Incentivizing Negative Emissions Through Carbon Shares[*]

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I show that commonly proposed emission taxes are not optimal for controlling climate change: they can achieve zero emissions but cannot induce negative emissions. The first-best policy charges firms period by period for leaving a stock of carbon in the atmosphere, not just for injecting carbon into the atmosphere. I propose a feasible version of this policy that requires emitters to post an upfront bond that finances a transferable asset (a "carbon share"). The regulator reduces this asset's face value as damages accumulate and pays out the asset's remaining face value once its holder removes the underlying unit of carbon from the atmosphere. I show that the optimal bond is equal to the maximum possible marginal damage from climate change, with the carbon share paying a dividend as long as the worst-case is not realized. Quantitatively, a bond that is double the optimal emission tax is sufficient to provide optimal carbon removal incentives in 95% of cases.

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1 Introduction

Countries and corporations are increasingly adopting ambitious goals of eliminating or offsetting all greenhouse gas emissions by the middle of the century, if not sooner.[1] What happens once these targets are reached? Harm from carbon dioxide will not cease at this point, as old emissions will remain in the atmosphere. The role for policy should also not cease: it is unlikely that the optimal policy would stop precisely at zero emissions, forgoing use of the several technologies for removing carbon dioxide from the atmosphere.[2] In fact, many models suggest that achieving global temperature targets will require negative emissions over the latter part of the century (e.g., Clarke et al., 2014; Rogelj et al., 2015, 2018; Hilaire et al., 2019; Realmonte et al., 2019). Yet there has been little analysis of market-based mechanisms for implementing such targets.

The standard economic prescription for climate change requires taxing emissions (or, equivalently for present purposes, capping emissions) so that market actors account for the external costs that their emissions cause through global climate change.[3] However, it typically escapes notice that an emission price contains a sharp discontinuity: it incentivizes emission reductions up to the point at which there are no further emissions from the present period, but it does not incentivize the further emission reductions that would offset emissions from past periods. A policymaker constrained to using emission taxes is constrained from ever implementing negative emissions without additional direct government procurement, regardless of what the policymaker learns about the severity of climate change impacts or about the evolving cost of removing carbon dioxide from the atmosphere. This constraint is not costly if there is no chance that the policymaker would choose to incentivize negative emissions. But the increasing adoption of zero emission targets and the ongoing progress in carbon dioxide removal technologies combine to suggest that negative emissions are in fact

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[1] For instance, France and the United Kingdom have passed laws requiring carbon neutrality by 2050. The European Union subsequently adopted the same target in its Green Deal. For some of the corporate commitments, see `https://sciencebasedtargets.org/2020/06/04/corporate-commitments-to-1-5c-and-net-zero-ramp-up-on-world-environment-day/` and `https://www.majorityaction.us/netzero`.

[2] Recognizing this, Microsoft has committed not just to eliminating ongoing emissions by 2030 but also to removing all of its historical emissions from the atmosphere by 2050.

[3] Much work has also discussed how the first-best policy is in fact a portfolio that also includes policies such as R&D subsidies that account for other market failures (e.g., Fischer and Newell, 2008; Nordhaus, 2008; Acemoglu et al., 2012; Lemoine, 2020). Most see Pigouvian emission pricing as critical to that portfolio. I here emphasize that corrective emission price.

quite plausible.[4]

Policy should recognize that the social harm from carbon dioxide follows not from its emission but from the choice to leave it in the atmosphere. A carbon stock tax ("atmospheric rental policy") explodes the Pigouvian emission tax into its constituent strip of period-by-period marginal damages. It charges firms period-by-period for renting atmospheric storage, with the charges based on same-period damages.[5] This policy treats current and past emissions symmetrically, so it maintains first-best abatement incentives up to the point at which all covered emissions have been removed from the atmosphere. This constraint is unlikely to bind if the policy is implemented soon.

However, the carbon stock tax requires today's firms to survive over many decades until removing near-term emissions becomes likely, and it also requires today's firms to anticipate surviving that long so that they internalize future charges when choosing current emissions. In reality, substantial market churn is likely over a 50-year-plus horizon, even for large energy firms.[6] This problem is a version of judgment-proofness (Shavell, 1986). I show that the regulator can restore optimal emission incentives under the simplest form of bankruptcy risk by making each period's charge a weighted average of the Pigouvian emission tax and the optimal rental charge described above. But such a policy still fails to incentivize the removal of bankrupt firms' emissions from the atmosphere.

