otherwise the economy could substitute capital and labor for energy).[30]

If $\sigma > 1$, we again obtain a different outcome, but now only if energy resources are *too* productive. EROI declines towards $1 + g_L$ as $A_E Q_D$ increases to χ. As described following Proposition 4, making energy resources more productive induces substitution towards energy inputs in deposit-tapping that works to reduce EROI. As $A_E Q_D$ becomes large (upper-right shaded region in Figure 1), demand for energy as an input to deposit-tapping eventually outstrips the economy's ability to generate that energy from scarce deposits.[31] Appendix A shows that, in this region, a balanced growth path exists in which energy crowds out all other inputs to deposit-tapping and output per capita shrinks at a constant rate.[32] This is a case of *deposit-constrained degrowth*, in which the high productivity of energy resources makes energy abundant but that very abundance means that the pace of deposit-tapping proceeds faster than deposits are found.

Formally, the cutoff χ on $A_E Q_D$ arises for $\sigma \neq 1$ due to an inability to maintain incentives for deposit-tapping without violating the economy's energy constraint. A balanced growth path must have deposits being tapped. For that case to be optimal, the marginal value of a tapped resource needs to exceed the marginal value of a deposit in the ground. In equilibrium, the premium for tapped vs untapped deposits depends on the marginal productivity of energy inputs to deposit-tapping. For $\sigma < 1$, the equilibrium marginal productivity of energy inputs to deposit-tapping is large when $A_E Q_D$ is small, and for $\sigma > 1$, the equilibrium marginal productivity of energy inputs to deposit-tapping is large when $A_E Q_D$ is large. On the other hand, the equilibrium marginal productivity of energy inputs to tapping tends to be small when a greater share of energy is used in deposit-tapping, via the logic of diminishing returns.

[30]Some premodern societies managed quite sophisticated drilling operations without modern energy inputs (see Kuhn, 2004). This suggests that energy may not be truly essential to deposit-tapping and that society would return to energy-enabled growth before collapse was complete. However, this premodern production had relatively low productivity, so that energy-enabled growth would sustain a relatively low level of consumption.

[31]In the case of the mined resource, labor and capital inputs constrain energy production from already-opened mines and limit demand for energy as an input to mine-opening.

[32]In particular, such a path exists when ρ is close to Ω, Ω is not too close to g_L, and $A_E Q_D > X$, where $X \leq \chi$, χ/X is strictly decreasing in σ, and $\lim_{\sigma\to\infty} X = \lim_{\sigma\to\infty} \chi$. At any given finite σ, there is a region of $A_E Q_D$ just below χ in which multiple balanced growth paths coexist when ρ is close to Ω and Ω is not too close to g_L. One path has energy-enabled growth, and the other path has deposit-constrained degrowth.

To maintain a given premium for tapped vs untapped deposits, the share of energy used in deposit-tapping must move opposite to $A_E Q_D$ when $\sigma < 1$ and must move with $A_E Q_D$ when $\sigma > 1$. But the share of energy used in deposit-tapping is bounded above by 1. Therefore $A_E Q_D$ can only become so small if a balanced growth path is to exist for $\sigma < 1$ and $A_E Q_D$ can only become so large if a balanced growth path is to exist for $\sigma > 1$.[33]

Figure 2 simulates outcomes for $\sigma = 0.5$ and $\sigma = 2$ over 300 years, using a model discretized with an annual timestep and calibrated to prior literature (details in Appendix B).[34] The top left panel plots output per capita, the top right panel plots tapped deposits per capita (which are proportional to energy per capita), the lower left panel plots untapped deposits per capita, and the lower right panel plots EROI. The solid lines show cases of energy-enabled growth, calibrated to the current EROI of tapped resources. The black dashed line reduces $A_E Q_D$ to half of χ with $\sigma = 0.5$, putting the economy in a regime of energy-scarce degrowth. The gray dashed line increases $A_E Q_D$ to twice χ with $\sigma = 2$, putting the economy in a regime of deposit-constrained degrowth.

In line with the theory, output per capita approaches a constant level for both cases of energy-enabled growth. When $\sigma = 0.5$, the economy does so by maintaining enough resources per capita to generate the energy needed to tap new deposits and by maintaining enough deposits to tap in the future. EROI becomes large because the need to preserve deposits keeps the marginal returns to energy inputs in deposit-tapping high. In contrast, when $\sigma = 2$, the economy substitutes away from the tapped resource as deposits become scarce. Both tapped and untapped deposits approach zero. EROI is negative while the deposits are being run down but eventually stabilizes well above $1 + g_L = 1.01$ (and well below its value when $\sigma = 0.5$).

In the other two cases, output per capita declines at a constant rate over much of the interval. In the case of energy-scarce degrowth, that decline rate is 0.03% per year. The productivity of deposits is too low to sustain a significant rate of deposit-tapping: the stock of tapped deposits initially declines rapidly over an interval with negative EROI before declining more slowly over an interval with EROI above $1 + g_L$, albeit with EROI small and declining. Ironically, this case of energy-scarce degrowth has the most deposits per capita, because deposit-tapping is constrained by the availability of energy. And this case arises even with EROI above the net energy cliff of $1 + g_L$.

