Campaign Finance in the Age of Super PACs

Christian Cox*

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Abstract

The United States Supreme Court 2010 decision in *Citizens United v. Federal Election Commission* led to a major deregulation of election campaign finance law. A new political action committee emerged, known as the Super PAC, with a relatively unfettered ability to raise and spend money in elections. I characterize the influence of Super PACs on U.S. Congressional elections by estimating a novel election model. The model provides a comprehensive and tractable framework to analyze the effects of multiple heterogeneous players on election outcomes, incorporating endogenous candidate entry, candidate policy, committee entry, and committee spending across both the primary and general elections. I allow for unobserved heterogeneity in candidate quality and committee costs. Results indicate that Super PACs have small effects on voting outcomes and did, on average, slightly help Republicans. Super PACs also have modest effects on committee behavior, candidate policy platforms, and entry.

*Jackson School of Global Affairs, Yale University; christian.cox@yale.edu.
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1 Introduction

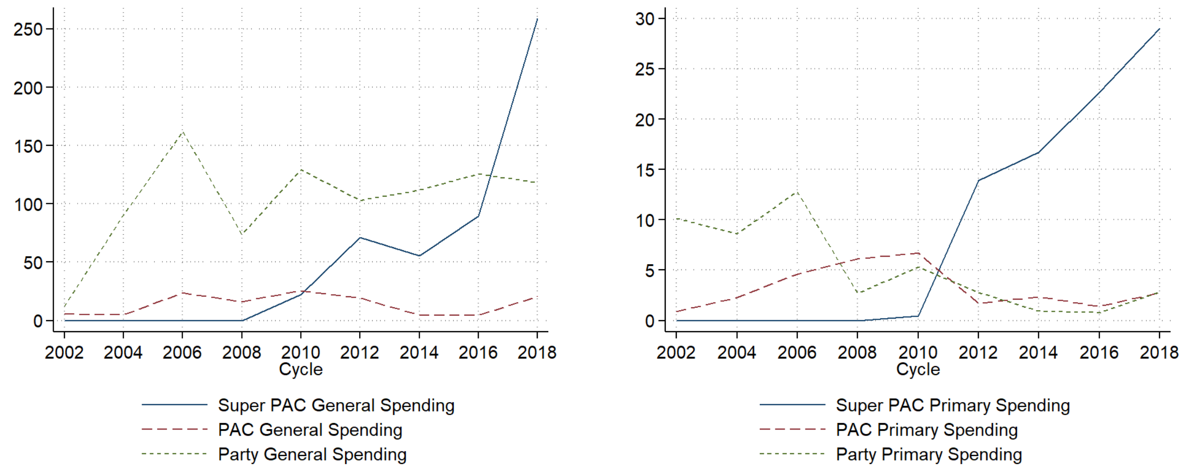
Campaign contributions are an integral part of U.S. elections and allow citizens to support candidates. The rules that govern these contributions, such as limits per donor and restrictions on corporate giving, were upended in the 2010 decisions *Citizens United v. Federal Election Commission* (FEC) and *SpeechNow v. FEC*. The latter case, relying on the former, created a new kind of political action committee (PAC), the "Super PAC", which could receive unlimited contributions per donor.[1]

Super PACs started spending soon after their creation. Long-time Democratic incumbent John Spratt of South Carolina's fifth district was defeated in his 2010 general election with opposition spending of $2,839,419, a third of which came from newly formed Super PACs. These groups also spent in primaries, with the Super PAC named "Campaign For Primary Accountability" spending $136,277 to help defeat Ohio's second district Republican incumbent Jean Schmidt in her 2012 primary. Super PACs may not only have influenced who won the election, but also candidate positions. For example, Republican incumbents post-2010 have been almost twice as likely to position themselves further to the right than to the left. While not necessarily causal, those who chose a more moderate position faced almost twice as much primary Super PAC opposition spending as others. Super PAC spending, shown in Figure 1 below, reveal their potential impact; Super PACs have been a major force in general elections, and dominate primary spending by non-candidate committees.

The data indicate that Democrats and incumbents have faced the brunt of this new spending, and Democratic members of Congress are looking to get the *Citizens United* decision overturned (Carney 2019). Proponents of both Court decisions argued that election spending is akin to free speech and that "outside money" provides a counterweight to established political parties. Opponents feared corporations and wealthy individuals would flood elections with outside money. Both sides have seen their arguments partially materialize. First, Super PACs have supported House challengers with more than $377 million since

[1] Part of this ruling also allowed them to accept corporate and union donations; the exceptions are foreign nationals, federal contractors, national banks, and federally chartered corporations.

Figure 1: House Election “Outside Committee” Ad Spending (in Millions)



The left (right) graph shows total general (primary) election ad spending by Super PACs, PACs, and party committees from 2002-2018.

2018, but Super PACs helping incumbents have also spent just over $202 million.[2] Second, while corporate political spending has not significantly increased since 2010, the substantial spending by Super PACs in Figure 1 is largely due to donations by wealthy individuals.

In this paper, I analyze how Super PACs affect Congressional primary and general elections. I investigate how their spending influences voting behavior, spending by other committees, candidate platforms, and candidate entry decisions. I model a multistage game for the primary and general elections, incorporating the collective efforts of candidates, parties, traditional PACs, and Super PACs.[3] I allow for heterogeneity along multiple dimensions, such as spending effectiveness and fundraising constraints.[4] I first estimate the effect of candidate and committee decisions on voters and then estimate the equilibrium conditions for those decisions using backward induction to incorporate forward-looking behavior. It is vital to include the actions prior to general elections, such as primary elections and candi-

[2] They sometimes support the party, fringe groups, or just one candidate (Dwyre and Braz 2015; Chen and Fang 2017; Kolodny and Dwyre 2018; Miller 2018; Herrnson, Heerwig, and Spencer 2018).

[3] Many analyze only one part of the election with one player per side (Strömberg 2008; Shachar 2009; Gordon and Hartmann 2016; Incerti 2018; Limbocker and You 2020).

[4] This heterogeneity differentiates the analysis from those that exploit symmetry (Strömberg 2008); sources of asymmetry include parameters, timing, and donors (Meirowitz 2008).

date entry, as any counterfactual scenario studying Super PACs should not hold these fixed.[5] Simply estimating the general election marginal spending effect of Super PACs on vote share outcomes is insufficient to fully characterize their impact on the election; they operate in a strategic environment and one should estimate their direct and indirect influence on entry, policy, primaries, and the spending of others to get an accurate counterfactual.

A key challenge is dealing with candidate unobservables. The general election winner, general election loser, primary election losers, and potential candidates who did not enter may differ in the eyes of voters in unobserved ways (Dal Bó and Finan 2018). To account for the unobserved heterogeneity across candidates that faced each other in an election, I use exogenous variation in donor information that affects committee spending. To deal with the unobserved match-ups between candidates that influence forward-looking behavior, I exploit the dynamic model structure. Finally, to proxy for the unobserved selection of candidate entry, I compare entrants and non-entrants based on state legislature election records.[6]

Results indicate that Super PACs slightly increase overall spending and help Republicans in general elections, with substantial heterogeneity. I also find nontrivial changes to candidate entry and platforms; Super PACs promote Republican challenger entry and have moderating effects on Democratic incumbents. Post *Citizens United* spending exhibits a closely matched arms race on both sides, largely canceling out effects. Thus, I also simulate a ban on Super PACs that only affects one political party; this leads to lopsided effects, indicating that failing to match opposition spending is a legitimate concern for candidates and donors. Finally, I quantify the bias from ignoring equilibrium adjustment and discuss possible welfare effects.

I contribute to the literature by estimating a comprehensive campaign finance model that differentiates between candidate and "outside" spending, includes within-election dynamics, and allows for entry and policy alongside spending. I provide analysis of Super PACs in national elections using a novel approach with counterfactual simulations on their effects. I

[5] The set of candidates in the general election is not random; many races are largely determined in the primaries; ignoring the primaries omits the decision making that precedes and informs spending in the general (Albert, Desmarais, and La Raja 2016; Boatright, Malbin, and Glavin 2016).

[6] Other approaches include using lagged advertising prices as instruments (Stratmann 2009; Chung and Zhang 2020; Gordon and Hartmann 2016), discontinuities of district/media market (Strömberg and Snyder 2010; Spenkuch and Toniatti 2018; Wang 2018), repeat challengers (Levitt 1994), lagged votes/spending (Green and Krasno 1988), and competitiveness measures (Erikson and Palfrey 2000).

also contribute to estimation methodology for games with a contest structure. First, I provide a tractable approach for estimating collective contests (Nitzan 1991) with endogenous effort and entry. Second, I show how to recover unobservables from match-ups not observed in the data; one can exploit the model structure and effort levels of losers in earlier rounds to infer expectations of performance shocks in hypothetical later rounds. This paper relates to the work on spending in elections, primaries and candidate entry, "outside" influence and donors, and the new literature on *Citizens United* and Super PACs.[7]

There is a rich literature on election spending, primary elections, and political selection (Albert, Desmarais, and La Raja 2016; Anagol and Fujiwara 2016; Carson 2016; Fowler 2016; Dal Bó and Finan 2018; Lim and Snyder 2021). This paper broadens the literature by incorporating Super PACs and a novel model that considers strategic behavior across multiple stages of an election. The dynamics extend Adams and Merrill (2008).[8] I allow for heterogeneous players on both sides of each election, private information, unobserved characteristics of candidates and committees, and endogeneity in spending, policy, and entry.

There is little work on Super PACs in national elections,[9] and state election evidence suggests they helped Republicans win more state legislature seats (Klumpp, Mialon, and Williams 2016). The large effects found in the state-level literature are not necessarily predictive of what will happen on the national stage. There are differences in spending, policy issues, and other variables that affect each environment. For example, state-level candidates raise substantially less money than federal candidates, which allows outside groups like Super PACs to more easily affect the outcomes of the former. For the state-level analysis, identification stems from variation prior to 2010 in state campaign finance laws (Werner and Coleman 2014).[10] That strategy is not feasible with national elections, and my integrated approach

[7]This includes the drivers of candidate ideology (Boleslavsky and Cotton 2015; Baker 2016b).

[8]This is distinct from other within-election games (Klumpp and Polborn 2006; Denter and Sisak 2015; Roos and Sarafidis 2017; Ellickson, Lovett, and Shachar 2019; Acharya, Grillo, and Sugaya 2018) or between-election dynamics (Kawai and Sunada 2015; Polborn and Snyder 2017). Kawai and Sunada (2015) is a hybrid model with between-election war-chest building and some within-election facets (abstracting away from policy, donors, outside spending, and primary-contested incumbents).

[9]There is a growing body of descriptive work (Hansen, Rocca, and Ortiz 2015; Baker 2016a; Barutt and Schofield 2016; Miller 2017).

[10]Many use that same variation (Hamm, Malbin, Kettler, and Glavin 2014; Spencer and Wood 2014; Abdul-Razza, Prato, and Wolton 2020; Harvey and Mattia 2019; Petrova, Simonov, and Snyder 2019; Gilens, Patterson, and Haines 2021).

controls for unobservables and equilibrium adjustment across multiple dimensions.[11]

My methodology contributes to and builds on the structural estimation of election campaigns and political contests literature (Coate and Conlin 2004; Diermeier, Keane, and Merlo 2005; Strömberg 2008; Bombardini and Trebbi 2011; Kawai and Sunada 2015; Gordon and Hartmann 2016; Kang 2016; Sieg and Yoon 2017; Iaryczower, Moctezuma, and Meirowitz 2017; Garcia-Jimeno and Yildirim 2017; Huang and He 2021). A model-based approach is important for evaluating how elections would unfold without Super PACs. A reduced form approach using pre and post 2010 variation is not ideal as each election has different candidates, committees, donors, and voters. Furthermore, entry, policy, and spending are functions of election specific unobservables. Finally, estimating a single stage in isolation or not controlling for strategic responses ignores equilibrium effects and biases counterfactuals.

The paper continues as follows: I start with detailing the empirical environment and data in section 2. I follow with the model in section 3, describing each stage of the game. I discuss the identification and estimation in section 4. The model has parameters that I estimate stage by stage, including the general election voter preferences, committee preferences in the general election, primary election voter preferences, additional committee parameters in the primary election, and the parameters that govern candidate entry and policy decisions. Section 5 discusses the parameter estimates and considers a counterfactual on how the elections would change if Super PACs never existed. I run this simulation by solving the estimated model but excluding Super PACs from spending in the election. I conclude in section 6.

2 Data

The two principal groups in this environment are candidates and voters: candidates choose policy platforms and voters choose their preferred candidate. The two broad groups in the background are election committees and donors: committees spend money to help candidates win and donors supply these committees with campaign contributions. The main committees are the campaign committees, political party committees, traditional political action

[11] They rely on a difference-in-difference method to estimate reduced form effects of Citizens United on various electoral and policy outcomes. This is distinct from an approach that estimates marginal effects alongside equilibrium behavior throughout the entire election cycle allowing for time-varying unobservables.

committees (PACs), and Super PACs, each with spending and fundraising limitations.[12]

2.1 Voting and Candidate Data

One way to measure the effects of committee spending on the election is through the share of votes a candidate receives. The primary, runoff, general, and general runoff election data are from the FEC, and I use data from the 2002-2018 cycles for House elections. In primaries, incumbents win re-election with a high success rate and uncontested primaries were the norm prior to 2010. The number of contested primaries increased during 2010 and stayed high afterwards, as shown in Figure 2. The 2010 surge was largely driven by the "Tea-Party" movement in which establishment Republicans faced a higher rate of contested primaries.

My measure for candidate policy/position/platform/ideology comes from Bonica (2014). This commonly used measure is based on a spatial model of donors where they contribute to candidates to whom they are ideologically aligned. Bonica uses correspondence analysis to construct the "CF-scores" based on the network of donors and recipients.[13] See Appendix A.1 for more details on the voting data and CF-scores.

These scores are available for all Congressional candidates from 1980-2018. It is well defined for most candidates that received donations. Practically all candidates fit between -4 and 4, where -4 is most liberal, 0 is in the middle, and 4 is most conservative. Figure 3 shows the distribution of these scores for pre and post (including) 2010. Note that the distribution is slightly wider in post-2010, indicating higher polarization. The twin peaks around -1 and 1 are due to most candidates not going beyond a moderate position. There is a local trough at 0 as most candidates are at least slightly positioned to one political side.

Republican incumbents who face a primary challenger are slightly more extreme than those who are unopposed. For all Republican candidates, less extreme candidates are generally more likely to win the primary. The average position for Republican incumbent primary winners is more extreme than for incumbent losers, but there are very few incumbent pri-

[12]For political party committees, I include federal, state, and "Leadership PAC" type committees. I group hybrid/Carey PACs with Super PACs.

[13]An alternative measure is based on Congressional voting records (DW-NOMINATE scores), and is insufficient for this analysis as it is only observed for incumbents with a voting record. I find that the correlation between DW-NOMINATE and CF-scores is 93.46% among House incumbents.

Figure 2: Primary Entry

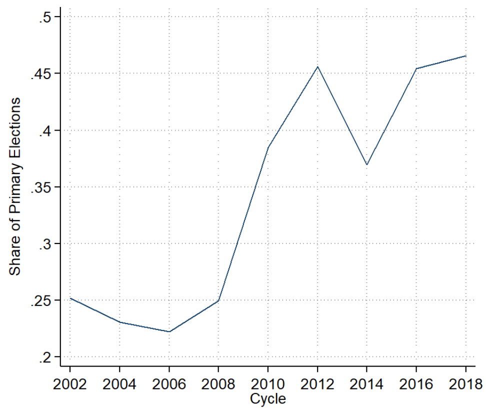


Figure 3: Candidate Position Distribution

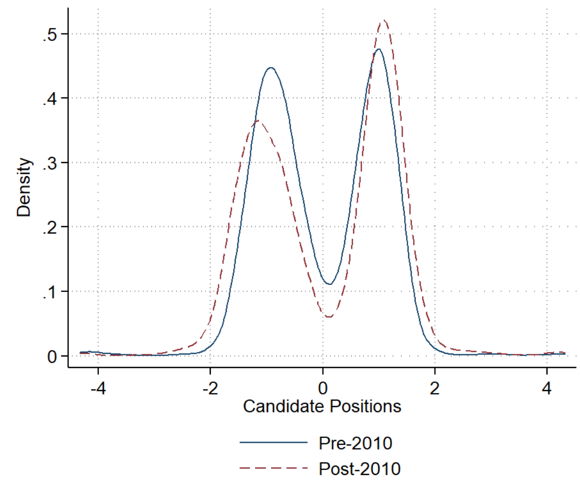


Figure 2 shows the share of contested elections from 2002-2018: at least one primary opponent in a primary election divided by all of the races in that election cycle. Figure 3 shows the distribution of candidate positions for elections prior to 2010 and post (including) 2010, based on Bonica's score. -4 is most "left-wing" (liberal) and 4 is most "right-wing" (conservative).

mary losers. Candidates that are outspent are more likely to lose and the variance increases with position; for more extreme candidates, large spending gaps may be necessary to win.

2.2 Committee Data

Political action committees are formal entities, regulated by the FEC, that can raise and spend money in elections. PACs support candidates through multiple channels: they donate money to the candidate's campaign committee, rally supporters, and spend on "communications" in support or opposition of a candidate. Direct contributions to a given candidate have strict upper limits that prevent a single PAC from "buying" too much influence. Also, an individual can only give a few thousand dollars to a PAC (or party) per election cycle.[14]

Prior to 2010, non-PAC groups such as corporations, nonprofits, unions, and trade associations were limited in their ability to spend in elections. They could form their own PAC, but they could not donate money directly nor make ads targeting candidates. Ads targeting candidates but not coordinated with the candidate or party are called "independent expenditures" (IEs). The 2010 case *Citizens United v. FEC* allowed these non-PAC groups to make independent expenditures. A following 2010 case *SpeechNOW v. FEC* allowed individuals

[14]PACs can spend a lot in direct contributions by donating to many candidates. Some can coordinate with a campaign on ads, but this has restrictions. Party limits are higher (see Supplement Table S1).

Table 1: Total General (Ad) Spending (in Millions)

	Democrat		Republican		
	Challenger	**Incumbent**	**Challenger**	**Incumbent**	**Total**
	Pre, Post	**Pre, Post**	**Pre, Post**	**Pre, Post**	**Pre, Post**
Candidate	238, 400	148, 261	152, 225	284, 345	823, 1230
Party	135, 175	26, 122	80, 179	98,113	339, 589
PAC	24, 23	6, 21	6, 13	14, 18	50, 75
Super PAC	0, 202	0, 48	0, 107	0,139	0, 497
Total	397, 800	178, 451	238, 524	397, 615	

This table show pre and post (including) 2010 total general election ad spending by candidate election committees and general election independent expenditures by parties, PACs, and Super PACs, separated by whether the committee is aligned with a Democrat or Republican candidate and whether the candidate is an incumbent or challenger.

and corporations to donate unlimited amounts to IE-only PACs (coined Super-PACs).[15] I link each "outside" committee (PAC, Super PAC, party) to the candidates they support, combine that with donor data (discussed below) per district in which the committee is active. I distinguish between spending targeted in the primary and general. I combine ads supporting the candidate and attacking the opponent. See Appendix A.2 for data details.

Table 1 displays total general election ad spending in House election pre and post (including) 2010 for four committee types based on the party and incumbency status of the candidate they support. Presidential election cycles often have more Congressional spending as there are donor spillovers, and there are two sets of Presidential and non-Presidential cycles in both pre/post periods. Candidates consistently spend the most, and this is because there is candidate spending in every single race, whereas parties and Super PACs spend sporadically. Total spending increased since 2010 across all committee types, with the new \$497 million in Super PAC spending neary matching the total increase of \$681 million by candidates, parties, and PACs. Super PACs spend more on challengers than on incumbents and Republican incumbents have seen the smallest increase in spending since 2010.

While there are more Republican incumbents after 2010, the 2010 Congressional redistricting may have favored Republicans (Eguia 2021), leading to less competitive districts and less spending by incumbents to defend their seat. The large increase in Democratic incumbent spending is mirrored by the increase in Republican challenger spending as that is a common match-up for competitive races. In these races, candidates, parties, and Super

[15]See Supplement S.1 for additional information on this case and related campaign finance issues.

PACs have large spending expenditures relative to pre-2010. The substantial increase in total spending for Democratic challengers is largely driven by the 2018 elections, which saw unprecedented levels of fundraising for Democratic House challengers.

Traditional PACs (called PACs) are distinct from parties and Super PACs as they spend relatively little on independent expenditures and their main method is through giving money directly to candidates, especially incumbents. Despite their limitations in fundraising, their role has not necessarily diminished with the rise of Super PACs (Baker 2018), and thus I include their ad spending in the analysis. A major concern for parties, beyond retaining majorities, is re-electing incumbents. Their spending patterns align with these goals and they often focus on competitive races, such as districts with weak opposition incumbents and open seats in swing states. Super PACs are similar in that they spend large amounts in few but highly competitive races. Both will also occasionally spend in a safe race, often to challenge an important incumbent. Parties and Super PACs differ most in primary elections.

Table 2 shows total primary election ad spending for races with an incumbent. Prior to (and including) 2010, candidate committees dominated spending. This changed after 2010, when Super PACs started to spend; while their average is low, they can outspend candidates when they participate. Party and PAC spending have seen a downward trend in primaries. One explanation of this behavior is that parties are relatively ineffective spenders or have high primary costs. There may be some substitution from party to Super PAC spending as the decrease in spending to support Democratic incumbents by parties is closely matched with an increase by Democratic Super PACs. Republican Super PACs spend more in primaries than their Democratic counterparts, and there has been an increase in spending across all candidate types. The changes for primaries without an incumbent (open races) are even larger; candidate spending increased from \$167 to \$245 million, party spending decreased from \$22 to \$3 million, and Super PACs spent \$54 million since 2010.

The total spending statistics do not tell us about the strategic responses between committees, such as whether or not they mirror each other in which races they enter. When a committee helps a candidate, the opposing committees often match their spending. For example, if at least one Super PAC spends during the general, then in 94% of those races, at least one party committee or PAC would also spend. Also, Super PACs outspend parties in

Table 2: Total Non-Open Race Primary (Ad) Spending (in Millions)

	Democrat		Republican		
	Challenger	**Incumbent**	**Challenger**	**Incumbent**	**Total**
	Pre, Post	**Pre, Post**	**Pre, Post**	**Pre, Post**	**Pre, Post**
Candidate	9, 12	45, 79	5, 22	52, 108	112, 222
Party	0, 2	8, 3	0, 1	4, 3	12, 10
PAC	2, 1	2, 5	1, 1	0, 3	5, 10
Super PAC	0, 3	0, 6	0, 11	0, 9	0, 29
Total	11, 19	55, 94	8, 34	56, 124	

This table show pre and post (including) 2010 total non-open race primary election ad spending by candidate election committees and primary election independent expenditures by parties, PACs, and Super PACs, separated by whether the committee is aligned with a Democrat or Republican candidate and whether the candidate is an incumbent or challenger. The terminology "Open Race Primary" is used to not confuse races without incumbents to "Open Primaries", a term commonly used for primaries in which party affiliation is not required.

66% of the races in which they spend. In primaries, Super PACs are the lone non-candidate spenders 43% of the time. Prior to 2010, parties were alone 73% of the time, which decreased to 37% after 2010. The primaries are becoming more crowded, but this could be due to either increased levels of participation or simply lower number of primaries spent in. Parties spent in about 7% of primaries before and after, and Super PACs spent in 13% after 2010.

2.3 Donor Data

Donors supply committees with campaign contributions. A committee's ability to spend is affected by how much they raise, which itself is influenced by variation in their donors' financial well-being and partisanship. Super PACs are particularly sensitive because they can receive large contributions from a single individual; contribution limits for candidates/PACs (≈$5,000) and parties (≈$35,000) force these committees to have a broader set of donors. All committees are still vulnerable to shocks in the income or wealth of their donors. The strength of this variation is based on the elasticity of campaign contributions, and the wealth elasticity of contributions by billionaires is significant (Bonica and Rosenthal 2015).

Donors are known because all political committees (those regulated under the FEC) are required to disclose the identities of their individual donors, including the donation amount, date, name, address, and employment information. I do not observe financial information of donors directly and instead consider IRS zip code level incomes (Gimpel, Lee, and Kaminski 2006). To gage donor ideology, I use donor Bonica CF scores from their historical donation record.[16] The donor data are primarily used to get variation in a committee's budget. The various ways these data factor into estimating the effects of Super PACs can be illustrated by first going through the model.

3 Model

A theoretical framework is useful to estimate the effects of Super PACs on electoral competition and analyze the counterfactual of how the elections outcomes could change without Super PACs existing. This model should capture the different direct and indirect channels through which Super PACs could influence the election, including the within-election dynamics of each stage of the election, from the initial entry and policy-platform decisions by candidates to the general election voters' decisions. I will estimate the various parameters from the model, such that the endogenous decisions can be re-solved for in the counterfactual, holding these estimated parameters fixed.

3.1 Model Setup

The game environment is as follows: There are two sides, Republican and Democrat, competing to win a Congressional seat. Candidates make policy and entry decisions prior to the election and committees raise and spend money to help the candidates win. For exposition, let there be a Republican incumbent.

There are four classes of players: First candidates: $\{R_1, R_2, D_1, D_2\}$, where R_1 is the Republican Incumbent, R_2 is the Republican Primary Challenger, D_1 is the first Democratic Primary Challenger, and D_2 is the second Democratic Primary Challenger. Let $c \in \{R_1, R_2, D_1, D_2\}$ denote an arbitrary candidate. Second there are committees (campaign, parties, PACs, and Super PACs) aligned to each candidate: let $i_c \in N_c$ refer to a committee aligned with candidate c; N_c is the set of committees aligned to candidate c. Next, there are many voters v for each side in the primary and the general, and finally donors $m \in M$ that donate to committees based on fundraising.

[16] In Supplement S.3 I discuss other sources of donor variation, including address level housing characteristics of individual donors and billionaire donors' wealth.

The actions take place over four stages. Actions from previous stages are observed by players. First, the incumbent chooses a (policy) position in a discrete one-dimensional space with $d_I \in \{0, ..., \Theta\}$ and $d_I = 0$ indicates they will not seek re-election. The positions can be interpreted as a political scale of left-to-right or moderate-to-extreme. They capture how voters and donors perceive candidates, such as "a moderately liberal Democrat" versus "an extremely liberal Democrat". Second, the challengers decide whether to enter the election or not and choose a position $d_c \in \{0, ..., \Theta\}$. Non-entry is $d_c = 0$. Let $\mathbf{d} = \{d_{R_1}, d_{R_2}, d_{D_1}, d_{D_2}\}$.

Third, committees (other than the candidate's committee) make primary entry decisions $a^P_{i_c} \in \{0, 1\}$. Let $\mathbf{a}^P = \{a^P_{i_c}\ \forall i_c\}$. Then the committees decide how much effort to exert in raising money in the primary election $e^P_{i_c} \in \mathbb{R}_+$ (zero for non-entrants). Then, donors make their primary election donations $y^P_{mi_c} \in \mathbb{R}_+$, which gets converted into spending (ads) $S^P_{i_c}$. Then, the primary voters (on each side) vote and a winner is decided $w^P_c \in \{0, 1\}$ for both Republican and Democratic primaries; let $\mathbf{w}^P$ denote the set of primary winners.

