Conversations*

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Abstract

In a *conversation* interlocutors talk without constraints on talking order or duration. The contents of simultaneous messages are assumed to get lost. By equating messages with disclosures of singleton subsets of interlocutors' *possibility sets* we endow them with literal meanings. *Literal-meaning strategies* minimize inferences from disclosures. All other strategies are *pragmatic-meaning strategies.* With common knowledge of possibility sets' sizes, optimal literal-meaning strategies let only the better-informed player talk. Optimal pragmatic-meaning strategies are strictly better and generally require that with positive probability both players talk simultaneously. With uncertain sizes of possibility sets, both players talk also in optimal literal-meaning strategies.

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This conversation would have been hard enough even with two people talking.

Taylor in Barbara Kingsolver's *The Bean Trees*

1 Introduction

Decision relevant information is frequently distributed among multiple parties ("players" in the sequel). Even if their interests are fully aligned, physical and language constraints may limit how much information they can convey to each other at any given point in time. They then have an interest in engaging in a *conversation*, i.e., to share information incrementally by talking to each other, without constraints on talking order or duration. We are interested in the form such conversations take when players are impatient. We ask: who talks, about what, and when?

We let players' private information take the form of "possibility sets." A player's possibility set contains the states she cannot rule out.[1] We endow messages with literal meanings by requiring them to be disclosures of subsets of players' possibility sets. Physical constraints allow players only to move at discrete points in time. Moreover, the contents of simultaneous disclosures are lost. We also impose a language constraint by limiting players to disclosures of only singleton subsets of their possibility sets. That is players have names for single states but not for collections of them. If they want to describe a collection of states they can do so by continuing to talk for multiple periods.

We investigate a game in which two impatient players face multiple issues and need to decide which, if any, action to take for each issue. Among all available actions for an issue, there is one that matches the state for that issue, which we refer to as the "true state." That action yields a positive payoff for both players for that issue, whereas taking any other action yields a negative payoff. Player's do not individually know the state for an issue but would know the state if they pooled their information.

Each issue is a potential *topic* for a conversation. Each player's information, her type, takes the form of a *possibility profile* composed of one *possibility set* for each topic. Each possibility set for a topic is a non-trivial subset of the set of states for that topic. A player's possibility set for a topic

[1] We use "state" to refer to payoff states rather than states of the world. A full description of the state of the world would also include players' private information.

contains the set of states for that topic that the player considers possible. All players' possibility sets for a topic contain the true state for that topic, making it distributed knowledge among them.

In each period, players can communicate information about their possibility sets. They face two principal frictions: (1) at any given time they are only able to communicate partial information about a single possibility set and (2) the contents of simultaneous disclosures are lost. That is, players cannot reveal all their information all at once and, to be effective, may have to engage in some form of turn taking. We assume that in any period a player can only disclose a singleton element of one of her possibility sets. When players make simultaneous disclosures, they observe that fact but not the content of the disclosure of their counterpart. We also assume that players must complete topics that they have started before disclosing in another topic – that is, we assume *no switching*.

It may help to have the following situation in mind: Ann and Bob plan a vacation. Where to go is the (lone) topic of their conversation. They consider a fixed number of possible destinations. Each cares about a different characteristic of a potential destination. For each possible destination Ann only knows about the characteristic she cares about – she investigated each destination for that feature. Likewise for Bob. They are only interested in visiting a destination that matches both of the characteristics they are interested in. A friend of theirs told them, with both present, that there is exactly one destination that meets both of their requirements but forgot to mention which one. A state is the identity of this ideal destination. The state space is the set of all these possible identities. Ann's possibility set is the set of destination with the characteristic she cares and knows about, and likewise for Bob. They have no words for these possibility sets or any nontrivial subset, but can name every individual destination. Since their interests are fully aligned, they have no reason to be untruthful and hence, without loss of generality, whatever they say satisfies a truth-telling constraint and might as well be viewed as a disclosure.

We consider primarily the case in which the sizes of players' possibility sets are commonly known. In that environment, we begin by examining (perfect Bayesian Nash) equilibria that capture the difficulty of coordinating on attaching meanings to disclosures beyond what is disclosed: a player's strategy is a *literal-meaning strategy* if it is invariant to exchanging states everywhere in that player's private history and for all choices the strategy prescribes.[2] In a *literal-meaning equilibrium* players use (but are not restricted to) literal-meaning strategies.

If a player uses a literal-meaning strategy and the sizes of possibility sets are commonly known then, conditional on her not knowing the truth, the timing of her disclosure and listening choices is independent of her type and whenever she discloses she randomizes uniformly over the undisclosed elements in her possibility set. As a result, conditional on the speaker not knowing the truth, the only information the listener learns from the speaker's disclosure is the identity of the disclosed element. Hence, in a literal meaning equilibrium, the meaning of a disclosure, aside from possibly indicating the speaker's ignorance of the truth, coincides with what is disclosed – meaning (very nearly) equals saying.

We refer to any equilibrium that is not a literal-meaning equilibrium as a *pragmatic-meaning equilibrium*. Strategies in a pragmatic-meaning equilibrium encode additional information into a disclosure – if, for example, states can be put in an order, a player's strategy may prescribe always to disclose the minimal undisclosed element of her possibility set. In a pragmatic-meaning equilibrium, meaning transcends literal meaning. This relates our exercise to the linguistics literature on pragmatics that was started by Grice [19].

For the case that the sizes of players' possibility sets are commonly known, we show that every talking order can be supported by a literal-meaning equilibrium while in optimal literal-meaning equilibria only the better informed player talks. In optimal literal-meaning equilibria interlocutors address different topics in the order of who is better informed on each topic. Optimal pragmatic-meaning equilibria strictly improve on optimal literal-meaning equilibria and generally require that both players talk with positive probability and do so simultaneously, even if at the outset one is better informed than the other.

If we let the sizes of possibility sets be private information, even in literal-meaning strategies players can encode significant information into disclosures beyond what is disclosed. They can use the choice between listening and disclosing as a means of providing information about how well informed they are. As result, it is no longer the case that optimal literal-meaning strategies prescribe that only the ex ante better-informed player discloses. Any

[2]This condition is a symmetry constraint in the spirit of Crawford and Haller's [10] notion of an "attainable strategy" in repeated games with absence of labels for actions. Blume [7] considers a richer set of symmetry constraints.

such strategy is payoff dominated (and remains so for multiple periods) by a strategy in which players alternate having the choice between listening and disclosing while their partner listens. Early disclosures can be used to signal that one's possibility set is small and that therefore one should continue disclosing. A player with a large possibility set then has an incentive to delay disclosing in order to find out whether their partner is better informed at the interim stage.

2 Model

Two players, $i = 1, 2$, converse in periods $t = 1, 2, \ldots$, with an infinite time horizon. Their goal is to uncover the true state in a finite K-dimensional state space $\Omega = \prod_{k=1}^{K} \Omega_k$ with typical element $\omega = (\omega_1, \ldots, \omega_K) \in \Omega$. The true state $\omega^* \in \Omega$ is generated by each state component ω_k^* being independently drawn from a uniform common-prior distribution over the set Ω_k.

In period 0, before the start of the conversation, each player i privately learns a *possibility profile* $S_i = \prod_{k=1}^{K} S_{ik} \subset \Omega$ that is composed of *possibility sets*, S_{ik}. Player i's possibility profile is her "type." In each dimension k, player i's possibility set indicates the set of state components ω_k that player i considers possible. The size of player i's possibility set S_{ik} in each dimension k is exogenously fixed at $|S_{ik}| = n_{ik}$ and commonly known (we relax this constraint in Section 5). The sizes of players' possibility sets satisfy $n_{ik} > 1$ and $n_{1k} + n_{2k} - 1 \leq |\Omega_k|$. That is, in every dimension neither player individually knows the truth, while it is possible for players to know it jointly. Conditional on the true state ω^* each pair (S_{1k}, S_{2k}) is independently drawn from a uniform distribution over the set

$$\{(\tilde{S}_{1k}, \tilde{S}_{2k}) \subset \Omega_k^2 | \tilde{S}_{1k} \cap \tilde{S}_{2k} = \{\omega_k^*\}, |\tilde{S}_{ik}| = n_{ik}, i = 1, 2\}.$$

Hence, the true state is distributed knowledge. We refer to each dimension k as a *topic*.

As long as the conversation has not ended, in each period each player chooses between listening, taking an action for one of the topics, disclosing an element of one of her possibility sets, or ending the entire conversation. Denote player i's choice to listen by ℓ_i. For each $\omega_k \in \Omega_k$, player i has a corresponding action $a_i^{\omega_k}$. Her set of actions for the k-th topic is $A_{ik} := \{a_i^{\omega_k}\}_{\omega_k \in \Omega_k}$ and the set of all of the actions available to her is $A_i := \bigcup_{k=1}^{K} A_{ik}$. Denote player i's choice to disclose element $\omega_k \in S_{ik}$ in her kth possibility set by $d_i^{\omega_k}$. Let $\mathcal{S}_i$ denote the set of all S_i that satisfy $|S_{ik}| = n_{ik}$ for all $k = 1, \ldots, K$. For each $S_i \in \mathcal{S}_i$ define $D_{ik}(S_i) := \{d_i^{\omega_k}\}_{\omega_k \in S_{ik}}$ and $D_i(S_i) := \bigcup_{k=1}^{K} D_{ik}(S_i)$. The former is the set of disclosure choices in topic k and the latter the entire set of disclosure choices available to player i. Define $D_i := \bigcup_{S_i \in \mathcal{S}_i} D_i(S_i)$. Denote player i's choice to end the conversation by e_i. Then prior to the beginning of game, the set of choices that player i may have available is $C_i := D_i \cup A_i \cup \{e_i\} \cup \{\ell_i\}$.