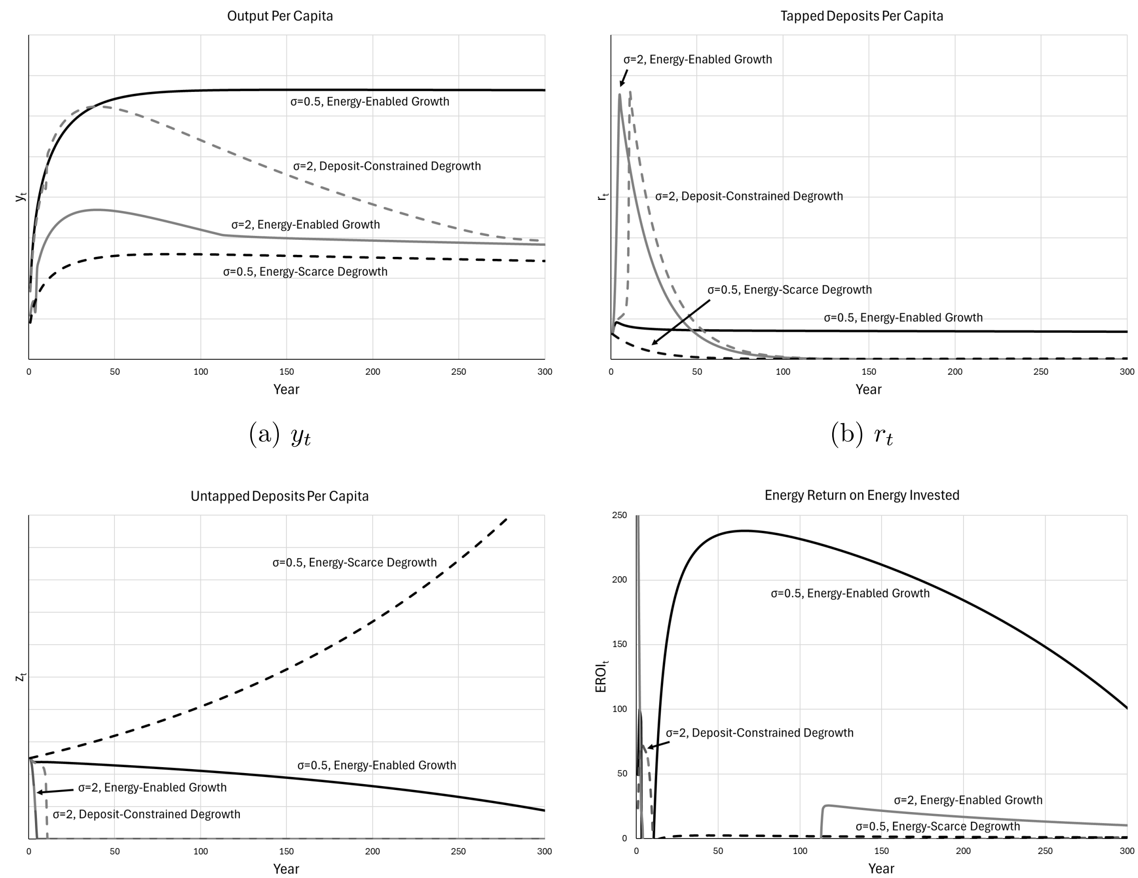
In the case of deposit-constrained degrowth, output per capita is initially high but soon

[33] From equation (A-31) and $e_{ss} = Q_D r_{ss}$ (with subscript ss indicating evaluation at a steady state),

$$\frac{e_{Rss}}{e_{ss}} = \left(\frac{\phi_{MR}\kappa_E}{\lambda + \rho} \frac{(1-\omega)(\rho - \Omega)}{(1-\omega)(\rho-\Omega) + \omega(\rho - g_L)} \right)^{\sigma} (\lambda + g_L)(\Omega - g_L)^{\frac{\omega}{1-\omega}(1-\sigma)} [A_E Q_D]^{\sigma - 1}.$$

The bounds on $A_E Q_D$ derive from recognizing that $e_{Rss}/e_{ss} \leq 1$. There were no such bounds in the case of the mined resource because, from equation (2), the endogenous mining inputs imply $e_{ss} \neq Q_M r_{ss}$.

[34] I solve the economy over 400 years but cut the plots off at 300 years in order to minimize effects of the terminal horizon.



(a) y_t (b) r_t

(c) z_t (d) $EROI_t$

Figure 2: Simulated outcomes for a tapped resource.

declines by around 0.3% per year. Untapped deposits quickly dwindle to essentially zero, at which point EROI becomes negative and the stock of tapped deposits begins dwindling. The dwindling energy supplies eventually cause output per capita to start declining.