Fourth, the committees (including those who may not have entered the primary) make general entry decisions $a^G_{i_c}$. Let $\mathbf{a}^G = \{a^G_{i_c}\ \forall i_c\}$. Then they choose fundraising efforts for the general election $e^G_{i_c}$. Then donors make their general election donations $y^G_{mi_c}$, which gets converted into spending $S^G_{i_c}$. Finally, voters vote to determine a general election winner w^G_c.

3.2 Model Parameterization

I describe the payoffs in this section, going through each stage, starting at the end. The various distributional and functional forms chosen throughout are common in the discrete choice literature, and make the model tractable for estimation.

3.2.1 Voter Decision and Election Outcomes

Consider the final stage; a general election voter v chooses candidate R, D, or not to vote. Their utility from voting for candidate c, U_{vc}, is given in equation (3.1) and inspired by Gordon and Hartmann (2016). It is a function of campaign spending, exogenous observables, and private information. The spending $S^G_{i_c} \geq 0$ is by committees $i_c \in N_c$ supporting candidate c and has corresponding effectiveness parameters $\beta_{ic} \geq 0$ and $\phi \in (0, 1)$. The

$\phi = 1$ case leads to perfect substitutability; only one player per side ever spends. The utility per candidate is also affected by k observed exogenous district-candidate characteristics $X_R^G \in \mathbb{R}^k$ and the position choice d_c, with corresponding parameters δ_1 (of dimension k) and δ_2. The unobservables include unobserved candidate-election characteristics $\xi_c^G \in \mathbb{R}$ and voter private information idiosyncrasies $\varepsilon_{vc} \in \mathbb{R}$. The utility of abstention is $U_{v0} = u_0^G + \varepsilon_{v0}$. It is standard to set u_0^G to zero, but other normalizations may be appropriate, provided they do not affect the identification of ξ_c^G or the equilibrium properties of the game.

$$U_{vc} = \underbrace{\sum_{i_c \in N_c} \beta_{i_c} (S_{i_c}^G)^\phi + X_c^G \delta_1 + d_c \delta_2}_{u_c^G} + \xi_c^G + \varepsilon_{vc} \tag{3.1}$$

Voters observe everything except other voters' idiosyncrasies. Committees do not observe $\{\xi_c^G, \varepsilon_{vc}\}_{\forall v,c}$, but know their distributions. Voters observe ξ_c^G because it includes how voters perceive candidates and shocks that occur during the election up to election day that affect the voter's decision. While a voter does not know what their neighbor thinks, captured in the private information ε_{vc}, it is reasonable to let them know the district-candidate level local information. Committees and candidates make their spending and policy decisions early enough in the election such that ξ_c^G is not exactly known at the time.

The voter has priors on each candidate $\{X_c^G, \xi_c^G, \varepsilon_{vc}\}$; policy and spending gives them new information. To pivot from the voter's perspective to the committee's, construct the share of votes and the probability of winning. Let the voter's private idiosyncrasies ε_{vc} be independently and identically distributed (iid) Type 1 Extreme Value with location zero and scale one, T1-EV(0,1).[17] Then the share of votes s_c^G is the following for $\aleph$ number of candidates (see Supplement Lemma 1 for details):

$$s_c^G = \frac{\exp(u_c^G + \xi_c^G)}{\exp(u_0^G) + \sum_{\iota=1\ldots\aleph} \exp(u_\iota^G + \xi_\iota^G)}. \tag{3.2}$$

Then, under a plurality rule, candidate c wins if $s_c^G > s_n^G \; \forall n \neq c$. For two candidates, the win indicator for candidate R is $\mathbb{1}[s_R^G > s_D^G]$, which is equivalent to $\mathbb{1}[u_R^G + \xi_R^G > u_D^G + \xi_D^G]$. Now

[17] The standard Type 1 Extreme Value distribution (special case of Gumbel) is a continuous distribution with pdf $f(x) = \exp(x)\exp(-\exp(x))$. The difference in two T1-EV(0,1) follows a logistic distribution.

committees may not perfectly know how voters will perceive candidates and thus have beliefs over the unobserved candidate shocks. Let $\xi_c^G \overset{iid}{\sim}$ T1-EV(ψ_c^G,σ_ξ). Rewrite ξ_c^G in terms of a T1-EV(0,1) random variable $\xi_c^* = (\xi_c^G - \psi_c^G)/\sigma_\xi$, meaning $\xi_c^G = \xi_c^* \sigma_\xi + \psi_c^G$, then rewrite the indicator: $\mathbb{1}[u_R^G + \xi_R^* \sigma_\xi + \psi_R^G > u_D^G + \xi_D^* \sigma_\xi + \psi_D^G] \implies \mathbb{1}[(u_R^G + \psi_R^G)/\sigma_\xi - (u_D^G + \psi_D^G)/\sigma_\xi > \xi_D^* - \xi_R^*]$. Then the expected value of this function is the win probability $P(w_R^G = 1|\mathbf{w}^P)$ from the committee's perspective:[18]

$$P(w_R^G = 1|\mathbf{w}^P) = \frac{\exp((u_R^G + \psi_R^G)/\sigma_\xi)}{\sum_{c \in \{D,R\}} \exp((u_c^G + \psi_c^G)/\sigma_\xi)}. \tag{3.3}$$

3.2.2 Committees: Donors, Spending, and Entry

To construct the committee payoff, I model donors to map committee fundraising efforts $e_{i_c}^G$ into spending $S_{i_c}^G$. The general election donor m maximizes the utility from giving to the political causes they support. Their program is given in (3.4) and they choose how much to give to committee i_c with $y_{mi_c}^G$. Whether or not they give is based on the donor's political alignment with the committee to which they are donating, α_{mi_c}, which is function of the candidate's policy. The benefit is also a function of how much they give and the committee fundraising effort $e_{i_c}^G$. This setup is inspired by the "naive" donors specification from Bouton, Castanheira, and Drazen (2020).[19] Their costs are a function of their donation, weighted by their wealth α_m^0 and the committee i_c's fundraising ability $\alpha_{i_c}^F$.[20]

$$\max_{y_{mi_c}^G \in \mathbb{R}_+} \alpha_{mi_c} y_{mi_c}^G e_{i_c}^G - \frac{(y_{mi_c}^G)^2}{2\alpha_m^0 \alpha_{i_c}^F} \tag{3.4}$$

Solving program (3.4) and regrouping leads to the following interpretation: the donor

[18]Note this is the "contest success function" for the general election. Also, this is only for a plurality voting rule. A majority rule could use $P = \exp(-\exp(s_c - 0.5))$ with a runoff, and a top-two primary CSF would be the density of the 2nd order statistic for winning. Two states have majority rules for the general; 11 have it for the primary. Three states (varying across time) use open primaries. I exclude the races with unique designs (like Louisiana) and use the run-off as the "main" election when applicable.

[19]In their main model, donors internalize their influence over the election outcome, which in my model is done by the committees. My approach also differs from Schnakenberg and Turner (2020), who model the donor's decision between two kinds of candidates based on policy preference.

[20]The weighting by fundraising limits is an alternative to a strict limit per donor. The interpretation is clearer once one looks at the donation production function and think of these donors as classes of donors. It is easy for a Super PAC to raise a lot of money with little effort: they can get $1 million from one wealthy donor. For a candidate to raise that much, they would have to raise the maximum of $5,000 from 200 people.

supplies campaign contributions $y^G_{mi_c}$ to political committees i_c by choosing their contribution level based on their preference/ability $\gamma_{mi_c} = \alpha_{mi_c}\alpha^0_m\alpha^F_{i_c}$ and the political committee's fundraising efforts $e^G_{i_c}$. Their optimal donation function can be thought of as the fundraising production technology (from that donor) for the committee $y^G_{mi_c} = \gamma_{mi_c}e^G_{i_c}$.[21]

One may want to model the donors as "strategic", meaning they directly take into account the effects of their donation on the election. In my model, equilibrium donor behavior results in outcomes basically equivalent to those with strategic donors. This is because the committee's fundraising effort is strategic. For example, a Super PAC communicates to donors the importance of a race, convincing them to give. If donors are influenced by fundraising effort, then their objective in (3.4) is appropriate. If they are not, then the donors effectively act like committees, and treating committees as separate is superfluous. The committees are vessels for donor money, but the two are distinct agents, and previous work has focused on one of them. I capture how committees strategically raise money from impressionable donors, which then becomes election spending that influences voters.

The donations translate into spending: $S_{i_c} = \sum_{m\in M} y^G_{mi_c}$.[22] Thus $S^G_{i_c} = \sum_{m\in M}\gamma_{mi_c}e^G_{i_c}$, where γ_{mi_c} is the fundraising yield (inverse cost) from donor m for committee i_c. A committee's value associated with winning is $V_{i_c} \geq 0$. The general election committee effort program is $\max_{e^G_{i_c}\in\mathbb{R}_+} V_{i_c}\cdot P(w^G_c = 1|w^P_c = 1, \mathbf{w}^P_{-c}) - e^G_{i_c}$ *s.t.* $S^G_{i_c} = \sum_{m\in M}\gamma_{mi_c}e^G_{i_c}$, and written in (3.5) in terms of spending. Let $g_{i_c} = (\sum_{m\in M}\gamma_{mi_c})^{-1} \geq 0$, where g_{i_c} can be interpreted as spending constraints; spending on ads has a marginal cost associated with raising the sufficient funds.[23] Entry cost are sunk at this point so I omit them in the expression below.

$$\max_{S^G_{i_c}\in\mathbb{R}_+} V_{i_c}\cdot P(w^G_c = 1|w^P_c = 1, \mathbf{w}^P_{-c}) - g_{i_c}\cdot S^G_{i_c} \tag{3.5}$$

Before the general election, the committees make entry decisions. I allow for private

[21] To make this model of spatial donors that are also influenced by fundraising efforts consistent with Bonica (2014), an interpretation is that the individual donor is not influenced by fundraising (only policy), and rather just the number of donors is affected by fundraising efforts.

[22] This lets donors give to specific races per committee and abstracts away from dynamic fundraising.

[23] Using implicit costs to capture contribution limits is an alternative from explicitly modeling constraints (Avis, Ferraz, Finan, and Varjao 2019; Maloney and Pickering 2018). Also, I directly estimate g_{ic}, and thus am in fact agnostic to the exact donor framework. The key is that the committee's payoff can be written as (3.5), which only places restrictions on the donors in so far as the committee's marginal cost of fundraising effort being an exogenous function of committee and donor characteristics.

information in payoffs, $\lambda_{ic}^{G} \overset{iid}{\sim} \text{Logistic}(0,1)$. Committees then have beliefs over the entry decisions of others. Let $\pi_{i_c}^{G} = V_{i_c} \cdot P(w_c^G = 1|\cdot) - e_{i_c}^{G}$. The expected payoff for a given entry decision conditional on private information, $u_{i_c}^{G}(a_{i_c}^{G}|\cdot) - \lambda_{ic}^{G} a_{i_c}^{G}$, integrates over these beliefs. $N = \dim\{N_c\}$. The summation is across all 2^{N-1} combinations of decisions $\mathbf{a}_{-i_c}^{G}$; denote the belief by committee i_c in the probability of committee j choosing a_j^G from the decision profile $\mathbf{a}_{-i_c}^{G}$ with $p_j(\mathbf{a}_{-i_c}^{G})$, where $-i_c$ refers to committees except i_c. The entry program is given in (3.6), where $\mathbf{S}^*$ is the vector of optimal spending for a given entry profile. Note that spending has a public good aspect; a committee can have a non-entry positive payoff, as the win probability is not necessarily zero if a given committee does not enter.[24]

$$\max_{a_{i_c}^{G} \in \{0,1\}} u_{i_c}^{G}(a_{i_c}^{G}|\mathbf{p}_{-i_c}) - \lambda_{ic}^{G} a_{i_c}^{G} \quad s.t. \;\; u_{i_c}^{G} = \sum_{\mathbf{a}_{-i_c}^{G} \in \{0,1\}^{2N-1}} \pi_{i_c}^{G}(\mathbf{S}^*|a_{i_c}^{G}, \mathbf{a}_{-i_c}^{G}) \prod_{j \neq i_c} p_j(\mathbf{a}_{-i_c}^{G}) \tag{3.6}$$

The previous stages are repeated in the primary election, but the committees now use the expected outcome of the general election: $E[P_c^G|\mathbf{w}^P] = \sum_{\mathbf{a}^G \in \{0,1\}^{2N}} P_c^G(\mathbf{a}^G) \prod_j p^*(a_j^G)$, where $P_c^G(\mathbf{a})$ is the win probability from equation (3.3) evaluated at the equilibrium spending levels $\mathbf{S}^*$ for a given entry profile and $p^*(a_j^G)$ is the equilibrium probability of that entry profile. For the Republican side, the program is given in (3.7), where $c \in \{R_1, R_2\}$.

$$\begin{aligned} \max_{S_{i_c}^{P} \in \mathbb{R}_+} \; V_{i_c} \cdot \big[& P(w_c^P = 1) E[P(w_c^G = 1 | w_c^P = 1 \cap w_{D_2}^P = 1)] \cdot P(w_{D_2}^P = 1) + \\ & P(w_c^P = 1) E[P(w_c^G = 1 | w_c^P = 1 \cap w_{D_1}^P = 1)] \cdot P(w_{D_1}^P = 1) \big] - g_{i_c}^{P} S_{i_c}^{P} \end{aligned} \tag{3.7}$$

Before the primary election, the committees make entry decisions. The committee's primary private information is $\lambda_{ic}^{P} \overset{iid}{\sim} \text{Logistic}(F_{i_c}^{P}, 1)$, where $F_{i_c}^{P} \geq 0$ is a common knowledge entry cost mean.[25] Let $\pi_{i_c}^{P} = V_{i_c} \cdot E[P(w_c^G = 1|\cdot)] - g_{i_c}^{P} S_{i_c}^{P}$ and $u_{i_c}^{P} - \lambda_{ic}^{P} a_{i_c}^{P}$ be the expected payoff. Then the program for this entry stage is $\max_{a_{i_c}^{P} \in \{0,1\}} u_{i_c}^{P}(a_{i_c}^{P}|a_{-i_c}^{P}) - \lambda_{ic}^{P} a_{i_c}^{P}$.

[24] This is not innocuous; under a favor-buying framework, the committee who does not support the candidate receives nothing. I am implicitly assuming committees just want the candidate to win and they do not care if that is through their spending or others'.

[25] Committees in the primary do not observe the private shock for the general election, and a committee does not observe its own private shock in the general until reaching it.

3.2.3 Candidate Policy and Entry

Prior to the primary, the potential challengers make entry decisions alongside discrete policy positions. I write the program for all challengers in (3.9), based on the probability of winning the overall election minus their costs. Let V_c be the value to candidate c of winning, V_c^0 be the outside option, and $\bar{\theta}_c$ be the ideal position point. Let η_{d_c} be private variation in payoffs per choice, where $\eta_{d_c} \overset{iid}{\sim}$ T1-EV(0,1). The probability of winning the general election from the challenger's perspective, $E[P_c^G|\mathbf{d}]$, is an expectation over both the general and primary election committee equilibrium entry.

$$E[P_c^G|\mathbf{d}] = \sum_{\mathbf{a}^P \in \{0,1\}^{4N}} \left[\sum_{\mathbf{a}^G \in \{0,1\}^{2N}} P_c^G(\mathbf{a^P}|\mathbf{a^G}, \mathbf{d}) \prod_j p_j^*(a_j^G) \right] \prod_j p_j^*(a_j^P). \tag{3.8}$$

$$\max_{d_c \in \{0,\ldots\Theta\}} V_c \cdot E[P_c^G|\mathbf{d}] + V_c^0 \cdot (1 - E[P_c^G|\mathbf{d}]) - (d_c - \bar{\theta}_c)^2 \cdot \mathbb{1}[d_c > 0] + \eta_{d_c} \quad \forall c \in \{R_2, D_1, D_2\} \tag{3.9}$$

Finally there is the first stage in which the incumbent I chooses a position. The expected win probability is now defined as $E[P_I^G|d_I] = \sum_{\mathbf{d}'_C \in \dim\{\Theta\}^{|\mathbf{d}_C|}} E[P_I^G(d_I|\mathbf{d}'_C)] \prod p(\mathbf{d}'_C)$, taking an expectation over the equilibrium distribution of challenger decisions $\mathbf{d}_C$. The notation for valuations and costs is similar, with private information $\eta_{d_I} \overset{iid}{\sim}$ T1-EV(0,1).

$$\max_{d_I \in \{0,\ldots\Theta\}} V_I \cdot E[P_I^G|d_I] + V_I^0 \cdot (1 - E[P_I^G|d_I]) - (d_I - \bar{\theta}_I)^2 \cdot \mathbb{1}[d_I > 0] + \eta_{d_I} \tag{3.10}$$

The extent to which committees, like Super PACs, affect policies as opposed to simply electing candidates of an unchanged policy (Lee, Moretti, and Butler 2004) can be separated by observing various aspects of the model. First, one can map out the equilibrium response of candidate policy with respect to committee influence parameters, such as the spending effectiveness. Second, the gap between the incumbent's ideal and their chosen policy tells us how far they deviated; counterfactual analysis can parse out whether the voters, challengers, or committees drove that policy gap.

3.3 Model Discussion

I solve the game with backward induction. I characterize results from the model, including Proposition 1 and uniqueness conditions, in Supplements S.2 and S.9.

Proposition 1. *There exists a pure strategy Bayesian Nash equilibrium in which all agents condition on payoff relevant actions.*

The *Citizens United* case affected this environment in multiple ways, and I focus on how Super PACs entered the game with possibly different valuations, costs, and effectiveness. Super PAC spending not only affects others' spending, but can also change the candidates' decisions and the election outcome. The campaign finance laws that each committee is subjected to show up in the model through heterogeneous costs and effectiveness. One may expect Super PACs to have lower costs since they can raise without restrictions, but that need not always be the case: fundraising efficacy is a function of not only the donation limit, but a variety of other factors. In fact, many candidates raise more money than some Super PACs despite having strict limits per donor. Super PACs are simply a new entity that have the potential to raise and spend well beyond what was previously possible.

One concern is that Super PACs may be playing a "long game" across election cycles, and ignoring this may affect the results. While they may have long-term policy goals, their spending decisions per election cycle are still aimed at affecting immediate election outcomes. My framework captures these aspects of campaign finance. Next, I estimate the model to understand the magnitude and direction of the effects.

4 Identification and Estimation

I estimate the parameters that govern preferences for voters (parameters from equation (3.1) for the general and primary elections), committees (parameters from programs (3.5) and (3.7)), and candidates (parameters from programs (3.9) and (3.10)). The purpose is to recover the equilibrium mappings in order to evaluate counterfactual choices. Let $\mathcal{X}$ be the set of exogenously given observables and $\mathcal{P}$ be the set of parameters. The main equilibrium objects are general spending: $S^G(\mathbf{a}^G, \mathbf{w}^P, \mathbf{d}_C, d_I|\mathcal{X}, \mathcal{P})$, general entry: $Pr[a^G(\mathbf{w}^P, \mathbf{d}_C, d_I|\mathcal{X}, \mathcal{P})]$, primary spending: $S^P(\mathbf{a}^P, \mathbf{d}_C, d_I|\mathcal{X}, \mathcal{P})$, primary entry: $Pr[a^P(\mathbf{d}_C, d_I|\mathcal{X}, \mathcal{P})]$, challenger position: $Pr[d_C(d_I|\mathcal{X}, \mathcal{P})]$, and incumbent position: $Pr[d_I(\mathcal{X}, \mathcal{P})]$. I assume that the observed data are in equilibrium and are selected from the same equilibrium across observations.

The main estimation steps are: 1. estimate voter preferences for general and primary elections; 2. estimate valuations and general election costs with general election first order condition and entry conditions; 3. estimate primary election costs using primary first order conditions, inverted equilibrium general election win probabilities, and entry conditions; 4. estimate and draw simulated valences for non-entrants, and then estimate challenger valuations and costs using entry and position variation; 5. following step 4, estimate incumbent valuations, costs, and ideal points using position variation.

Each estimation step is robust to multiple equilibria and the uniqueness conditions required for calculating counterfactual outcomes can be checked ex-ante, discussed more later. Due to the across-stage dependencies, I estimate confidence intervals for counterfactuals and committee/candidate parameters with non-parametric bootstrap, accounting for all steps.

4.1 Estimation Of General Election Voter Preferences

Voter preferences are captured by the spending effectiveness parameters β, observed candidate characteristic parameters δ, and unobserved candidate characteristics ξ; each of these varies across the general and primary election. The last term captures election day shocks and unobserved heterogeneity, which I collectively call candidate valence, and which committees and candidates know in expectation ψ_c^G for the general and ψ_c^P for the primary.

The influence of campaign spending on votes has been extensively studied in the pre-2010 environment (Carson 2016; Stratmann 2017), and a major source of endogeneity is the unobserved valence which affects the degree of competitiveness.[26] To be specific, the main threat to identifying the effects of observables on voter preferences is the unobserved election shock ξ. This influences voters directly and affects committees and candidates through

[26] Races that are not competitive do not exhibit large spending on either side. A weak incumbent combined with a strong challenger often result in a competitive race (Erickson and Palfrey 1998). In such instances, the challenger is able to spend more, so then the incumbent spends more, and outside groups become interested. Failing to isolate these competitive races precisely can result in biased estimates. Evidence for this problem can be seen in Supplement Table S3 with weaker Super PAC spending effect.

their endogenous choices. Thus identification of spending effectiveness β and candidate characteristics δ is contaminated by ξ. I use instrumental variables to extract the variation in the endogenous variables that exogenously predicts voting behavior.

An ideal instrument for spending would be a shock to a committee's budget unrelated to the election in question; I use variation from their donor base from outside the state. The intuition for how a donor-based instrument works is illustrated in the model. Donors are influenced by committee fundraising effort, and the donations only affect the voters indirectly through election spending. A shock to a committee via their donors changes the committee's ability to raise funds, which exogenously varies how much they spend.[27]

I differentiate between donors in and out of the state in which the committee is spending. Within-district or state donor variation may correlate with a given district's electoral outcome, and factors affecting out-of-state donors may be less related to the characteristics of a given district (Gimpel, Lee, and Pearson-Merkowitz 2008; Rhodes, Schaffner, and La Raja 2018).[28] Variation based on outside donors is only conditionally exogenous as variables that affect the overall economy or political climate will affect all donors. The key is that conditional on the pre-spending controls, the variation explained by the instruments is only related to the given election through spending.[29]

The excluded donor instruments I use to predict spending but not vote share include the change in out-of-state donor income and the variance in out-of-state donor ideology scores for all committee types. The latter affects spending ability as a high variance in donor ideology provides a fundraising challenge; a homogeneous donor base is easier to corral. It is important to control for the within-state versions of these to better justify the exclusion restriction; some of the within-state donors are the voters in that district and thus directly influence the election. Since there may be a concern about behavior by large donors, I also

[27]Shocks to donors may affect all of the committees to which they give, and that can be correlated across donors who are in similar areas or professions. These overlapping donors do not pose an identification problem, as long as the shocks are uncorrelated with unobservables and only influence voters via spending.

[28]We do not observe a committee that was interested in spending but did not. Endorsement data could reveal this, but many committees do not report this information. To define the IV in such cases, I use the average for committees that are aligned with the same party-incumbency status in that state.

[29]A concern is that some committees do not rely on out-of-state donors; there may be heterogeneity in IV strength across non-excludable dimensions such as the committee's size or scope. The median number of states in which they receive donations is 23 and 3.16% have donors from only one state. The average (dollar amount) share coming from each state is 10%, and the maximum share across all states is on average 52%.

consider an inverse-donation weighted version of the IVs, which captures the variation in smaller donors, who have distinct behavior (Bouton, Cagé, Dewitte, and Pons 2022).

To instrument for candidate position, I use lagged Senate incumbent positions from the state. The inclusion of lagged incumbent success, district characteristics, expected competitiveness, and state fixed effects controls for pertinent variables within the district such that out of district (within state) variation in the historical partisanship of other incumbents correlates with the candidate's policy choice, but does not otherwise affect their election odds in the House election in question.[30] Then, the exogenous variation in candidate positions and spending across differential vote shares (accounting for turnout) identify β and δ, with ξ as the residual.

The spending effectiveness parameters β_{i_c} are pooled across committee types (candidate, Super PAC, and party/PAC), meaning there are three distinct spending effectiveness coefficients for the general election. To construct the estimating equation, transform equation (3.1).[31] Recall the general election voter utility for choosing candidate c, $U_{vc} = u_c^G + \xi_c^G + \varepsilon_{vc}$, where $u_c = \sum_{i_c \in N_c} \beta_{i_c} (S_{i_c}^G)^\phi + \mathbf{X}_c^G \delta_1 + d_c \delta_2$, and the vote shares $s_c^G = \exp(u_c^G - u_0^G + \xi_c^G)/(1 + \sum_{c=1}^C \exp(u_c^G - u_0^G + \xi_c^G))$. The log vote share can be written as $\ln(s_c^G) = u_c^G - u_0^G + \xi_c^G + \ln(s_0^G)$, where s_0^G is the share of absenteeism.[32] I estimate this as a regression with the excluded donor instruments $\mathbf{Z}$, $\phi = 1/2$, and normalized outside abstention mean utility u_0^G.

$$\ln(s_c^G/s_0^G) = \sum_{i_c \in N_c} \beta_{i_c} (S_{i_c}^G)^{1/2} + \mathbf{X}_c^G \delta_1 + d_c \delta_2 - u_0^G + \xi_c^G \tag{4.1}$$

Since I use the ratio of candidate vote share to turnout in the dependent variable, differential turnout can have large effects on ξ_c^G. In an election with an expectation of a lopsided

[30] This approach is similar to Iaryczower, Moctezuma, and Meirowitz (2017). I choose Senate as that is less sensitive to local district variation; a downside is that it does not vary between districts or candidates within the state. Results are not sensitive to using average outside-of-district by-party lagged position of House candidates within the same state.

[31] Some alternative specifications include interacting choices with covariates (which is difficult to instrument for) or random coefficients to allow for more flexible substitution patterns; Gordon and Hartmann (2016) note that the latter specification does not significantly change results (Dow and Endersby (2004) make a similar point on the usefulness of multinomial logit in voting research).

[32] The outside share $s_0^G = 1 - \sum_{c=1}^C s_c^G = 1 - \frac{\sum_{c=1}^C \exp(u_c^G - u_0^G + \xi_c^G)}{1 + \sum_{c=1}^C \exp(u_c^G - u_0^G + \xi_c^G)}$, which is equivalent to $s_0^G = 1/(1 + \sum_{c=1}^C \exp(u_c^G - u_0^G + \xi_c^G))$; then taking logs yields $\ln(s_0^G) = -\ln(1 + \sum_{c=1}^C \exp(u_c^G - u_0^G + \xi_c^G))$.

outcome, one may posit that voter turnout would be low. To formally incorporate such an aspect, one would have to specify a model where pivotality affects the voter's turnout decision, which has been shown to not match the data well (Coate, Conlin, and Moro 2008). I include a variety of controls that are predictive of lopsided outcomes.