The choices available to player i are further constrained by being history-dependent subsets of C_i. Player i only has a choice in period t if neither player j has ended the conversation prior to that period by choosing e_j. An action $a_i^{\omega_k}$ is only available to player i in period t if neither player has taken an action for the kth topic prior to that period. That is, any player's action for a topic, even a wrong one, *completes* that topic. A disclosure choice $d_i^{\omega_k}$ is only available to player i in period t if $\omega_k \in S_{ik}$, neither player disclosed ω_k prior to period t in a period in which they were the only discloser, neither player j took an action in A_{jk}, and for every topic $k' \neq k$ for which some player $\tilde{\imath}$ made a disclosure prior to period t some player $\hat{\imath}$ (possibly the same as $\tilde{\imath}$) took an action $a_{\hat{\imath}k'}$ prior to period t. That is, a player cannot disclose information that is not available to her, disclose information that has already been disclosed, disclose for a topic that has been completed by some player's action, or switch from disclosures having been made in one topic to disclosing in another prior to the former having been completed with an action.[3] A player only understands a disclosure by her counterpart if she herself is listening. If either she also discloses or acts, she observes the fact that her counterpart disclosed but not the content of the disclosure. Denote the fact of a disclosure by player i, stripped of its content, by d.

For a detailed description of the game, players' private information, and the choices available to them at every stage, consider each player i's private histories. For every S_i, let (S_i) denote a length-0 private history of player i. Given any finite $t \geq 1$, a sequence $(S_i, (c_i^1, \tilde{c}_{-i}^1), (c_i^2, \tilde{c}_{-i}^2), \ldots, (c_i^t, \tilde{c}_{-i}^t))$ is a length-t (private) history, h_i^t, for player i if and only if the following conditions hold:

1. $S_i \in \mathcal{S}_i$ (player i's type satisfies the commonly known size constraint $|S_{ik}| = n_{ik}$ in every dimension);

[3]One effect of the latter requirement is that if players try to simultaneously start conversing about different topics, they foreclose further disclosures on those topics.

2. $c_i^\tau \in C_i$ for all $\tau \leq t$ (player i's length-t history records her own choices up to and including time t);

3. $\tilde{c}_{-i}^\tau \in C_{-i} \cup \{d\}$ for all $\tau \leq t$ (player i's length-t history records her observations of the choices of her counterpart up to and including time t, where in some instances she sees only the fact, d, that $-i$ disclosed, and not what $-i$ disclosed);

4. For all $\tau \leq t$, $\tilde{c}_{-i}^\tau = d$ if $c_i^\tau \neq \ell$ and $c_{-i}^\tau \in D_{-i}$ and otherwise, $\tilde{c}_{-i}^\tau = c_{-i}^\tau$ (player i is only prevented from seeing the content of her counterpart's disclosure if either she herself discloses or takes an action);

5. $c_i^\tau \neq e_i^\tau$ for all $\tau < t$ (player i did not end the conversation prior to period t);

6. $\tilde{c}_{-i}^\tau \neq e_{-i}^\tau$ for all $\tau < t$ (player i did not observe player $-i$ ending the conversation prior to period t);

7. for all $\tau \leq t$, if $c_i^\tau = d_i^{\omega_k}$ then $\omega_k \in S_{ik}$ (player i can only disclose elements of her possibility sets);

8. for all $\tau \leq t$, if $\tilde{c}_{-i}^\tau = d_{-i}^{\omega_k}$ then $\omega_k \in S_{-i,k}$ (if player i observes the content of a disclosure by player $-i$, the disclosed element has to belong to player $-i$'s possibility set);

9. for all $\tau < t$, if $c_i^\tau = d_i^{\omega_k}$ and $\tilde{c}_{-i}^\tau \neq d$, then $c_j^{\tau+s} \neq d_j^{\omega_k}$ for $j = 1, 2$ and all $s \geq 1$ (if player i made a successful disclosure, then neither player can disclose that element again);

10. for all $\tau < t$, if $\tilde{c}_{-i}^\tau = d_{-i}^{\omega_k}$, then $c_j^{\tau+s} \neq d_j^{\omega_k}$ for $j = 1, 2$ all $s \geq 1$ (if player i observed a successful disclosure by player $-i$, then neither player can disclose that element again);

11. for all $\tau < t$, if $c_i^\tau \in D_{ik'}(S_i)$ and $c_j^s \notin A_{jk'}$ for both $j = 1$ and $j = 2$ and all s satisfying $\tau \leq s < t$, then $c_i^t \notin D_{ik}(S_i)$ and $\tilde{c}_{-i}^t \notin D_{-i,k}(S_{-i})$ for all $k \neq k'$ (if player i disclosed in some topic neither player can switch to disclosing in some other topic before the former topic has been completed by an action);

12. for all $\tau < t$, if $\tilde{c}_{-i}^\tau \in D_{-i,k'}(S_{-i}) \cup \{d\}$, and $c_j^s \notin A_{jk'}$ for both $j = 1$ and $j = 2$ and all s satisfying $\tau \leq s < t$, then $c_i^t \notin D_{ik}(S_i)$ and

$\tilde{c}^t_{-i} \notin D_{-i,k}(S_{-i})$ for all $k \neq k'$ (if player $-i$ disclosed in some topic neither player can switch to disclosing in some other topic before the former topic has been closed by an action);

13. for all $\tau < t$ and $s \geq 1$, if either $c^\tau_i \in A_{ik}$ or $c^\tau_{-i} \in A_{-i,k}$, then $c^{\tau+s}_i \notin A_{ik} \cup D_{ik}$ and $\tilde{c}^{\tau+s}_{-i} \notin A_{-i,k} \cup D_{-i,k}$ (if player i took an action for some topic, that topic has been completed, no more actions can be taken in that topic, and no more disclosures in that topic can be made);

14. there exists $S_{-i} \in \mathcal{S}_{-i}$ such that $|S_i \cap S_{-i}| = 1$ and conditions (1) - (13) hold for player $-i$ (player i's private history is consistent with some private information for player $-i$).

Given any length-t private history h^t_i of player i, a choice $c_i \in C_i$ is *available* to player i if there exists a continuation history h^{t+1}_i with $c^{t+1}_i = c_i$. Denote the set of choices available to player i following history h^t_i by $C_i(h^t_i)$. A sequence $(S_i, (c^1_i, c^1_{-i}), (c^2_i, c^2_{-i}), \dots)$ is an infinite-length (private) history, h^∞_i, for player i if and only if $(S_i, (c^1_i, c^1_{-i}), (c^2_i, c^2_{-i}), \dots, (c^t_i, c^t_{-i}))$ is a length-t (private) history for every t.

In every period t the players' common payoff $U(c^t_i, c^t_{-i}, \omega)$ depends on their profile of choices $(c^t_i, c^t_{-i}) \in C_i \times C_{-i}$ and the state $\omega \in \Omega$. If player i ends the entire conversation by choosing e_i, both players receive the (continuation) payoff $\beta > 0$.[4] This is the always available outside option payoff from not engaging in a conversation. If both players either disclose or listen in period t, i.e., if $c^t_i \in D_i(S_i) \cup \{\ell_i\}$ for $i = 1, 2$, their payoff is zero for that period. If the true state is ω, player i chooses $c^t_i = a^{\omega_k}_i$, i.e., takes the action that matches the component of state ω in the kth topic and player $-i$ chooses $c^t_{-i} \in \{a^{\omega_k}_{-i}\} \cup D_{-i}(S_{-i}) \cup \{\ell_{-i}\}$, i.e., either chooses the same matching action, discloses or listens, the common payoff is $\alpha > \beta$. That is, solving the issue for one of the topics is worthwhile, at least when it can be done without delay. For every other profile of choices (c^t_i, c^t_{-i}), the common payoff is $\gamma < 0$, where $\alpha < -\gamma$. Hence, it is prohibitively costly to take an action that does not match a component of the state ω in some topic k.

To summarize, for any state $\omega \in \Omega$, the common period-t payoff from

[4] This implies that by choosing to end a conversation, a player vetoes any current and future action for the topics under consideration.

choice profile (c_i^t, c_{-i}^t) equals

$$U(c_i^t, c_{-i}^t, \omega) = \begin{cases} 0 & \text{if } c_i^t \in D_i(S_i) \cup \{\ell_i\} \text{ and } \mathrm{c}_{-i}^t \in D_{-i}(S_{-i}) \cup \{\ell_{-i}\}, \\ \alpha & \text{if } c_i^t = a_i^{\omega_k} \text{ and } c_{-i}^t \in \{a_{-i}^{\omega_k}\} \cup D_{-i}(S_{-i}) \cup \{\ell_{-i}\} \\ & \text{for some } k \in \{1, \ldots, K\} \\ \beta & \text{if } c_i^t = e_i \text{ or } c_{-i}^t = e_{-i} \\ \gamma & \text{otherwise} \end{cases},$$

where $\alpha > \beta > 0 > \gamma$ and $\alpha < -\gamma$. For any $\omega \in \Omega$ and $\delta \in (0, 1)$, the payoff from a conversation equals the present discounted value of the per-period payoffs

$$\sum_{t=1}^{T} \delta^{t-1} U(c_i^t, c_{-i}^t, \omega),$$

where T is finite if the conversation terminates in period T or it is infinite if the conversation never terminates.