6 Manufactured Resources: Solar Photovoltaics

A solar resource is similar to oil resources in that, once installed, solar photovoltaic cells produce energy for a period of time without ongoing inputs of labor or capital. However, a solar resource is different in that photovoltaic cells are manufactured, not found. The transition equation for the resource base becomes:

$$\dot{R}(t) = F(L_R(t), K_R(t), E_R(t)) - \lambda R(t). \tag{9}$$

Energy production is analogous to (8):

$$E(t) = Q_S R(t), \tag{10}$$

with $Q_S > 0$. Equation (9) is the same as equation (3) with $\omega = 0$ (i.e., in the special case of abundant resources). Solar panels do require surface area, whether on Earth or in space, so the solar resource might in principle become scarce, but the stock of sites with high-quality solar fluxes is likely so large that resource scarcity may be a second-order concern for a long time (Jacobson and Delucchi, 2011; Rogner et al., 2012).

The following corollary describes the potential for constant output per capita in this economy:

Corollary 6. *Let Assumption 1 hold and define*

$$\chi \triangleq (\lambda + g_L)\left(\frac{\lambda + \rho}{\kappa_E(\lambda + g_L)}\right)^{\frac{\sigma}{\sigma-1}}.$$

Any interior balanced growth path has all real variables growing at the rate of population and the price of solar panels growing at rate $\rho - g_L$.

i If $\sigma = 1$, such a balanced growth path exists. The elasticity of output per capita on the balanced growth with respect to Q_S is strictly positive and constant in Q_S.

ii If $\sigma < 1$, such a balanced growth path exists if and only if $A_E Q_S > \chi$.

iii If $\sigma > 1$, such a balanced growth path exists if and only if $A_E Q_S < \chi$.

iv Along such a balanced growth path,

$$EROI(t) = 1 + \left(\frac{A_E Q_S}{\chi}\right)^{1-\sigma} g_L.$$

Proof. Follows proof of Proposition 5, with $\omega = 0$. □

Figure 3 depicts the set of possible outcomes described by the corollary. These results are largely familiar from the case with a tapped resource. We again have a balanced growth path with constant output per capita in the Cobb-Douglas case. If inputs to solar panel manufacturing are instead complementary and solar panels are not sufficiently productive, then we again risk energy-scarce degrowth. Many analysts have expressed concern about the lower EROI of solar photovoltaics compared to fossil energy resources (e.g., Trainer, 2018; Capellán-Pérez et al., 2019). We here see that while a decline in the EROI achievable along a balanced growth path may affect the level of consumption per capita, it does not affect the long-run growth rate of consumption as long as EROI remains above $1 + g_L$. However,

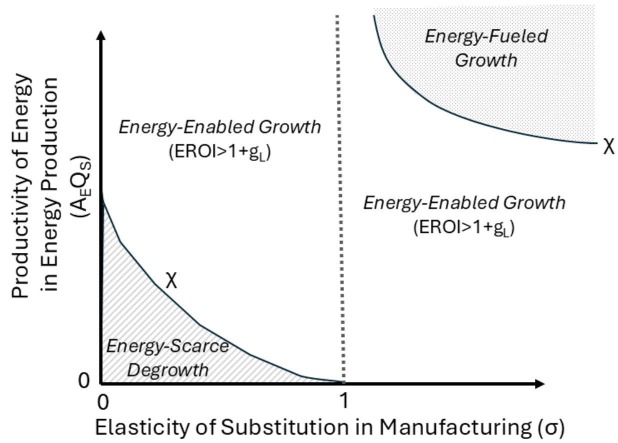


Figure 3: Schematic of long-run outcomes for manufactured resources.

if the productivity of solar panels is low enough and inputs to solar panel manufacturing are complementary, then the economy does not generate enough additional energy to have energy and output both keep pace with population growth in the absence of technical change.

A new possibility arises if energy inputs are substitutable in solar panel manufacturing and $A_E Q_S$ is sufficiently large, as in the upper-right shaded region of Figure 3:

Assumption 3 (Log Utility)**.**

$$u(Y_t/L_t) = \ln (Y_t/L_t) \, .$$

Proposition 7 (Energy-Fueled Growth with Manufactured Resources)**.** *Let Assumptions 1 and 3 hold and fix $\sigma > 1$. Define g_e as the growth rate of energy per capita, g_y as the growth rate of output per capita, g_k as the growth rate of capital per capita, and χ as in Corollary 6. If $A_E Q_S > \chi$, then there exists a balanced growth path with $L_R(t), K_R(t) = 0$ and with $R(t)/L(t)$, $E_R(t)/L(t)$, and $E_Y(t)/L(t)$ all growing at rate $g_e > 0$. Along such a path:*

i $g_y = g_k = \frac{\alpha_E}{\alpha_E + \alpha_L} g_e$.

ii $g_e = A_E Q_S \kappa_E^{\frac{\sigma}{\sigma-1}} - (\rho + \lambda) > 0$.