Unobserved competitiveness distorts the effects of spending and policy. Thus it is important to include control variables $\mathbf{X}_c^G$ related to a candidate's expected election performance through the voters' preferences. The results are not sensitive to some of the controls, but their inclusion is meant to alleviate concerns about the exclusion restrictions.

I control for donor (zip) income variation within the state as it may correlate with the out-of-state income shocks for each committee and can affect voters. I also include economic factors that may affect voters such as district unemployment rate, district income, and district total unemployed. I interact these with incumbency status as the state of the economy affects incumbents and challengers differently. I also include the percentage of district that graduated high school, district average age, district racial and gender demographics, and city precipitation (rainfall inches) on election day (all interacted with party).[33]

Political controls include incumbency status, party, Republican vote share from the last presidential race (interacted with party), the vote share of the last incumbent in the district (interacted with incumbency), the number of Senate candidates running in the state, an open-race indicator, whether the governor has the same party as candidate, within-state donor ideology variance, and Cook's competitiveness ratings (interacted with incumbency/party).[34]

To account for relative costs of advertising in different markets, I divide expenditures by local ad prices.[35] I also include a cost estimate per committee type for ads in a given media market (from the Wesleyan Media Project) to control for heterogeneity in prices faced by committees. Finally, I include state and election cycle fixed effects, with the latter interacted

[33] Weather has been found to affect turnout. Liao and Junco (2020) also show that news-worthy extreme weather events affect voter opinions.

[34] These ratings are based on assessments of incumbency weakness and the "safety" of the seat for the general election. Some years scraped from Cooks website and other years generously shared by Jim Campbell. I have not included polling data given availability (see Supplement S.4).

[35] Measured as the cost-per-point. Generously shared by Gregory Martin for 2000-2008 from Martin (2019). I use SRDS for 2010 onward and impute some missing years. I use the off-election year lagged prices. There is variation in prices between committees (Moshary 2019) and heterogeneous coefficients absorb the mean.

with party and incumbency status.[36] Some election structures, such as nonpartisan blanket primaries, are not well approximated with the model framework, and so I drop all districts in Louisiana, California after 2012, and Washington state after 2008.

Super PAC ads are predominately negative in tone and the data suggests their spending may depress turnout in certain races. In general, attack ads may affect turnout (Malloy and Pearson-Merkowitz 2016) and some ads in the primary may be divisive. Thus to best fit the data, I allow the mean utility of abstaining u_0^G to be affected by Super PAC spending in the general election and Democratic primary (and party/PAC spending in Republican primaries). Since the turnout effect must be normalized to identify valences, I let Super PAC spending depress turnout and help the candidate equally. This normalization does not affect the equilibrium properties of the game, as the probability of winning P_c^G from the committee's perspective remains unchanged. This specification is equivalent to using the negative of the opponent Super PAC's spending in the regression.

4.2 Estimation Of General Election for Committees

The estimated parameters from equation (4.1) tell us the elements that influence voters directly. Next I estimate the remaining objects relevant to committees, namely the committee's valuation for winning the overall election and a cost function that may vary across the general and the primary elections. Recall the general election post-entry committee payoff:

$$\pi_{i_c}(S_{i_c}^G, \mathbf{S}_{-i_c}^G) = V_{i_c} P_c^G - g_{i_c} \cdot S_{i_c}^G. \tag{4.2}$$

This is a function of V_{i_c}: the value to committee i of candidate c winning, P_c^G: the probability of candidate c winning the general election defined in equation (3.3) and a function of voter utility u_c^G and expected valence ψ_c^G for all general election candidates, and g_{i_c}: the marginal cost of spending (fundraising constraints and donor preferences). I let the committee's expectation of a candidate's valence equal the (estimated) realized valence, $\psi_c^G = \hat{\xi}_c^G$. Without more assumptions, I cannot separately identify a committee's expectation of a given candidate's valence; we only observe their single spending decision and a single election out-

[36] Summary statistics for these variables are reported in Supplement Table S4.

come (Gordon and Hartmann 2016).[37] This is not restrictive as committees observe the full set of controls. The probability of winning P_c^G can then be calculated for the observed pair of candidates in the general election with an additional normalization on the variance of uncertainty of candidate quality: I let $\sigma_\xi = 1$ (see Supplement S.7 for a discussion).

Valuations and costs are not immediately separately identified as low committee spending could signal either low valuations or high costs. Separate identification is achieved by exploiting spending and entry variation. I let them be functions of data and parameters, allowing the cost to vary with candidate positions d_c: $V_{i_c} = \exp(X_{i_c}^V \varrho_c)$ and $g_{i_c} = \exp([X_{i_c}^g, d_c]^\top \varphi_c^G + \gamma_{i_c}^G)$, where $\gamma_{i_c}^G$ is unobserved cost heterogeneity. The vector $X_{i_c}^V$ includes a constant, incumbency status of the candidate, year, lagged presidential votes, and the incumbent's tenure length relative to the state average, with all variables interacted with committee type, incumbency, and party fixed effects. Allowing the coefficients to vary across party is important as there is asymmetry in motivations and behavior (Grossmann and Hopkins 2016).

There are aspects about the value specification to note. First, the valuation is exogenous, but this is not too restrictive as costs are a function of candidate policy and unobserved heterogeneity. Second, one may argue that the value of winning one race is affected by outcomes of other races, particularly if a given race will swing the majority control of Congress. This concern is reduced by Incerti (2018), who studies party spending in House races with majority-seeking and total-seat-seeking models; he finds evidence for the latter. Also, I control for aspects of seat importance such as its safety and the incumbent's tenure.

The vector $X_{i_c}^g$ includes a constant, the number of senate candidates in the state (to measure competition for resources and state political activity), the voting age population of that district, and average ad prices in the state that year, all interacted with committee type, incumbency status, and party fixed effects. I construct a regression based on the derivative of equation (4.2) for a given set of entrants: $V_{i_c} \partial P_c^G / \partial S_{i_c}^G - g_{i_c} = 0$. I take this first order condition, rearrange it to set marginal benefit to marginal cost, and then isolate the marginal probability of winning as a function of the log valuations, log costs, and the error term. The observed candidate decision is a function of the error term $\gamma_{i_c}^G$ and I instrument for it using

[37] To separately identify them, we could assume time-invariant valences or normalize more parameters. One could also use polling data: track how spending changes with new polls that allow committees to update expectations. This is difficult for House races given polling data quality (see Supplement S.4).

the lagged Senate incumbent positions in the state. I estimate equation (4.3) as a regression, and recover the unobserved marginal cost shock $\gamma_{i_c}^G$ for entrants.[38]

$$\log\left(\frac{\partial P_c^G}{\partial S_{i_c}^G}\right) = -X_{i_c}^V \varrho_c + [X_{i_c}^g, d_c]^\top \varphi_c^G + \gamma_{i_c}^G \tag{4.3}$$

The term $\partial P_c^G/\partial S_{i_c}^G$ is equal to $\beta_{i_c}^G \phi (S_{i_c}^G)^{\phi-1} P_c^G(1-P_c^G)$. This regression identifies the ratio of valuations to costs with variation in the marginal effect of spending on the probability of winning for different levels of the instruments. This can only identify valuation coefficients that are excluded from costs, meaning it cannot separately identify variables in both.

Next I use entry variation to separately identify valuations and costs. The intuition is that with an identified V/g, another equation that can identify V given g will identify $(V/g)g$ and hence g. The costs are identified using the variation in estimated entry probabilities p_{i_c} and expected win probability from entry for a given value to cost ratio and expenditure (across levels of the covariates). The expression below stems from evaluating equation (3.6), where the expectations are over the equilibrium entry profiles. See Appendix A.3 for details.

$$g_{i_c} = \frac{\log\left[p_{i_c}/(1-p_{i_c})\right]}{(V_{i_c}/g_{i_c})\left(E[P_{i_c}^G|a_i=1] - E[P_{i_c}^G|a_i=0]\right) - E[S_{i_c}^G|a_i=1]} \tag{4.4}$$

4.3 Primary Election Estimation

I estimate the primary election analogs to general election parameters, except the valuation for winning the overall election. The key challenge for the primary is the presence of an additional unobservable, namely unobserved general election valences for primary losers. For the primary voter preferences, I mirror the general election approach to estimate spending effectiveness, candidate position effects, and primary valences ξ^P (letting $\psi_c^P = \hat{\xi}_c^P$).[39] I estimate the Republican and Democratic primaries separately. The outside committee instruments

[38]For non-entrants, I impute it by averaging across party and committee type. Note relying on spending introduces a selection issue: the first order conditions do not hold with equality for non-entrants. Thus there is the assumption that $\gamma_{i_c}^G$ do not systematically differ in unobserved ways across entrants and non-entrants. For an alternative approaches, see Erikson and Palfrey (1998) or Box-Steffensmeier and Lin (1996).

[39]Note that the committees have two distinct expectations on candidate quality: the primary valence and the general valence. The committees form an ex-ante expectation based on each electorate. I do not model learning where the committee forms a more accurate estimate for the general election valence based on the primary election outcome; identifying parameters related to this updating would be difficult.

are weaker in the primary because there are fewer donors, less predictable spending, and more zeros; I consider both OLS and IV specifications.

The remaining steps are involved. I start by exploiting the model dynamics to isolate the unobserved general election valence for primary losers. Then I leverage variation in which primaries are contested to simultaneously recover this valence alongside primary marginal costs and cost shocks. Finally, I use entry variation to estimate primary fixed entry costs. I discuss the setup and intuition, leaving the details on each step to Appendix A.4.

To utilize the committee's first order condition, one must deal with the unobserved (counterfactual) general election outcomes. For example, the R_1 candidate aligned committee considers both general election outcomes of R_1 facing either D_1 or D_2 when they choose their primary efforts. To be precise, rewrite a Republican committee's payoff as follows with two candidates per side, where the expected probability of winning the general election for a given set of primary winners is defined as $E[P_c^G|\mathbf{w}^P] = \sum_{\mathbf{a}^G \in \{0,1\}^{2N}} P_c^G(\mathbf{a}^G|\mathbf{w}^P) \prod_j p_j^*(a_j^G|\mathbf{w}^P)$, $E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_1}^P = 1)]$ is the expected probability of winning the general election against D_1, and $P_{D_1}^P = P(w_{D_1}^P = 1)$ is the probability of D_1 beating D_2 in the primary:

$$V_{i_c} P_c^P \cdot \Omega_c - g_{i_c}^P S_{i_c}^P \quad s.t. \;\; \Omega_c = E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_2}^P = 1)] \cdot P_{D_2}^P + E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_1}^P = 1)] \cdot P_{D_1}^P \quad . \tag{4.5}$$

In the Ω_c expression, only one object is unobserved for candidates that won their primary, namely the general election probability against the candidate on the other side that lost their primary (for example, the general election probability of Trump vs Sanders in 2016). For candidates that lost their primary, both general election probabilities are unobserved. I already backed out the general election expected valence ψ_c^G for candidates that made it to the general election in the data, but one does not observe it for the primary election losers. This valence term affects the decisions of committees in the primary (and candidates decisions before that), and thus identification of the remaining parameters hinges on recovering it.

I recover ψ_c^G for primary losers using variation in the general and primary that exploits beliefs revealed by equilibrium spending. This approach relies on inverting the equilibrium win probability to solve for the primary loser ψ_c^G as a function of observed objects and primary costs. The intuition is that a committee takes the probability of their preferred candidate winning the general election into account when choosing primary spending; their behavior reveals information about their underlying expectations for an unobserved outcome.

The logic of how to recover this counterfactual ψ_c^G can be seen through the available variation. Since I estimated the primary voter preferences, the effects of primary spending on election outcomes are known, allowing us to isolate how costs affect spending. The cost function in the primary $g_{i_c}^P$, conditional on a known valuation V_{i_c}, shifts a committee's willingness to spend. Thus variation in primary spending and observed expected outcomes in the realized match-ups for a given cost implies a single expected probability of winning the general election for the counterfactual match-up. Then, given the probability functional form and exogenous inputs, it implies a single counterfactual expected valence.

Since I also allow for unobserved heterogeneity in costs, there are additional steps needed to isolate marginal costs, based on exploiting the structure of single-contested primaries. Finally, I estimate mean fixed entry costs using entry variation with a primary election version of equation (4.4).[40] Now we have recovered valuations and costs of committees and valences across both the primary and general elections for candidates that entered the election. For candidates who did not enter, we must exploit a different source of variation.

4.4 Estimation Of Candidate Stages

Now that the general and primary elections are characterized, we can calculate a candidate's probability of winning for any combination of opponents and positions, conditional on valence. Using this, I estimate the candidate stages. Recall the candidate's objective in equation (4.6): V_c: value to candidate c of winning, V_c^0: outside option, $\bar{\theta}_c$: ideal position point, and η_c: private variation in payoffs. The probability of winning is now the expected probability pre-entry, where the candidate positions $\mathbf{d}$ are now written as explicit arguments: $E[P_c^G|\mathbf{d}] = \sum_{a_i^P \in \{0,1\}^{4N}} E[P_c^G|\mathbf{d}, \mathbf{a}^P] \prod_j p_j^*(a_j^P|\mathbf{d})$.

$$\Pi_c = V_c \cdot E[P_c^G|\mathbf{d}] + V_c^0 \cdot (1 - E[P_c^G|\mathbf{d}]) - (d_c - \bar{\theta}_c)^2 \cdot \mathbb{1}[d_c > 0] + \eta_c(d_c) \qquad (4.6)$$

[40]This is not possible for the general election as I use spending and entry variation there to separately identify valuations from costs. Separating them is required to characterize general election equilibrium entry.

The unknowns $\{V_c, V_c^0, \bar{\theta}_c\}$ must be restricted as candidate decisions can be rationalized by a variety of combinations (Diermeier et al. 2005, Tillmann 2014). I allow the value from office and the outside option value to vary only at the district-party level. Specifically, $V_c = \exp(W_c \lambda)$, where W_c is a data vector including the incumbent's tenure length to date, election cycle, and district income (all interacted with party). I specify $V_c^0 = \exp(W_c^0 \lambda^0)$, where W_c^0 includes party and election cycle fixed effects. I restrict the ideal points to vary at the election cycle-party level for incumbents and set them for challengers to be their observed choices; non-entrant ideal points cannot be separately identified from valuations.

For a given candidate that entered, I observe their entry decision and their policy position, and thus there are two sources of variation to compare across candidates. To estimate the entry stage among candidates, one needs to know the identity of each potential entrant in the event that they do not enter.[41] I construct potential entrants, with as many potential entrants as there are "empty" spots with two candidates per race: two candidates per side per primary. This approach is feasible because the variables I use to predict parameters do not rely on individual characteristics. However, there may be selection on unobservables.

The general election and primary election valences of candidates that never ran, meaning the potential entrants that chose $d_c = 0$, are not recoverable from Congressional election data (without more assumptions).[42] The identification of candidate preferences requires an estimate of these valence terms as one needs to calculate their expected probability of winning. I let the committee and candidate expectations of these valences ψ_c for the non-entrants follow a distribution: $\psi_c^{NE} \overset{iid}{\sim} N(\mu_{NE}, \sigma_{NE})$. The variance σ_{NE} is estimated with maximum likelihood using the variation in estimated entrant valences. The average expected valence for non-entrants, μ_{NE}, is likely different from that of entrants.

To allow for this selection, I use a proxy to estimate the difference in means of the valences for entrants and non-entrants. State legislature members are a significant source

[41] Tillmann (2014) estimates a Congressional candidate entry model and generates a list of potential entrants; because he has their identities, he uses their characteristics to predict entry.

[42] In addition, any valence for a candidate in an uncontested race in which the total number of votes is zero (or party convention where turnout cannot be measured like CT and UT) is also under-identified; this occurs for 20% of primary incumbents and 12% of primary non-incumbents. Since 67% of uncontested primaries still have ballots (albeit with likely distinct voting behavior), I draw valences for those unidentified uncontested primary winners from the estimated primary winner valence distribution from balloted uncontested primaries.

of the candidate pool for Congressional elections (over 40% of current members of Congress since 2010). I compare the state legislature election (general and primary) valences for state legislature incumbents who ran for Congress and those who did not. This tells us how different entrants are from similar non-entrants. I estimate state legislature vote share regressions and then calculate the mean difference in valence of the potential entrants, μ_{NE}.[43]

For a given vector of valences for all candidates $(\psi_c^G, \psi_c^P)\ \forall c$, either estimated or drawn from the proxy distribution, I calculate $E[P_c^G|\mathbf{d}]$ for every combination of candidate decisions. I allow for two positions beyond non-entry (0), namely moderate (1) and extreme (2).[44] As in the committee entry stages, I define the system of equilibrium challenger choice probabilities $p_c(d_c = \theta)$ in equation (4.7), where $\pi_c = \Pi_c - \eta_{d_c}$. I construct estimating equations based on this equilibrium probability. Variation in the estimated probability (across levels of the covariates), controlling for differential expected win probabilities, identifies the valuations. The incumbent's estimating equation is similar. See Appendix A.5 for details.

$$p_c(d_c = \theta) = \frac{\exp(E[\pi_c(d_c = \theta|\mathbf{p}_{-c})])}{\sum_{w=0}^{\Theta} \exp(E[\pi_c(d_c = w|\mathbf{p}_{-c})])}. \tag{4.7}$$

4.5 Estimation Discussion

Before proceeding to the results, I review the estimation and discuss possible biases. Voter preferences are estimated from vote share regressions. I deal with unobserved candidate quality with out-of-state donor shocks. Committee preferences are estimated from spending first order conditions and entry conditions. The main unobservable here is the general election valence of primary election losers, and I leverage the dynamic structure to recover it. Candidate preferences are estimated from entry and policy conditions. I address the selection bias of unobserved non-entrant quality using state legislature variation. Each stage feeds into the next, capturing how each stage influences the rest of the election.

How do estimates in one stage affect the others? Suppose the marginal effects of spending are biased. This would in turn cause an additional bias in the valuation/cost estimates as

[43] I get the election results for state legislatures from ICPSR, campaign spending from the National Institute on Money in State Politics, and donor records and ideology scores from the state-level election DIME dataset.

[44] I normalize Bonica CF-score positions by dividing by the max of all absolute-positions and then set cutoff points at the $\leq$ 60th percentile across the position distribution for moderate and $>$ 60th for extreme.

those attempt to rationalize observed spending behavior conditional on a known spending effect. Thus if the spending coefficient is upwardly biased, the valuation to cost ratio would be downwardly biased. In this case, one stage's bias partially negates the other, leading to a smaller bias in the counterfactual prediction relative to a method that ignores or normalizes one of the stages. This is an advantage of jointly considering all stages of the election.

Similarly, voter and committee parameters affect candidates through the win probability. A downwardly biased estimate on the influence of candidate policy on their election odds causes the candidate's observed choice to be rationalized with an upwardly biased valuation of winning (relative to the outside option and cost). This affects the counterfactual in that one would downplay how much candidates react, attenuating policy change effects.

5 Results and Counterfactuals

5.1 Parameter Estimates

Table 3 reports the committee spending and candidate position coefficients from the voter preferences estimation for the general elections of House races from 2010 to 2018. The dependent variable is the difference in log share of votes the candidate received and the log share of absentees. I estimate spending effects by candidates, Super PACs, and combined party and traditional PAC spending. Supplement Table S5 shows the controls. The vote share regressions show robust errors.

I find that candidates are statistically more effective per dollar in converting spending into votes than other committees. Super PACs are weaker but precise, whereas parties and PACs have noisy effects. The candidate position coefficient reflects how voters respond to more extreme positions, measured here as 1 for a moderate position and 2 for extreme (binning CF scores at the 60th percentile). The coefficient is negative, implying general election voters prefer moderate candidates, but the effect is noisy.[45] A reduced form interpretation would

[45] Due to concerns about large donors affecting the IV validity, I also consider a inverse-weighted (by donation amount) version which captures the donor shocks of many small donors. While the strength of the instrument is noticeably weaker, the coefficients and standard errors are similar in magnitude and in relative terms: candidate spending has a coefficient estimate and standard error of 0.0381 & 0.0251; Super PACs have 0.0137 (0.0081), party & PAC are 0.0166 (0.0277), and the candidate position is -0.2871 (0.1891).

be that for a candidate, a one standard deviation increase in spending ($437,846) at the average ad price leads to a 21% increase in vote share relative to absenteeism. Due to the likely anticipated response by others, the best way to interpret these coefficients is in the context of the whole equilibrium, which I do in the counterfactual analysis.

Table 4 reports the regression results from the primary elections. I use OLS for the main estimates and report the (weak) IV regressions to show that the results do not qualitatively change; slight changes in coefficient magnitudes do not affect equilibrium outcomes if the changes are similar across committee types; in addition, the other parameters, like valuations, would adjust to rationalize the data, leading to similar equilibrium predictions. In the Republican primary, candidates still have the largest effect but Super PACs are close behind. Parties and PACs again have small and imprecise effects. In the Democratic primary, candidates clearly dominate and Super PACs have a smaller noisy effect. Thus Super PACs play an outsized role in Republican primaries. Also, primary voters reward extreme candidates with a precise positive coefficient on position in both primaries.[46]

Table 5 reports the estimated valuations and costs for committees and candidates, averaged for different committee types, elections, and parties. I use non-parametric bias-corrected percentile bootstrap confidence intervals with 600 draws.[47] Valuations are not sufficient to indicate how much a committee will spend as their spending effectiveness and costs also influence their decision. For example, PACs have high valuations, large marginal costs, moderate fixed costs, and low effectiveness. This aligns with their behavior of spending small amounts in many races. The estimates and confidence intervals on party spending indicate noisy effects, which is a byproduct of the limited variation in party spending.

Challenger valuations are quite high as there is often an entrant against an incumbent despite the large incumbency advantage that results in frequent challenger losses. Costs for challengers are typically higher, which may indicate their weaker fundraising abilities. Valuations for Republican challengers are on average larger than Democratic challengers; this mirrors Gordon and Hartmann (2016) who find a similar result for Presidential candidates. Republicans are willing to spend more in races in which they are more likely to lose, implying

[46]Supplement Table S6 shows the controls. Estimates using donor real estate shock IVs (see Supplement S.3) are similar in relative terms but the IVs are weaker and the data are unavailable for the full sample.

[47]Since spending in the model is only rationalized by $\beta > 0$, I constrain it in the bootstrap.

Table 3: General Election Voter Parameters: Spending/Position and Excluded IVs

	Candidate Spending	-Opp SPAC Spending	Party/PAC Spending	Candidate Position	Log-Diff Vote Share
Candidate Spending					0.0445**
					(0.0167)
Negative Opponent					0.0153**
Super PAC Spending					(0.0054)
Party & PAC Spending					0.0183
					(0.0155)
Candidate Position					-0.2772
					(0.1636)
Inc=0 × Out Of District	0.5043	1.0552	0.8647	-0.0146	
Lagged Position	(0.4257)	(1.1839)	(0.4749)	(0.0435)	
Inc=1 × Out Of District	-0.7804	0.1419	1.4956*	0.4922***	
Lagged Position	(0.5918)	(1.3974)	(0.7419)	(0.0580)	
Out-Of-State Candidate	0.4036**	0.1159	0.1444	0.0377*	
Change in Donor Zip Income	(0.1539)	(0.2571)	(0.1567)	(0.0178)	
Out-Of-State Party	13.5133***	-29.7319***	20.9792***	0.1144	
Donor Ideology Variance	(1.0704)	(2.7011)	(1.3650)	(0.1065)	
Out-Of-State Candidate	3.9278***	0.4817	2.1193***	-0.1800**	
Donor Ideology Variance	(0.6500)	(1.1375)	(0.6167)	(0.0688)	
Out-Of-State Super PAC	5.1858***	-14.6325***	3.4582**	0.2379*	
Donor Ideology Variance	(1.0324)	(2.4457)	(1.0779)	(0.0995)	
Out-Of-State Party	3.0916***	-4.7728***	3.7194***	0.0355	
Change in Donor Zip Income	(0.6652)	(1.1107)	(0.7759)	(0.0435)	
Out-Of-State Opp. Candidate	0.3278**	-0.1814	0.0190	0.0288**	
Change in Donor Zip Income	(0.1025)	(0.2895)	(0.1051)	(0.0102)	
Out-Of-State Opp. Spac	3.4386***	-11.5922***	4.0074***	-0.0356	
Donor Ideology Variance	(0.3429)	(0.8775)	(0.4438)	(0.0259)	
Observations	3514	3514	3514	3514	3514
R^2	0.528	0.340	0.421	0.330	0.619
F-statistic of excluded IVs	65.25	48.41	50.25	11.79	
F-statistic	30.5905	8.4761	11.6747	35.7321	80.4733

Robust standard errors in parentheses. Controls in Supplement Table S5. The KP under-identification test rejects the null with an LM-statistic of 51.390 and p-value of 0.000. The Hansen J-statistic for over-identification fails to reject with 4.93 and a p-value of 0.49. The first stage F-tests of the excluded instruments all have p-value<0.000 and their F-values are 65.25, 48.41, 50.25, and 11.79; the SW F-values are 10.16, 15.19, 8.77, and 11.54 respectively (p-value<0.000).

Table 4: Primary Election Voter Parameters: Spending/Position

DV: Log Diff vote Share	Republican	Democratic	Republican	Democratic
Candidate Spending	0.0272***	0.0415***	0.1539	0.1723***
	(0.0049)	(0.0065)	(0.0843)	(0.0432)
Super PAC spending	0.0208**		0.1834	
	(0.0071)		(0.1144)	
Negative Opponent	0.0039		0.1786	
Party and PAC Spending	(0.0065)		(0.2027)	
Candidate Position	0.1935***	0.2475***	0.2709	0.8216
	(0.0371)	(0.0461)	(0.5386)	(0.4555)
Negative Opponent		0.0043		0.0201
Super PAC Spending		(0.0051)		(0.0472)
Party and PAC Spending		0.0233		0.2373
		(0.0166)		(0.1339)
Model	OLS	OLS	2SLS	2SLS
Observations	2385	2190	2385	2190
R^2	0.578	0.492	0.328	0.159
F-statistic	50.37	36.92	33.36	20.68

Robust standard errors in parentheses. Controls for columns 1 and 2 are in Supplement Table S6. The 2SLS columns use the primary election equivalents of the instruments from Table 3.

Democrats may be more risk averse, with Democratic PACs as the exception.

Recall that I allow the candidate's position to affect the committee cost function. The coefficient is slightly positive (negative) for Republican (Democratic) committees in the general, and negative in the primaries. A positive coefficient implies that as the candidate becomes more extreme, the implicit cost of spending increases. Thus, primary donors seem to prefer more partisan candidates. There is no significant heterogeneity across committees.

I also find primary election losers have a lower average and higher variance of general election valences than primary winners. This indicates that the pool of candidates that successfully make it to the general are not necessarily always the highest "quality" in unobserved dimensions, a result that corroborates Tillmann (2014). Finally, I find that state legislature incumbents who did not run for Congress have on average 14% lower quality, conditional on controls, than the state legislature incumbent Congressional race entrants.

I calculate the means and correlations for the observed and estimated model outcomes.[48]

[48] See Supplement Table S7. I discuss the approximating function specifications and fit in Supplement S.5.