A private history h_i^T is a *terminal history* if $T = \infty$ or T is finite and there is a player j for whom $c_j^T = e_j^T$. All private histories that are not terminal histories are *nonterminal* histories. Denote the set of nonterminal histories by H_i. A strategy $\sigma_i : H_i \to \Delta(C_i)$ for player i maps the set of player i's non-terminal histories into the set of probability distributions over player i's choice set C_i, where σ_i satisfies $\sigma_i(h_i^t) \in \Delta(C_i(h_i^t))$ for every nonterminal history h_i^t.

Much of the paper will be concerned with literal-meaning strategies and equilibria in which players use but are not restricted to those strategies. This is meant to capture some of the difficulty of encoding meaning into a disclosure beyond what is immediately implied by it. The literal-meaning condition is a symmetry property. Elements of a topic that have not been distinguished by history have to be treated identically. In terms of a single topic, for simplicity denoted by Ω, it says that a player's strategy σ_i is a literal meaning strategy if and only if

$$\sigma_i(h_i^t)(c) = \sigma_i(\pi(h_i^t))(\pi(c)), \forall i, \forall h_i^t, \forall c, \forall \pi, \tag{1}$$

where each π is a permutation of the elements of Ω.[5]

[5] Blume and Park [8] refer to this as a *no-common-labeling* condition. If players lack common labels for the elements of Ω, for each player their counterpart's strategy has to be a literal-meaning strategy. Here, I have in mind that players may have common labels but find it difficult to make use of them.

To illustrate one of the key effects of requiring a strategy σ_i of player i to be a literal-meaning strategy, suppose that after some private history h_i^t the strategy prescribes that player i disclose one of the elements of her remaining possibility set. Denote the set of undisclosed elements in player i's possibility set following history h_i^t by S_i^t and consider a permutation $\tilde{\pi}$ with the property that $\tilde{\pi}(\omega) \in S_i^t, \forall \omega \in S_i^t$ and $\tilde{\pi}(\omega) = \omega, \forall \omega \in \Omega \setminus S_i^t$. Notice that with any such permutation, $h_i^t = \pi(h_i^t)$. Suppose that c is a disclosure d_i^ω of an element ω in S_i^t. Then from the literal-meaning condition (1) we get

$$\sigma_i(h_i^t)(d_i^\omega) = \sigma_i(h_i^t)(d_i^{\tilde{\pi}(\omega)}), \forall i.$$

That is, there is equal probability of player i disclosing each of the elements of her remaining possibility set.

3 Literal-meaning strategies: equilibria and optimality

In a literal-meaning equilibrium the interlocutors use strategies that minimize what can be inferred from a disclosure. All the listener learns from a disclosure, aside from the fact that the speaker does not yet know the truth, is the identity of the disclosed element of the speaker's possibility set. This minimizes the burden of having to coordinate on message meanings.

In this section, we first show that there still is a significant coordination problem. In general, there is a large number of Pareto ranked literal-meaning equilibria. If an issue is sufficiently important and can be satisfactorily resolved in a reasonable amount of time, then every talking order can be supported by a literal-meaning equilibrium. All of the following talking orders, as well as many others, are supported by a literal-meaning equilibrium: a social hierarchy determines who always talks; the less well informed player always talks; the better informed player always talks; players alternate, following a politeness norm. This result is driven by the strong complementarity between talking and listening. Given that the contents of simultaneous talk are lost, it is optimal to listen when the other is talking and vice versa.

Restricting attention to optimal literal-meaning equilibria drastically reduces the coordination problem. In an optimal literal-meaning equilibrium, the better informed player talks exclusively. At any given instant the better informed player is more likely to name the true state and if that fails, the informational advantage of the better informed player increases, reinforcing the rationale for letting the better informed player keep talking.

We first derive these results for a single topic and then, using the assumption that rules out topic switching, extend it to multiple topics. For multiple topics, we show in addition that in an optimal literal-meaning equilibrium players converse about topics in the order of how well informed they are about them and that they may skip topics about which they have little information.

3.1 A single-topic example

The example in this section provides a simple illustration of literal-meaning equilibria when there is only a single topic. It also demonstrates that (1) in every literal-meaning equilibrium there is a residue of pragmatic inference, (2) in some literal-meaning equilibria players leverage that pragmatic inference by acting on it, and (3) there are multiple optimal literal-meaning equilibria with different behaviors on path.

Consider the case of a single topic with a state space Ω of size $|\Omega| = 4$. Suppose that the size of player 1's possibility set is $|S_1| = 2$ and the size of player 2's possibility set is $|S_2| = 3$. Let $\alpha = 100$, $\beta = 0.1$, $\gamma = -500$ and $\delta = 0.9$ (the example is robust to significant variations in the values of these parameters). Then there is a literal-meaning equilibrium with the following behavior on path: In period 1 player 1 discloses and player 2 listens. In period 2 player 1 listens. Player 2 acts in period 2 if player 1 disclosed an element of S_2 in period 1 and otherwise discloses. If period 3 is reached, then player 1 acts and player 2 listens. Following a period in which a player has acted, some player i (or both) ends the game by choosing e_i.

Notice that if period 3 is reached, player 1 is justified to act regardless of what player 2 disclosed in period 2. This is the case because period 3 is only reached if player 2 did not act in period 2. Player 2 would have acted, had the state disclosed in period 1 been the true state. Since player 2 chose to disclose rather than to act in period 2, player 1 infers that the member of her possibility set that she disclosed in period 1 cannot be the true state.[6]

[6]It is player 2's failure to act that matters, not that player 2 is making an irrelevant disclosure. There is a similar equilibrium in which instead of disclosing player 2 stays silent in response to player 1 having failed to disclose the true state. Inferences from inaction are reminiscent of those in familiar logic puzzles like the dirty-faces puzzle that is described in Littlewood [26] or the unfaithful-wives puzzle, an early version of which

Hence, since one the elements of her possibility set is the true state, it has to be the undisclosed one.

In this equilibrium, the information conveyed to player 1 through a disclosure by player 2 is the identity of the disclosed element of player 2's possibility set but also the fact that the state initially disclosed by player 1 is not the true state. Player 1's initial disclosure creates context that helps shape the meaning of player 2's subsequent disclosure. This will be true for every disclosure or silence in a literal-meaning equilibrium that follows a disclosure by the other player. What is peculiar about this equilibrium is not that there is minimal pragmatic inference but that players take advantage of the difference between saying, i.e., the content of the disclosure by player 2, and meaning, which also includes the indication implied by player 2's disclosure that the state disclosed earlier by player 1 is not the true state – they are leveraging pragmatic inference.

There are other literal-meaning equilibria that do not leverage pragmatic inference. In one such equilibrium player 1 discloses in every period, player 2 listens until player 1 discloses the true element, then player 2 acts, and finally either player ends the game. In this equilibrium, there is also a difference between what is said (or not said) and what is meant – silence in response to a disclosure indicates that the disclosed element was not the true state. As in the equilibrium described earlier, player 1 gains information from player 2's silence, but in this case that inference is irrelevant for her continuation play – as long as player 2 has not acted, she continues disclosing. In this equilibrium players make but do not leverage pragmatic inferences.

Both of the equilibria we described are optimal in the class of literal-meaning equilibria. We will later return to this example to show that once we drop the literal-meaning requirement, there are equilibria that strictly improve on optimal literal-meaning equilibria. In those equilibria leveraging pragmatic inference is necessary for optimality.

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appears in Gamow and Stern [17]. Similar inferences play a role in Geanakoplos and Polemarchakis [18], where it can occur that agents keep refining their information while repeatedly announcing the same conflicting posteriors until they finally agree. In their words, there is "no evident revision."

3.2 A class of literal-meaning equilibria for a single topic

In this section we show that whenever the outside option is not too attractive, we can arbitrarily assign players to periods and have a literal-meaning equilibrium in which in every period the player who has been assigned to that period discloses until the true state has been discovered.

We focus on the case in which $K = 1$, i.e., there is a single topic. To economize on notation, we suppress the index k and write ω for ω_k and n_i for n_{ik}. To describe the strategies we use to prove our result it is convenient to introduce a classification of histories. For every player i and every state $\omega \in S_i$, let $H_i^{\omega t}$ denote the set of all private length t histories of player i for which $c_i^\tau \notin A_i, \tilde{c}_{-i}^\tau \notin A_{-i}$ for all $\tau \leq t$, and there is a time $\tau' \leq t$ with $\tilde{c}_{-i}^{\tau'} = d_{-i}^\omega$. Histories in $H_i^{\omega t}$ are those in which no action has been taken and player i has learned that the true state is ω through a disclosure by player $-i$. The union of all those histories is $H_i^{1t} := \bigcup_{\omega \in S_i} H_i^{\omega t}$. This is the set of all histories in which player i has learned the true state through a disclosure of her counterpart and the topic has not yet been completed with an action.

For every private length-t history h_i^t of player i, use $D_i(h_i^t)$ to denote the set of all of player i's successful disclosures made in periods $1, \ldots, t$. Use S_i^t to denote those elements in player i's possibility set that neither player successfully disclosed prior to period t. Let H_i^{2t} denote the set of all private length-t histories of player i for which $c_i^\tau \notin A_i, \tilde{c}_{-i}^\tau \notin A_{-i}$ for all $\tau \leq t$ and $|S_i^t| = 1$. Histories in H_i^{2t} are those in which player i can infer the true state for any strategy of player $-i$ that has player $-i$ act whenever she knows the truth: If $|S_i^t| = 1$, and no action has been taken, then player i must have disclosed all but one state in their possibility set prior to period t and if period $t+1$ has been reached it is the case that $-i$ did not act. Therefore, if player $-i$ acts whenever she knows the truth, following any history in H_i^{2t}, player i knows the true state.