iii $EROEI(t) = 1 + \frac{A_E Q_S \kappa_E^{\frac{\sigma}{\sigma-1}}}{A_E Q_S \kappa_E^{\frac{\sigma}{\sigma-1}} - (\rho - g_L)} (g_e + g_L)$

Proof. See Appendix H. □

We have *energy-fueled growth*: the economy features strictly positive long-run growth in output per capita, despite the absence of technical progress. When energy crowds out other resources in solar panel manufacturing, the production function for solar panels becomes, from (5),

$$F(L_R(t), K_R(t), E_R(t)) = A_E \kappa_E^{\frac{\sigma}{\sigma-1}} E_R(t).$$

Differentiating (10) with respect to time and using the previous equation and (9), the instantaneous change in energy production at time t is:

$$\dot{E}(t) = A_E Q_S \kappa_E^{\frac{\sigma}{\sigma-1}} E_R(t) - Q_S \lambda R(t).$$

If $E_R(t)$ is a constant share s_E of $E(t)$, this becomes:

$$\dot{E}(t) = \left[A_E Q_S \kappa_E^{\frac{\sigma}{\sigma-1}} s_E - \lambda \right] E(t).$$

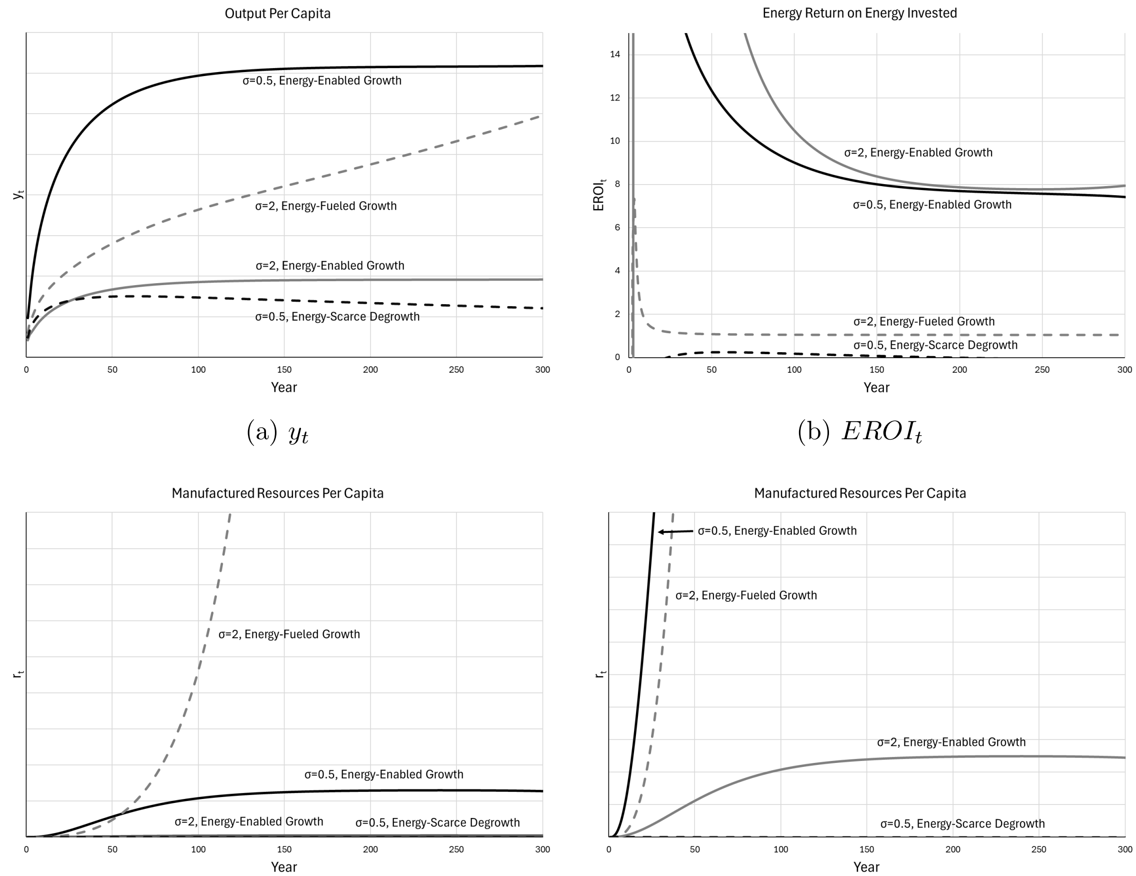
We have a linear accumulation equation. The economy can grow by reinvesting energy in making energy as long as the return on that investment is sufficiently large. The growth rate of output per capita is proportional to the growth rate of energy per capita (part i). That growth rate of energy per capita increases in the productivity Q_S of solar panels and the productivity A_E of energy in manufacturing solar panels (part ii). Economy-wide EROI increases in both productivity parameters and, consistent with Proposition 2, is greater than $1 + g_L + g_e$ (part iii).