Table 5: Committee/Candidate Valuations and Costs Estimates

Committee Valuations (in Thousands)		
Inc R Candidate	0.5259	[0.1126, 1.5227]
Inc R Super PAC	0.5185	[0.2629, 9.4834]
Inc R Party	0.3663	[0.1876, 1.3556]
Inc R PAC	1.4161	[0.3967, 23.3659]
Cha R Candidate	0.4877	[0.1052, 1.3812]
Cha R Super PAC	0.5850	[0.3000, 12.7342]
Cha R Party	0.4829	[0.2726, 1.9711]
Cha R PAC	1.9914	[0.6138, 34.2989]
Inc D Candidate	0.2794	[0.1388, 1.6633]
Inc D Super PAC	0.4314	[0.1470, 2.0259]
Inc D Party	0.2865	[0.1389, 2.3298]
Inc D PAC	3.3444	[1.4334, 47.1684]
Cha D Candidate	0.1877	[0.1009, 1.1518]
Cha D Super PAC	0.5721	[0.4725, 7.5908]
Cha D Party	0.2796	[0.2310, 3.9818]
Cha D PAC	3.8027	[3.0020, 132.2389]

Candidate Entry Valuations		
Inc R	21.3663	[0.0048, 39.3725]
Cha R	79.3051	[72.7964, 83.9186]
Inc D	38.9213	[0.4686, 89.1583]
Cha D	81.7723	[79.4525, 87.8268]

Candidate Non-Entry Valuations		
Inc R	28.0700	[0.0000, 57.6772]
Cha R	72.7875	[70.7003, 73.8260]
Inc D	0.010	[0.0000, 47.1628]
Cha D	76.9479	[75.0249, 78.0273]

Committee Primary Election Fixed Costs		
R Super PAC	1.7536	[1.6901, 1.8225]
R Party	2.4530	[2.3059, 2.6158]
R PAC	1.6913	[1.6368, 1.7464]
D Super PAC	1.9491	[1.8719, 2.0499]
D Party	2.6726	[2.4481, 2.9395]
D PAC	1.9310	[1.8370, 2.0460]

Committee General Election Costs (in Tens)		
Inc R Candidate	0.0135	[0.0207, 0.3972]
Inc R Super PAC	0.0097	[0.0137, 0.9496]
Inc R Party	0.0014	[0.0038, 0.0547]
Inc R PAC	0.0325	[0.2236, 17.4309]
Cha R Candidate	0.0208	[0.0149, 0.2917]
Cha R Super PAC	0.0104	[0.0112, 0.6100]
Cha R Party	0.0598	[0.0038, 0.0520]
Cha R PAC	0.0322	[0.1516, 10.4116]
Inc D Candidate	0.0122	[0.0208, 0.2075]
Inc D Super PAC	0.0013	[0.0113, 0.2583]
Inc D Party	0.0077	[0.0079, 0.2203]
Inc D PAC	0.7761	[0.5153, 40.3875]
Cha D Candidate	0.0161	[0.0219, 0.1979]
Cha D Super PAC	0.0019	[0.0143, 0.3184]
Cha D Party	0.0073	[0.0055, 0.1453]
Cha D PAC	0.6784	[0.5874, 38.0509]

Committee Primary Election Costs		
Inc R Candidate	0.0918	[0.0563, 1.1269]
Inc R Super PAC	0.0663	[0.0467, 2.5689]
Inc R Party	0.0100	[0.0077, 0.0872]
Inc R PAC	0.2212	[0.1553, 9.6182]
Cha R Candidate	0.1156	[0.0552, 1.0158]
Cha R Super PAC	0.0628	[0.0340, 1.7726]
Cha R Party	0.1510	[0.0096, 0.1291]
Cha R PAC	0.2015	[0.1133, 6.2827]
Inc D Candidate	0.0759	[0.0464, 0.4991]
Inc D Super PAC	0.0079	[0.0059, 0.1388]
Inc D Party	0.0467	[0.0330, 0.7367]
Inc D PAC	4.7774	[2.7755,154.9467]
Cha D Candidate	0.0989	[0.0577, 0.6661]
Cha D Super PAC	0.0121	[0.0087, 0.1930]
Cha D Party	0.0450	[0.0303, 0.8642]
Cha D PAC	4.0673	[2.5704,120.1414]

Incumbent Position Costs		
Inc R	1.1544	[0.7649, 1.4991]
Inc D	1.1528	[0.5684, 1.2981]

The 95% confidence intervals are bias corrected percentile bootstrap. This shows valuations and costs for committees and candidates. ‘Inc’ refers to incumbent. ‘Cha’ refers to challenger. R and D refer to Republican and Democrat aligned groups.

The candidate positions, entry decisions, and election outcome means fit especially well, indicating that, on average, the model's first and final stages reach outcomes similar to the data; this provides us some reassurance that the modeled dynamics mirror the data generating process. Entry totals differ for some committees despite similar spending means; this occurs because the model occasionally predicts more entry with less spending per entry decision. The model cannot fully replicate some of the asymmetries in the data.[49]

5.2 Counterfactuals

I consider the counterfactual scenario of Super PACs never existing (or a law banning them). To evaluate the 2010-2018 elections in this setting, I first use the parameter estimates to fully solve the model under the observed data with Super PACs, and then solve the model with the same parameter estimates but now excluding Super PACs.[50] There are two sources for changes in a given stage: first the change in behavior conditional on the same outcomes from the previous stage, and then the change in behavior given a different outcome from a previous stage. Comparing the differences in equilibrium outcomes between the observed and counterfactual scenarios incorporates both. I am holding un-modeled variables, like exogenous covariates, constant in the counterfactual, which is not innocuous. For example, ad prices could readjust without Super PACs as spending decreases. In this case there would be a second order equilibrium adjustment, which could attenuate the overall effect.

I separate the analysis for elections with an incumbent and without; the main discussion is on the former, and I highlight notable similarities and differences with the latter. Only 11% of races in the sample are open (meaning no incumbent in the primary or general), and the existence of an incumbent can create differences in equilibrium outcomes. Finally, across the counterfactual distributions, there is a pile-up near zero, which is driven by the fact that

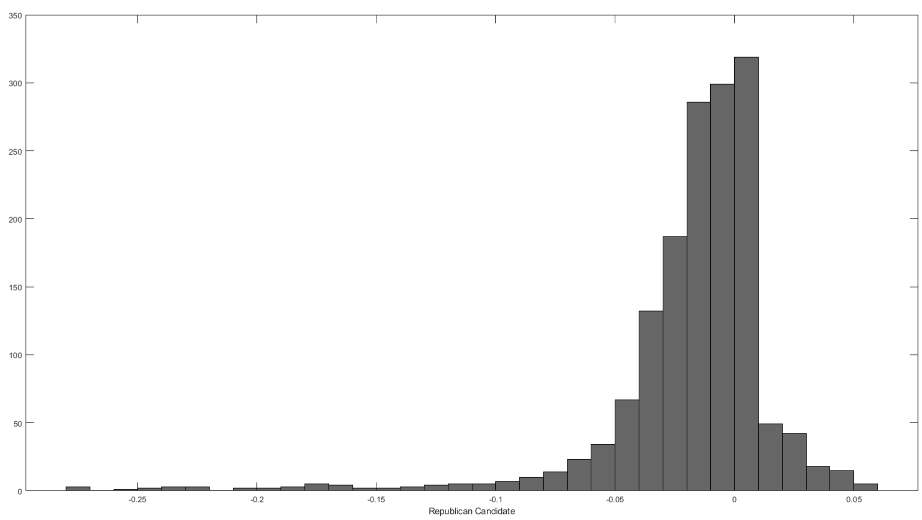
[49] Also, there are so few instances of Democratic parties spending in primaries that there is little overlap in which districts those committees enter across the data and model, leading to a poor fit (particularly for a single draw as shown). The differences in spending means for certain committees are driven by a few outliers.

[50] The fixed point algorithm needs to use the same equilibrium function across the two scenarios, and sufficient for that is a unique equilibrium; uniqueness conditions can be checked ex-ante (see Lemmas 3, 4, 6, and 7); results are not sensitive to starting values. I consider one simulation and private information draw to study actual choices instead of just probability distributions (and due to computational constraints).

Super PACs did not spend anything or very little in many races.[51] I report the mean and median effects for each distribution in the table notes.

In Figure 4, I report the percent change in Republican general election vote share (excluding abstention) without Super PACs. The average Republicans vote share changes by -2.2% [-3.03, 0.21] with substantial variation. Republican incumbent shares increase by 1.3 percentage points without Super PACs and Republican challengers see their chances decrease by 1.2 points. This is intuitive as Super PACs typically help challengers more than incumbents. Furthermore, the change is slightly larger in Democratically leaning states. Thus Super PACs may provide higher benefits in competitive but difficult environments.

Figure 4: Percent Change in Republican General Election Vote Share without Super PACs



This plots the histogram of percent changes in Republican general election vote share (excluding abstention) with and without Super PACs. I compare the simulated equilibrium and counterfactual shares if Super PACs cannot enter. The mean is -2.17% and the median is -1.09%.

The large left tail indicates that Super PACs may have provided a lifeline for some Republican candidates that otherwise would have performed quite poorly. Overall, Republicans may lose on average 2.1% of House seats in the counterfactual analysis sample (6 to 7 seats of the 325 districts on average per cycle studied) without Super PACs, which also represents

[51] Super PACs spent in 48% of general elections and 13% of primaries overall (20% of contested primaries). For the histograms, I trim the bottom and top 1.5% of observations as a few outliers skew the graphs. There are also large tails on some of the distributions, and these outliers are partially a natural consequence of the data; for example, the distribution of observed spending has a large right tail.

an average 3.5% decline of their currently held seats in the sample; this is significant in close Congresses. The result is similar for open races, where Republicans see a -2.6% change in vote share. However, Democratic Super PACs are continually gaining on Republicans each new election cycle, so the trend could change. Further study on the U.S. Senate may be illuminating as Super PAC spending represents a larger share of total spending in Senate races (relative to House races) and there are fewer seats.

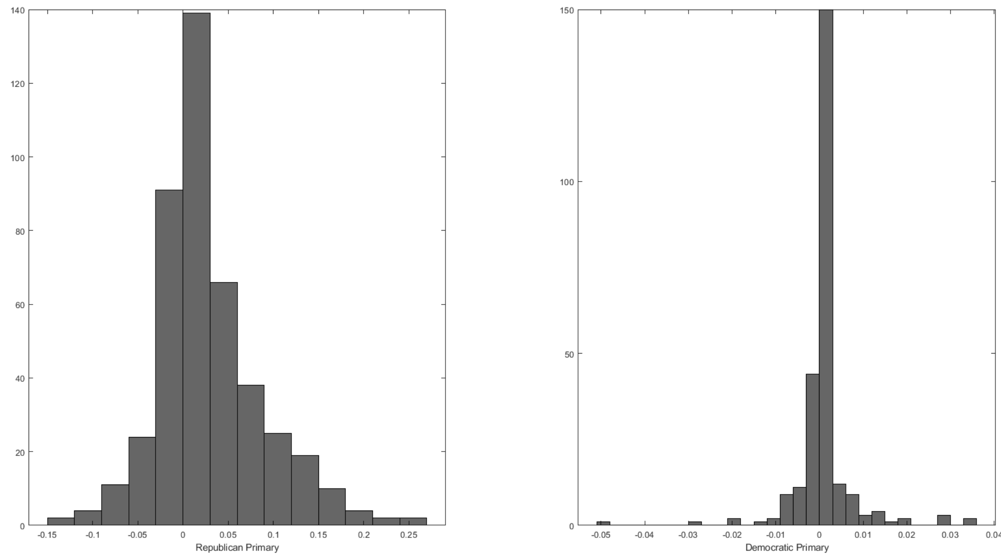
Next, suppose one held the spending, policy, and entry decisions constant and simply evaluated the change to general election candidate vote shares without Super PACs existing. In this case, the counterfactual prediction would be biased by an average of -92% with a 40% sign reversal rate across districts. In other words, the counterfactual would be biased towards zero and predict that Super PACs on average slightly helped Democrats. This illustrates the importance of allowing all agents to optimally respond to regime changes in predictions.

Super PACs also increased general election spending, as total spending (in races with incumbents) would change by -3.0% [-14.76 -0.05] if Super PACs did not exist. The remaining committees see a total 8.5% [2.75, 12.55] spending increase in the counterfactual compared to reality. Many races have increased spending as candidates cannot rely on Super PACs to spend on their behalf. The lack of large Super PAC expenditures, not sufficiently compensated for with spending elsewhere due to contribution limits and ineffectiveness, depresses total spending. [52] In open races, general election spending decreases by 7.5% in the counterfactual; Super PACs are likely more influential in open races as challengers cannot rely on self-funded war-chests like incumbents.

The changes in vote share and spending in the general election are due not only to the absence of Super PACs within this stage, but also to changes in previous stages of the election. The set of candidates that enter the general election from the primary, and their policies, are affected by the absence of Super PACs. Figure 5 shows the percent change in incumbent primary election vote share without Super PACs for contested primaries; incumbents are generally helped as the distribution skews to the right. The effect for Democrats is relatively

[52] Supplement Figure S2 shows the counterfactual distribution of the percent change in general election spending without Super PACs for different committee types; the median is a 1% increase with large right tails for some committees. In Supplement Figure S4, I report the percent change in party committee and PAC entry probability into general elections without Super PACs.

Figure 5: Percent Change in Incumbent Primary Vote Share Without Super PACs



This plots the histogram of percent changes in incumbent primary election vote share (excluding abstention) with and without Super PACs. I compare the simulated equilibrium and counterfactual shares if Super PACs cannot enter. For Republicans, the mean is 3.16% and the median is 1.63%. For Democrats, the mean is 0.30% and the median is 0.02%.

small, with most of the distribution falling between ± 1 percentage point changes, whereas Republican incumbents see slightly larger increases. Super PACs mainly help challengers, with a 3.2% [-0.22, 4.23] change in vote share for Republican incumbents without them.

Total primary spending (in races with incumbents) changes without Super PAC by -11.1% [-38.0, -4.69] in the absence of Super PACs. Total candidate spending changes by -1.2% [-1.90, 3.47] without Super PACs whereas party spending increases 3.7% [-5.20, 19.83], and PAC spending changes by -5.9% [-12.05, 29.42]. Thus Super PACs seem to play heterogeneous roles in the primaries with complementing some spending and crowing out others. The relatively small number of bins for Democratic party committees is a function of the fact that they are selective in spending and there are many uncontested primaries.[53] The effects on open races are much larger: a total spending decrease of 23%. This again suggests that Super PACs play a major role in creating competition in open races.

Super PACs have out-sized effects on primaries relative to general elections. This reiterates the importance of accounting for the primary election in the analysis. Not only does

[53]Supplement Figure S3 displays the counterfactual distribution of the percent change in primary election spending without Super PACs. In Supplement Figure S5, I report the percent change in party committee and PAC entry probability into primary elections without Super PACs.

ignoring the primary miss out on these direct changes, but this spills over into predictions on the general election; the set of candidates that make it there are affected by Super PACs. For this same reason, it is important to study changes to candidate choices.

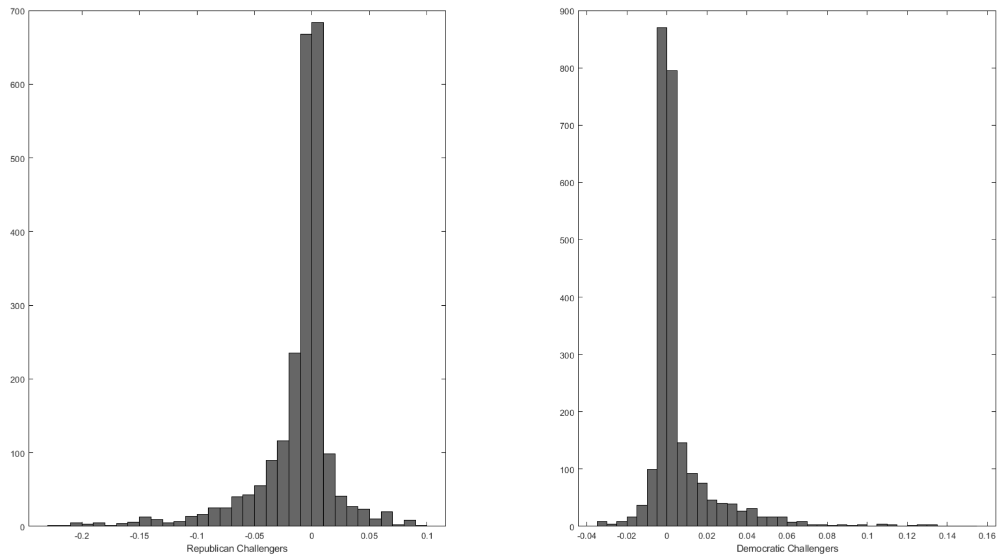
Figure 6 reports the percent change in challenger entry probability without Super PACs, defined as one minus the probability of $d_c = 0$. There is a concentration near zero for both Republican and Democratic challengers, but with a left tail for Republicans and right tail for Democrats. The tail indicates that Republican challenger entry decreases without Super PACs. The average change in Republican challenger entry without Super PACs is -1.5% [-1.64, -0.64]; the average change for Democrats is 0.9% [0.27, 0.63]. The effect is also larger in states that are dominated by the candidate's party compared to opposition states. Overall, we see that Super PACs encouraged Republican challenger entry and slightly depressed Democratic challenger entry.[54] The median effects are much smaller, -0.15% and 0.02% for Republican and Democratic challengers respectively. This reflects the data: most districts have non-competitive primaries, and only a few capture the interest of Super PACs.

Figure 7 displays the percent change in challenger extreme position probability without Super PACs. Both types of challengers barely change on average without Super PACs. Average Republican challenger change in extreme position is -0.2% [-0.01, 0.81]. The average extreme change for Democrats is 0.13% [-0.06, 0.03]. Super PACs are more likely to support challengers and increase their chances of winning the primary, and thus challengers now have higher expected probabilities of winning the general, and since general election voters have a preference for moderation, challengers could increase their general election chances through a more moderate position. However Republican Super PAC also have a slight preference for extremism in the general election. These countervailing forces combine into null effects.

Figure 8 reports the percent change in incumbent extreme position probability without Super PACs; the average change is 1.4% [1.21, 7.29] for Democratic incumbents and 0.5% [0.01, 2.71] for Republicans. Thus Super PACs seem to be a slight moderating force for Democratic incumbents. One explanation is that since Super PACs helped Republicans

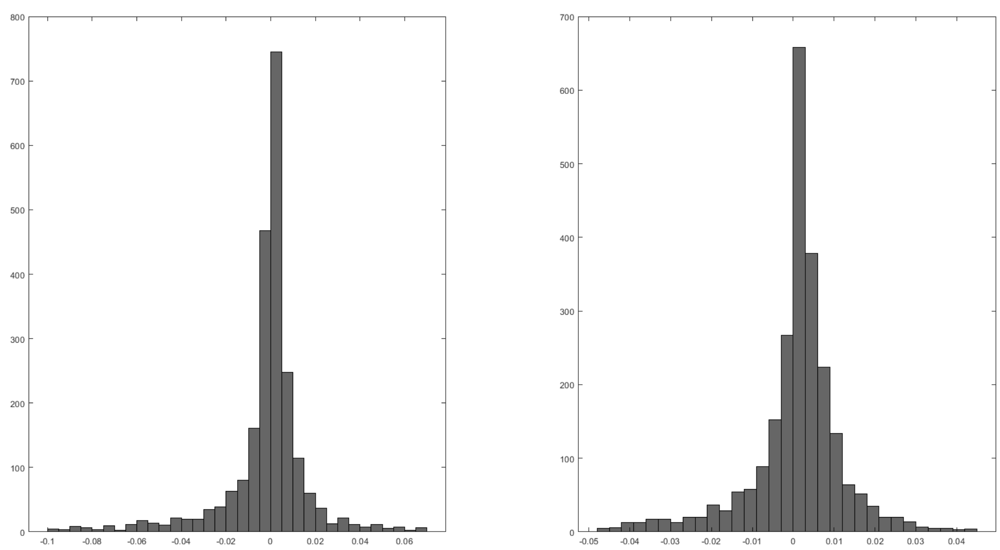
[54] The model may over-assign credit to Super PACs in 2010 given the large Tea-Party induced entry with only fledgling Super PAC primary activity; the election cycle fixed effects and interactions soak up some of this. It should also be noted that if Super PACs never existed, then the set of possible entrants may change beyond what the observables and estimated valence ranges considered here can capture.

Figure 6: Percent Change in Challenger Entry Without Super PACs



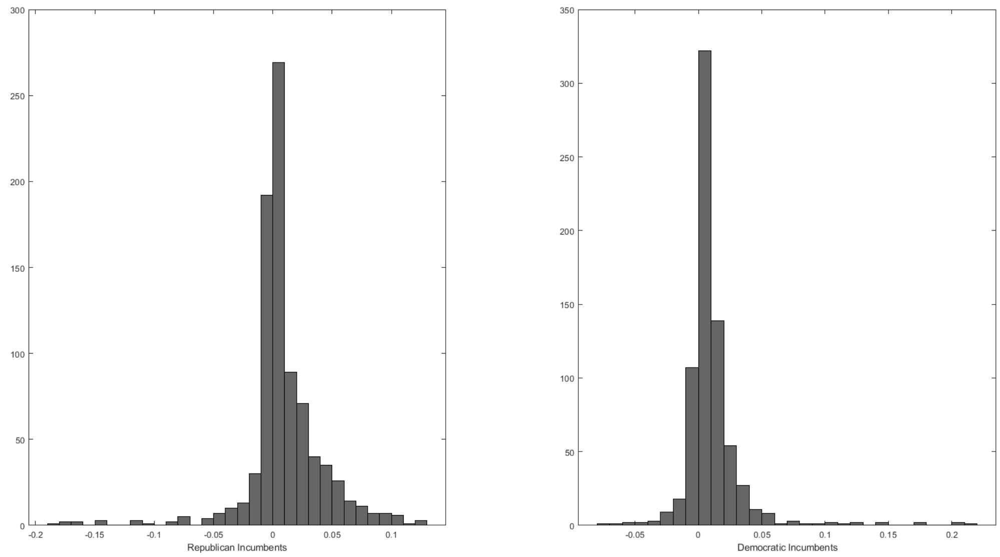
This plots the histogram of percent changes in challenger entry with and without Super PACs. I compare the simulated equilibrium and counterfactual challenger entry probabilities if Super PACs cannot enter, for Republican and Democrats. For Republicans, the mean is -1.48% and the median is -0.15%. For Democrats, the mean is 0.90% and the median is 0.02%.

Figure 7: Percent Change in Challenger Extreme Position Without Super PACs



This plots the histogram of percent changes in challenger extreme position with and without Super PACs. I compare the simulated equilibrium and counterfactual challenger extreme position probabilities if Super PACs cannot enter, for both Republican and Democratic candidates. For Republicans, the mean is -0.19% and the median is 0.05%. For Democrats, the mean is 0.13% and the median is 0.17%.

Figure 8: Percent Change in Incumbent Extreme Position Without Super PACs



This plots the histogram of percent changes in incumbents extreme position with and without Super PACs. I compare the simulated equilibrium and counterfactual incumbent extreme position probabilities if Super PACs cannot enter, for both Republican and Democratic candidates. For Republicans, the mean is -0.52% and the median is 0.26%. For Democrats, the mean is 1.41% and the median is 0.56%.

in the general, their absence relieves general election pressure on Democrats, and so the incumbent focuses on the primary. This is backed up by the fact that the moderating effect for Democrats is stronger in districts where the Republican candidate fairs well in the general election with Super PAC support.[55] How should one reconcile this slight moderating effect with the trend towards polarization since 2010 as seen in Figure 3? Super PACs may not be part of the cause, and their subtle influence is likely not affecting the overall trend.[56]

The descriptive statistics allude to possibly large Super PAC effects, but the counterfactuals indicate more muted influence. The simultaneous rise of the “Tea-Party” movement, spending cannibalization, and candidate selection reiterate the need to control for confounders and equilibrium responses. The small effects of special interest spending on pol-

[55] Also, Democratic Super PACs are more likely to support challengers and thus without them, challengers are more vulnerable; an incumbent can deter entry by going more extreme in the primary. This move is less effective when Super PACs support the challenger as they are less deterred. Without Super PACs, Democratic challenger entry decreases a few percentage points (conditional on races with Super PACs).

[56] I measure policy in one dimension, which may miss out on heterogeneity: for example, Gilens et al. (2021) find state-level evidence that *Citizens United* changed corporate tax policy but not necessarily other issues. The extent to which a change along one dimension in a multidimensional policy framework would show up in changes to a one dimensional measure is unclear, but it may suggest that a null finding using a one dimensional scale could conceal nuanced effects that multi-dimensional analysis could capture.

icy is also not an uncommon result (Besley and Coate 2001; Kang 2016). Beyond providing an affirmation in the context of Super PACs, my analysis shows the influence of spending throughout the entire dynamic process within a single election, including direct and indirect effects. This rich environment provides novel insights into which channels are most affected, and how the influence dissipates into only moderate changes to policy outcomes.

The cannibalization of spending efforts is evident in both the data and the model; spending by one side is often matched with spending by the other. The counterfactual considered so far removes Super PACs from both sides, resulting in often small changes since the Super PACs were canceling each other out in the first place. Thus one may wonder why donors even bother funding all of this spending if it has no effect? One explanation is the fear of being swamped in opponent spending with no response, which could result in large election outcome changes. To study this, I consider a one-sided counterfactual: suppose the laws changed such that only one party was affected and only their Super PACs could not spend.

With a ban on Republican Super PACs, Republican general election vote share decreases by an additional 1.03 percentage points beyond the symmetric ban, indicating that Super PAC spending is helpful on its own and counteracts opponent spending. Challengers suffer slightly as well as Super PACs were a nontrivial source of support.[57] The party without Super PACs has their incumbents perform better in primaries; this helps in the general election due to incumbency advantage but is simultaneously counteracted by opponent general election Super PAC spending. Overall, the asymmetric ban on Republican Super PACs alters outcomes (compared to the symmetric ban) more than a ban on Democratic Super PACs. This is intuitive as, so far, Republican Super PACs have been relatively more influential.

Finally, it is useful to discuss possible welfare effects from the hypothetical Super PAC ban. Given the small to moderate effects on policy, the zero-sum election spending may be seen as "social waste". Formally, the model has voters receiving utility from this spending; without Super PACs, general election voter utility decreases by 3.4%, largely because there is less spending. However, if we ignore this component and only consider the utility from policy and candidate characteristics, utility increases by 0.4% without Super PACs; thus

[57] Democratic (Republican) challengers are less likely to enter under a Democratic (Republican) Super PAC ban as not only do they have less support, but opposing Super PACs spend unfettered.

Super PACs may slightly "distort" the set of candidates who win.

6 Conclusion

In this paper I tackle the role of unregulated money in national level campaigns. I focus on Super PACs and their effect on House elections. I solve a novel campaign model that incorporates a variety of important factors and estimate the model using joint variation in spending and donor data. I find that while Super PACs have noisy effects that largely cancel each other out, their presence has changed the campaign finance environment. The slight changes in spending, entry, and policy may have unforeseen consequences and could affect eventual legislation. The result in the literature that Republicans are, on average, helped in the general election by Super PACs is corroborated. Super PACs seem to help challengers in primaries, and the effects on candidate positions are nuanced and varied. Incumbents rarely lose their primary, and Super PACs have only slightly changed that.