Define $H_i^{*t} := H_i^{1t} \cup H_i^{2t}$. This is the set of player i's private histories in which neither player has yet taken an action and player i either has learned the true state directly or can infer it from player $-i$'s inaction, as just described.

Let H_i^{at} be the set of all of player i's private length-t histories for which there exists a period $\tau \leq t$ with either $c_i^\tau \in A_i$ or $\tilde{c}_{-i}^\tau \in A_{-i}$. This is the set of histories in which either player took an action. Finally, define $H_i^{0t} := H_i^t \setminus \{H_i^{*t} \cup H_i^{at}\}$. This is the set of all length-t private histories of player i in which no action has been taken and player i remains uncertain about the identity of the true state ω^*.

Our first result refers to a class of strategy profiles that make use of an assignment of players to time periods. To every period $t = 1, 2, \dots$ assign a player $i_t \in \{1, 2\}$. Refer to this as a "player assignment." For every finite set X, let $\mathcal{U}(X)$ denote the uniform distribution on X. Given a player assignment, call a strategy profile $\sigma = (\sigma_1, \sigma_2)$ a "player-assignment profile" if it satisfies

1. For every private history $h_i \in H_i^{\omega t}$ of player i, let $\sigma_i(h_i) = a^\omega$.
2. For every private history $h_i \in H_i^{2t}$ of player i with $S_i^t = \{\omega\}$, let $\sigma_i(h_i) = a^\omega$.
3. For every private history $h_i \in H_i^{at}$ of player i, let $\sigma_i(h_i) = e_i$.
4. For every private history $h_i \in H_i^{0t}$ of player i, let

$$\sigma_i(h_i) = \begin{cases} \mathcal{U}(D_i(S_i) \setminus D_i(h_i)) \text{ if } i = i_t, \text{and} \\ \ell \text{ otherwise.} \end{cases}$$

Proposition 1 *There exists $\overline{\beta} > 0$ such that for all $\beta \in (0, \overline{\beta})$, every player-assignment profile σ is a literal-meaning equilibrium profile.*

Proof: For every private history h_i^t of player i with $|S_i^t| > 0$, let player i's belief in period $t+1$ about the identity of the true state be given by

$$\mu(\omega = \omega^* | h_i^t) = \begin{cases} 1 \text{ if } h_i^t \in H_i^{\omega t} \\ 1/|S_i^t| \text{ if } \omega \in S_i^t \text{ and } h_i^t \notin H_i^{1t} \\ 0 \text{ otherwise} \end{cases}$$

That is, player i only updates her beliefs about the true state in response to new evidence and only in accordance with the evidence provided. Given that player $-i$'s strategy prescribes

$$\sigma_{-i}(h_{-i}) = \begin{cases} \mathcal{U}(D_{-i}(S_{-i}) \setminus D_{-i}(h_{-i})) \text{ if } -i = i_t \\ \ell \text{ otherwise} \end{cases}$$

for every history $h_{-i} \in H_{-i}^{0t}$ of player $-i$ and that player $-i$ ends the game after every other history, on the equilibrium path these are the beliefs obtained using Bayes' rule.

On the path of play that is induced by σ, every player assignment profile has players at worst conduct an exhaustive search over a set with n_1+n_2-1 states that contains the true state, examining a new state at every turn. Hence, the expected payoff from any player assignment profile is at least $\delta^{n_1+n_2-1}(\alpha+\delta\beta)>0$. Off the path of play that is induced by σ, the same is true, except that the lower bound on the expected continuation payoff may be higher for histories during which some states have already been examined. If we choose $\overline{\beta}$ so that $\delta^{n_1+n_2-1}(\alpha+\delta\overline{\beta})\geq\overline{\beta}>0$, then for every $\beta\in(0,\overline{\beta})$ we have $\delta^{n_1+n_2-1}(\alpha+\delta\beta)>\beta>0$. Therefore, for all $\beta\in(0,\overline{\beta})$ and for every private history of player i, player i prefers to conform with σ_i rather than to end the conversation by choosing e_i.

Following any history $h_i^t\in H_i^{1t}$, player i knows the identity of the true state ω^* from a disclosure of player $-i$. Taking action $a_i^{\omega^*}$ results in immediately receiving the maximal payoff α provided player $-i$ does not take an action a_{-i}^{ω} with $\omega\neq\omega^*$ or ends the conversation with e_{-i}. Unless player $-i$ takes action $a_{-i}^{\omega^*}$, making any other choice results at best in the payoff $\delta\alpha<\alpha$. If player $-i$ takes an action a_{-i}^{ω} with $\omega\neq\omega^*$ or chooses e_{-i}, then the game ends and player i cannot influence the outcome. Hence, following any history $h_i^t\in H_i^{1t}$, the strategy σ_i is optimal.

In any history $h_i^t\in H_i^{2t}$, player i disclosed all but one element of her possibility set prior to period t. Had any of these disclosures been equal to $d_i^{\omega^*}$, then, given the specification of strategy σ_{-i}, player $-i$ would have taken the action $a_{-i}^{\omega^*}$ and ended the game. Since the game has not ended, player i can infer that none of her disclosures were equal to $d_i^{\omega^*}$, and hence the remaining element in her possibility set must equal ω^*. Therefore, taking action $a_i^{\omega^*}$ in period $t+1$ as prescribed by strategy σ_i is optimal.

Given our specification of beliefs, for any private history $h_i^t\in H_i^{0t}$, player i's posterior probability that $\omega=\omega^*$ is no larger than $\frac{1}{2}$ for all $\omega\in S_i$. Therefore, since $\alpha<-\gamma$, it is not optimal for player i to deviate to any strategy σ_i' that prescribes taking action a_i^{ω} in period $t+1$ for any $\omega\in\Omega$.

Given any private history $h_i^t\in H_i^{0t}$ for which $i_{t+1}=i$, consider a deviation to a strategy σ_i' that prescribes choosing ℓ in period $t+1$ rather than making a disclosure in $D_i(S_i)\setminus D_i(h_i^t)$. Given that player $-i$'s strategy prescribes to listen in period $t+1$ and that player i's deviation does not affect in which periods player $-i$ listens and in which periods she discloses, at best this deviation misses one opportunity to learn about one of the elements in S_i. Hence, there can be no gain from any such deviation.

Given any private history $h_i^t\in H_i^{0t}$ for which $i_{t+1}=-i$, consider a deviation to a strategy σ_i' that prescribes making a disclosure in $D_i(S_i) \setminus D_i(h_i^t)$ in period $t+1$ rather than listening. Given that player $-i$'s strategy prescribes to disclose in period $t+1$ and that player i's deviation does not affect in which periods player $-i$ listens and in which periods she discloses, at best this deviation misses one opportunity to learn about one of the elements in S_{-i}. Hence, there can be no gain from any such deviation. □

3.3 Optimal literal-meaning strategies for a single topic

Optimal profiles of literal-meaning strategies maximize players' (common) expected payoff at the beginning of the game within the class of literal meaning strategies. On path, they remain optimal. Consider the case in which β is sufficiently small for it not to be optimal to immediately end the conversation.

If player i disclosed ω^* in the last round, optimal continuation play is trivial: player $-i$ takes the optimal action and ends the game. Otherwise, the only information relevant for optimal continuation play is the number of elements in each player's remaining possibility set and who disclosed last. This follows from the fact that if players use literal-meaning strategies, then after any on-path history the only information each player has about the location of ω^* is their remaining possibility set, that all elements in that set are equally likely, and who disclosed in the previous round. Who disclosed in the last round is relevant in the event that player i has only one element left in their possibility set and may be able to infer that this is ω^* from player $-i$ having chosen to disclose or be silent rather than to act in the previous period.

Define $V(m,n,i)$ as the (common) expected continuation payoff on path when players use optimal literal-meaning strategies, have reached a history in which player 1 has m elements left in their possibility set, player 2 has n elements left, player i disclosed in the previous period and player i failed to disclose ω^*. Here, let $i = 0, 1, 2$, where $i = 0$ indicates that neither player disclosed in the previous period.

Notice that $V(1,n,2) = \alpha + \delta\beta$: Since player 2 disclosed last, and therefore did not act, player 1 infers that it cannot be the case that any of the states she disclosed are equal to ω^*. Since ω^* is in S_1, she can conclude that the single state in her remaining possibility is ω^*. Therefore, it is optimal for her to take the action a^{ω^*}, following which it is optimal for either player i to end the game with the action e_i, which results in a payoff $\alpha + \delta\beta$ for both players.

Similarly, $V(1,n,0) = \alpha + \delta\beta$: Since player 2 chose to be silent in the previous round rather than to act, player 1 infers that it cannot be the case that any of the states she disclosed are equal to ω^*. Since ω^* is in S_1, she can conclude that the single state in her remaining possibility is ω^*. Therefore, it is optimal for her to take the action a^{ω^*}, which results in a payoff $\alpha + \delta\beta$ for both players.

Observe next that $V(1,n,1) = \delta(\alpha + \delta\beta)$ when $n > 1$: Since player 1 failed to disclose ω^*, $n > 1$, and player 1 has not yet observed player 2's response, both players remain uncertain about ω^* and hence won't act in the current period. Player 2's inaction, however, will reveal to player 1 that the sole remaining element in his possibility is ω^*. Hence, in the next period player 1 will take the action a^{ω^*}.

Whenever $m \geq 2$ and $n \geq 2$, which includes the beginning of the game, neither player has enough information to justify taking an action and therefore the information about who disclosed in the last period is irrelevant. Hence, for $m \geq 2$ and $n \geq 2$, we can suppress i in the triple (m,n,i) and, slightly abusing notation, write $V(m,n) = V(m,n,i)$. Also, for $n \geq 2$ define $V(1,n) \coloneqq V(1,n,1) = \delta(\alpha + \delta\beta)$.