Figure 4 simulates outcomes for $\sigma = 0.5$ and $\sigma = 2$ (details again in Appendix B). The top left panel plots output per capita over time, the top right panel plots EROI over time, and the lower panels plot manufactured resources per capita over time, with the lower right panel a zoomed-in version of the lower-left panel. The solid lines again show cases of energy-enabled growth, calibrated to the current EROI of solar photovoltaic resources. The black dashed line again reduces $A_E Q_D$ to half of χ with $\sigma = 0.5$, putting the economy in a regime of energy-scarce degrowth. And the gray dashed line again increases $A_E Q_D$ to twice χ with $\sigma = 2$, now putting the economy in a regime of energy-fueled growth.

Output per capita again approaches a constant level for both cases of energy-enabled growth, in line with the theory. As before, the economy maintains more resources per capita when energy is an essential input to resource production. EROI steadily declines but remains well above the cutoff $1 + g_L = 1.01$.

In the case of energy-scarce degrowth, output and resources per capita steadily decline and resources per capita become very scarce. The small $A_E Q_D$ means that EROI is actually negative at first. EROI increases to around 0.25 before declining, so that it is well below 1 even at its peak. In this case, the small EROI reflects the low productivity of energy resources.

In the case of energy-fueled growth, output per capita steadily increases, with growth of 0.2–0.3% per year. Resources per capita explode, as the high productivity of the energy resource and the substitutability of energy for other inputs to resource production create a



(a) y_t (b) $EROI_t$

(c) r_t (d) r_t, zoomed-in

Figure 4: Simulated outcomes for a manufactured resource.

self-sustaining loop of creating energy to make energy. EROI is quite high in early periods and declines to around 1.05 as energy becomes abundant. In this case, EROI declining towards a level only slightly above $1 + g_L$ reflects ongoing growth in energy and output per capita rather than a scarcity of energy. The distance of EROI from the net energy cliff represented by $1+g_L$ is not on its own a good guide to whether energy is fueling the economy or will continue fueling the economy.

7 Self-Replicating Resources: Roboticized Solar

I now assess the potential for interactions between solar panels and robotics to generate a qualitatively different set of outcomes. Let energy production follow (10). The resource base of solar photovoltaics is now manufactured from energy $E_R(t)$ and robots $B_R(t)$:

$$\dot{R}(t) = A_E B_R(t)^{\kappa_B} E_R(t)^{\kappa_E} - \lambda R(t),$$

with $\kappa_E, \kappa_B > 0$ and $\kappa_E + \kappa_B = 1$. Robots themselves are produced by combining energy $E_B(t)$ with robots $B_B(t)$. The total stock $B(t)$ of robots evolves as:

$$\dot{B}(t) = A_B B_B(t)^{\beta_B} E_B(t)^{\beta_E} - \Psi B(t),$$

with $A_B > 0$, $\beta_E, \beta_B > 0$, and $\beta_E + \beta_B = 1$. Robots depreciate at rate $\Psi > 0$. The robot market clears when $B_R(t) + B_B(t) = B(t)$, and the energy market clears when $E_Y(t) + E_R(t) + E_B(t) = E(t)$.

The following proposition describes outcomes in this economy:

Proposition 8 (Self-Replicating Resources)**.** *Let Assumptions 1 and 3 hold. Define g_e as the growth rate of energy per capita, g_y as the growth rate of output per capita, and g_k as the growth rate of capital per capita. There exists a balanced growth path with $R(t)/L(t)$, $E_R(t)/L(t)$, $E_Y(t)/L(t)$, and $B(t)/L(t)$ all growing at rate $g_e > 0$ if and only if*

$$A_E Q_S > \chi_0, \text{ where } \chi_0 \triangleq \frac{\rho + \lambda}{\kappa_E} \left(\frac{\kappa_E \beta_B}{\kappa_B \beta_E} \left(\frac{\rho + \Psi}{\beta_B A_B} \right)^{\frac{1}{\beta_E}} \right)^{\kappa_B}.$$

Along such a path:

i $g_y = g_k = \frac{\alpha_E}{\alpha_E + \alpha_L} g_e$.

ii g_e *increases in* Q_S, A_E, *and* A_B *and decreases in* ρ, λ, *and* Ψ.

iii There exists $\chi_1 > \chi_0$ *such that the prices of solar panels and robots decrease over time if and only if* $A_E Q_S > \chi_1$.

iv $EROI(t)$ *increases in* A_E, Q_S, *and* A_B.

v $EROI(t) > 1 + \frac{1}{\kappa_E} \frac{\rho + \lambda}{g_L + \lambda} g_L$.

Proof. See Appendix I. □

We have *energy-fueled growth* when solar photovoltaic panels are sufficiently productive: the economy features strictly positive long-run growth in output per capita, despite the absence of technical progress. Moreover, in contrast to Section 6, this outcome arises even when the elasticities of substitution in solar panel production and robot production are each unity (i.e., even with Cobb-Douglas production functions).