I do not study direct effects on legislative outcomes, but Super PAC funded Republican state legislature electoral gains (Klumpp, Mialon, and Williams 2016) have not necessarily resulted into major state level policy changes (Grossmann 2019). Furthermore, the literature on campaign contributions has largely found mixed effects, indicating that donors may primarily target their contributions to simply help get their preferred candidates into office (Ansolabehere, De Figueiredo, and Snyder 2003; Fowler, Garro, and Spenkuch 2020).

Social welfare 501(c)(4) nonprofits, known as "dark money" groups, are absent in this study. They do not report their spending or donors to the FEC as they do not engage in "express advocacy" for a candidate. Their ads can be tracked with advertising data, but their influence can be indirect; a growing number of Super PAC donations come from these nonprofits, providing a discreet alternative for Super PAC donors (see Supplement S.1).

Finally, Super PACs, and elections overall, are gradually shifting away from television towards the Internet. For example, the Super PAC "The Lincoln Project" focused on creating "viral" content, and the Super PAC "America First Action" operated a news website called "American Herald", spreading their content on social media. Modeling these strategies and their network effects is important for understanding 21st century campaign finance.

A Appendix

A.1 Voting and Candidate Data Details

The FEC has votes and parties for all balloted candidates in Congressional elections which had general elections occur on election day. Non-election day special elections are added from the FEC's non-prepared reports and the CQ election database. On turnout data: to measure turnout, I use population data from Census. For separate closed primaries, the population to use for turnout is different than the district total VAP as the voting population is split based on political affiliation. Since I want the relevant population for that party's primary, I adjust the population using party affiliation percentages at the state level from Gallup.

Bonica (2014) constructs a contingency table of all donor-recipient committee matches with the dollar values in each cell, then converts the dollars into counts using contribution limits (see Bonica (2014) Appendix A). He then performs a singular value decomposition on the normalized matrix. The final positions are then defined based on the eigenvalues of square of that decomposition. The measure has limitations with capturing within-party dynamics (Tausanovitch and Warshaw 2017). There is a concern with using a contribution based measure. It is primarily based on the existence of a contribution, and thus many small donors (who were less influenced by the court decision) provide the bulk of the variation. Recall the high correlation with DW-Nominate scores, which do not suffer from these problems.[58]

A.2 FEC Committee Data Details

The FEC provides committee expenditures at the transaction level for everything over $200. The groups engaged in independent expenditures (IEs) must disclose to which candidate that expenditure was targeted and whether it was for or against the candidate. The date is for when the "communication is publicly distributed or otherwise publicly disseminated" (FEC), and committees often note whether a given expenditure is aimed at the primary or general election. Campaign committee advertising spending is calculated from itemized expenditure reports. I use the self-reported transaction codes and augment that with string-matching in the description field to determine which transactions are related to ad spending. Summing over all transactions for candidates is inappropriate as the IEs are ad-spending and the expenditure files include other spending.[59] Dollars are inflation adjusted to 2015.

I do not consider Senate races as, given the limited time window of analysis, there are insufficient observations (estimation would need to be separate between House and Senate). The main downside of omitting Senate races is that the majority of Super PAC spending in Congressional races is targeted at Senate races, so the overall effect of Super PACs on Congressional outcomes cannot be fully captured in this analysis.

[58] Other methods (Ramey 2016) provide measures disconnected from contributions, but again are difficult to calculate for every challenger per district each year.

[59] About 30% of candidates running are not listed in either the FEC committee independent expenditure or candidate expenditure datasets, which is not surprising given that most candidates without spending receive trivial votes. Also there are candidates with spending that did not run during the given cycle; those are often post-cycle house-cleaning or early future fundraising (neither are large).

A.3 General Election Estimation Details

The committee's expected payoff for a given entry decision conditional on their private information $\lambda_{i_c}^G$ is denoted with $\mathcal{U}_{i_c}^G$, and p_j is i_c's belief about committee j's choice.

$$\mathcal{U}_{i_c}^G(a_{i_c}^G|\mathbf{p}_{-i_c}) = \sum_{\mathbf{a}_{-i}^G \in \{0,1\}^{2N-1}} \pi_{i_c}^G(\mathbf{S}^*|a_{i_c}^G, \mathbf{a}_{-i_c}^G) \prod_{j \neq i} p_j(\mathbf{a}_{-i}^G) + \lambda_{i_c}^G a_{i_c}^G \tag{A.1}$$

Then the probability of entry, where $u_{i_c}^G = \mathcal{U}_{i_c}^G - \lambda_{i_c}^G a_{i_c}^G$, is $p_{ic}(a_{i_c} = 1)$=$Prob[u_{i_c}^G(1|\mathbf{p}_{-i_c}) + \lambda_{i_c}^G > u_{i_c}^G(0|\mathbf{p}_{-i_c})]$, and with the Logistic distribution leads to conditional choice probabilities.

$$p_{i_c} = \frac{\exp(u_{i_c}^G(1|\mathbf{p}_{-i_c})/\sigma)}{\exp(u_{i_c}^G(1|\mathbf{p}_{-i_c})/\sigma) + \exp(u_{i_c}^G(0|\mathbf{p}_{-i_c})/\sigma)} = f(p_{-i_c}) \tag{A.2}$$

This system defines a fixed point $\mathbf{p} = f(\mathbf{p})$; note that calculating $u_{i_c}^G \forall i_c$ requires solving the general election spending stage for all combinations of entry. Rather than solving the system for $\mathbf{p}$, I flexibly estimate P_c^G and $\mathbf{p}$ based on the sufficient set of inputs (see Supplement S.5), letting $\sigma = 1$. Next, one could estimate costs comparing observed entry to the model prediction $E\left[\mathbf{X}^\top (a_{i_c} - p_{i_c})\right] = \mathbf{0}$. However we can also construct a linear regression that illustrates the variation that is identifying the parameters. Consider the log-odds ratio:

$$\log\left(\frac{p_{i_c}}{1 - p_{i_c}}\right) = \log\left[\frac{\exp(u_{i_c}^G(1|\mathbf{p}_{-i_c}))}{\exp(u_{i_c}^G(0|\mathbf{p}_{-i_c}))}\right]. \tag{A.3}$$

This can be rewritten, where the expectations for the win probability and spending are over the equilibrium entry profiles: $E[P_{i_c}^G|a_i] = \sum_{a_{-i} \in \{0,1\}^{2N-1}} P_{i_c}^G(a_1, ., a_i, ., a_N) \prod_{j \neq i} p_j^*(a_{-i})$.

$$\log\left(\frac{p_{i_c}}{1 - p_{i_c}}\right) = V_{i_c}\left(E[P_{i_c}^G|a_i = 1] - E[P_{i_c}^G|a_i = 0]\right) - g_{i_c} E[S_{i_c}^G|a_i = 1] \tag{A.4}$$

Next, isolate g and then take the logarithm of both sides. This yields the linear regression that identifies the parameters common to valuations and costs.

A.4 Primary Election Estimation Details

Consider a committee's spending first order condition in terms of the ex-ante expected probability of winning the general election $\Omega_c = \frac{\omega_{i_c}^P (S_{i_c}^P)^{1-\phi}}{\phi P_c^P (1 - P_c^P)} \equiv K_c$, where $\omega_{i_c}^P = g_{i_c}^P / (\beta_{i_c}^P V_{i_c})$. For committees whose candidates won their primary election, one can rewrite this in terms of the main unobservable: the general election probability of beating the other candidate that lost their primary (let D_1 be the opponent who won their primary):

$$E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_2}^P = 1)] = \frac{K_c - E[P(w_c^G = 1|w_c^P = 1 \cap w_{D_1}^P = 1)] P_{D_1}^P}{1 - P_{D_1}^P}. \tag{A.5}$$

This left hand side probability, denoted as EP_{CF}, takes into account general election equilibrium committee entry for the hypothetical match-up between candidates R_1 and D_2, and is thus just a function of the exogenously given objects at the start of the general election, including the unknown $\psi^G_{D_2}$. I invert this probability with respect to $\psi^G_{D_2}$:[60]

$$\psi^G_{D_2} = EP^{-1}_{CF}\left(\frac{K_c - E[P(w^G_c = 1|w^P_c = 1 \cap w^P_{D_1} = 1)]P^P_{D_1}}{1 - P^P_{D_1}}\right). \tag{A.6}$$

This approach works when a candidate considers only two potential general election opponents; this is not restrictive as many races have only two candidates that receive many votes and the vast majority of races only have two that spend non-trivially.[61] Then a fully contested primary has four candidates: two of them move on to the general and I only have to recover the general valences for the two primary losers. I recover $\psi^G_{D_2}$ from an R_1 aligned committee's spending first order condition (FOC) and $\psi^G_{R_2}$ from a D_1 committee's FOC. The logic of this approach and the sources of variation can be seen in Figure 9, which shows the inputs to the primary first order condition.

We could now estimate the following moment $E[\mathbf{X}^\top \psi^G_{D_2}|S^P_{i_{R_1}} > 0] = \mathbf{0}$, but that would require normalizing the unobserved heterogeneity cost shock $\gamma^P_{i_c}$ from the primary cost function $g^P_{i_c} = \exp(X^P_{i_c}\varphi^P_c + \gamma^P_{i_c})$. Since there is significant variation in primary spending across committee types, allowing for heterogeneous $\gamma^P_{i_c}$ is important. I exploit the structure of single-contested (only one party) and double-contested primaries to partially recover both.

When a single primary is contested, the primary committee spending first order condition system has only one counterfactual general election matchup probability (EP_{CF}) and it appears only in the primary loser's FOC.[62] We can thus rearrange the primary winner's FOC to isolate the costs, where $EP^G_c = E[P^G_c|\mathbf{w}^P]$ for the observed primary election winners:

$$g^P_{i_c} = (EP^G_c \beta^P_{i_c} V_{i_c} \phi P^P_c \cdot (1 - P^P_c))/((S^P_{i_c})^{1-\phi}). \tag{A.7}$$

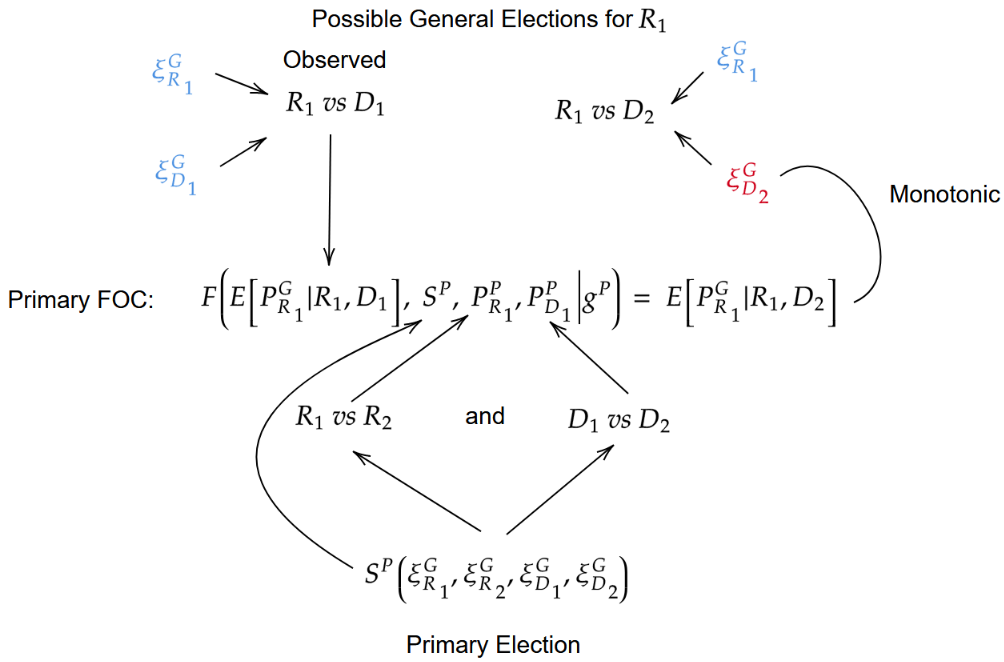
I estimate the cost function parameters φ^P_c and back out the unobserved cost shocks $\gamma^P_{i_c}$

[60] The non-closed form nature of $E[P^G_c|\theta, \mathbf{w}^P]$ makes a direct proof of invertibility difficult. Since I estimate the general election parameters first (and $E[P^G_c|\theta, \mathbf{w}^P]$ only depends on those), one can check the inversion condition per observation beforehand. Graphing the function across the range of estimated general election valences at the values for the estimated general election parameters shows that it has a sigmoid shape across observations (see Supplement Figure S1). I approximate the inverse function using a flexible polynomial (see Supplement S.5).

[61] Of primaries 2010-2016 from my sample (House elections ignoring third party), 74% have fewer than three candidates, but this is because 42% are not even contested. 55% of contested primaries have only two candidates. Among contested races with at least three, 66% have only two dominants candidates, defined as where the sum of the non-top two candidates by vote share is less than 25% of the total vote, and in 90% of races three plus, the top two receive 60% of or more of the vote. Furthermore, among primary races with three plus, in 96% of races, 90% or more of the ad-spending by candidate committees is done by the top two candidates and in 99.7% of races, 75% or more of ad-spending is done by the top two. 98% of races have the top two receiving 75% or more of outside spending. Thus in most of these elections, the smaller candidates are not in the same strategic environment as major candidates and can be added to the absenteeism count.

[62] For example, suppose the Republican primary has two candidates, R_1, R_2, but the Democratic primary is uncontested with just one candidate, D_1. Then the Republican candidates know who they will face in the general election and they know D_1's expected general election valence (since $\psi^G_{D_1}$ was already estimated). Thus R_1 can formulate their expected chances against D_1 with the only unobservable being primary cost shocks. For candidate R_2 however, since they lost the primary, their chance against D_1 was never observed, and hence is a counterfactual, EP_{CF} that is a function of the unobserved $\psi^G_{R_2}$.

Figure 9: Diagram of Primary FOC Estimation



This diagram shows the intuition behind the estimation using the primary FOC: the main inputs and the source of their identification alongside the backed-out valence (blue) and the unobserved valence (red) that forms the basis of the moment. I omit the additional arguments present in the primary spending function for notational ease.

for primary winners in single-contested races by logging the equation above. Then I use the estimated φ^P_c as known for primary winners in dual-contested primaries to partially identify their cost shocks. By plugging in φ^P_c into the FOC of a dual-contested winner, we are left with two unobservables: the unobserved general election valence of their hypothetical general election opponent who lost the primary (ψ_{CF}) and their own cost shock ($\gamma^P_{i_c}$). We can rewrite their FOC in terms of the counterfactual match-up probability (as in equation (A.5)):

$$EP_{CF}(\psi_{CF}, \psi_c|\cdot) = K_c(g^P_{i_c}(\gamma^P_{i_c}, \varphi^P_c|\cdot)|\cdot)/(1 - P^P_c) - (EP^G_c P^P_c)/(1 - P^P_c). \tag{A.8}$$

The left hand side of equation (A.8) is bounded between 0 and 1. I refine its bounds by exploiting valences estimated from the vote share regressions. Consider the following valence bounds: $\psi^{LB}_{obs} = \min(\psi_{obs}) - \text{std}(\psi_{obs})$ and $\psi^{UB}_{obs} = \max(\psi_{obs}) + \text{std}(\psi_{obs})$. I combine these realistic bounds with the data and estimated parameters to calculate significantly tighter bounds on EP_{CF}. I solve the general election (equation (3.6)) for the pair of candidates in question, substituting in the unobserved valence ψ_{CF} for each bound to calculate bounds $EP_{CF} \in [EP_{CF}(\psi^{LB}_{obs}, \psi_c|\cdot), EP_{CF}(\psi^{UB}_{obs}, \psi_c|\cdot)]$.

I plug these bounds into equation (A.8) and back out cost shock bounds: $(\gamma^{P,LB}_{i_c}, \gamma^{P,UB}_{i_c})$. I generate an estimated cost function by drawing cost shocks from a uniform distribution based on the estimated bounds per committee $\gamma^P_{r,i_c} \in U[\gamma^{P,LB}_{i_c}, \gamma^{P,UB}_{i_c}]$: $g^P_{i_c} = \sum_r \exp(X^P_{i_c}\varphi^P_c + \gamma^P_{r,i_c})$.

This provides substantially more information than normalizing the cost shock.[63]

Next, I plug the estimated $g^P_{i_c}$ for the primary winners into equation (A.8) to recover ψ_{CF} for the losers in the double-contested primary. For this we need the inverse function EP^{-1}_{CF} from equation (A.6). We can then estimate the cost function parameters and unobserved cost shocks for the double-contested primary losers. This follows the exact same method as for the single-contested primary winners, but now we estimate the cost function parameters φ^P_c and back out the unobserved cost shocks $\gamma^P_{i_c}$ for primary losers in double-contested races.

Finally, I estimate the cost function and ψ^G_{CF} for losers in single-contested primaries. They have one unknown EP_{CF}, and so we can mirror the approach used for primary winners in double-contested primaries: plug in the estimated φ^P_c from double-contested losers, construct bounds on ψ^G_{CF}, solve for the bounds on EP_{CF}, back out bounds on the unobserved cost shocks, average across draws from a distribution to recover the cost function $g^P_{i_c}$, and finally use the function EP^{-1}_{CF} to recover the unobserved valence for single-contested primary losers.

Next I estimate a fixed entry cost: construct the log-odds ratio for equilibrium entry and isolate the cost term, as in equation (A.9). This is possible because all other terms are known from previous estimation steps.[64] I estimate fixed costs for non-candidate committees with heterogeneity across party, committee type, and incumbency status.

$$F^P_{i_c} \cdot a^P_{i_c} = V_{i_c}\left(E[P^G_c|a^P_i = 1] - E[P^G_c|a^P_i = 0]\right) - g^P_{i_c} E[S^P_{i_c}|a^P_i = 1] - \log\left(\frac{p^P_{i_c}}{1 - p^P_{i_c}}\right) \tag{A.9}$$

A.5 Candidate Stages Estimation Details

Next, recall the equilibrium probability equation (4.7). This leads to a simulated equation, shown for a given draw (I use 100 non-entrant valence draws for results), using semi-parametric estimates for $\mathbf{p}$ (see Supplement S.5).[65] Consider the expected payoff for a given choice, where $P^G_c = E[P^G_c|\mathbf{d}]$:

$$E[\pi_c(d_c|\mathbf{p}_{-c})] = \sum_{\mathbf{d}_{-c}\in|\theta|^{2N}} \left(V_c P^G_c + V^0_c(1 - P^G_c) - (d_c - \bar{\theta}_c)^2 \cdot \mathbb{1}[d_c > 0]\right) \prod_j p^*_j(\mathbf{d}_{-c}) \tag{A.10}$$

$$= (V_c - V^0_c) \sum_{\mathbf{d}_{-c}\in|\theta|^{2N}} \left(P^G_c\right) \prod_j p^*_j(\mathbf{d}_{-c}) + V^0_c - (d_c - \bar{\theta}_c)^2 \cdot \mathbb{1}[d_c > 0]. \tag{A.11}$$

The log-odds ratio is then based on the difference, $\Delta_{\theta,\theta'}(\cdot)$, in "benefits" and costs: $\log\left(\frac{\hat{p}(d_c=\theta)}{\hat{p}(d_c=\theta')}\right) = (V_c - V^0_c) \cdot \Delta_{\theta,\theta'}\left(\sum_{\mathbf{d}_{-c}\in|\theta|^{2N}} \left(P^G_c\right) \prod_j p^*_j(\mathbf{d}_{-c})\right) - \Delta_{\theta,\theta'}((d_c - \bar{\theta}_c)^2)$.

For challengers, the cost term is known. Also, we can calculate $E[P^G_c|\mathbf{d}]$ for any position and $\mathbf{p}$ is already estimated. I estimate the nonlinear equation below, where $\Delta\mathcal{R} \equiv$

[63] The large set of parameters and number of estimation steps remaining make set inference infeasible. For non-entrant committees/candidates, I draw from $N(\text{mean}(\gamma^P), \text{std}(\gamma^P))$.

[64] To utilize entry variation, we need to calculate all possible primary elections for different committee entry profiles; I flexibly estimate post-entry win probability per primary P^P_c and the committee equilibrium entry probabilities $\mathbf{p}^p$ based on the (nearly) sufficient set of inputs to predict them (see Supplement S.5).

[65] I use the cost function parameters φ^P_c from the primary loser truncated regressions for non-entrants.

$\Delta_{\theta,\theta'}(\sum_{\mathbf{d}_{-c}\in|\theta|^{2N}}(P_c^G)\prod_j p_j^*(\mathbf{d}_{-c}))$. Since the left hand side in (A.12) is the difference in valuations, W and W^0 cannot share covariates.

$$V_c - V_c^0 = \left(\log\left(\frac{\hat{p}(d_c=\theta)}{\hat{p}(d_c=\theta')}\right) + \Delta_{\theta,\theta'}((d_c-\bar{\theta}_c)^2)\right)/(\Delta\mathcal{R}) \tag{A.12}$$

Finally, I consider the incumbent's decision. I estimate their valuations and ideal point. I use maximum likelihood and average across challenger draws to construct the incumbent's expected win probability per choice. Their summation is over policies excluding non-entry.[66]

$$\sum_{w=1}^{\Theta}(d_I-1)\log\left(\frac{\exp(\pi_I(d_I=\theta))}{\sum_{w=1}^{\Theta}\exp(\pi_I(d_I=w))}\right) \tag{A.13}$$

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[66] There are very few instances in the data of an incumbent un-expectantly deciding not to re-run (as opposed to a previously announced retirement for example); from 2011-2020, of the 243 non-rerunning incumbents, 70% announced it before election-year and of the remaining 30% who mentioned it during election-year, 65% mentioned it within January and February, well before their primaries (Ballotpedia 2020). Thus I omit their re-entry choice and consider the race open when in the data they do not re-run.

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S Supplementary Material

S.1 Background

Tables S1 and S2 detail the spending limits and restrictions.

Independent expenditures (IEs) were created by the 1976 *Buckley v. Valeo* case that allowed unlimited spending on political messaging (by individuals or PACs).[1] *SpeechNOW v. FEC* (not a Supreme Court ruling, but a DC court of appeals), ruled that individuals could contribute unlimited funds to committees that make IEs. The SpeechNOW committee wanted to raise funds for IEs without forming a PAC (to avoid limiting itself to receiving at most $5,000 per person). The court ruled that if the organization is IE only (not a PAC that can make both direct contributions and IEs), then it has no restrictions on fundraising (still no foreign funding however). This allowed individuals to basically pool IEs through Super PACs, making large sums more coordinated. Before *SpeechNow* individuals could either donate to a PAC (subject to contribution limits) or act on their own (not with a PAC).

Prior to *Citizens United v. FEC*, corporations had to form their own PACs. The case allowed corporations and unions to use their general treasury funds to make IEs. This was partially a response to the 2002 "Bipartisan Campaign Reform Act". Part of this act was the banning of "electioneering communications" (EC) [TV ads mentioning candidates 60 days prior to general or 30 days prior to primary] by non-PACs. It also prohibited corporations and unions from spending on ECs. See Prato and Wolton (2017) for a discussion.

While most committees fall cleanly between independent expenditure-only (Super) or traditional PACs, the district court case *Carey v. FEC* allowed for the formation of the "hybrid PAC" (Carey Committee), which is a single PAC that operates as both a traditional PAC and Super PAC, with the requirement that the funding for each activity stems from two separate bank accounts. Unlimited donations aimed at independent expenditures originate from one and none of that money can be used in coordination expenses, and vice-versa.

Spending on ads that do not support/oppose a candidate (issue advocacy) are less regulated. If the issue ad mentions a candidate and is within 60(30) days of a general (primary) election, then the ad must be disclosed (called an electioneering communication). Furthermore, prior to 2010, corporations could not make ECs; *Citizens United* overturned that.[2]

"Hard money" is money donated with a donation limit. "Soft" money has no cap and has been limited to parties ever since the Federal Election Campaign Act (FECA) of 1971 and were subsequently upheld in the 1976 *Buckley v. Valeo* case and were further limited in the 2002 "Bipartisan Campaign Reform Act". There is substantial legal scholarship on IEs and soft money.[3] Some consider IEs to be the new form of soft money (Tokaji and Strause 2014). The 2014 *McCutcheon v. FEC* case overruled some of the 2002 "Bipartisan Campaign Reform Act" (The BCRA was upheld in the 2003 case *McConnell v. FEC*), removing "aggregate contribution limits" made to national parties and federal candidate

[1] An IE is for a communication "expressly advocating the election or defeat of a clearly identified candidate that is not made in cooperation, consultation, or concert with, or at the request or suggestion of, a candidate, a candidate's authorized committee, or their agents, or a political party or its agents" [11 CFR 100.16(a)].

[2] The 2007 case *Wisconsin Right to Life v. FEC* loosened the restrictions on what classified ads to be EC, allowing more politically charged non-EC ads.

[3] Parties are nontrivial in spending even after 2002 (also noted in Lax, Phillips, and Zelizer (2019)).

committees (the total amount one can give across all contributions in a cycle). This made it possible for individuals to give to many more candidates.[4]

Social welfare nonprofits (501(c)(4)s), known as "dark money" groups, used to be limited in that they could not directly do IEs. They could still lobby and make non-EC issue ads. *Citizens United* allowed them to make political expenditures; they still cannot spend the majority ($> 50\%$) of their operating budget on these funds. But they do not need to disclose their donors and they can raise unlimited amounts (see Oklobdzija (2018) for a network analysis). 501(c)(4) spending totaled 257 million in 2012 ($\approx 20\%$ of outside spending), but declined to 106 mil. in 2018.[5] Also, as the Center for Responsive Politics reports, they often spend earlier in the cycle, they often do not target individual candidates, and most of their spending occurs well before Super PAC spending. Finally, when a 501(c)(4) donates to Super PAC, the original donor is undisclosed. This is allowed as long as the donor does not instruct the 501(c)(4) to give to the Super PAC; otherwise they risk being a "straw donor".

Table S1: Campaign Contribution Limits for 2017-2018 Federal Elections

		Recipient				
		Candidate committee	**PAC† (SSF and nonconnected)**	**Party committee: state/district /local**	**Party committee: national**	**Additional national party committee accounts‡**
Donor	**Individual**	$2,700* per election	$5,000 per year	$10,000 per year (combined)	$33,900* per year	$101,700* per account, per year
	Candidate committee	$2,000 per election	$5,000 per year	Unlimited transfers	Unlimited transfers	
	PAC: multicandidate	$5,000 per election	$5,000 per year	$5,000 per year (combined)	$15,000 per year	$45,000 per account, per year
	PAC: nonmulticandidate	$2,700* per election	$5,000 per year	$10,000 per year (combined)	$33,900* per year	$101,700* per account, per year
	Party committee: state/district/local	$5,000 per election	$5,000 per year	Unlimited transfers	Unlimited transfers	
	Party committee: national	$5,000 per election**	$5,000 per year	Unlimited transfers	Unlimited transfers	

Source: Federal Election Commission https://transition.fec.gov/info/contriblimitschart1718.pdf. This table describes the various campaign contribution limits by the different combinations of donor and recipient. The footnotes are defined as follows: $*$:"Indexed for inflation in odd-numbered years." $**$: "Additionally, a national party committee and its Senatorial campaign committee may contribute up to $47,400 combined per campaign to each Senate candidate." †: " "PAC" here refers to a committee that makes contributions to other federal political committees." ‡: "The limits in this column apply to a national party committee's accounts for: (i) the presidential nominating convention; (ii) election recounts and contests and other legal proceedings; and (iii) national party headquarters buildings. A party's national committee, Senate campaign committee and House campaign committee are each considered separate national party committees with separate limits."