Proposition 2 *If $\beta \leq \frac{\delta(1-\delta^m)}{1-\delta}\frac{\alpha+\delta\beta}{m}$, then for all $n \geq m \geq 2$,*

$$V(m,n) = \frac{1}{m}\delta(\alpha + \delta\beta) + \frac{m-1}{m}\delta V(m-1,n),$$

Furthermore, in every optimal literal-meaning profile the player with the lower number of remaining elements in their possibility set must disclose one of their elements following every on-path history in which she has two or more undisclosed elements left in her possibility set.

Proof: We know from Proposition 1 that for sufficiently small $\beta > 0$ there are strategy profiles for which neither player i has an incentive to end the game by choosing e_i. Suppose that this condition on β is satisfied – we will verify the more specific condition in the statement of Proposition 2 at the the end of this proof. Then we can restrict attention to strategy profiles that search for and eventually discover ω^* rather than immediately end the game.

Consider $V(2,n)$. With $n > 2$, have

$$V(2,n) = \max\left\{\frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta V(1,n,1), \frac{1}{n}\delta(\alpha+\delta\beta) + \frac{n-1}{n}\delta V(2,n-1)\right\}$$
$$= \max\left\{\frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta^2(\alpha+\delta\beta), \frac{1}{n}\delta(\alpha+\delta\beta) + \frac{n-1}{n}\delta V(2,n-1)\right\}.$$

Since $V(2,n-1) \leq \delta(\alpha+\delta\beta)$, it follows that for $n > 2$

$$V(2,n) = \frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta V(1,n,1) = \frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta^2(\alpha+\delta\beta)$$
$$= \frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta V(1,n)$$

When $n = m = 2$, then

$$V(2,2) = \max\left\{\frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta V(1,2,1), \frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta V(2,1,2)\right\}$$
$$= \frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta^2(\alpha+\delta\beta) = \frac{1}{2}\delta(\alpha+\delta\beta) + \frac{1}{2}\delta V(1,2).$$

Hence, we have verified the expression for $V(m,n)$ in the statement of the proposition for $m = 2$ and $n \geq 2$. The expression for $V(m,n)$ evidently holds whenever $n = m \geq 2$. Therefore, consider $n > m \geq 2$. We proceed by induction on m.

Suppose the expression for $V(m,n)$ is correct for $m = k$ and all $n > k$. We want to show that it holds for $m = k+1$ and all $n > k+1$.

$$V(k+1,n) =$$
$$\max\left\{\frac{1}{k+1}\delta(\alpha+\delta\beta) + \frac{k}{k+1}\delta V(k,n), \frac{1}{n}\delta(\alpha+\delta\beta) + \frac{n-1}{n}\delta V(k+1,n-1)\right\}$$

Notice that $V(k,n-1) \geq V(k+1,n-1)$. Hence,

$$V(k+1,n) \leq$$
$$\max\left\{\frac{1}{k+1}\delta(\alpha+\delta\beta) + \frac{k}{k+1}\delta V(k,n), \frac{1}{n}\delta(\alpha+\delta\beta) + \frac{n-1}{n}\delta V(k,n-1)\right\}$$

Since, by assumption $n > k+1$, it is the case that $n-1 > k$, and therefore by the induction hypothesis we have that $V(k, n-1) = V(k, n)$.

This implies that

$$V(k+1, n) = \frac{1}{k+1}\delta(\alpha + \delta\beta) + \frac{k}{k+1}\delta V(k, n).$$

This verifies the expression for the value function in the statement of the proposition.

From inspecting the value function, it follows immediately that to realize these continuation payoffs, the player with the lower number of remaining elements must disclose one of their elements following every on-path history in which they have 2 or more undisclosed elements.

A soon as the player with the lower number of remaining elements reaches the point where they have disclosed all but one of their elements, we saw that there are different optimal continuations. There is, however, one among these in which that player discloses their sole remaining element. It follows that, for sufficiently small β the expected payoff from any optimal literal-meaning profile equals

$$\begin{aligned}
&\frac{1}{m}\delta(\alpha + \delta\beta) + \frac{m-1}{m}\left(\frac{1}{m-1}\delta^2(\alpha + \delta\beta) + \frac{m-2}{m-1}\left(\frac{1}{m-2}\delta^3(\alpha + \delta\beta) + \dots\right)\right. \\
=&\frac{(\alpha + \delta\beta)}{m}\sum_{t=1}^{m}\delta^t \\
=&\frac{\delta(1-\delta^m)}{1-\delta}\frac{(\alpha + \delta\beta)}{m},
\end{aligned}$$

which confirms the explicit bound on β in the proposition. □

Thus in any optimal strategy profile, only the better informed player talk and expected payoff only depends on how well informed that player is. As a consequence reducing the information of the less well informed player does not impact payoffs at the optimum.

Corollary 1 *Whenever $|S_i| \leq |S_{-i}|$, there is no loss from making player $-i$ less informed by enlarging her possibility set.*

3.4 Optimal literal-meaning strategies for multiple topics

Suppose now that there are $K \geq 2$ topics. Recall that we assumed *no switching* – a topic that has been started with a disclosure for that topic must be completed with an action for that topic before another topic can be started. Given our finding in Proposition 2 for optimal play with a single topic, we can then ask how to optimally arrange the order in which to converse about multiple topics.

At any given time, we say that players *converse about topic k* if either player makes a first disclosure in Ω_k or has disclosed an element in Ω_k and neither player $i = 1, 2$ has ended the conversation by choosing e_i or completed the topic by taking an action $a_i \in A_{ik}$. For every topic k, select a *best informed player* $i_k \in \arg\min_{i \in \{1,2\}} \{|S_{i,k}|\}$ and define $m_k := |S_{i_k,k}|$ as the *size of topic k*. Assume that there is at least one topic k that satisfies $\frac{\delta(1-\delta^{m_k})}{1-\delta} \frac{(\alpha+\delta\beta)}{m_k} > \beta$. Define a topic k' as being *simpler* than topic k'' if $m_{k'} \leq m_{k''}$. Order topics by decreasing simplicity, so that for all $k = 1, \ldots, K-1$ it is the case that $m_k \leq m_{k+1}$. Say that *players converse about topics in the order of their simplicity* if (i) they always converse about a topic k with $\frac{\delta(1-\delta^{m_k})}{1-\delta} \frac{(\alpha+\delta\beta)}{m_k} > \beta$, (ii) they never converse about a topic k with $\frac{\delta(1-\delta^{m_k})}{1-\delta} \frac{(\alpha+\delta\beta)}{m_k} < \beta$, and (iii) for any two topics k' and k'' with $m_{k'} < m_{k''}$ and $\frac{\delta(1-\delta^{m_{k''}})}{1-\delta} \frac{(\alpha+\delta\beta)}{m_{k''}} \geq \beta$, they converse about topic k' before conversing about topic k''.

Proposition 3 *In every optimal literal-meaning profile (i) players converse optimally about every topic they converse about; (ii) players immediately either start a new topic or end the conversation after completing a topic; and, (iii) players converse about topics in the order of their simplicity.*

Proof: Property (i) in the proposition follows immediately from our no-switching assumption: No switching implies that it is never optimal for players to simultaneously start conversing about different topics - they would be trapped there without gaining any information. Conditional on starting a topic k, players are constrained to converse about that topic before moving on to another topic and the conversation about topic k does not constrain how they converse about other topics. Thus optimally conversing about topic k, once a conversation about that topic has been started, is both directly beneficial and, in addition, avoids delaying transitions to other topics k' that satisfy $\frac{\delta(1-\delta^{m_{k'}})}{1-\delta}\frac{(\alpha+\delta\beta)}{m_{k'}} > \beta$, if they exist, or toward getting the positive benefit β from ending the conversation if no such topic remains.

Property (ii) in the proposition is a simple consequence of the fact that any length of time during which both players are silent only serves to defer optimal continuation payoffs and thus reduces overall payoffs.

To establish the property (iii), suppose that players converse about topics with sizes $m_1, \ldots, m_K$ following the order of topics $k = 1, 2 \ldots$, and end the conversation following topic k^*, allowing for the possibility that $k^* < K$ (that is, before there has been a conversation about every topic). By property (ii), players immediately either start a new topic or end the conversation after completing a topic. Use V_k to denote the continuation payoff from optimally conversing about topics $k, k+1, \ldots, k^*$ in that order. Then,

$$V_{k^*} = \frac{\delta(1-\delta^{m_{k^*}})}{1-\delta}\frac{(\alpha+\delta\beta)}{m_{k^*}}$$

and for $k < k^*$

$$\begin{aligned} V_k &= \frac{1}{m_k}(\delta\alpha + \delta^2 V_{k+1}) + \frac{m_k - 1}{m_k}\delta\left(\frac{1}{m_k - 1}\left(\delta\alpha + \delta^2 V_{k+1}\right) + \frac{m_k - 2}{m_k - 1}\delta(\ldots)\right) \\ &= \frac{1}{m_k}\frac{\delta(1-\delta^{m_k})}{1-\delta}(\alpha + \delta V_{k+1}) \end{aligned}$$

Hence,

$$V_1 = \alpha \sum_{k=1}^{k^*} \delta^{k-1}\left(\prod_{j=1}^{k}\frac{\delta(1-\delta^{m_j})}{1-\delta}\frac{1}{m_j}\right) + \delta^{k^*-1}\left(\prod_{j=1}^{k^*}\frac{\delta(1-\delta^{m_j})}{1-\delta}\frac{1}{m_j}\right)\delta\beta$$

Consider two topics k' and $k'+\ell$, where $k'+\ell \leq k^*$ and suppose that $m_{k'} > m_{k'+\ell}$. Then, if we switch the position of these two topics in the order of the conversation, not changing the position of any other topics, this strictly increases the values of the kth summand in the first term of the expression for V_1 for all k with $k' \leq k < k'+\ell$, leaving the values of all other summands and the value of the second term unchanged. Hence, for all topics that players converse about, it is optimal to converse about simpler topics prior to conversing about more complex topics.