Proposition 8 shows that the growth rate of output per capita is proportional to the growth rate of energy per capita (part i). The growth rates of energy per capita and output per capita increase in the productivity of the energy resource and in the productivity of robot production and decrease in each depreciation rate and in the discount rate (part ii). The prices of solar panels and robots increase over time if solar panels are not too productive but decrease over time if solar panels are sufficiently productive (part iii). EROI increases in the productivities of the energy resource and robot production (part iv), and strictly positive growth in energy per capita and output per capita requires that EROI exceed $1 + g_L$ by a sufficiently large margin (part v). Solar photovoltaics must generate enough surplus energy to outpace population growth.[35]

To see why long-run growth is possible in the absence of technical progress, consider the dynamic equations governing the growth rates of the solar resource and of robots. The growth rate of the solar photovoltaic resource base is:

$$\frac{\dot{R}(t)}{R(t)} = A_E Q_S \left[\frac{B_R(t)}{E_R(t)}\right]^{\kappa_B} \frac{E_R(t)}{E(t)} - \lambda. \tag{11}$$

If robots could not keep up with the growth rate of energy, then the term in brackets would go to zero over time and the growth rate of energy must become negative. If the shares s_R^B and s_R^E of robots and energy allocated to solar panel production are constant, then equation (11) becomes:

$$\frac{\dot{R}(t)}{R(t)} = A_E Q_S \left[\frac{s_R^B}{s_R^E}\frac{E(t)}{B(t)}\right]^{\kappa_B} s_R^E - \lambda. \tag{12}$$

$\dot{R}(t)$ is linear in $R(t)$ when energy and robots grow at the same rate.

The growth rate of robots is:

$$\frac{\dot{B}(t)}{B(t)} = A_B \left[\frac{E_B(t)}{B_B(t)}\right]^{\beta_E} \frac{B_B(t)}{B(t)} - \Psi. \tag{13}$$

If energy could not keep up with the growth rate of robots, then the term in brackets would go to zero over time and the growth rate of robots must become negative. Combining this result with the result from (11) that robots must grow at least as fast as energy when growth is positive, solar photovoltaics and robots must grow at the same rate on a balanced growth path with positive growth. If the shares s_B^B and s_B^E of robots and energy allocated to robot production are constant, then equation (13) becomes:

$$\frac{\dot{B}(t)}{B(t)} = A_B \left[\frac{s_B^E}{s_B^B}\frac{B(t)}{E(t)}\right]^{\beta_E} s_B^B - \Psi.$$

[35] Defining the robot return on robot investment (RROI) by analogy to EROI in (6), we have, from (A-65), that $g_e > 0$ only if $RROI(t) > 1 + \frac{1}{\beta_B}\frac{\rho+\Psi}{g_L+\Psi}g_L$.

$\dot{B}(t)$ is linear in $B(t)$ when energy and robots grow at the same rate. So, combining with (12), when energy and robots grow at the same rate and maintain constant shares, we have an equilibrium with linear accumulation equations for solar panels and robots and therefore featuring the potential for strictly positive growth.

Combining equations (11) and (13) (by way of equation (A-58)), we find:

$$\frac{\dot{R}(t)}{R(t)} = A_E Q_S \left(\frac{A_B \frac{B_B(t)}{B(t)}}{\frac{\dot{B}(t)}{B(t)} + \Psi} \right)^{\kappa_B/\beta_E} \left(\frac{\beta_E \kappa_B}{\beta_B \kappa_E} \right)^{\kappa_B} \frac{E_R(t)}{E(t)} - \lambda.$$

The growth rate of the solar photovoltaic resource base declines in the growth rate of robots, because an increasing share of energy must be devoted toward robot production in order to sustain its high growth rate. The common growth rate g_e solves:

$$g_e = A_E Q_S \left(\frac{A_B \frac{B_B(t)}{B(t)}}{g_e + \Psi} \right)^{\kappa_B/\beta_E} \left(\frac{\beta_E \kappa_B}{\beta_B \kappa_E} \right)^{\kappa_B} \frac{E_R(t)}{E(t)} - \lambda.$$

If the solar resource is sufficiently productive, then the right-hand side is greater than zero when evaluated at the optimized factor allocation, in which case the unique g_e that solves the equation at that allocation is strictly positive. In line with this analysis, Proposition 8 shows that the growth rate of energy per capita is strictly positive if and only if the solar resource is sufficiently productive, where the threshold is lower when robot production is more productive and robots depreciate more slowly.

In this economy, the solar resource becomes self-replicating. Its energy creates the robots that combine with solar energy to produce more solar photovoltaics. Those additional solar panels generate additional energy that, once combined with robots, helps produce more robots, more photovoltaics, and more output. If the solar resource is sufficiently productive, then its energy can generate economic growth. This story requires highly automated processes for fabricating solar photovoltaics, but once these exist, it does not require any ongoing technical change in any part of the economy. It also does not assume that robots directly affect final good production. Energy production here fuels its own growth and thereby fuels the growth of output per capita.