[4]The 2011-2012 limits: $46,200 for federal candidates + $70,800 for national parties = $117,000 limit.

[5]See CRP. This spending is predominantly issue ad based. Any IEs or ECs must be reported to the FEC. Few 501(c)(4)s file reports with the FEC so either these groups stick to non-EC issue ads or do not properly disclose. 501(c)(5) unions and 501(C)(6) trade associations have similar rules but spend much less.

Table S2: Contribution and Spending Regulations

Can Make IEs	Pre-2010	Post-2010
Individuals	yes	yes
Corporations & Unions	no	yes
Traditional PACs & Parties	yes	yes
Super PACs	n/a	yes
Can Contribute to Any Committee	**Pre-2010**	**Post-2010**
Individuals	yes	yes
Corporations & Unions	no	no
Traditional PACs & Parties	yes	yes
Super PACs	n/a	no
Can Contribute to Super PACs	**Pre-2010**	**Post-2010**
Individuals	n/a	yes
Corporations & Unions	n/a	yes
Traditional PACs & Parties	n/a	yes
Super PACs	n/a	no
Fundraising Limits for IEs	**Pre-2010**	**Post-2010**
Individuals	n/a	n/a
Corporations & Unions	n/a	n/a
Traditional PACs & Parties	yes	yes
Super PACs	n/a	no

This table gives the different rules prior to and after the major 2010 campaign finance law changes for independent expenditures and contributions by the main entities spending in elections. Independent expenditures (IEs) are communications not coordinated with the candidate or party. Those who can contribute can also makes Coordinated expenditures, which are communications that can be coordinated with the candidate. "Can Make IEs" refers to whether or not those entities are allowed to spend their own money on an IE. "Can Contribute to Any Committee" refers to whether or not those entities can give money directly to a candidate's election committee. "Can Contribute to Super PACs" refers to whether or not those entities can give money directly to a Super PAC. "Fundraising Limits for IEs"refers to whether or not those who can make IEs have limits on fundraising (for those that legally engage in fundraising).

S.2 Model Characterizations

As noted in the proposition, I focus on equilibria where agents condition on payoff relevant actions: recall that I allow players to observe all actions from previous stages (hidden actions complicate defining the equilibrium). Thus for example, voters observe fundraising effort and an equilibrium might exist with voters conditioning on effort. Committee effort is not payoff-relevant to voters conditional on spending.

The general election spending stage has a solution, but uniqueness is not guaranteed given the convexity of the exp function in P_c^G, but the equilibria can be characterized. Existence and uniqueness of the entry stage is easier to demonstrate. Existence of the primary spending stage is straightforward, but uniqueness is not. However simulation evidence suggests that a sufficient expression can be empirically validated in the voter preferences estimation before needing to solve the model, thus we can check uniqueness before relying on it in a given observation for the counterfactual simulation. Existence and uniqueness for the primary entry stage is similar to the general election argument. Challenger decisions are a generalization of the entry stages and the incumbent's decision is straightforward. The equilibrium properties of each stage are discussed in Lemmas 2-7 in Supplement S.9.

S.3 Wealthy Donor Variation

Donors can give unlimited amounts to Super PACs, so if there are donors who want to spend a large amount in a given race, their most efficient option is to go through Super PACs. Thus a Super PAC's incentive to invest in a race is largely influenced by whether there are such donors who will support that. Since Super PACs raise significant funds from these donors, they are especially vulnerable to a downward shock in how much that donor gives.

While Super PACs are arguably more sensitive to large swings in donor incomes, donor variation may be weakened by the fact that reported incomes are right censored and the wealthy are less sensitive to local economic shocks; their contributions respond to a variety of factors (Larreguy and Teso 2018; Broockman, Ferenstein, and Malhotra 2019). Other sources of donor variation are how much they give unrelated to a given race, how much their "network"/neighbors give, or lagged giving. Conceptually, it is difficult to define mega-donor pre-2010 without defining them via multiple candidates (due to contribution limits), and even that is limited before the 2014 case *McCutcheon v. FEC*.

I consider variation in individual large donor housing values, real-estate prices, taxes, zip code level mortgage information, and other financial indicators that are proxies for their financial well-being. The address level real-estate transaction data are from Corelogic's nationwide database on deeds and taxes (first used to study donors in Zhao (2019)).[6] The zip code data on mortgage performance and origination are also from Corelogic. I track the financial well-being changes for that individual and zip code over time, which may affect how much the donors give. I weight each shock by the amount that the citizens in that zip code gave to same-party candidates in the previous election; since I average the shocks across donors/zip codes for a given committee, equal weight on each means the locations

[6]Corelogic's database (via Princeton's Data and Statistical Services) on deeds contains every assessment from county offices dating back to the 1990s; the tax data goes back to 2005; the residential mortgage performance data are for 65% of all active mortgages with their originations back to 1998.

with few donors with a big shocks have a larger effect on the committee than locations with many donors with small shocks. This is likely not the case and may weaken the instrument. Results for these IVs with pre-2018 data are similar.[7]

I also scraped the Forbes list of U.S. citizen billionaires since 2010 (following Bonica and Rosenthal (2015)). Forbes states their wealth that year but does not retain historical records. I used the Internet Archive's "Wayback Machine" to find archived versions of the page to get variation over time. I matched this list of names (601 in total) to campaign contributions (2010-2016 cycles) using a fuzzy-matching algorithm with a Levenshtein-distance cutoff of 99.2%. In total, 24.6% of them gave over $100,000 to Super PACs, but that only represented 20.6% of $100,000+ donations to Super PACs. They have a higher degree of repeated giving than non-billionaires (average 9.46 instances of giving compared to non-billionaire population of 5.05), but only 137 Super PACs out of the 309 that received $100,000+ had a billionaire donor. Family members would not be matched. Many committees do not receive any money from a billionaire, so there are too many zeros to rely on this measure.[8]

S.4 Polling Data

Polling data has issues, such as more polling for competitive races, some races only having polling late into the election, and others having early in the election. Polling data variation includes between and within elections; there is variation across time within an election for some races, but the intervals are not uniform. I do not include polling data recorded throughout the election. One could track the spending effects on each poll up until election day, turning the cross-section into a panel, but this is complicated as the presence of a poll is endogenous. Many Congressional races will only have a couple polls throughout the entire election cycle, and the most competitive races have the most polling done. For example, in 2010 only 8 races had 7 polls on different days (Incerti 2018); with the vast majority having less than 2. Presidential and Senate races have significantly more polling coverage.

S.5 Approximations

The main object needed to calculate the equilibrium is the expected win probability (EPG) from the candidate's perspective; below it is written for challengers and incumbents have another layer integrating of challenger decisions $\sum_{\mathbf{d}'_C} E[P_I^G(d_I|\mathbf{d}'_C)] \prod p(\mathbf{d}'_C)$.

$$E[P_c^G|\mathbf{d}] = \sum_{a_i^P \in \{0,1\}^{4N}} \left[\sum_{a_i^G \in \{0,1\}^{2N}} P_c^G(\mathbf{a}^\mathbf{P}, \mathbf{a}^\mathbf{G}|\mathbf{d}) \prod_j p_j^*(a_j^G) \right] \prod_j p_j^*(a_j^P).$$

I approximate the P and p functions given the observed d_c. Approximating p is straightforward (Bajari, Hong, Krainer, and Nekipelov 2010). Next is the general election probability

[7] I use the change in: zip code level income, house sale price, house value, house tax, zip mortgage balance, zip mortgage interest rate, zip foreclosure rate, zip days delinquent, and zip max days delinquent.

[8] We also have information on the donor's profession through their self-reported occupation. Looking at the industry performance of their profession may provide indicator's of that donor's financial well-being, but this does not work for some donors, such as the retired. Also it is missing for a non-trivial number of donors.

of the candidate winning conditional on a given entry profile P.[9] To solve for each $u^G_{i_c}$, we need to know the equilibrium spending per entry profile.[10] I estimate P^G_c as a function of entry and use the first order condition to get the implied spending; if one knows the equilibrium probability of winning for that given entry profile, spending is the implicit function: $S^G_{i_c} = ([V_{i_c}/g_{i_c}]\beta_{i_c}\phi P^G_c(1-P^G_c))^{1/(\phi-1)}$.[11]

The general election probability of winning as a function of entry P^G_c: regression of the log-odds of the probability of winning on the sufficient inputs to characterize post-entry decision-making: the ratio of effectiveness times valuations divided by costs for all committees of the two general election candidates, the sum of observed and unobserved district/candidate characteristics (including candidate's policy) times their coefficients from the voter regression, and indicators for whether the committee entered. The adjusted R^2 is 0.98 with just linear terms and there are 1656 observations. An additional indicator of the fit: if no outside committee enters and all variables are identical, then in theory one should get a perfect 50% win probability. I find it to be 0.5050 for the Republican in the general election, so the approximation to the equilibrium function is quite close.

The general election equilibrium conditional (choice) entry probabilities (CCP) for the 6 different kinds of committees with entry decisions (Republican and Democratic Super PACs, parties, and PACs) p^G: polynomial logit regressions of the entry decision on the near-sufficient inputs to predict entry. These include the same inputs from predicting P^G above (except entry of course) but now fully interacted with different coefficients for each of the 6 committees; I include non-collinear polynomial combinations. The fit for R-SPAC is 0.81, R-party 0.81, R-PAC 0.90, D-SPAC 0.80, D-party 0.84, D-PAC 0.71; 1656 observations each.

The general election valence inversion function: this approximates the EP^{-1}_{CF} function of how the EPG changes with expected valence, conditional on all sufficient inputs to predict the EPG. It is a polynomial (non-collinear interactions and squares) of the EPG, all inputs from predicting the CCP, with the slight alteration of separating out the opponent valence as a separate input from the candidate characteristics term (necessary for future steps of estimation). The adjusted R^2 is 0.97 with 1656 observations.

The primary election probability of winning as a function of entry P^P: the primary versions of the same set of inputs for predicting P^G alongside the EPGs for each match-up combination for all entrant candidates. For Republican primaries, the adjusted R^2 is 0.78 with linear terms and there are 958 observations. For Democratic primaries, the adjusted R^2 is 0.83 with linear terms and there are 758 observations (both only contested primaries).

[9] This approximation simply is a shortcut to solving all combinations of general election spending stages with a single function that takes the entry profile and the minimal set of exogenously given variables at that point to give a prediction of the probability.

[10] A direct approach would be to solve for the spending stage equilibrium per entry profile. The downside of using this approach is that when one wants to consider counterfactual scenarios of committee entry, it is necessary to solve for the spending level for that given entry profile. An alternative approach is to use entry rather than spending as the input. This approach however then requires one to include additional inputs that enter the probability indirectly through spending (thus they were not required to be inputted before). These include the data from estimating committee parameters. In the case of the primary election, this additionally includes the expected general election probabilities of the counterfactual match-ups, which are functions of the unobserved valences for the candidates that did not win their primary.

[11] This requires assuming that the counterfactual choices based on the same primitives from the observed data use the same equilibrium.

The primary election equilibrium conditional (choice) entry probabilities (CCP) for the 12 different kinds of committees with entry decisions (Incumbent and challenger - Republican and Democratic Super PACs, parties, and PACs): p^p. The (non-entry) same inputs used for predicting P^P but separating out valuations from costs as distinct inputs. Given the limited variation, I group the estimation and use limited polynomials. The fit between prediction and data are: 0.66, 0.26, 0.86, 0.51, 0.30, 0.63, 0.92, 0.29, 0.85, 0.73, 0.32, 0.50 for Inc R SPAC, Inc R Party, Inc R PAC, Cha R SPAC, Cha R Party, Cha R PAC, Inc D SPAC, Inc D Party, Inc D PAC, Cha D SPAC, Cha D Party, Cha D PAC, respectively.

The challenger choice probabilities for Republicans and Democrats, with three choices each (do not enter, moderate position, and extreme position): inputs are EPGs for candidate position combinations and valuation predictors. For Republican challengers, the fit is 0.76, 0.49, and 0.45 for three positions respectively, and for Democrats it is 0.75, 0.48, 0.53.

S.6 Estimation Routine

```
#1 Estimate General & Primary voter equations
#2 Esimate General first order conditions
#3 Approximate General Probability & CCP
#4 Construct General Probability prediction and EPG pre-general
#5 Estimate General entry conditions
#6 Using fully estimated general, approximate inverse EPG pre-general w.r.t. valence
#7 Estimate Primary FOC first order conditions
        #7.1 Single-contested primary winner FOC
        #7.2 Double-contested primary winner FOC
        #7.3 Double-contested primary loser FOC
        #7.4 Single-contested primary loser FOC
#8 Estimate Primary entry conditions
#9 Construct all entrant valences and match-up EPG pre-general and EPG pre-primary
#10 Approximate Primary Probability & CCP
#11 Estimate Non-entrant Valence distribution
#12.1 Loop over non-entrant Valence draws
        #12.2 Loop over all incumbent decisions
                #12.3 Loop over all challenger decisions
                        #12.4 Construct EPG pre-challenger across policy
                #12.5 Approximate challenger CCPs
                #12.6 Construct incumbent EPG pre-incumbent
#13 Estimate challenger policy conditions using observed incumbent decision
#14 Estimate incumbent policy conditions
```

S.7 Normalization of σ_ξ

With a value for ψ_c^G and σ_ξ, one can plug in observed spending and candidate characteristics to calculate the probability of winning. The probability from the committee's perspective will be biased towards 0.5 (meaning closer to the observed vote share excluding abstention) from above and below if the specified uncertainty is too high, and biased towards the corners if the specified uncertainty is too low. While I find that $\mathrm{Var}(\hat{\xi}_c^G) = 0.52$ and $\mathrm{Var}(\hat{\xi}_c^P) = 1.04$, it is likely not equal to σ_ξ^2 (the committee's uncertainty about candidate quality).[12] To study the sensitive, I re-run the estimation and look at the general election vote share counterfactuals, but changing uncertainty normalizations of σ_ξ. Average results are not

[12] Gordon and Hartmann 2016 estimate σ_ξ from the FOC by exploiting pre-spending race competitiveness ratings. I include those as covariates in the vote share regression and they have a normalized cost function.

significantly different across 10%, 25%, and 50% reductions. For a 50% change, the center and tails of the distribution become more prominent, due to the higher degree of certainty.

S.8 Additional Tables

Table S3: General Election Voter Regression: OLS

DV: Log Share Of Votes Minus Log Abstention Share			
Candidate Spending	0.0034	Average Ad Cost Per Committee	0.0299
	(0.0019)		(0.0176)
-(Opponent Super PAC Spending)	0.0002	District White Percentage	0.3073**
	(0.0008)		(0.1016)
Party and PAC Spending	0.0083***	District Male Percentage	-2.0899
	(0.0020)		(1.4911)
Candidate Position	-0.0145	R x District High-School Rate	0.0042
	(0.0186)		(0.0031)
Within-State Candidate Donor Zip Income Variation	-0.0584*	R x District Median Age	-0.0006
	(0.0287)		(0.0056)
Within-State Party Donor Zip Income Variation	-0.1408*	R x Lagged Republican Presidential Votes	2.8101***
	(0.0684)		(0.2143)
Within-State Super PAC Donor Zip Income Variation	-0.0409*	R x District White Percentage	0.3581*
	(0.0205)		(0.1435)
Within-State Candidate Donor Ideology Variance	0.2865***	R x District Male Percentage	0.9904
	(0.0585)		(1.9331)
District Unemployed Rate	0.0092	Incumbent x District Unemployed Number	0.0124***
	(0.0111)		(0.0035)
District Income	0.0974***	Incumbent x District Unemployed Rate	-0.0011
	(0.0126)		(0.0122)
District Unemployed Number	-0.0373***	Incumbent x Lagged Incumbent Votes	0.1822**
	(0.0034)		(0.0605)
Lagged Republican Presidential Votes	-1.1542***	Incumbent x District Income	-0.0349*
	(0.1330)		(0.0137)
Incumbent	5.6512	Inc=0 x Party=D x Cook's Competitiveness	-0.0064
	(19.6501)		(0.0150)
Party=Republican	93.3354***	Inc=0 x Party=R x Cook's Competitiveness	0.0641***
	(11.2374)		(0.0107)
Lagged Incumbent Votes	-0.2199***	Inc=1 x Party=D x Cook's Competitiveness	0.0533*
	(0.0464)		(0.0234)
Number Of Senate Candidates	-0.0013	Inc=1 x Party=R x Cook's Competitiveness	-0.0324
	(0.0012)		(0.0355)
Contested Primary	-0.0398	Inc=0 x Party=D x Cycle Time Trend	0.0500***
	(0.0352)		(0.0108)
Governor Same Party	0.0041	Inc=1 x Party=D x Cycle Time Trend	0.0474***
	(0.0175)		(0.0056)
District High-School Rate	-0.0142***	Inc=0 x Party=R x Cycle Time Trend	0.0026
	(0.0031)		(0.0097)
District Median Age	0.0318***	Constant	-1.0e+02***
	(0.0044)		(21.6954)
District Election Day Precipitation	-0.0129		
	(0.0707)		
Fixed effects	State and cycle	F	105.4564
Observations	3514	R^2	0.697

Robust standard errors in parentheses. This shows a regression of the different in general election log vote share from absenteeism share on election spending, candidate position, and various controls. Both columns are from the same regression.

Table S4: General Election Voter Regression Summary Statistics

Variable	Mean	Std. Dev.	Min.	Max.
Log Share Of Votes Minus Log Abstention Share	-0.757	0.773	-7.435	1.099
Candidate Spending	4.520	5.94	0	50.221
Super PAC Spending	1.421	4.116	0	45.695
Party Spending	1.758	4.852	0	36.874
PAC Spending	0.553	1.664	0	20.813
Candidate Position	1.458	0.498	1	2
Out Of District Lagged Position	0.505	0.232	0	2.198
Out-Of-State Candidate Donor Zip Income Variation	0.48	0.543	-1.456	5.704
Out-Of-State Party Donor Ideology Variance	0.133	0.091	0	0.556
Out-Of-State Candidate Donor Ideology Variance	0.258	0.186	0	1.605
Out-Of-State Super PAC Donor Ideology Variance	0.136	0.091	0	0.815
Out-Of-State PAC Donor Zip Income Variation	0.201	0.231	-0.768	3.503
Out-Of-State Opponent Donor Zip Income Variation	0.678	0.861	-2.432	7.049
Out-Of-State Opponent S-PAC Donor Ideo. Var.	0.144	0.316	-2.452	2.872
Within-State Candidate Donor Zip Income Variation	0.301	0.41	-1.806	5.517
Within-State Party Donor Zip Income Variation	0.161	0.197	-0.373	3.783
Within-State Super PAC Donor Zip Income Variation	0.254	0.498	-3.26	5.05
Within-State Candidate Donor Ideology Variance	0.331	0.167	0	1.531
District Unemployed Rate	6.428	2.479	2.142	16.869
District Income	8.049	1.435	5.267	15.369
District Unemployed Number	9.034	6.58	1.271	29.548
Lagged Republican Presidential Votes	0.484	0.152	0.03	0.825
Incumbent	0.46	0.498	0	1
Party=Republican	0.501	0.5	0	1
Lagged Incumbent Votes	0.581	0.249	0	1
Number Of Senate Candidates	7.95	7.385	0	27
Contested Primary	0.885	0.32	0	1
Governor Same Party	0.485	0.5	0	1
District High-School Rate	29.17	6.143	11.2	46.757
District Median Age	40.237	3.454	29.306	51.269
District Election Day Precipitation	0.088	0.141	0	1.052
Average Ad Cost Per Committee	6.648	0.701	4.659	9.000
District White Percentage	0.754	0.174	0.16	0.968
District Male Percentage	0.491	0.01	0.457	0.537
Cook's Competitiveness	0.446	2.787	-3	3
N	3536			

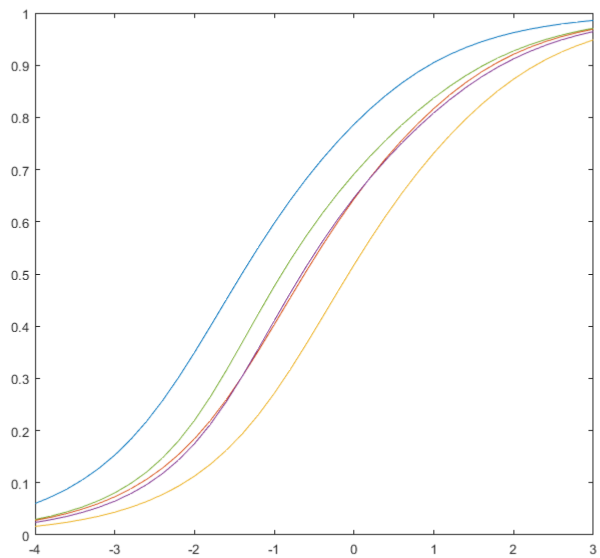
This table shows the summary statistics for the variables used in the estimation of general election voter preferences. Spending by each committee, district income, district unemployment number, and precipitation is scaled as followed: $(X/1e3)^{0.5}$.

Table S5: General Election Voter Parameters: Controls

	1st Stage 1	1st Stage 2	1st Stage 3	1st Stage 4	2nd Stage
Within-State Candidate Donor Zip Income Variation	0.2803	-0.0127	-0.1524	0.1074***	-0.0381
	(0.1822)	(0.4531)	(0.2202)	(0.0226)	(0.0357)
Within-State Party Donor Zip Income Variation	-0.3212	1.4126	1.0325	-0.1147	-0.2416*
	(0.4860)	(1.0111)	(0.5982)	(0.0725)	(0.1051)
Within-State Super PAC Donor Zip Income Variation	-0.0251	-0.6206	0.1295	-0.0117	-0.0329
	(0.2113)	(0.5285)	(0.2945)	(0.0202)	(0.0235)
Within-State Candidate Donor Ideology Variance	0.3643	-0.4461	-0.8091	-0.0526	0.1596*
	(0.4828)	(0.9210)	(0.5097)	(0.0639)	(0.0673)
District Unemployed Rate	0.1507	-0.2538	0.0459	0.0058	0.0073
	(0.1000)	(0.1921)	(0.1199)	(0.0104)	(0.0123)
District Income	0.0405	-0.0711	-0.0658	0.0075	0.0990***
	(0.0718)	(0.2151)	(0.0766)	(0.0101)	(0.0136)
District Unemployed Number	-0.1882***	0.0329	-0.1232***	-0.0035	-0.0292***
	(0.0249)	(0.0530)	(0.0318)	(0.0027)	(0.0041)
Lagged Republican Presidential Votes	1.9836	7.5329*	-1.4218	-0.3375**	-1.4231***
	(1.3873)	(3.1590)	(1.6470)	(0.1252)	(0.1745)
Incumbent	177.4192	-8.3e+02*	3.2859	25.7424	18.2572
	(210.2473)	(373.3865)	(206.8778)	(18.7635)	(22.9349)
Party=Republican	71.1369	158.5742	-51.9656	56.7934***	95.1111***
	(111.2091)	(270.9655)	(115.7772)	(10.3234)	(16.3418)
Lagged Incumbent Votes	-1.7525***	3.4291***	-1.9051***	0.0170	-0.1654**
	(0.4336)	(0.9164)	(0.5061)	(0.0445)	(0.0515)
Number Of Senate Candidates	-0.0201	0.0246	-0.0012	0.0001	-0.0008
	(0.0109)	(0.0246)	(0.0132)	(0.0012)	(0.0013)
Contested Primary	0.0709	1.3445	0.1133	0.1239***	-0.0182
	(0.3782)	(0.9754)	(0.4501)	(0.0345)	(0.0441)
Governor Same Party	-0.0562	0.4028	-0.2985	0.0625***	0.0179
	(0.1637)	(0.3779)	(0.1866)	(0.0157)	(0.0229)
District High-School Rate	-0.0358	0.1437*	0.0369	-0.0106***	-0.0175***
	(0.0293)	(0.0621)	(0.0311)	(0.0029)	(0.0040)
District Median Age	0.0034	-0.0873	0.0030	0.0289***	0.0394***
	(0.0434)	(0.0943)	(0.0500)	(0.0042)	(0.0067)
District Election Day Precipitation	0.7545	0.5074	-0.3822	0.0101	-0.0394
	(0.5947)	(1.4060)	(0.5781)	(0.0592)	(0.0768)
Average Ad Cost Per Committee	-0.7612***	-0.4803	-0.3520	-0.0710***	0.0478
	(0.1840)	(0.3760)	(0.2350)	(0.0184)	(0.0255)
District White Percentage	0.5022	-4.4618**	0.4368	0.0902	0.3666**
	(0.7982)	(1.6983)	(0.8387)	(0.0834)	(0.1153)
District Male Percentage	27.8203*	6.9216	25.2859	0.4257	-3.6735*
	(12.4005)	(26.5611)	(13.4319)	(1.2855)	(1.6944)
R x District High-School Rate	0.0271	-0.0079	0.0266	0.0062*	0.0040
	(0.0278)	(0.0615)	(0.0311)	(0.0028)	(0.0036)
R x District Median Age	0.0476	-0.1460	0.0821	-0.0361***	-0.0099
	(0.0526)	(0.1079)	(0.0569)	(0.0050)	(0.0092)
R x Lagged Republican Presidential Votes	-3.3104	3.3585	-0.1849	1.1498***	3.1559***
	(1.8061)	(4.0597)	(2.1593)	(0.1618)	(0.3057)
R x District White Percentage	-0.6410	2.1613	-0.4032	-0.1437	0.3243*
	(1.0394)	(2.1356)	(1.1015)	(0.1166)	(0.1558)
R x District Male Percentage	-12.3622	-23.6431	-2.4519	2.1511	2.4989
	(15.2209)	(34.3866)	(17.9240)	(1.7503)	(2.1762)
Incumbent x District Unemployed Number	-0.0739***	0.0325	0.0161	0.0032	0.0160***
	(0.0220)	(0.0571)	(0.0236)	(0.0025)	(0.0041)
Incumbent x District Unemployed Rate	0.0426	0.0260	0.0733	-0.0360**	-0.0131
	(0.1203)	(0.1903)	(0.1249)	(0.0110)	(0.0142)
Incumbent x Lagged Incumbent Votes	0.7437	-1.7591	0.4937	-0.2239***	0.1128
	(0.5849)	(1.1819)	(0.6999)	(0.0596)	(0.0811)
Incumbent x District Income	-0.1833*	-0.0451	0.0877	-0.0032	-0.0308*
	(0.0862)	(0.2566)	(0.0917)	(0.0112)	(0.0155)
Inc=0 x Party=D x Cook's Competitiveness	-0.4828***	0.1140	-0.4302*	0.0291*	0.0303
	(0.1386)	(0.4330)	(0.1762)	(0.0120)	(0.0194)
Inc=0 x Party=R x Cook's Competitiveness	0.5159***	-0.7325**	0.1405	-0.0009	0.0437**
	(0.0983)	(0.2351)	(0.1244)	(0.0101)	(0.0137)
Inc=1 x Party=D x Cook's Competitiveness	2.4223***	-0.9685	2.5524***	0.0236	-0.0512
	(0.3392)	(0.5462)	(0.4856)	(0.0225)	(0.0418)
Inc=1 x Party=R x Cook's Competitiveness	-1.7402**	-0.4174	-1.4279	0.0008	0.0692
	(0.5427)	(0.9390)	(0.8953)	(0.0296)	(0.0575)
Inc=0 x Party=D x Cycle Time Trend	0.1171	-0.3421	-0.0236	0.0412***	0.0576***
	(0.1277)	(0.2619)	(0.1230)	(0.0105)	(0.0140)
Inc=1 x Party=D x Cycle Time Trend	0.0330	0.0720	-0.0229	0.0283***	0.0485***
	(0.0552)	(0.1339)	(0.0574)	(0.0051)	(0.0081)
Inc=0 x Party=R x Cycle Time Trend	0.0844	-0.4130*	0.0011	0.0129	0.0090
	(0.1041)	(0.1848)	(0.1026)	(0.0093)	(0.0113)
State & Cycle Fixed Effects	Yes	Yes	Yes	Yes	Yes

Robust standard errors in parentheses. These are the controls for the regressions in Table 3.