To avoid such problems, I propose a new type of policy that I call carbon shares. Each emitter posts a bond and receives a carbon share attached to the unit of emission. The emitter can choose to retain or sell its carbon share. Initially, the face value of the carbon share is the bond. In each subsequent period, the regulator pays a dividend to the holder of the share and deducts both that dividend and a damage charge from the face value of the share. If the owner of a share ever removes the unit of carbon attached to it, then the owner

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[4] Carbon dioxide removal, or negative emission, strategies include chemically separating carbon dioxide from air ("direct air capture"), capturing the emissions from power plants that burn biomass ("bioenergy with carbon capture and storage"), accelerating the weathering of rocks, enhancing uptake of carbon by forests or oceans, and more. See National Research Council (2015), Fuss et al. (2018), and National Academies of Sciences, Engineering, and Medicine (2018) for recent reviews.

[5] A conventional emission tax charges emitters for transferring their carbon dioxide to the public for storage in the atmosphere. The public bears the responsibility for removing that carbon dioxide from the atmosphere if damages turn out to be large or removal turns out to be cheap. I explore policies that do not require direct government procurement or net outlays from the public purse.

[6] Plus there could be principal-agent problems preventing firms from fully internalizing charges that will arise many decades down the line.

receives the remaining face value and the share is retired. In essence, the share is an option to recover the remaining face value, with the strike price being the cost of carbon removal. This policy converts past emissions into a valuable asset that investors want to own, whether or not the emitter continues to exist.

I show that the optimal carbon share policy combines the first-best emission and removal incentives of the stock tax with the judgment-proof upfront payments of the emission tax. The regulator should set the initial bond at least equal to the worst-case social cost of carbon emissions based on information available at the time of emission.[7] In each period, the deducted damage charge should be equal to the current period's marginal damage estimate, which was also the optimal stock tax. The dividends return the difference between an updated estimate of the worst-case social cost of carbon and the previous estimate. The shareholder thus receives substantial dividends if climate change turns out to impose small costs and few dividends if climate damages turn out to be large. Shareholders remove their carbon in order to recover the stream of future damage charges, which leads them to weigh the cost of carbon removal against the expected remaining marginal harm from atmospheric carbon. Emitters' incentives are also first-best: emitters pay the worst-case social cost of carbon but receive a valuable asset in return, and I show that their net outlays are equal to the expected social cost of carbon. Emitters' upfront net outlays are therefore exactly the same as under the Pigouvian emission tax.

I quantitatively assess the benefits of the proposed policy within a conventional economic model of climate change. I assume that the true costs of climate change are initially unknown but revealed in 2065. Their variance is determined by the results of a recent expert survey (Pindyck, 2019). The first-best policy nearly always uses negative emissions at some future time, and may even do so in 2065. If the policymaker required a bond double what the year 2015 emission tax would have been, then the policymaker could fund the ex post optimal series of charges in over 95% of damage realizations. By enabling negative emissions, the carbon share policy increases the benefits from implementing climate policy by nearly 10%.

The recommendation to address climate change through Pigouvian emission pricing dates back at least to Nordhaus (1977). In fact, Nordhaus (1977) observes that there are two strategies for controlling carbon dioxide: reducing emissions and cleaning it from the atmosphere

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[7] "Social cost of carbon" is sometimes used to indicate the marginal welfare loss from carbon emissions along a business-as-usual pathway. I here use it to refer to marginal welfare loss along the optimal pathway.

ex post. He restricts attention to the first in order "to avoid the odor of science fiction" (pg 343). More recently, Nordhaus (2019) decides to ignore carbon removal on the premise that it is not available at both scale and reasonable cost. The most recent (2016R) version of his benchmark DICE integrated assessment model does allow limited negative emissions after 2150, but versions as late as 2007 constrained emissions to be weakly positive in all periods.