8 Implications for Real-World Energy Transitions

At the turn of the nineteenth century, the global economy was driven by biomass, a harvested resource. Within the model of Section 3, technical change was necessary just to hold output per capita constant. Over the nineteenth century, the global economy came to be dominated by coal, a mined resource (Smil, 2010, Chapter 2). Within the model of Section 4, this transition generated a qualitative shift in behavior: energy-enabled growth became possible, with any degree of technical change now sufficient to induce growth in output per capita.

Over the course of the twentieth century, the global economy came to be driven by oil and gas (Smil, 2010, Chapter 2). Rather than an economy that produced energy from coal with substantial ongoing labor and capital inputs, the economy became one that required inputs upfront, at the point of tapping an oil or gas well. These inputs were plausibly essential to the tapping process. Within the model of Section 5, the high EROI of oil and gas kept the economy within a regime of energy-enabled growth. However, the EROI of oil and gas deposits has declined over time—and may decline further—as oil and gas deposits have become increasingly scarce (see footnote 7 above). And a further reduction in EROI could occur as power plants are paired with technology to capture carbon emissions, whether onsite or through direct air capture. Within the model of Section 5, the oil and gas economy risks entering a regime of energy-scarce degrowth if EROI declines a lot.

Solar energy may someday dominate the economy. There is currently no shortage of locations with high-quality solar resources. Once solar panels are produced, they generate energy with little to no ongoing inputs, much as oil and gas resources do. But producing solar panels does require energy, and some literature suggests that solar panels have a smaller EROI than do contemporary oil and gas resources (see footnote 8 above). Two qualitative shifts are possible for growth behavior in a solar economy.

First, if energy inputs are complementary to labor and capital inputs in the production of solar panels, then the reduction in EROI could shift the economy from a regime in which any technical change generates growth to a regime in which technical change would be necessary just to overcome the drag from energy-scarce degrowth (Section 6). This pessimistic scenario manifests the "net energy cliff" that some analysts fear will be induced by a transition to a solar economy (e.g., Trainer, 2018; Capellán-Pérez et al., 2019).

Second, if energy can substitute for other inputs in the production of solar panels, or if solar panels come to be produced with automatic robotic inputs, then solar panels may shift the economy from relying on resources with scarce deposits to relying on resources produced from ever-more abundant factors of production. In that case, a solar economy may experience energy-fueled growth, even in the absence of further technical change (Section 7). A fraction of the electricity generated from existing solar panels would be reinvested in producing more solar panels in factories, with potentially another fraction reinvested in producing more automated panel-making robots. Energy would become increasingly abundant, in line with some speculation (Economist, 2024). This optimistic scenario manifests a "net energy ramp", which is the energetic counterpart of the growth takeoff scenarios pondered by scholars of artificial intelligence (e.g., Sandberg, 2013; Nordhaus, 2021).

9 The Energy Ladder

The switch from biomass resources to coal (and then to oil and gas) may have already transformed energy from a drag on growth to an enabler of growth, and the optimistic net energy ramp scenario for solar envisions a further transformation in which energy could directly drive growth. However, even if the optimistic scenario comes to pass, that sequence of transitions had a real cost: the century-plus of intensive fossil fuel use loaded the atmosphere with carbon dioxide and thereby saddles a solar economy with climate change. I now consider the possibility that the world could not have avoided creating climate costs on the way to achieving the solar economy.

I demonstrate this possibility within a stylized model in which biomass (harvested), coal (mined), oil (tapped), and solar (whether manufactured or self-replicating) resources all exist. The economy can use only one type of resource at a time, as if its capital stock is tooled to a certain type of resource, and the economy reaches a balanced growth path when one exists. At time 0, only the biomass resource is available for energy production. At the time that coal resources are first accessed, their available deposits are strictly positive, and at the time that oil resources are first accessed, their available deposits are strictly positive.

Developing a new type of resource requires paying a one-time fixed cost per capita.[36] This fixed cost reflects the need to find the new resources, figure out how to extract them, innovate in how to use them, and retool the capital stock to use them. The fixed cost ξ_M of first accessing a mined resource captures the cost of innovating in the techniques and steam engines necessary to avoid water seeping into mines. The fixed cost ξ_D of first accessing an oil resource captures the cost of using steam engines and immature techniques in an unsuccessful sequence of initial drilling attempts. The fixed cost ξ_S of first accessing a solar resource captures the cost of the intensive R&D required to understand and commercialize the photoelectric effect. Reflecting the reliance of oil and solar on progressively more advanced knowledge and specialized capital, assume $0 < \xi_M < \xi_D < \xi_S$. Up to and including the moment of paying the fixed cost, the resource base for an undeveloped resource is zero. The fixed cost must be paid out of production fueled by already-developed types of energy resources. The new resource base then begins growing as energy, labor, and capital are devoted to opening mines, tapping deposits, or producing solar panels.

The following proposition considers the interaction between various resources:[37]

[36]Defining fixed costs in per capita terms sharpens the story. If costs are fixed in aggregate rather than per capita, then they decline in per capita terms. The main results go through, once expressed in terms of timing rather than in terms of whether a resource is accessed at all.