Figure S1: Invertibility of Expected Probability in Valence



This graphs the equilibrium expected probability of winning the general election (pre-general entry, post-primary) across the full range of estimated expected valences, for 10 random observations; the shape is consistent across observations.

Table S6: Primary Election Voter Parameters: Controls

	Republican	Democratic		Republican	Democratic
Within-State Candidate	0.0528	0.0730	Average Ad Cost	-0.0202	0.1131*
Donor Zip Income Variation	(0.0651)	(0.0503)	Per Committee	(0.0512)	(0.0454)
Within-State Party	-0.2804*	-0.5207**	District White %	0.9120***	0.2789
Donor Zip Income Variation	(0.1267)	(0.1901)		(0.2341)	(0.1874)
Within-State Super PAC	-0.0581	-0.0319	District Male %	-3.5479	-4.3066
Donor Zip Income Variation	(0.0590)	(0.0535)		(3.0807)	(2.8374)
Within-State Candidate	0.8698***	0.8377***	Incumbent x District	0.0237**	-0.0061
Donor Ideology Variance	(0.1208)	(0.1058)	Unemployed Number	(0.0087)	(0.0073)
District Unemployed Rate	0.0631**	0.0163	Incumbent x District	0.0245	0.0331
	(0.0213)	(0.0251)	Unemployed Rate	(0.0231)	(0.0301)
District Income	0.0394	0.0318	Incumbent x Lagged	0.4336**	0.0628
	(0.0373)	(0.0233)	Incumbent Votes	(0.1383)	(0.1218)
District Unemployed	-0.0262**	-0.0173*	Incumbent x	0.0040	0.0295
Number	(0.0082)	(0.0078)	District Income	(0.0414)	(0.0276)
Lagged Republican	1.6195***	-1.9798***	Inc=0 x Party=R x	-0.0060	
Presidential Votes	(0.2820)	(0.2573)	Cook's Competitiveness	(0.0131)	
Incumbency Status	40.7314	-1.3e+02**	Inc=1 x Party=R x	0.1373	
	(37.1421)	(46.7678)	Cook's Competitiveness	(0.0714)	
Lagged	-0.3604***	0.0125	Inc=0 x Party=R x	0.0202	
Incumbent Votes	(0.1021)	(0.0790)	Cycle Time Trend	(0.0184)	
Number Of Senate	-0.0013	-0.0048	Inc=0 x Party=D x		0.0703***
Candidates	(0.0025)	(0.0033)	Cook's Competitiveness		(0.0119)
Contested Primary	-0.1614*	-0.1475**	Inc=1 x Party=D x		-0.0668
	(0.0626)	(0.0569)	Cook's Competitiveness		(0.0462)
Governor Same Party	-0.0487	-0.1691**	Inc=0 x Party=D x		-0.0669**
	(0.0576)	(0.0605)	Cycle Time Trend		(0.0231)
District High-School Rate	0.0064	0.0078	Constant	-45.3193	130.8263**
	(0.0058)	(0.0057)		(37.0748)	(46.5863)
District Median Age	0.0279**	0.0429***	State & Cycle FE	Yes	Yes
	(0.0086)	(0.0098)	Observations	2385	2190
District Election	0.2763*	0.0713	R^2	0.578	0.492
Day Precipitation	(0.1346)	(0.1362)	F	50.3706	36.9291

Robust standard errors in parentheses. These are the controls for Table 4.

Table S7: Model Fit Statistics

Candidate Position			
Variable	Data	Model	Correlation
Inc R Candidate	1.377	1.2743	0.2815
Cha R Candidate	0.7128	0.6371	0.4270
Inc D Candidate	1.3475	1.2786	0.3861
Cha D Candidate	0.5590	0.4945	0.4852

Candidate Entry Totals		
Variable	Data	Model
Inc R Candidate	1545	1583
Cha R Candidate	787	801
Inc D Candidate	1539	1523
Cha D Candidate	632	625

General Election Spending			
Variable	Data	Model	Correlation
R candidate	295.2791	169.2999	0.7444
R Super PAC	115.9741	67.2293	0.7462
R Party	144.2056	142.2421	0.7516
R PAC	16.0309	8.2118	0.7770
D Candidate	335.3436	216.9161	0.6606
D Super PAC	124.8620	17.3993	0.2201
D Party	142.8750	89.3063	0.5127
D PAC	24.3584	10.2651	0.6937

General Election Entry Totals		
Variable	Data	Model
R candidate	1458	1543
R Super PAC	491	1289
R Party	326	1435
R PAC	816	1290
D Candidate	1458	1543
D Super PAC	435	403
D Party	473	1083
D PAC	718	1069

General Election Vote Share			
Variable	Data	Model	Correlation
R average	0.5060	0.5522	0.6452
R Inc	0.6703	0.6879	0.2687
R Cha	0.3118	0.3918	0.2650

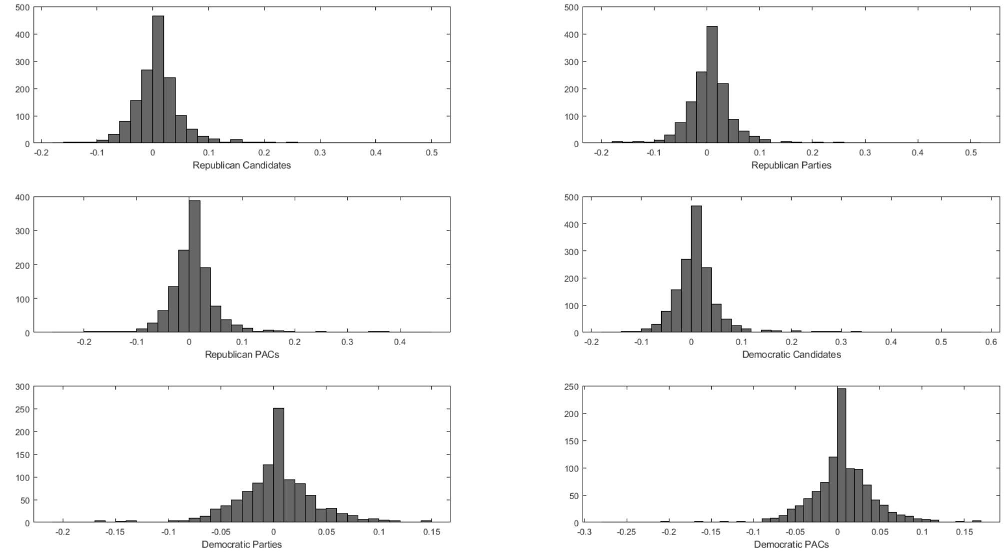
Primary Election Spending			
Variable	Data	Model	Correlation
Inc R Candidate	58.4044	57.9023	0.4863
Inc R Super PAC	6.4622	25.8916	0.3032
Inc R Party	1.0393	1.5921	0.5948
Inc R PAC	1.6691	1.3926	0.5953
Cha R Candidate	20.0280	41.4971	0.4814
Cha R Super PAC	6.8172	41.4624	0.1906
Cha R Party	0.9899	0.3564	0.0322
Cha R PAC	0.6338	1.1480	0.2315
Inc D Candidate	35.2046	26.0485	0.6443
Inc D Super PAC	3.8535	6.1009	0.3883
Inc D Party	0.6590	5.4525	0.0000
Inc D PAC	1.0470	0.8783	0.7884
Cha D Candidate	21.0192	14.2689	0.3625
Cha D Super PAC	3.3354	5.3948	0.6476
Cha D Party	1.4934	8.5719	0.0000
Cha D PAC	0.6270	0.2872	0.1041

Primary Election Entry Totals		
Variable	Data	Model
Inc R Candidate	787	780
Inc R Super PAC	64	509
Inc R Party	31	95
Inc R PAC	179	226
Cha R Candidate	787	780
Cha R Super PAC	87	505
Cha R Party	39	52
Cha R PAC	74	167
Inc D Candidate	632	583
Inc D Super PAC	57	124
Inc D Party	26	195
Inc D PAC	176	247
Cha D Candidate	632	583
Cha D Super PAC	44	82
Cha D Party	11	152
Cha D PAC	30	218

Primary Election Vote Share			
Variable	Data	Model	Correlation
Inc R Candidate	0.7605	0.7289	0.1212
Inc D Candidate	0.7865	0.7514	0.2711

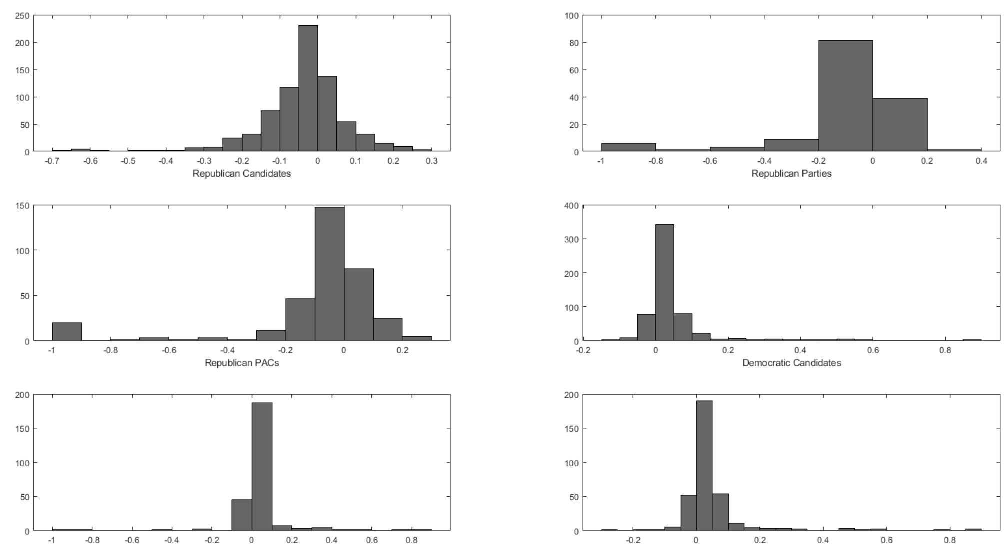
This table shows the data and model averages and correlations for the main choice variables. It also shows the total sum of binary entry decisions in the data and model (for one private draw). “Inc” refers to incumbent. “Cha” is shorthand for challenger. R and D are shorthand for Republican and Democrat aligned groups. The vote share is defined excluding abstention.

Figure S2: Percent Change in General Election Spending Without Super PACs



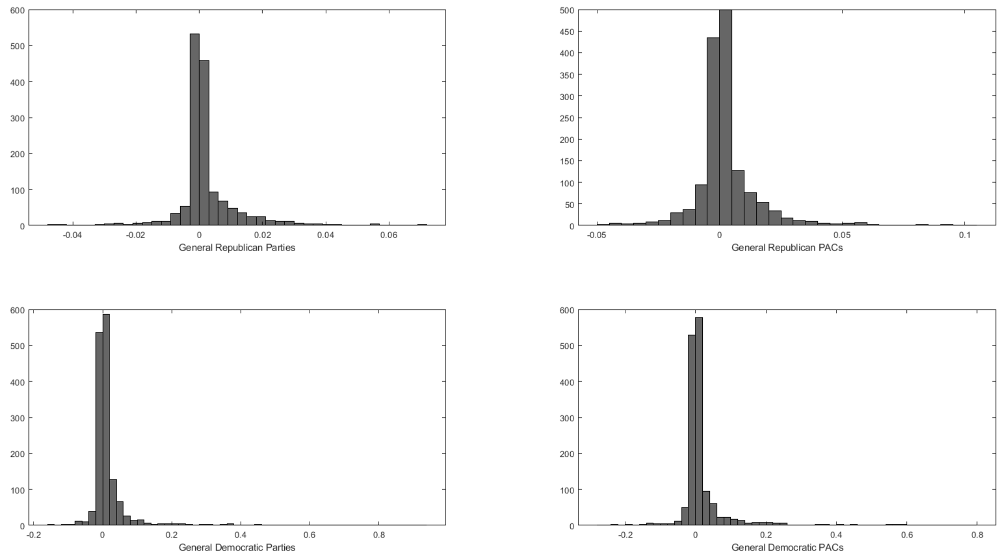
This plots the histogram of percent changes in general election spending with and without Super PACs. I compare the simulated equilibrium spending and counterfactual spending if Super PACs cannot enter for candidates, parties, and PACs for both Republicans and Democrats. For Republicans, the mean is 3.69% and the median is 0.03% for candidates, 3.98% & 0.00% for parties, and 14.37% & 0% for PACs. For Democrats, the mean is 16.68% and the median is 0.04% for candidates, 0.72% & 0.00% for parties, and 2.22% & 0.03% for PACs.

Figure S3: Percent Change in Primary Election Spending Without Super PACs



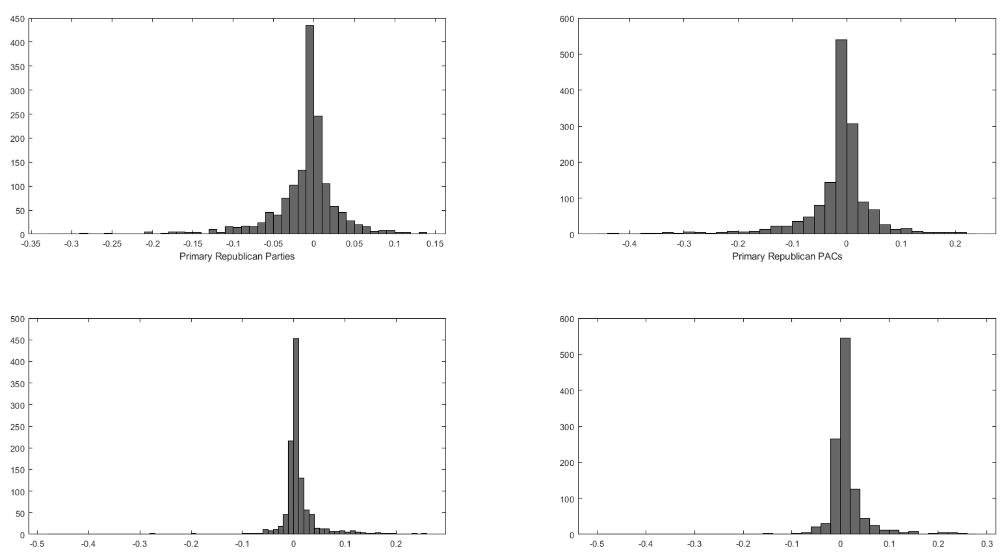
This plots the histogram of percent changes in primary election spending with and without Super PACs. I compare the simulated equilibrium spending and counterfactual spending if Super PACs cannot enter for candidates, parties, and PACs for both Republicans and Democrats. For Republicans, the mean is -4.39% and the median is -2.50% for candidates, -8.64% & -2.65% for parties, and -6.73% & -3.02% for PACs. For Democrats, the mean is 8.68% and the median is 2.15% for candidates, 7.78% & 1.18% for parties, and 8.86% & 2.22% for PACs.

Figure S4: Percent Change in General Committee Entry Without Super PACs



This plots the histogram of percent changes in general election committee entry with and without Super PACs. I compare the simulated equilibrium and counterfactual committee entry probabilities if Super PACs cannot enter for parties and PACs for both Republicans and Democrats. For Republicans, the mean is 0.43% and the median is 0.00% for parties, and 0.69% & 0.01% for PACs. For Democrats, the mean is 1.74% and the median is 0.07% for parties, and 1.86% & 0.03% for PACs.

Figure S5: Percent Change in Primary Committee Entry Without Super PACs



This plots the histogram of percent changes in primary election committee entry with and without Super PACs. I compare the simulated equilibrium and counterfactual committee entry probabilities if Super PACs cannot enter for parties and PACs for Republicans and Democrats. For Republicans, the mean is -1.98% and the median is -0.17% for parties, and -2.18% & -0.20% for PACs. For Democrats, the mean is 0.56% and the median is 0.22% for parties, and 0.50% & 0.30% for PACs.

S.9 Proofs

Lemma 1. *When voter i's indirect utility from choosing candidate j is expressed as:* $U_{ij} = u_j + \xi_j + \epsilon_{ij}$*, where* $\epsilon \sim$ *iid Type 1 EV with* $\psi = 0, \sigma = 1$*, then the share of votes can be written as the following (with utility of abstention* $U_{i0} = u_0 + \epsilon_{i0}$ *and number of candidates* J*):*

$$s_j = \frac{\exp(u_j + \xi_j)}{\sum_{k=0...J} \exp(u_k + \xi_k)}.$$

Proof of Lemma 1.

Consider the voter i with the following preferences over alternatives $j = 1...J$ with an outside option $j = 0$: $U_{ij} = u_j + \xi_j + \epsilon_{ij}, \quad \epsilon \sim$ iid Type 1 EV with $\psi = 0, \sigma = 1$. Then the probability that voter i, drawn at random from the population, votes for candidate j is: $P_{ij} = (u_j + \xi_j + \epsilon_{ij} > u_k + \xi_k + \epsilon_{ik}\ \forall k \neq j)$. Following Train (2009), given the distribution of the errors, $F(\varepsilon_{ij}) = \exp(-\exp(-\varepsilon_{ij}))$, and that the ε are independent, the cumulative distribution function over all alternatives different from j is the product of each CDF.

$$\begin{aligned} P_{ij} &= \int_{-\infty}^{\infty} \left(\prod_{k \neq j} \exp(-\exp\{-(u_j + \xi_j + \varepsilon_{ij} - u_k - \xi_k)\}) \right) \exp(-\varepsilon_{ij}) \exp(-\exp(-\varepsilon_{ij})) d\varepsilon_{ij} \\ &= \int_{-\infty}^{\infty} \exp \left(-(\exp(-\varepsilon_{ij})) \sum_j \exp\{-(u_j + \xi_j - u_k - \xi_k)\} \right) \exp(-\varepsilon_{ij}) d\varepsilon_{ij} \end{aligned}$$

Then define $x = \exp(-\varepsilon_{ij})$, which with the transformation of variables:

$$\begin{aligned} P_{ij} &= \int_{\infty}^{0} \exp \left(-(x) \sum_j \exp\{-(u_j + \xi_j - u_k - \xi_k)\} \right) (-1) dx \\ &= \frac{\exp\left(-(x) \sum_j \exp\{-(u_j + \xi_j - u_k - \xi_k)\}\right)}{-\sum_j \exp\{-(u_j + \xi_j - u_k - \xi_k)\}} \Bigg|_0^{\infty} = \frac{1}{\sum_j \exp\{-(u_j + \xi_j - u_k - \xi_k)\}}. \end{aligned}$$

Finally, we can rewrite out the choice probability: $P_{ij} = \frac{\exp(u_j + \xi_j)}{\sum_{k=0...J} \exp(u_k + \xi_k)}$. Note that this term is the same $\forall i$, meaning $P_{ij} = P_j$. Since choice probabilities are not observed, we can construct the share of votes for a given candidate based on an average of choices from a sample of the voters: $s_j = \frac{\sum \mathbb{1}[choice = j]}{n}$. For the market share to be consistent for the probability, we need $s_j \to_p P_j$ as the number of votes $n \to \infty$.[13] I assume we have sufficient number of votes to utilize the equivalence between shares and aggregate probability. □

Lemma 2. *The program in equation (3.5) has a strictly (finite) positive solution for strictly positive* $V_{i_c}\ \forall i_c \forall c$*, strictly positive* $\beta_{i_c}\ \forall i_c \forall c$*,* $\phi \in (0,1)$*, and strictly positive* $\sum_{j \in J} \gamma_{j i_c}\ \forall i_c \forall c$*.*

[13] As Gandhi, Lu, and Shi (2019) point out, this is not sufficient for the parameters in u_j to be identified given the nonlinearity in $\log(\cdot)$ and is not well defined for a candidate that receives 0 votes.

Proof of Lemma 2.

Rewrite the effort game as the spending game with the following grouping of variables: the cost of spending $g_{i_c} = \left(\sum_{j \in J} \gamma_{ji_c}\right)^{-1}$, heterogeneous spending: $\tilde{\beta}_{i_c} = \beta_{i_c}(1 + X_c^{G_1}\delta_1)$, and candidate characteristics $\Delta_c = h_c^G + \psi_c$.

$$\max_{S_{i_c}^G \in \mathbb{R}_+} V_{i_c} \left(\frac{\exp\left(\sum_{j_c \in N_c} \tilde{\beta}_{j_c}(S_{j_c})^\phi + \Delta_c\right)}{\sum_{c \in \{D,R\}} \exp\left(\sum_{j_c \in N_c} \tilde{\beta}_{j_c}(S_{j_c})^\phi + \Delta_c\right)} \right) - g_{i_c} S_{i_c}^G$$

First we must check whether a solution exists at all.[14] It is clear that the payoff is continuous in all arguments. The unrestricted strategy space is non-compact but without loss of generality we can consider a top-bounded space, despite the payoff not being globally concave. Intuitively this is clear as the payoff is a positive constant times a probability (bounded between 0 and 1) plus a linear strictly decreasing cost function. Thus at some point, the costs will overpower the benefits and any solution will be finite. The first order condition for player i_c of this program is:

$$V_{i_c}\tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1} \left(\frac{\prod_{c \in \{D,R\}} \exp\left(\sum_{j_c \in N_c} \tilde{\beta}_{j_c}(S_{j_c})^\phi + \Delta_c\right)}{\left(\sum_{c \in \{D,R\}} \exp\left(\sum_{j_c \in N_c} \tilde{\beta}_{j_c}(S_{j_c})^\phi + \Delta_c\right)\right)^2} \right) - g_{i_c} = 0.$$

Note that the derivative of the probability of winning function $P_c = P(w_c^G = 1 | w_c^P = 1, \mathbf{w}_{-\mathbf{c}}^{\mathbf{P}})=\left(\left(\frac{\exp\left(\sum_{j_c \in N_c} \tilde{\beta}_{j_c}(S_{j_c})^\phi + \Delta_c\right)}{\sum_{c \in \{D,R\}} \exp\left(\sum_{j_c \in N_c} \tilde{\beta}_{j_c}(S_{j_c})^\phi + \Delta_c\right)}\right)\right)$ is strictly positive and is increasing in $S_{i_c}^G$. Also note that we can write this first order condition more compactly:

$$V_{i_c}\tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1} P_c(1 - P_c) - g_{i_c} = 0.$$

The second order condition is the following, letting the probability be written as P_c:

$$V_{i_c}\tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1} \left((\phi - 1)(S_{i_c})^{-1} P_c \cdot (1 - P_c) + \frac{\partial P_c}{\partial S_{i_c}} \cdot (1 - P_c) + P_c \cdot (-\frac{\partial P_c}{\partial S_{i_c}}) \right).$$

To determine the sign of this expression, the following version is easier to work with, using the fact that $\frac{\partial P_c}{\partial S_{i_c}} = \tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1} P_c \cdot (1 - P_c)$ and combining terms:

$$V_{i_c}\tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1}[P_c \cdot (1 - P_c)] \left((\phi - 1)(S_{i_c})^{-1} + [\tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1} \cdot (1 - 2P_c)] \right).$$

The expression called $V_{i_c}\tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1}$ is strictly positive, and thus the sign is determined by the sum in the parentheses. Since we assumed $\phi \in (0, 1)$, the first term $(\phi - 1)(S_{i_c})^{-1}$ is strictly negative for any $S_{i_c} > 0$. Note that if $P_c > 1/2$ then the entire expression will be negative and thus the objective function will be concave. However, if $P_c < 1/2$, then it is unclear. The following

[14] Note that we cannot rely on the conditions from the Debreu, Glicksberg, and Fan Theorem: in an infinite strategic form game, if the strategy space is compact and convex, if the payoffs are continuous in other players' strategies, and if the payoff is continuous and concave in own strategies, then there exists a pure strategy Nash equilibrium. I cannot use this as the payoff is not globally concave. While a quasi-concave version of this theorem exists, I just directly show an equilibrium exists.

expression determines the sign of the second order condition $\frac{\partial \pi_{i_c}^2}{\partial S_{i_c}^2}$:

$$\text{sign}\left[\frac{\partial \pi_{i_c}^2}{\partial S_{i_c}^2}\right] = \text{sign}[(1-2P_c)\tilde{\beta}_{i_c}\sqrt{S_{i_c}} - 1].$$

Since P_c is strictly increasing in S_{i_c}, as S_{i_c} increases, the term $(1-2P_c)\tilde{\beta}_{i_c}\sqrt{S_{i_c}}$ will become larger and eventually negative. Thus the convexity of π_{i_c}, if any, is confined to some interval $[0, B]$ for $B > 0$. Whether or not any optimal $S_{i_c}^G$ is strictly positive can easily be seen by comparing the payoff from positive spending and zero spending, denoting the sum of others' spending on the same side, $\sum_{j_c \in N_c \setminus \{i_c\}} \tilde{\beta}_{i_c}(S_{i_c})^\phi$, with $\mathcal{S}_{-i_c}$. Note that the other side does not have an excluded player.

$$V_{i_c}\left(\frac{\exp\left(\tilde{\beta}_{i_c}(S_{i_c})^\phi + \mathcal{S}_{-i_c} + \Delta_c\right)}{\sum_{c\in\{D,R\}} \exp\left(\tilde{\beta}_{i_c}(S_{i_c})^\phi + \mathcal{S}_{-i_c} + \Delta_c\right)}\right) - g_{i_c}S_{i_c}^G - V_{i_c}\left(\frac{\exp\left(\mathcal{S}_{-i_c} + \Delta_c\right)}{\sum_{c\in\{D,R\}} \exp\left(\mathcal{S}_{-i_c} + \Delta_c\right)}\right)$$

At $S_{i_c} = 0$, this term is zero. Thus a positive solution will always dominate a zero if this expression is ever positive for all values of the other variables. To see whether this term is strictly positive for any $S_{i_c} > 0$, we can check its derivative at zero:

$$V_{i_c}\tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1}\left(\frac{\prod_{c\in\{D,R\}} \exp\left(\sum_{j_c\in N_c} \tilde{\beta}_{j_c}(S_{j_c})^\phi + \Delta_c\right)}{\left(\sum_{c\in\{D,R\}} \exp\left(\sum_{j_c\in N_c} \tilde{\beta}_{j_c}(S_{j_c})^\phi + \Delta_c\right)\right)^2}\right) - g_{i_c}.$$

Since $\phi \in (0,1)$ and the expression in parentheses is strictly positive, the limit from the right is positive infinity. Thus this function initially increases, starting from zero, and hence is somewhere positive. Now we need to check for the existence of a positive solution. First take the first order condition and rearrange it: $P_c(1-P_c) = \frac{g_{i_c}}{V_{i_c}\tilde{\beta}_{i_c}\phi(S_{i_c})^{\phi-1}}$. Since the right hand side is the same for all players in the game, the best responses are linear functions, letting $\omega_{i_c} = g_{i_c}/(V_{i_c}\tilde{\beta}_{i_c})$:

$$S_{i_c} = S_{j_c}\left(\frac{\omega_{j_c}}{\omega_{i_c}}\right)^{1/\phi} \quad \forall j_c\ \forall c.$$

Thus we can rewrite the first order condition in terms of one player, say player 1_R:

$$V_{1_R}\tilde{\beta}_{1_R}\phi(S_{1_R})^{\phi-1}\left(\frac{\prod_{c\in\{D,R\}} \exp\left((S_{1_R})^\phi \sum_{j_c\in N_c} \tilde{\beta}_{j_c}\left(\frac{\omega_{1_R}}{\omega_{j_c}}\right) + \Delta_c\right)}{\left(\sum_{c\in\{D,R\}} \exp\left((S_{1_R})^\phi \sum_{j_c\in N_c} \tilde{\beta}_{j_c}\left(\frac{\omega_{1_R}}{\omega_{j_c}}\right) + \Delta_c\right)\right)^2}\right) - g_{i_c} = 0.$$

We can show that this has a real and unique solution. From the preceding discussion, we know that any solution is nonzero and finite, so since the payoff function starts off positive, increases, and eventually becomes negative, we know a positive solution exists. $\square$

Lemma 3. *The equations that define whether there is a unique solution for the program (3.5) can be expressed as a single equation with two parameters and one variable. Sufficient for a unique solution are magnitude restrictions on the relative sizes of the two parameters.*

Proof of Lemma 3.