Much other literature is more optimistic about the costs and scalability of carbon removal technologies, with several climate-economy models even showing heavy use of such technologies around midcentury (e.g., Obersteiner et al., 2001; Azar et al., 2010; Clarke et al., 2014; Rogelj et al., 2015, 2018; Hilaire et al., 2019; Realmonte et al., 2019). Further, Microsoft and Stripe each recently committed to paying for carbon dioxide removal services. Despite the increasingly prominent discussion of carbon dioxide removal, I know of no work on market-based approaches to incentivizing optimal use of these technologies. Indeed, these technologies pose no special problems if aggregate emissions are guaranteed to remain positive.[8] I here investigate how to provide optimal incentives for carbon dioxide removal in the realistic case where we do not know in advance whether we should utilize these technologies to drive net emissions below zero.[9]

Although climate change policy has been almost exclusively focused on controlling the injection of pollution into the atmosphere, the broader environmental policy literature grapples with the need to clean up pollution that has already been released. This literature has discussed bonding and deposit-refund schemes as solutions to the problem of monitoring improper waste disposal.[10] I obtain a sharp result: the required bond (or deposit) should

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[8] Conventional emission pricing policies could incentivize use of carbon dioxide removal technologies up to the point at which net emissions are zero. However, the European Union's flagship cap-and-trade program does not in practice provide the credits for carbon dioxide removal that could provide even this limited incentive (Scott and Geden, 2018). Hilaire et al. (2019) conclude their review by noting that negative emissions would require large-scale government financing. My new mechanism obviates this need.

[9] In an earlier working paper, I informally described the possibility of charging emitters based on their greenhouse gas property left in the atmosphere (Lemoine, 2007). In a deterministic model, Yang and Davis (2018) show that incentivizing optimal mine remediation requires placing a Pigouvian tax on the stock of damaged land, not on the flow of damaged land. This result mirrors the advantage of atmospheric rental charges over Pigouvian emission taxes. Numerical examples in White et al. (2012) suggest that combining the mining stock tax with an assurance bond can improve welfare in the presence of bankruptcy risk. Goodkind and Coggins (2015) investigate the implications of corner solutions in abatement for the choice between price and quantity instruments but do not consider how to transcend corner solutions. I discuss quantity policies in the conclusion.

[10] Torsello and Vercelli (1998) review the history of bonding analyses. Bonding has been proposed for monitoring long-term underground storage of sequestered carbon (Klass and Wilson, 2008; Gerard and Wilson,

be set to the worst-case social cost of carbon. Others have proposed that fees on materials or products be set to the most harmful possible environmental fate, with the fee refunded in accord with the harmfulness of actual outcomes (e.g., Solow, 1971; Mills, 1972; Bohm and Russell, 1985; Costanza and Perrings, 1990; Boyd, 2002). These informal proposals rely on arguments ranging from ambiguity aversion to difficulties in monitoring pollution to judgment-proofness. The long timespans over which carbon emissions affect the atmosphere make the judgment-proofness argument especially salient here. I formally show how the worst-case bond can be used to finance a transferable asset that reduces the bond's upfront cost to emitters, does not burden the regulator with cleaning up past emissions in the event that emitters forsake the bond, and provides first-best incentives for both emission and cleanup.

The next section contains the theoretical analysis. Section 3 quantifies the welfare gains from using carbon shares instead of emission taxes. The final section concludes. The appendix details the numerical model.

2 Theoretical Analysis

Consider a world with many small firms and infinitely many periods. Index firms by i, and normalize total firms to be of measure 1. Firm i's business-as-usual emissions in period t are $e_{it} > 0$. Firm i can choose to eliminate quantity $A_{it} \leq e_{it}$ of time t emissions. Abatement cost $C_{it}(A_{it})$ is strictly increasing and strictly convex, with $C'_{it}(0) = 0$ for convenience (where primes indicate derivatives). Each firm can also fund the removal of quantity $Z_{it} \geq 0$ of emissions from the atmosphere. It purchases this emission removal from a competitive industry with aggregate cost curve $G_t(Z_t; \tilde{g}_t)$, with $Z_t \triangleq \int_0^1 Z_{it} \,\mathrm{d}i$ and $G_t(\cdot; \tilde{g}_t)$ strictly increasing and strictly convex in Z_t. The random variable $\tilde{g}_t$ shifts the marginal cost of emission removal, with $G'_t(Z_t)$ increasing in $\tilde{g}_t$. Its value is known to all firms in period t, and it has support between g^L and g^H, where $g^L < g^H$. Current emissions can be offset either by