Suppose that topics have been put in this order and examine the following alternative expression for V_1.

$$V_1 = \alpha \sum_{k=1}^{k^*-1} \delta^{k-1} \left(\prod_{j=1}^{k} \frac{\delta(1-\delta^{m_j})}{1-\delta} \frac{1}{m_j} \right) + \delta^{k^*-1} \left(\prod_{j=1}^{k^*} \frac{\delta(1-\delta^{m_j})}{1-\delta} \frac{1}{m_j} \right) (\alpha + \delta\beta)$$

If we dropped topic k^* from the conversation, the payoff would be

$$\begin{aligned}
\hat{V}_1 &= \alpha \sum_{k=1}^{k^*-1} \delta^{k-1} \left(\prod_{j=1}^{k} \frac{\delta(1-\delta^{m_j})}{1-\delta} \frac{1}{m_j} \right) + \delta^{k^*-2} \left(\prod_{j=1}^{k^*-1} \frac{\delta(1-\delta^{m_j})}{1-\delta} \frac{1}{m_j} \right) \delta\beta \\
&= \alpha \sum_{k=1}^{k^*-1} \delta^{k-1} \left(\prod_{j=1}^{k} \frac{\delta(1-\delta^{m_j})}{1-\delta} \frac{1}{m_j} \right) + \delta^{k^*-1} \left(\prod_{j=1}^{k^*-1} \frac{\delta(1-\delta^{m_j})}{1-\delta} \frac{1}{m_j} \right) \beta
\end{aligned}$$

Hence dropping topic k^* from the conversation raises the common expected payoff if and only if $\frac{\delta(1-\delta^{m_{k^*}})}{1-\delta} \frac{(\alpha+\delta\beta)}{m_{k^*}} < \beta$. An analogous argument shows that adding a topic $k^* + 1$ to the conversation raises the common expected payoff if and only if $\frac{\delta(1-\delta^{m_{k^*+1}})}{1-\delta} \frac{(\alpha+\delta\beta)}{m_{k^*+1}} > \beta$.

This concludes the proof of claim (iii) in the proposition that players converse about topics in the order of their simplicity. □

4 Pragmatic-meaning equilibria

Until now, we have focussed on equilibria in which there are restrictions on what disclosures can mean. In literal-meaning equilibria a disclosure only indicates (i) that the discloser did not (yet) know the truth and (ii) what was disclosed. In this section we remove this restriction. We focus on the single-topic case since it highlights the distinction between literal-meaning equilibria, in which meaning closely matches saying and pragmatic-meaning equilibria, which allow for significant gaps between saying and meaning. Throughout, we assume that $\beta > 0$ is sufficiently small relative to α that terminating the conversation is not optimal.

Proposition 4 *There exists a pragmatic-meaning equilibrium with a strictly higher expected payoff than from any literal-meaning equilibrium.*

The idea for proving this result is simple – pick an optimal literal meaning-equilibrium, which by definition makes no distinction among the elements of Ω, and compare it to an alternative equilibrium that takes advantage of ordering the states in Ω from lowest to highest. We highlight the key steps, leaving out those details that closely follow the pattern in the proof of Proposition 1.

Propositions 1 and 2 imply that in an optimal literal-meaning equilibrium the player with the lower number of elements in her possibility set discloses in every round prior to the truth being discovered, with the possible exceptions of the last and the penultimate round. Furthermore, the optimal literal-meaning equilibria in which the player with the lower number of undisclosed elements does not disclose in the last and the penultimate round are payoff equivalent to an optimal equilibrium in which that player does disclose in every round.

Hence, if, without loss of generality, we let $n_1 \leq n_2$, there is an optimal literal-meaning equilibrium with strategy profile σ in which player 1 discloses in every round and player 2 listens in every round until she learns the truth. Given that player 1 uses a literal-meaning strategy, she makes no distinctions among the elements of her remaining possibility set whenever she discloses, and hence player 2 only learns the identity of the disclosed state from player 1's disclosure.

Compare this to enumerating the elements of Ω from ω^1 to ω^n, replacing player 1's strategy by the strategy σ_1' that specifies

$$\sigma_1'(h_1) = \arg\min_j \{d^{\omega^j} \in (D_1(S_1) \setminus D_1(h_1))\}$$

for every private history of player 1 and having player 2, as before, listening in every period until she learns the truth, following which player 2 acts. The key is that with strategy σ_1' of player 1, player 2 is able to infer the truth faster in expectation than with σ_1 : whenever player 1, following the strategy σ_1' discloses ω^ℓ and $\{\omega^\ell, \omega^{\ell+1}, \ldots, \omega^n\} \cap S_2$ is a singleton set $\{\omega\}$, player 2 can infer that ω is the true state.

We can reflect this in the specification of player 2's beliefs as follows. for every state $\omega \in S_2$, let $H_2^{\omega t}$ denote the set of all private length-t histories of player 2 for which there is a time $\tau \leq t$ with $c_1^\tau = d_1^{\omega^\ell}$ and $\{\omega^\ell, \omega^{\ell+1}, \ldots, \omega^n\} \cap S_2 = \omega$. Histories in $H_2^{\omega t}$ are those in which player 2 can infer the true state ω through a disclosure by player 1. The union of all those histories is $H_2^{1t} := \bigcup_{\omega \in S_2} H_2^{\omega t}$. This is the set of all histories in which player 2 has learned the true state through a disclosure of player 1.

For every private history h_2^t of player 2 with $|S_2^t| > 0$ and ω^ℓ the maximal element disclosed by player 1 according to history h_2^t, let player 2's belief in period $t+1$ about the identity of the true state be given by

$$\mu(\omega = \omega^*|h_2^t) = \begin{cases} 1 \text{ if } h_2^t \in H_2^{\omega t} \\ \frac{1}{|\{\omega^\ell,\ldots,\omega^n\}\cap S_2^t|} \text{ if } \omega \in \{\omega^\ell,\ldots,\omega^n\} \cap S_2^t \text{ and } h_2^t \notin H_2^{1t} \\ \frac{1}{|S_2^t|} \text{ if } \omega \in S_2^t, \{\omega^\ell,\ldots,\omega^n\} \cap S_2^t = \emptyset \text{ and } h_2^t \notin H_2^{1t} \\ 0 \text{ otherwise.} \end{cases}$$

That is, player 2 only updates her beliefs about the true state in response to new evidence and does so in accordance with what she can infer from player 1's strategy, unless the inference is inconsistent with what she knows. In the latter case, she assigns equal probability to those states in her possibility set that she cannot rule out based on past disclosures alone.

Our next result shows that (i) there is room for improving on the better informed player always disclosing the minimal element of their remaining possibility set and (ii) optimality requires simultaneous talk with positive probability. The latter is true despite the fact that we assumed that simultaneous disclosures erase the content of what is disclosed. It relies on players being able to make inferences from simultaneous talk even if they do not understand what is being said.

Proposition 5 *Suppose that* $1 < |S_i| < |\Omega| - 1, i = 1, 2$ *and* $|\Omega| > 4$*. Then in any optimal pragmatic-meaning equilibrium of a single-topic conversation there is positive probability that both players talk and that they do so simultaneously.*

Proof: In any optimal pragmatic-meaning equilibrium at least one player must talk with positive probability in period 1. Otherwise, it would be possible to accelerate the discovery of the true states by treating every period t as if it were period $t+1$. We will show that in any optimal pragmatic-meaning equilibrium both players talk with positive probability in period 1. To arrive at a contradiction, suppose that there is an optimal pragmatic-meaning equilibrium in which only one player talks with positive probability in period 1. Wlog let that be player 1. If in the hypothesized equilibrium player 1 uses a strategy that reveals S_1 with probability 1 in period 1, then there must be at least as many messages as there are sets S_1. Since the maximal number of messages available to player 1, i.e., the number of possible disclosures plus silence, satisfies $|S_1| + 1 < |\Omega|$, this implies that we need

$$|\Omega| > \binom{|\Omega|}{|S_1|}.$$

And, because $1 < |S_1| < |\Omega| - 1$, $\binom{|\Omega|}{|S_1|}$ is minimized by $|S_1| = 2$ and by $|S_1| = |\Omega| - 2$. But $n \geq \binom{n}{2} = \binom{n}{n-2}$ is violated for all $n \geq 4$. Hence, given the antecedent in the statement of the proposition, it is impossible for player 1 to use a strategy that reveals S_1 with probability 1 in period 1.

Therefore, the maximal probability with which player 2 learns the truth from player 1's disclosure in period 1 is bounded away from one. Denote that maximal probability by $p < 1$. In the postulated pragmatic-meaning equilibrium, for every realization of the pair of of possibility sets, the expected payoff is no larger than δ and with probability $1 - p > 0$ it is no larger than δ^2.