Continuing from the proof of Lemma 2, the question now is multiplicity. It will be useful to denote terms with simpler notation: $A_c = \sum_{j_c \in N_c} \tilde{\beta}_{jc} / (\omega_{jc})$, and express the solution in terms of $X = \omega_{1_R} (S_{1_R})^{\phi}$, with shorthand $e_c = \exp(XA_c + \Delta_c)$. Then we can rewrite:

$$(1/\phi)X = \frac{e_R e_D}{(e_R + e_D)^2}.$$

The goal is to show that these two functions intersect once. First note that the term on the left is strictly increasing linear function starting at 0. The term on the right starts above zero and eventually decreases (which can be seen because the denominator is strictly larger than the numerator and increases at a faster rate). As shown below, this function may initially increase or decrease, but a single intersection with the left hand size function is guaranteed. Consider the derivative of the second term after some combining of terms:

$$\frac{e_R e_D (e_D - e_R)(A_R - A_D)}{(e_R + e_D)^3}.$$

The equation that determines the sign: $\operatorname{sign}[(\exp(XA_D + \Delta_D) - \exp(XA_R + \Delta_R))(A_R - A_D)]$. If $(A_R - A_D)$, then eventually this will be negative. However for low values of X, if $\Delta_D > \Delta_R$, this can be positive. Thus it either starts off positive then goes strictly negative, or is negative throughout. Since the left hand side function starts below the right hand side function, the only possibility of more than one intersection is when the right hand side function increases at a slow enough rate to cross the left hand side and subsequently cross two more times: the bell shape curve can lead to either 1 crossing or three. This can occur when there are extreme differences on opposite ends: if the effective influence of one side $\sum_{i_c \in N_c} \beta_{ic}^2 V_{ic}/g_{ic}$ is much higher than the other side while simultaneously the other side has an extreme effective valence $h_d + \psi_d$ relative to the initial side (however if too extreme then again a single crossing), then 3 equilibria arise. The only possibility of 2 equilibria are when the increasing part of the bell curve function intersects the left hand side straight line with a tangent before coming back down with another intersection.

Note that we can fully characterize the right hand side in terms of just two parameters (fixing ϕ), where we define $\varpi = A_R - A_D$ and $\varrho = \Delta_R - \Delta_D$:

$$\frac{e_R e_D}{(e_R + e_D)^2} = (\exp(\varpi X + \varrho) + \exp(-(\varpi X + \varrho)) + 2)^{-1}.$$

Then uniqueness can be characterized from the relative magnitude of those two parameters, namely $(\sum_{i_c \in N_c} \beta_{ic}^2 V_{ic}/g_{ic} - \sum_{i_d \in N_d} \beta_{id}^2 V_{id}/g_{id})$ and $(h_d + \xi_d - h_c - \xi_c)$ for candidates c and d. The derivative of this expression is as follows: $\frac{\varpi(\exp(-(\varpi X + \varrho)) - \exp(\varpi X + \varrho))}{(\exp(\varpi X + \varrho) + \exp(-(\varpi X + \varrho)) + 2)^2}$. Thus the function increases when $\varpi(\exp(-(\varpi X + \varrho)) - \exp(\varpi X + \varrho)) > 0$, but if that increasing rate is small enough, it will cross $(1/\phi)X$ while it is increasing: meaning when $\varpi(\exp(-(\varpi X + \varrho)) - \exp(\varpi X + \varrho)) < (1/\phi)$. We can find when the slopes are equal: $\log\left(\frac{-(1/\phi)(1/\varpi) \pm \sqrt{((1/\phi)(1/\varpi))^2 - 4}}{2}\right)/\varpi - \varrho/\varpi = x$.

∎

Lemma 4. *The program in equation (3.6) has a pure strategy solution for strictly positive V_{i_c} $\forall i_c \forall c$, strictly positive β_{i_c} $\forall i_c \forall c$, $\phi \in (0, 1)$, and strictly positive $\sum_{j \in J} \gamma_{ji_c}$ $\forall i_c \forall c$, and this solution is unique for sufficiently large σ (conditional on a unique program (3.5)).*

Proof of Lemma 4.

This proof follows the approach from Cox (2022). Denote any second stage Nash equilibrium vector of spending given an entry profile $(a_1, ..., a_N)$ as $(S_1^*, ..., S_N^*)$. The committee's interim expected payoff for a given entry decision conditional on their private information is denoted with U_i and given in equation (S.1). The summation is across all 2^{N-1} combinations of opponent decisions a_{-i}; the term $p_j(a_{-i})$ is the belief by player i in the probability of player j choosing a_j from the decision profile a_{-i}. The term $p_{i,j}(e_{-i})$ is the belief by player i of the probability of player j choosing the a_j from the decision profile a_{-i}; the term p_{-i} is the vector of opponent probabilities of $a = 1$; the term ε_i is private information:

$$U_i(S_1^*, ..., S_N^*, a_1, ..., a_N, p_{-i}) = \sum_{a_{-i} \in \{0,1\}^{2N-1}} \pi_i^*(S_1^*, ..., S_N^* | a_1, ..., a_N) \prod_{j \neq i} p_j(a_{-i}) + \varepsilon_i \cdot a_i. \quad \text{(S.1)}$$

First I show that there exists a pure strategy (Perfect Bayesian equilibrium for this stage) in cutoff strategies. Let the first part of the payoff be denoted with u_i so that $U_i = u_i + \varepsilon_i$. Given the iid distribution of ε, the beliefs are symmetric, meaning player i's belief about player j equals player k's belief about player j: $p_{i,j} = p_{k,j} = p_j$. Thus one can write out any player's belief about player i choosing $a_i = 1$ as $p_i(a_i = 1) = Prob[u_i(1, p_{-i}) + \varepsilon_i > u_i(0, p_{-i})]$. Which given the scaled Logistic distribution of ε yields the functional form below:

$$p_i = \frac{\exp(u_i(1, p_{-i})/\sigma)}{\exp(u_i(1, p_{-i})/\sigma) + \exp(u_i(0, p_{-i})/\sigma)} = f(p_{-i}).$$

This is a continuous system of choice probabilities $\mathbf{p}$ that defines an equilibrium if one exists: $\mathbf{p} = f(\mathbf{p})$. Note that $\mathbf{p} \in [0,1]^N$ and $f(\mathbf{p}) : [0,1]^N \to [0,1]^N$. Thus f is a continuous function over a compact convex set. As noted in Bajari et al. (2010), applying Brouwer's fixed point theorem to this system yields a pure strategy equilibrium for finite values of π.

The system $\Phi(\mathbf{p}) = \mathbf{p} - f(\mathbf{p}) = 0$ will have one zero if the matrix of partial derivatives of Φ with respect to p is a positive dominant diagonal matrix, meaning:

$$|\frac{\partial \Phi_i}{\partial p_i}| > 0 \ \forall i \quad \& \quad |\frac{\partial \Phi_i}{\partial p_i}| \geq \sum_{j \neq i} |\frac{\partial \Phi_i}{\partial p_j}| \ \forall i.$$

Given the functional form, the first is satisfied with value of unity. The second can be satisfied for a sufficiently large σ. To see this, first write out the expression for a given i:

$$\sum_{j \neq i} |\frac{\partial \Phi_i}{\partial p_j}| = \frac{\exp(u_i(1)/\sigma - u_i(0)/\sigma)}{(1 + \exp(u_i(1)/\sigma - u_i(0)/\sigma))^2} \sum_{j \neq i} |\frac{\partial u_i(1)}{\partial p_j} - \frac{\partial u_i(0)}{\partial p_j}| \frac{1}{\sigma}.$$

$$\frac{\partial u_i(1)}{\partial p_j} = \sum_{a_{-\{i,j\}}} [\pi_i(a_i = 1, a_j = 1, a_{-\{i,j\}}) - \pi_i(a_i = 1, a_j = 0, a_{-\{i,j\}})] \prod_{k \neq \{i,j\}} p_k(a_{-\{i,j\}})$$

with a complementary expression for $\frac{\partial u_i(0)}{\partial p_j}$. Note that $\frac{\partial u_i(1)}{\partial p_j}$ is less than the maximum difference in payoffs for entering M, with an analogous bounding for $\frac{\partial u_i(0)}{\partial p_j}$, equal to m. Both M and m are well-defined given the interior solution to the second stage game.

$$\frac{\partial u_i(1)}{\partial p_j} \leq \max_{a_{-\{i,j\}}} [\pi_i(a_i = 1, a_j = 1, a_{-\{i,j\}}) - \pi_i(a_i = 1, a_j = 0, a_{-\{i,j\}})] = M_{ij}$$

$$\frac{\partial u_i(0)}{\partial p_j} \geq \min_{a_{-\{i,j\}}} [\pi_i(a_i = 0, a_j = 1, a_{-\{i,j\}}) - \pi_i(a_i = 0, a_j = 0, a_{-\{i,j\}})] = m_{ij}$$

The expression $\frac{\exp(u_i(1)/\sigma - u_i(0)/\sigma)}{(1+\exp(u_i(1)/\sigma - u_i(0)/\sigma))^2}$ can also be bounded above by noting that the function $\frac{\exp(x/\sigma)}{(1+\exp(x/\sigma))^2}$ achieves its maximum at $x = 0$ for any positive σ with a function value of $1/4$ at that point. Thus one can bound the sum of the absolute cross-partials:

$$\sum_{j \neq i} |\frac{\partial \Phi_i}{\partial p_j}| \leq \frac{1}{4\sigma} \sum_{j \neq i} |\frac{\partial u_i(1)}{\partial p_j} - \frac{\partial u_i(0)}{\partial p_j}| \leq \frac{1}{4\sigma} \sum_{j \neq i} |M_{ij} - m_{ij}|.$$

Thus sufficient for uniqueness (conditional on also satisfying uniqueness from the spending stage in Lemma 2) is $\sigma \geq \max_{i \in \mathcal{I}} \{\sum_{j \neq i} |M_{ij} - m_{ij}|/4\}$. □

Lemma 5. *The program in equation (3.7) has a strictly (finite) positive solution for strictly positive V_{i_c} $\forall i_c \forall c$, strictly positive β_{i_c} $\forall i_c \forall c$, $\phi \in (0, 1)$, and strictly positive $\sum_{j \in J} \gamma_{ji_c}$ $\forall i_c \forall c$.*

Proof of Lemma 5.

$$\max_{e_{i_c}^P \in \mathbb{R}_+} \; V_{i_c} P(w_c^P = 1) P(w_c^G = 1 | w_c^P = 1 \cap w_{D_2}^P = 1) \cdot P(w_{D_2}^P = 1) +$$
$$V_{i_c} P(w_c^P = 1) P(w_c^G = 1 | w_c^P = 1 \cap w_{D_1}^P = 1) \cdot P(w_{D_1}^P = 1) - e_{i_c}^P$$

Which can be rewritten as below, where $\Omega_c = P(w_c^G = 1 | w_c^P = 1 \cap w_{D_2}^P = 1) \cdot P(w_{D_2}^P = 1) + P(w_c^G = 1 | w_c^P = 1 \cap w_{D_1}^P = 1) \cdot P(w_{D_1}^P = 1)$.

$$\max_{e_{i_c}^P \in \mathbb{R}_+} \; V_{i_c} P(w_c^P = 1)(\Omega_c) - e_{i_c}^P$$

The arguments for the existence of a solution follow from the proof for the general election contest, as the payoffs have the same shape in own arguments, but are just scaled by the probabilities from the other primary election.

□

Lemma 6. *The solution to equation (3.7) is determined by just two variables in two equations. Sufficient conditions for uniqueness can be expressed in terms of 4 exogenous terms.*

Evidence for Lemma 6.

Continuing from the proof of Lemma 5.

$$\Omega_c V_{i_c} \tilde{\beta}_{i_c} \phi (S_{i_c})^{\phi - 1} \left(\frac{\prod_{d \in \{R_1, R_2\}} \exp\left(\sum_{j_d \in N_d} \tilde{\beta}_{j_d} (S_{j_d})^{\phi} + \Delta_d\right)}{\left(\sum_{d \in \{R_1, R_2\}} \exp\left(\sum_{j_d \in N_d} \tilde{\beta}_{j_d} (S_{j_d})^{\phi} + \Delta_d\right)\right)^2} \right) - g_{i_c} = 0$$

Define the term $\omega^P_{i_c} = g_{i_c}/(V_{i_c}\tilde{\beta}_{i_c})$. Note that the best response functions are linear with respect to the other players from your direct primary (not with respect to players from the other primary, whose actions are contained in Ω_c).

$$S_{i_c} = S_{j_d}\left(\frac{\omega_{j_d}}{\omega_{i_c}}\cdot\frac{\Omega_c}{\Omega_d}\right)^{1/\phi} \qquad \forall j_d\ \forall c \in \{R_1, R_2\}$$

We have two sets of these for both sides of the primary. This mirrors the general election just now with two sets with the exception of the Ω terms which capture the forward-looking nature of committees during the primary. Thus we can write out the primary election first order condition for the Republican side as just a function of spending of a single Republican committee (from either side) and the spending from the Democratic primary (with the analogous case for the Democratic spending). Thus the solution is characterized by two sets of equations:

$$\begin{aligned}(1/\phi)\omega_{1_R}S^{\phi}_{1_{R_1}} =&\Omega_{R_1}\cdot P_{R_1}([\Omega_{R_2}/\Omega_{R_1}]\omega_{1_R}S^{\phi}_{1_{R_1}})\cdot(1-P_{R_1}(\cdot))\\(1/\phi)\omega_{1_D}S^{\phi}_{1_{D_1}} =&\Omega_{D_1}\cdot P_{D_1}([\Omega_{D_2}/\Omega_{D_1}]\omega_{1_D}S^{\phi}_{1_{D_1}})\cdot(1-P_{D_1}(\cdot)).\end{aligned}$$

Recall from the proof for the general election, that each equation can have a unique solution (assumed here) so that we can write out the best responses as functions (not correspondences): $S_{1_{R_1}} = BR_R(S_{1_{D_1}})$, and $S_{1_{D_1}} = BR_D(S_{1_{R_1}})$. We can write out the two equations with simpler notation, letting $X = \omega_{1_R}S^{\phi}_{1_{R_1}}$ and $Y = \omega_{1_D}S^{\phi}_{1_{D_1}}$. Let G^R_{11} be the equilibrium expected general election probability of candidate R_1 beating candidate D_1, with similar notation for the other terms. Note that $G^D_{11} = 1 - G^R_{11}$, $G^R_{12} = 1 - G^D_{21}$, etc.

$$\begin{aligned}(1/\phi)X =&[G^R_{11}P_{D_1}(Y) + G^R_{12}(1-P_{D_1}(Y))]\cdot\\&P_{R_1}\left(X\cdot\left[\frac{G^R_{21}P_{D_1}(Y)+G^R_{22}(1-P_{D_1}(Y))}{G^R_{11}P_{D_1}(Y)+G^R_{12}(1-P_{D_1}(Y))}\right]\right)\cdot(1-P_{R_1}(\cdot))\end{aligned}$$

$$\begin{aligned}(1/\phi)Y =&[G^D_{11}P_{R_1}(X) + G^D_{12}(1-P_{R_1}(X))]\cdot\\&P_{D_1}\left(Y\cdot\left[\frac{G^D_{21}P_{R_1}(X)+G^D_{22}(1-P_{R_1}(X))}{G^D_{11}P_{R_1}(X)+G^D_{12}(1-P_{R_1}(X))}\right]\right)\cdot(1-P_{D_1}(\cdot))\end{aligned}$$

We must establish the curvature of the best responses. First take the derivative of the best response for X in terms of Y by differentiating the first equation by Y and re-arranging, where it will be useful to define the a new term which is derived from to the derivative of the ratio $\Omega_{R_2}/\Omega_{R_1}$ with respect to Y: $\Omega^R_\delta = \frac{(G^R_{21}-G^R_{22})\Omega_{R_1}-(G^R_{11}-G^R_{12})\Omega_{R_2}}{(\Omega_{R_1})^2}$.

$$\frac{\partial BR_X(Y)}{Y} = \frac{\frac{\partial P_{D_1}}{\partial Y}\left(G^R_{11}-G^R_{12}\right)P_{R_1}(1-P_{R_1}) + \Omega_{R_1}\frac{\partial P_{R_1}}{\partial X\cdot[\Omega_{R_2}/\Omega_{R_1}]}BR_X(Y)\frac{\partial P_{D_1}}{\partial Y}\Omega^R_\delta(1-2P_{R_1})}{1/\phi - [\Omega_{R_1}]\frac{\partial P_{R_1}}{\partial X\cdot[\Omega_{R_2}/\Omega_{R_1}]}[\Omega_{R_2}/\Omega_{R_1}](1-2P_{R_1})}$$

To determine the curvature of the best responses, consider the G terms.[15] If $G^R_{11} = G^R_{12}$, then the best response curve is flat because player 1_{R_1} is indifferent to which Democratic candidate wins. In this case the solution from the general election contest suffices to show a unique solution.

[15]If we assume $\Omega_{R_2} = \Omega_{R_1}$, then it is straightforward to establish curvature. The case of $\Omega_{R_2} \neq \Omega_{R_1}$ revolves around similar terms but involves significantly more algebra.

Similarly, if either of the probabilities for the opposing side are equal to 1, meaning the other candidate did not enter, then we again reach the degenerate best response. To consider the other cases, we must establish the curvature of the best responses. First take the derivative of the best response for X in terms of Y by differentiating the first equation, which is an implicit function of the best response function, by Y and re-arranging:

$$\frac{\partial BR(Y)}{Y} = \frac{\frac{\partial P_{D_1}}{\partial Y}\left(G_{11}^R - G_{12}^R\right) P_{R_1}(1 - P_{R_1})}{1/\phi - [G_{11}^R P_{D_1} + G_{12}^R(1 - P_{D_1})]\frac{\partial P_{R_1}}{\partial X}(1 - 2P_{R_1})}.$$

The sign of the numerator is based on the following, where $A_{D_c} = \sum_{j_{D_c} \in N_{D_c}} \tilde{\beta}_{j_{D_c}} / \left(\omega_{j_{D_c}}\right)$.

$$\text{sign}\left[\frac{\partial P_{D_1}}{\partial Y}\left(G_{11}^R - G_{12}^R\right)\right] = \text{sign}\left[\left(A_{D_1} - A_{D_2}\right)\left(G_{11}^R - G_{12}^R\right)\right]$$

The A terms are the aggregate effective spending influence of the democratic committees for the Democratic primary. The G terms are the equilibrium expected probability of the Republican winning against either Democrat. Thus the sign is positive if Democrat 1 candidates are more effective at spending and the Republican 1 has a better chance against Democrat 1 than Democrat 2 in the general. The sign of the denominator is determined by the following condition, where for shorthand $\theta = G_{11}^R P_{D_1} + G_{12}^R(1 - P_{D_1})$, and $\exp_{1_{R_c}} = \exp(A_{1_{R_c}} X + \Delta_{R_c})$:

$$\text{sign}[denom] = \text{sign}\left[1/\phi - \theta \cdot \frac{(\exp_{R_1} \exp_{R_2})(A_{R_1} - A_{R_2})(\exp_{R_2} - \exp_{R_1})}{(\exp_{R_1} + \exp_{R_2})^3}\right].$$

Note that $1/\phi$ is strictly greater than one and Ξ is strictly between zero and one. Also note that if the sign of this term ever changed, then it necessarily crosses 0 (as it is continuous) and the derivative would be undefined at that point. If $A_{R_1} - A_{R_2}$ is sufficiently large and $\Delta_{R_2} - \Delta_{R_1}$ is sufficiently large, then the sign can be positive for small X'; thus the question remains of whether there exists a Y' such that $X' = BR(Y')$. The best response is a Sigmoid function (with the convex-concave turning point being based on the difference in candidate characteristics for the opposite primary), either increasing if the product $\left(A_{D_1} - A_{D_2}\right) \cdot \left(G_{11}^R - G_{12}^R\right)$ is positive, decreasing if strictly negative, or flat if zero.

□

Lemma 7. *The program in equation (3.9) has a pure strategy solution for strictly positive V_c, V_c^0 $\forall c$. Furthermore, the solution to program in equation (3.9) is unique for sufficiently large σ_C.*

Proof of Lemma 7.

Using the same logic as from the Proof of Lemma 4, Brouwer's fixed point theorem for the multinomial logit case guarantees existence for finite payoff values. The sufficient conditions for uniqueness in the Proof of Lemma 4 have multinomial Logit analogs. However now there are additional equations, namely three per player (one for each decision). Thus player i has probability p_{i_d}: specifically p_{i_0}, p_{i_1}, and p_{i_2} such that $p_{i_0} + p_{i_1} + p_{i_2} = 1$; for example i_0 refers to the $d_c = 0$ decision for candidate i.

$$p_{i_d} = \frac{\exp(u_{i_d}(d, p_{j_d} \forall j \forall d)/\sigma_C)}{\sum_{f=\{0,1,2\}} \exp(u_{i_f}(f, p_{j_d} \forall j \forall d)/\sigma_C)} = f(p_{-i_d}).$$

The system $\Phi(\mathbf{p}) = \mathbf{p} - f(\mathbf{p}) = 0$ will have one zero if the matrix of partial derivatives of Φ with respect to $\mathbf{p}$ is a positive dominant diagonal matrix, meaning:

$$|\frac{\partial \Phi_{i_d}}{\partial p_{i_d}}| > 0 \; \forall i \; \forall d \quad \& \quad |\frac{\partial \Phi_{i_d}}{\partial p_{i_d}}| \geq \sum_{(j_d \forall j \forall d)\setminus(i_d)} |\frac{\partial \Phi_{i_d}}{\partial p_{j_d}}| \; \forall i_d.$$

The summation in the second inequality, namely $(j_d \forall j \forall d) \setminus (i_d)$, includes all of i's probabilities other than their choice for d and each other player j's full set of choice probabilities.

The own-derivative condition is satisfied with value of one. The second is satisfied with own cross-choice probability with a value of zero. The second for cross-player derivatives can be satisfied for a sufficiently large σ_C. To see this, first write out the expression for i_0:

$$\sum_{j_d \forall j \neq i \forall d} |\frac{\partial \Phi_{i_0}}{\partial p_{j_d}}| = \sum_{e=\{1,2\}} \left(\frac{\exp((u_{i_e} - u_{i_0})/\sigma_C)}{(1 + \sum_{f=\{1,2\}} \exp([(u_{i_f} - u_{i_0})/\sigma_C])^2} \sum_{j_d \forall j \neq i \forall d} |\frac{\partial u_{i_e}}{\partial p_{j_d}} - \frac{\partial u_{i_0}}{\partial p_{j_d}}| \frac{1}{\sigma_C} \right).$$

Following the logic from the Proof of Lemma 4, each cross partial of u_{i_d} with respect to p_{j_d} can be bounded; let that maximum be denoted with M_{i_d, j_d}. Then similarly, we can rewrite that first term on the right hand side:

$$\frac{\exp((u_{i_e} - u_{i_0})/\sigma_C)}{(1 + \sum_{f=\{1,2\}} \exp([(u_{i_f} - u_{i_0})/\sigma_C])^2} = p_{i_1} p_{i_0}.$$

This product is strictly bounded between 0 and 1. Thus one can bound the sum of the absolute cross-partials for i_0 and by extension every other choice and player:

$$\sum_{j_d \forall j \neq i \forall d} |\frac{\partial \Phi_{i_0}}{\partial p_{j_d}}| < \frac{1}{\sigma_C} \sum_{e=\{1,2\}} \left(1 \cdot \sum_{j_d \forall j \neq i \forall d} |M_{i_e, j_d} - M_{i_0, j_d}| \right)$$

Thus sufficient for uniqueness (conditional on uniqueness of the spending stage from Lemma 6) is $\sigma_C \geq \max_{i_D \forall i} \{ \sum_{e=\{1,2\}} \sum_{j_d \forall j \neq i \forall d} |M_{i_e, j_d} - M_{i_D, j_d}| \}$.

□

Proposition 1. *There exists a pure strategy Bayesian Nash equilibrium in which all agents condition on payoff relevant actions.*

Proof of Proposition 1.

The proof is by backward induction, and all steps are based on conditioning on payoff relevant only actions. By Lemma 2, the general election spending stage has a pure strategy Nash equilibrium. By Lemma 4, the general election entry stage has a pure strategy Bayesian Nash equilibrium. By Lemma 5, the primary spending stage has a pure strategy Nash equilibrium. By re-applying Lemma 4 to the primary stage, the primary entry stage has a pure strategy Bayesian Nash equilibrium. Then by Lemma 7, the challenger entry stage has a unique pure strategy Bayesian Nash equilibrium. The incumbent's discrete choice single-agent environment will have a unique pure decision rule given the discrete set of actions. Thus the game has a Bayesian Nash equilibrium in pure strategies. □

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