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2009). Such a policy could be important for ensuring the success of carbon removal. I here instead focus on incentives to undertake carbon removal. Deposit-refund schemes have been justified as a means to disincentivize illegal, hard-to-monitor dumping of sulfur emissions (Bohm, 1981), hazardous waste (Russell, 1987), or municipal waste (Fullerton and Kinnaman, 1995). Deposit-refund schemes have also been understood as means to avoid the fiscal costs of subsidies and the distributional costs of taxes (Bohm, 1981). Here the motivation is to overcome an inefficiency in conventional tax policies without incurring additional fiscal costs from using the public purse to directly fund carbon removal.

abatement or by removal, but abatement is the cheaper option for the first unit of emissions: $G_t'(0;g^L) > C_{it}'(0)$. Firms seek to minimize their costs, subject to current and anticipated policies.

A policymaker begins to implement policy in period 0. Let cumulative emissions up to time t be $M_t = M_{t-1} + \int_0^1 \left[e_{i(t-1)} - A_{i(t-1)} - Z_{i(t-1)}\right] \, \mathrm{d}i$, with pre-policy cumulative emissions $M_0 \geq 0$ given. Time t warming is $T_t = \alpha \left[M_t + \int_0^1 (e_{it} - A_{it} - Z_{it}) \, \mathrm{d}i\right]$.[11] This representation recognizes that carbon dioxide is a globally mixed pollutant and follows recent scientific findings that global temperature is approximately a linear function of cumulative emissions (see Dietz and Venmans, 2019, among others). Social damages from warming in period t are $D_t(T_t; \tilde{d}_t)$, with $D_t(\cdot; \tilde{d}_t)$ strictly increasing and weakly convex in T_t and $D_t'(0; \tilde{d}_t) = 0$. The random variable $\tilde{d}_t$ shifts marginal social damage, with $D_t'(T_t)$ increasing in $\tilde{d}_t$.[12] Its value is known to all firms in period t, and it has support between d^L and d^H, where $d^L < d^H$. For convenience, let the random variables $\tilde{g}_t$ and $\tilde{d}_t$ be jointly Markovian.

The regulator chooses period t policy to minimize expected discounted social costs, with knowledge of the random variables' realizations up to and including time t. The per-period discount rate is r. The time t regulator correctly understands the distribution of the random variables and rebates any tax revenue lump-sum.

2.1 Optimal Emissions

Consider the first-best allocation of abatement and emission removal. In period t, a regulator who can directly prescribe firms' decisions solves

$$\begin{aligned} V_t^{opt}(M_t, \tilde{g}_t, \tilde{d}_t) = \min_{A_{it} \leq e_{it}, Z_{it} \geq 0} \Bigg\{ \int_0^1 C_{it}(A_{it}) \, \mathrm{d}i + G_t(Z_t; \tilde{g}_t) + D_t(T_t; \tilde{d}_t) \\ + \frac{1}{1+r} E_t \left[V_{t+1}^{opt}(M_{t+1}, \tilde{g}_{t+1}, \tilde{d}_{t+1}) \right] \Bigg\}, \end{aligned}$$

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[11] Allowing time t emissions to affect temperature only with a lag would not qualitatively change the results.

[12] For exposition, I sometimes suppress dependence of G_t and D_t on the random variables.

where E_t indicates expectations at the time t information set. Repeatedly applying the envelope theorem,

$$\frac{\partial V_t^{opt}(M_t, \tilde{g}_t, \tilde{d}_t)}{\partial M_t} = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})].$$

At an interior solution, standard first-order conditions imply

$$C'_{it}(A_{it}) = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})], \tag{1}$$

$$G'_t(Z_t) = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})]. \tag{2}$$

These conditions equate the marginal private cost of abatement and emission removal to their marginal social benefits, as is familiar.[13] These conditions and the constraints implicitly define unique $A_{it}^{opt}(M_t, \tilde{g}_t, \tilde{d}_t)$ and $Z_t^{opt}(M_t, \tilde{g}_t, \tilde{d}_t)$.[14] Emission removal would be used while aggregate emissions are still positive if and only if there exists a firm i such that $C'_{it}(e_{it}) > G'_t(0)$.