Fix a possibility set $\widetilde{S}_2$ of player 2. There is positive probability that $S_2 = \widetilde{S}_2$. Modify the postulated optimal pragmatic-meaning equilibrium by having player 2 make a disclosure in period 1 if and only if $S_2 = \widetilde{S}_2$. Then, in the event that $S_2 = \widetilde{S}_2$ player 1 learns the truth in period 2 from the fact that player 2 made a disclosure (where the nature of the disclosure is irrelevant). Hence in the event that $S_2 = \widetilde{S}_2$, the modified strategy profile achieves a payoff equal to δ, which is never lower than the payoff from the postulated optimal pragmatic-meaning equilibrium and is strictly higher with probability $1 - p > 0$. □

Hence, politeness is suboptimal.[7] It is optimal for one of the players sometimes to interrupt the other to make an important point.

4.1 An example of an optimal pragmatic-meaning equilibrium

Recall our example from Section 3.1 with state space $|\Omega| = 4$, and players having possibility sets with sizes $|S_1| = 2$ and $|S_2| = 3$, respectively. We showed that given the values of the parameters α, β, γ, and δ chosen there,

[7] In Aumann and Hart's [5] study of long cheap talk, polite talk, i.e., "talk without simultaneous messages," is generally less effective than impolite talk because polite talk cannot take advantage of jointly controlled lotteries. In the common-interest environment of the present paper there is nothing gained from the use of jointly controlled lotteries.

there is an optimal literal-meaning equilibrium with the following behavior on path: In period 1 player 1 discloses and player 2 listens. In period 2 player 1 listens. Player 2 acts in period 2 if player 1 disclosed an element of S_2 in period 1 and otherwise discloses. If period 3 is reached, then player 1 acts and player 2 listens. Following a period in which a player has acted, some player i (or both) ends the game by choosing e_i. In this equilibrium there is positive probability that the first disclosure fails to reveal the truth. The expected payoff from this equilibrium is $\frac{1}{2}(\delta + \delta^2)(\alpha + \delta\beta)$.

There is a pragmatic-meaning equilibrium in which single disclosure suffices to identify the true state. To see this, consider a strategy for player 2 that prescribes for player 2 to disclose in period 1 according to the following rule:

$$\begin{aligned}\{\omega^1, \omega^2, \omega^3\} &\mapsto d^{\omega^1}\\ \{\omega^1, \omega^2, \omega^4\} &\mapsto d^{\omega^2}\\ \{\omega^1, \omega^3, \omega^4\} &\mapsto d^{\omega^3}\\ \{\omega^2, \omega^3, \omega^4\} &\mapsto d^{\omega^4}\end{aligned}$$

This strategy fully reveals player 2's possibility set for each of its possible realizations in the first period. Hence, the payoff from this equilibrium equals $\delta(\alpha + \delta\beta) > \frac{1}{2}(\delta + \delta^2)(\alpha + \delta\beta)$. The equilibrium is optimal since it identifies the true state at the earliest possible opportunity with probability 1.

There is no strategy for player 1 that would reveal player 1's possibility set for every possible realization in the first period. Hence, in this example it is the case that in every optimal literal-meaning equilibrium player 1 talks in the first period, whereas in every optimal pragmatic-meaning equilibrium player 2 talks in the first period.

Notice that this example also suggests that we need to rethink what it means to be "better informed." Player 1 would be better informed in a decision problem in which players had to pick an action based on their individual information. Here, in contract, one can think of player 2 as being better informed. Player 2 knows more about what she does not know: she can rule out exactly one state from the set of possible states. This allows her to signal the remaining states in her possibility set with a single disclosure. This observation straightforwardly generalizes to give us the following result.

Proposition 6 *There exists a pragmatic-meaning strategy that lets player i* *learn the identity of ω^* with certainty in or before period*

$$t^* := \min\{|S_{-i}|, |\Omega| - |S_{-i}|\} + 1.$$

Notice that the same conclusion would also hold for literal-meaning strategies if we added the ability to negate statements to players' languages. With negation agent j could name the $|\Omega| - |S_j|$ missing elements in her possibility set, which would be an efficient way of identifying her possibility set whenever S_j is large in relation to Ω.

5 Uncertainty about informedness

Until now, we have maintained the assumption that the sizes of players' possibility sets in each topic, n_{ik}, are commonly known. In this section we drop this assumption. Focussing on the case of a single topic (and therefore dropping the index k), we now assume that the size of each player i's possibility set n_i is randomly and privately drawn from a set $M = \{m_1, \ldots, m_L\}$ with $1 < m_1 < \cdots < m_L$. To isolate the effect of uncertainty about informedness, we consider literal-meaning strategies.

Denote player 1's distribution on M by p and player 2's distribution by q. We will have occasion to vary the set M while keeping its cardinality, L, and the distributions p and q fixed. For that reason, we assign probabilities p_ℓ and q_ℓ to the indices ℓ rather than directly to sizes m_ℓ, with the understanding that for any fixed M, $p_\ell = p(m_\ell)$ and $q_\ell = q(m_\ell)$ are the probabilities that player 1's and player 2's possibility sets are of size m_ℓ. Assume that $p_\ell > 0$ and $q_\ell > 0$ for all $\ell = 1, 2, \ldots, L$. Say that player 1 is *ex ante better informed* if player 2's informativeness distribution q first-order stochastically dominates player 1's distribution p, that is, player 1 is more likely to draw possibility sets of smaller sizes. To ensure that it is always worthwhile to converse, we let β vary with M and assume that for every M considered it is the case that $\beta < \frac{\delta(1-\delta^m)}{1-\delta}\frac{\alpha+\delta\beta}{m_L}$.

To state our next result, two definitions will be helpful. First we define what it means for one strategy profile to be better than another conditional on public histories. For any $\tau \geq 2$, strategy profile σ is *τ-sequentially superior* to strategy profile σ' if conditional on every on-path public history generated by σ the expected continuation payoff from σ is no less than from σ', and for every t less than τ there is an on-path public history of length t generated by σ for which the continuation payoff from σ strictly exceeds that from σ' (this also implies that strategy σ has a strictly higher ex ante expected payoff). Evaluating expected continuation payoffs conditional on public histories adopts the perspective of an outside observer, who does not know the realized sizes of players possibility sets.

Second, we define a class of strategy profile that, as we will see, permit players to exchange information about the sizes of their possibility sets. A strategy profile prescribes that *players alternate having the option to talk* if (i) conditional on no prior disclosure, one player is designated to listen while the other can choose whether to disclose, (ii) the roles of listener and talker switch every period prior to the first disclosure, (iii) the less well-informed player 2 is designated to listen in period 1, and (iv) once a player has disclosed, that player discloses for the remainder of the game.

Before, when a player was better informed and we restricted attention to literal meaning strategies, it was best to let that player do all the talking. Now, however, a player who is better informed ex ante need no longer be better informed at the interim stage, when the sizes of possibility sets are realized. The following proposition indicates how that tension is resolved. While for some distributions over possibility sets it may be nearly optimal to have only player 1 talk, there are also scenarios in which it is sequentially superior to let players alternate having the option to talk.

Proposition 7 *(i) For fixed M, $\delta > 0$ and every $\epsilon > 0$, there exist distributions p and q for which it is ϵ-optimal only to have player 1 talk. (ii) In contrast, for any fixed p, q and $\delta > 0$, there exist M and $\beta > 0$ for which letting players alternate having the option to talk is $2(L-1)$-sequentially superior to always continuing with only letting player 1 talk.*

Proof: Expected payoffs are bound from above by $\frac{\delta(1-\delta^{m_1})}{1-\delta}\frac{\alpha+\delta\beta}{m_1}$, the maximal expected payoff achievable if players were certain that the size of player 1's possibility set is minimal. If instead there is uncertainty about about the sizes of the possibility sets and their distributions are given by p and q, then the expected payoff from having player 1 always disclose until one of her disclosures matches the truth equals

$$\sum_{\ell=1}^{L} p_\ell \frac{\delta(1-\delta^{m_\ell})}{1-\delta}\frac{\alpha+\delta\beta}{m_\ell}. \tag{2}$$

Evidently, for any $\epsilon > 0$ and sufficiently large p_1, we have $\frac{\delta(1-\delta^{m_1})}{1-\delta}\frac{\alpha+\delta\beta}{m_1} - \sum_{\ell=1}^{L} p_\ell \frac{\delta(1-\delta^{m_\ell})}{1-\delta}\frac{\alpha+\delta\beta}{m_\ell} < \epsilon$. This establishes part (i).

Suppose that players adopt a strategy that makes the following prescriptions on path:

1. In period 1 player 1 discloses if $|S_1| = m_1$, and player 2 listens.
2. In period 2, if there has been no prior disclosure, player 2 discloses if $|S_2| = m_1$, and player 1 listens.
3. In period $t = \ell$, if there has been no prior disclosure, ℓ is odd, and $\ell < 2(L-1)$, player 1 discloses if $|S_1| = m_\ell$, and player 2 listens.
4. In period $t = \ell$, if there has been no prior disclosure, ℓ is even, and $\ell \leq 2(L-1)$, player 2 discloses if $|S_2| = m_\ell$, and player 1 listens.
5. Once a player has made a disclosure that player discloses until the other player acts.
6. A player acts following the period in which their conversation partner has disclosed the true state.

Then their expected payoff equals

$$\sum_{\ell=1}^{L-1}\prod_{i=1}^{\ell-1}[(1-p_i)(1-q_i)](p_\ell + \delta(1-p_\ell)q_\ell)\delta^{2(\ell-1)}\frac{\delta(1-\delta^{m_\ell})}{1-\delta}\frac{\alpha+\delta\beta}{m_\ell}$$
$$+\prod_{i=1}^{L-1}[(1-p_i)(1-q_i)]\delta\delta^{2(L-2)}\frac{\delta(1-\delta^{m_L})}{1-\delta}\frac{\alpha+\delta\beta}{m_L}, \tag{3}$$

using the convention $\prod_{i=1}^{\ell-1}[(1-p_i)(1-q_i)] = 1$ for $\ell = 1$.