[37]I study transitions in the absence of exogenous technical change, by imposing Assumption 1. If I instead assumed $g_A > 0$, then every resource could eventually be accessed, as in Hansen and Prescott (2002). However, to the extent that the new ideas embedded in $g_A > 0$ require energy, assuming $g_A > 0$ would beg the question of the feasibility of each transition.

Proposition 9 (Energy Ladder)**.** *Let Assumptions 1 and 3 hold, and assume that $\sigma = 1$ for each resource type. There exist ξ_D, ξ_S sufficiently large such that:*

i An economy that uses the biomass resource will not develop the oil or solar resources.

ii If ξ_M is sufficiently small and Q_M is sufficiently large, then the economy eventually develops the coal resource.

iii If, in addition to the conditions in part ii, ξ_D is not too large and Q_D is sufficiently large, then the economy subsequently develops the oil resource using energy from the coal resource.

iv If, in addition to the conditions in part iii, ξ_S is not too large and Q_S is sufficiently large, then the economy subsequently develops the solar resource using energy from the oil resource.

Proof. See Appendix J. □

The economy climbs an *energy ladder*, advancing from rung to rung of resources. Developing a first type of fossil energy resource generates the extra energy necessary to enable the development of a more productive type of fossil energy resource. As Smil (2017, 230) writes, "Every transition to a new form of energy supply has to be powered by the intensive deployment of existing energies and prime movers: the transition from wood to coal had to be energized by human muscles, coal combustion powered the development of oil, and. . . today's solar photovoltaic cells and wind turbines are embodiments of fossil energies. . . "

On the first rung, if the fixed costs of developing the oil and solar resources are large, then the economy with the biomass resource will not be able to produce the output required to begin developing those more advanced resources (part i). The biomass economy that constituted human history up to the early nineteenth century may not have been able to produce the capital stock and body of knowledge required to drill for oil resources or to understand and commercialize the photoelectric effect at the heart of solar panels. In the absence of a coal resource, the economy would have been stuck with the biomass resource and energy would always have constituted a drag on growth.

However, coal seams were in fact visible and readily available in places. As a result, the economy could transition from the biomass resource to the coal resource. Moreover, coal was found to offer a high energetic return relative to biomass. The economy began accessing coal in order to obtain its energetic return, and in so doing transitioned to a world of energy-enabled growth (part ii).

An economy with a sufficiently productive coal resource may raise output high enough to be able to afford the investments required to access subsurface oil deposits. Indeed, a steam engine powered the drill for the first oil wells (Smil, 2017, 247). If the oil resource has a high enough expected return, then the economy optimally pays that cost and transitions to using the oil resource (part iii).

An economy with a sufficiently productive oil resource may then raise output high enough to be able to afford the investments required to commercialize the photoelectric effect. In order to generate sufficient surplus energy to develop modern solar photovoltaics, the economy may have first needed to develop its fossil resources. In this way, the fossil age may have been necessary for the solar economy to emerge. If solar panels offer a sufficiently high energetic return, then the economy optimally pays that cost and transitions to using solar resources (part iv). The solutions to climate change thereby stand on the shoulders of the problem.

10 Conclusions

All economic activity requires energetic inputs. We have seen that the characteristics of energy resources used by an economy shape its growth possibilities. In particular, the characteristics of energy resources determine how fast technical change needs to be in order to achieve sustained growth. The important characteristics of energy resources are the nature of resource production and the productivity of energy in making additional energy, known in the literature as energy return on energy invested (EROI). We have seen that the EROI of a coal resource exceeds unity plus the growth rate of population regardless of the productivity of coal mining. And the EROI of oil or solar resources produced from complementary inputs must exceed unity plus the growth rate of population for the economy to avoid energy-scarce degrowth.

We have also seen that the step-change from tapped oil resources to manufactured solar resources may generate energy-fueled growth, even in the absence of further technical change, if either energy is substitutable for other inputs to solar panel manufacturing or there is an additional step-change in using robots to produce and install solar panels. The latter mechanism for sustained growth is consistent with robots requiring energy to operate. In reality, the production of solar panels is indeed increasingly automated, with further automation on the horizon. If these efforts succeed, the transition to low-carbon energy resources may end up stimulating an improvement in the economy's long-run growth prospects.

This analysis does not consider how energy may affect the pace of technical change, whether by directly fueling research (see Schurr, 1984) or by absorbing scientists who may otherwise work on improving general productivity (see Arkolakis and Walsh, 2024). If a solar resource makes energy cheaper and more abundant, then interactions with technical change could constitute a second growth dividend.

It may seem like a waste to have burned so much carbon when lower-carbon resources might actively drive growth, but we have also seen that accessing knowledge- and capital-intensive lower-carbon resources may have required burning fossil resources in order to raise output high enough. Future work should quantitatively assess historical transitions between resource types in order to understand how much earlier the world could have reached a solar economy. The emissions in excess of that point may have special significance when attributing historical responsibility for climate change.

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