2.2 Emission Tax Policy

Now consider a regulator seeking to control emissions through emission taxes. Firms report their emissions net of any removal they fund and pay τ_t per unit in period t. Firm i solves:

$$\pi_{it}^{tax}(\tau_t) = \min_{A_{it} \le e_{it}, Z_{it} \ge 0} \left\{ C_{it}(A_{it}) + p_t Z_{it} + \max\{0, \tau_t[e_{it} - A_{it} - Z_{it}]\} + \frac{1}{1+r} E_t\left[\pi_{i(t+1)}^{tax}(\tau_{t+1})\right] \right\},$$

where p_t is the cost of emission removal and where I suppress dependence of π_{it} on the random variables. Funding emission removal allows the firm to avoid paying a tax but does

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[13] One might wonder why the marginal cost of abatement is equated to the sum of future marginal damages out to an infinite horizon if there is a chance of removing a unit of today's emissions at some future time. The reason is that such removal is not free: by equation (2), optimal use of emission removal equates its marginal cost to the sum of marginal damage over all remaining periods. Equating current marginal abatement cost to the sum of all future marginal damage thus incorporates both expected realized marginal damage and expected future spending on emission removal.

[14] The first-best allocation does not specify which firms pay for emission removal because that allocation does not affect real outcomes. In contrast, the first-best allocation does specify that firms equalize marginal abatement costs.

not entitle the firm to a subsidy if total removal exceeds $e_{it} - A_{it}$. Firms therefore never choose $A_{it} + Z_{it} > e_{it}$. At an interior solution, the first-order conditions imply

$$\begin{aligned} C'_{it}(A_{it}) =&\tau_t, \\ p_t =&\tau_t. \end{aligned} \tag{3}$$

In equilibrium, $p_t = G'_t(Z_t)$, so the second condition implies:

$$G'_t(Z_t) =\tau_t. \tag{4}$$

Let A_{it}^{tax} and Z_{it}^{tax} indicate firms' choices. Both increase in τ_t. Because $A_{it}^{tax} + Z_{it}^{tax} \leq e_{it}$ for all τ_t, there is a tax $\bar{\tau}_{it}$ beyond which $A_{it}^{tax} + Z_{it}^{tax}$ is constant. Raising the tax above $\bar{\tau}_{it}$ does not affect firm i's net emissions because all emissions have either been eliminated or offset by emission removal. That maximum tax is the smallest τ_{it} such that

$$A_{it}^{tax} + Z_{it}^{tax} = e_{it}. \tag{5}$$

Z_{it}^{tax} weakly decreases in $\tilde{g}_t$, and either $A_{it}^{tax} + Z_{it}^{tax} = e_{it}$ or A_{it}^{tax} is independent of $\tilde{g}_t$ (conditional on τ_t). Therefore $\bar{\tau}_{it}$ weakly increases in $\tilde{g}_t$. It is independent of M_t and $\tilde{d}_t$. Let $\bar{\tau}_t$ denote $\sup_i \bar{\tau}_{it}$. Assume, for convenience, that some firm would find using emission removal to be cheaper than abating all of its emissions: $Z_{it}^{tax} > 0$ for some i when $\tau_t = \bar{\tau}_t$.

The time t regulator solves:

$$V_t^{tax}(M_t, \tilde{g}_t, \tilde{d}_t) = \min_{\tau_t} \left\{ \int_0^1 C_{it}(A_{it}^{tax}) \, \mathrm{d}i + G_t(Z_t^{tax}; \tilde{g}_t) + D_t(T_t; \tilde{d}_t) + \frac{1}{1+r} E_t[V_{t+1}^{tax}(M_{t+1}, \tilde{g}_{t+1}, \tilde{d}_{t+1})] \right\}.$$