In the expressions (2) and (3), set $\beta = 0$. Then both expressions are weighted sums of the same terms $\frac{\delta(1-\delta^{m_\ell})}{1-\delta}\frac{\alpha}{m_\ell}, \ell = 1, 2, \ldots, L$. Notice that the weight on $\frac{\delta(1-\delta^{m_1})}{1-\delta}\frac{\alpha}{m_1}$ in (3) is strictly larger than in (2). Therefore, by increasing $m_2, \ldots, m_L$, we can ensure that the weight on $\frac{\delta(1-\delta^{m_1})}{1-\delta}\frac{\alpha}{m_1}$ is decisive in the comparison of the payoffs in (2) and (3), and hence the payoff in (3) is strictly larger than the payoff in (2). By continuity, this remains true for sufficiently small $\beta > 0$. Hence, there exist M and $\beta > 0$ for which letting players alternate having the option to talk yields a strictly higher payoff than only letting player 1 talk. If player 1 discloses in period 1, this reveals that her possibility set if of minimal size, m_1, and therefore it is optimal for her to keep disclosing by Proposition 2. If player 1 fails to disclose in period 1, this demonstrates to the outside observer that the smallest player 1's possibility set can be is m_2, whereas there is still positive probability that player 2's possiblity is is of size $m_1 < m_2$. Hence, when we calculate the expected continuation payoffs from either continuing with the strategy in which players alternate having the option to talk or letting player 1 disclose until the truth is revealed, we get sums of $\frac{\delta(1-\delta^{m_\ell})}{1-\delta}\frac{\alpha}{m_\ell}, \ell = 1, 2, \ldots, L$. But now, only the payoff from letting players alternate having the option to talk has a positive weight on $\frac{\delta(1-\delta^{m_1})}{1-\delta}\frac{\alpha}{m_1}$. Hence, the same argument as before shows that there exist M and $\beta > 0$ for which letting players continue to alternate having the option to talk yields a strictly higher payoff than only letting player 1 continue to talk. If player 2 discloses in period 2, this reveals that her possibility set if of minimal size, m_1, and therefore it is optimal for her to keep disclosing by Proposition 2. If player 2 fails to disclose in period 2, this demonstrates to the outside observer that for both players the smallest a possibility set can be is m_2. This puts us back in essentially the same situation as at the beginning of the game: from the perspective of the outside observer, both players have possibility sets drawn from distributions $\tilde{p}$ and $\tilde{q}$, with support on $\{m_2, \ldots, m_L\}$, $\tilde{p}_\ell = \frac{p_\ell}{1-p_1}$, $\tilde{q}_\ell = \frac{q_\ell}{1-q_1}$, and $\tilde{q}$ first-order stochastically dominating $\tilde{p}$. Hence, fixing m_2, and increasing $m_3, \ldots, m_L$, we can find M and $\beta > 0$ such that continuing with letting players alternate having the option to talk yields a strictly higher payoff than yields a strictly higher payoff than only letting player 1 continue to talk. We can repeat this two-step argument $L - 1$ time (equivalently for $2(L - 1)$ periods). After $2(L - 1)$ periods, it has been revealed that both players' possibility sets are of size M_L and hence it is optimal for either player to start disclosing. □

Notice that with the sequentially superior strategy there is positive probability that for some number periods overtly nothing happens – the players remain silent. This, once again, is reminiscent of similar behavior in familiar logic puzzles (Littlewood [26], Gamow and Stern [17]) and the phenomenon in Bayesian dialogues (Geanakoplos and Polemarchakis [18]) of agents repeatedly announcing the same conflicting posteriors. In all of these case agents refine information on the basis of a form of inaction.

6 Discussion

This paper contributes to the literature on information sharing through cheap talk, Crawford and Sobel [9], and disclosure, Grossman [23] and Milgrom [28]. It also relates to the linguistics literature on pragmatics in the tradition of Grice [19], which is surveyed in Benz and Stevens [6].

The pragmatics literature is concerned with the difference between what is said and what is implicated. It notes that there is more to meaning than literal meaning. Grice postulates a *cooperative principle* that lets interlocutors work out what is meant on the assumption that they are rational and have a common goal. Following Grice and the bulk of the linguistics literature on pragmatics, we study information sharing in a common-interest environment.

There is a variety of approaches to understanding pragmatic reasoning in the linguistics literature, including, but not limited to, the iterative best response model of Franke [16], which is in similar spirit to Crawford's [11] application of level-k reasoning (Nagel [29]) to communication games, iterative deletion of weakly dominated strategies (Rothschild [32]), and the recently popular rational speech act framework of Frank and Goodman [15] (surveyed in Degen [12]). Common to the mentioned papers is their aim to exhibit a reasoning process that rationalizes pragmatic inference and their avoidance of the requirement that behavior be in equilibrium.

Departing from that literature, we study equilibria, which happen to exhibit pragmatic features to different degrees. Furthermore, we examine two-sided incomplete information. We allow interlocutors to assume both the role of speaker and that of listener and let them endogenously switch between those roles during the course of the conversation. While the pragmatics literature focusses on single communicative acts, modeled as two-stage games in which the speaker first sends a message which is then interpreted by the listener, we allow conversations to be open ended, with no definite termination point.

Open ended conversations have been considered in the cheap-talk literature by Forges [14], Aumann and Hart [5], and Krishna and Morgan [25]. Their results leverage jointly controlled lotteries, which play no role in our environment. As Aumann and Hart, we find that impolite talk is more effective than polite talk, but for different reasons. Antic, Chakraborty Harbaugh [2] consider open ended information exchange between interlocutors with common interests in the presence of a third party with misaligned interests who can overhear the conversation.

Unlike the cheap-talk literature, Crawford and Sobel [9], in which the meaning of messages is entirely endogenous, our messages have a literal meanings. This is consistent both with the disclosure literature in the tradition of Grossman [23] and Milgrom [28] and the game theoretic literature on pragmatics.

In order for communication in our environment not to be instantaneous and complete, and therefore non-interesting, we need a friction. In the cheap-talk literature, this friction comes from non-aligned interests. In the disclosure literature, both conflicts of interest and language constraints in the form of truth-telling conditions are sources of friction.

Glazer and Rubinstein [20][21][22], study pragmatic inference in environments in which the main friction is conflict of interests. They replace the role of Grice's cooperative principle in analyzing pragmatic inference with the goal of a third party, a mechanism designer or a judge. In the linguistics literature, Asher, Paul, and Venant [3] study conversations with conflict in zero-sum games, which like our environments have no determinate endpoints – one player's goal is to steer the conversation into a a winning set and the other player's goal is to prevent that. Pawlowitsch [30] explores incentives to strategize in "Bayesian dialogues" à la Geanakoplos and Polemarchakis [18] and links them to Gricean conversational implicatures (Grice [19]).

The principal friction in our model is a language constraint: at any given time interlocutors can only disclose a single state from the set of states they consider possible. We also impose two physical constraints. First, in keeping with most of the literature, players can only communicate at fixed discrete points in time. Second, we postulate that simultaneous talk is ineffective – player observes the fact of simultaneous talk but none of its content.

Our language constraint necessitates gradual information exchange. This is in contrast to Blume and Park [8] who model interlocutors' information the same way we do here but impose no constraint on which subsets from their possibility sets players can disclose. In Blume and Park's paper gradual information exchange arises endogenously for reasons similar to those in Stein [33], Dziuda and Gradwohl [13], Rosenberg, Solan, and Vieille [31], Hörner and Skrzypacz [24], Augenblick and Bodoh-Creed [4], and in the literature on incremental contributions to a public good (Admati and Perry [1] and Marx and Matthews [27]).

Departing from the pragmatics literature, which is primarily concerned with rationalizing the reasoning process that underlies pragmatic inferences, here the focus is on equilibrium behavior in conversations, which may vary in its reliance on pragmatic inference. This allows us to shed light on what players decide to talk about, who gets to talk when, and when they end their conversation. We note that even in equilibria that minimize pragmatic inference, some such inference is inevitable, which one might call *pragmatics creep.* We find that if talking is sufficiently valuable, a folk-theorem type result obtains: any talking order into which we can put players can be sustained in equilibrium. For optimal equilibria among those that minimize pragmatic inference, we can predict when potential interlocutors engage in a conversation, who talks in which order, and when interlocutors terminate a conversation. While there is a large set of literal-meaning equilibria, which minimize pragmatic inference, we find that they are never optimal.

In pragmatic-meaning equilibria, i.e., those that do not minimize pragmatic inference, interlocutors can make use of a variety of ways of taking advantage of the labeling of states. The revelation of a single state can serve to indicate an entire possibility set, by, for example always revealing the minimal state in one's possibility set. Or, it can be used to indicate which state or set of states does not belong to one's possibility set.

Rich conversation patterns also emerge when we stick with literal-meaning strategies but allow for uncertainty about the sizes of possibility sets. Specifically, players can gain from delaying disclosure when their possibility sets are relatively large, in the hope that their partner is better informed and for that reason starts disclosing early.

Our findings, by design, depend on agents being language constrained. Interlocutors in our conversations lack names for nontrivial subsets of the state space, are unable to use negation, and cannot name numbers. Enriching their language with any of these features would significantly simplify their task. This mirrors the distinction that Geanakoplos and Polemarchakis [18] make between direct and indirect communication. Their indirect communication limits agents to making statements about probabilities of events, whereas direct communication would allow them to declare their information. We believe that language constraints are real, that there is value in studying them in stylized settings, and that doing so can help illustrate the value of familiar features of language, like the ability to negate statements.

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