The regulator's first-order condition is

$$\begin{aligned} 0 =& \int_0^1 \frac{\partial A_{it}^{tax}}{\partial \tau_t} \left[C'_{it}(A_{it}^{tax}) - \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})] \right] \mathrm{d}i \\ &+ \int_0^1 \mathbb{1}_{Z_{it}^{tax}>0} \frac{\partial Z_{it}^{tax}}{\partial \tau_t} \left[G'_t(Z_t^{tax}) - \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})] \right] \mathrm{d}i, \end{aligned}$$

where $\mathbb{1}$ is the indicator function and where I substitute from the envelope theorem. The partial derivatives are zero for all firms i such that $\tau_t \geq \bar{\tau}_{it}$. Substituting the other firms' first-order conditions yields

$$\tau_t = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})]. \tag{6}$$

I denote this tax $\breve{\tau}_t$. This is the instrument familiar from previous literature (e.g., Nordhaus, 1982, 1991; Farzin, 1996). It is the unique optimal tax as long as $\breve{\tau}_t \leq \bar{\tau}_t$. If $\breve{\tau}_t > \bar{\tau}_t$, then any $\tau_t \geq \bar{\tau}_t$ is an optimum.

Comparing the resulting firm first-order conditions to (1) and (2), we see that, given whatever emission decisions were made prior to time t, two conditions must hold for the regulator to implement the first-best allocation in time t. First, $\breve{\tau}_t$ must be weakly less than $\bar{\tau}_{it}$ for all firms i, so that no firms merely eliminate their emissions instead of undertaking negative emissions. This in turn happens if either $\tilde{d}_t$ is sufficiently small or $\tilde{g}_t$ is sufficiently large. Second, $E_{t+s}[D'_{t+s}(T_{t+s})]$ must be as in first-best for all $s > 0$. This latter condition occurs if and only if either (i) $\bar{\tau}_{i(t+s)}$ cannot bind for any i at any $s \geq 0$ or (ii) $D_{t+s}(\cdot)$ is linear for all $s \geq j$, where j is the first time at which $\bar{\tau}_{ij}$ might bind for some i. If condition (i) does not hold, then there are states of the world in which some firms merely eliminate emissions, making T_{t+s} larger than first-best for all sufficiently large s. As a result, $E_t[D'_{t+s}(T_{t+s})]$ becomes larger than first-best for some $s > 0$ if condition (ii) also does not hold. When the regulator cannot implement first-best, the unconstrained-optimal tax $\breve{\tau}_t$ obtains more abatement than in first-best in order to compensate for the reality that some time t firms merely eliminate their emissions instead of undertaking negative emissions or for the possibility that future taxes will not obtain the future negative emissions potentially required by first-best.

The following proposition formalizes the foregoing results.

Proposition 1.

1. Ex-Ante Optimality: *Looking forward from time 0, $\{\breve{\tau}_t\}_{t=0}^{\infty}$ will achieve the first-best allocation in all states of the world if and only if $d^H \leq \bar{d}(g^L)$, for some $\bar{d}$ increasing in g^L.*

2. Ex-Post Optimality: *Looking backward from some time $s > 0$, $\{\breve{\tau}_t\}_{t=0}^{s}$ achieves the first-best allocation in periods 0 through s if and only if either (i) it achieves first-best ex-ante or (ii) each $D_t(\cdot)$ is linear for all t sufficiently large with each $\tilde{d}_j$ sufficiently*

small (equivalently, each $\tilde{g}_j$ *sufficiently large) that* $\breve{\tau}_j \leq \bar{\tau}_{ij}$ *for all firms* i *and all periods* $j \in [0, s]$.

Proof.

1. Follows from the foregoing analysis, defining $\bar{d}_t$ as the smallest $\tilde{d}_t$ such that $\breve{\tau}_t = \bar{\tau}_{it}$ for some firm i at $\tilde{g}_t = g^L$, recognizing that $\bar{\tau}_{it}$ is increasing in $\tilde{g}_t$, and defining $\bar{d}$ as the infimum of the $\bar{d}_t$.

2. Condition (i) follows by definition. For condition (ii), assume that $\{\breve{\tau}_t\}_{t=0}^s$ does not achieve first-best ex ante, which means that $\bar{\tau}_{it}$ binds in some state of the world and at some time t. Let k be the first time at which $\bar{\tau}_{it}$ binds for some i. $E_{t-j}[D'_t(T_t)]$ (for $j \in [0, t]$) is the same in equations (1), (2), and (6) if and only if each $D_t(\cdot)$ is linear for all $t \geq k$. And from equations (3) and (4), $\{\breve{\tau}_t\}_{t=0}^s$ implements A_{it}^{opt} and Z_t^{opt} from periods 0 through s if and only if, first, $E_{t-j}[D'_t(T_t)]$ (for $j \in [0, t]$) is the same in equations (1), (2), and (6) and, second, $\breve{\tau}_t \leq \bar{\tau}_{it}$ for all firms i and all $t \in [0, s]$. We know that $\breve{\tau}_t$ decreases in $\tilde{d}_t$ and that $\bar{\tau}_{it}$ increases in $\tilde{g}_t$, so $\breve{\tau}_t \leq \bar{\tau}_{it}$ if $\tilde{d}_t$ is sufficiently small or $\tilde{g}_t$ is sufficiently large. The proposition follows.

$\square$

We have detected a new inefficiency when high damages and/or cheap removal imply $\breve{\tau}_t > \bar{\tau}_{it}$ for some firm i. The cost of the tax policy relative to first-best depends on the probability of wanting some firms to undertake negative emissions and on the convexity of abatement and emission removal supply curves in regions with negative emissions. If abatement and removal costs are highly convex around each $\bar{\tau}_{it}$, then larger taxes would not substantially affect the climate even if firm i were incentivized to undertake negative emissions, so the loss from being unable to incentivize negative emissions is small.

The regulator could of course implement the optimal allocation if it could subsidize emission removal based on lump-sum taxation. However, such subsidies may be politically infeasible or may be financed only through distortionary taxation.[15] I henceforth consider

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[15] From equation (2), financing the subsidies through retained emission tax revenue can be insufficient to finance first-best negative emissions if marginal damage ends up greater than expected and negative emissions are substantial, two characteristics that are likely to be correlated. Also note that using emission tax revenue to fund emission removal is costly if this revenue could have been used to cut distortionary taxes (Goulder, 1995).

policies that can implement the optimal allocation without requiring net outlays from the public purse.

2.3 Atmospheric Rental Policy

Now consider taxing the stock of carbon rather than the emission of carbon. I refer to this stock tax as an atmospheric rental policy to differentiate it from standard carbon (emission) taxes: it charges firms period-by-period for their ongoing use of atmospheric storage. The regulator charges firms ψ_t for each unit of current or past emissions remaining in the atmosphere at the end of period t. Under familiar emission tax policies, firms pay a tax only in the period in which they emit; under the atmospheric rental policy, firms pay a tax in every period from the time of emission until the time of emission removal (should it occur). The optimal level of the charge will of course differ between the two policies.

Let M_{it} indicate firm i's cumulative emissions from time 0 up to time t:

$$M_{it} = \sum_{s=0}^{t-1} [e_{is} - A_{is} - Z_{is}].$$

At time t, firm i solves:

$$\pi_{it}^{rental}(\psi_t, M_{it}, M_t) = \min_{A_{it} \leq e_{it}, Z_{it} \geq 0} \Bigg\{ C_{it}(A_{it}) + p_t Z_{it} + \max\{0, \psi_t[e_{it} - A_{it} - Z_{it} + M_{it}]\} + \frac{1}{1+r} E_t[\pi_{i(t+1)}^{rental}(\psi_{t+1}, M_{i(t+1)}, M_{t+1})] \Bigg\},$$

where I again suppress dependence of π_{it} on the random variables. The maximization problem differs from that under the tax policy only in that payments here depend on the history of abatement and emission removal decisions. Firms now never choose $A_{it} + Z_{it} > e_{it} + M_{it}$. Repeatedly applying the envelope theorem and substituting for equilibrium p_t, the first-order conditions satisfied by an interior solution become:[16]

$$C'_{it}(A_{it}) = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\psi_{t+s}],$$
$$G'_t(Z_t) = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\psi_{t+s}].$$

Let A_{it}^{rental} and Z_{it}^{rental} indicate firms' choices. Each increases in each expected charge $E_t[\psi_{t+s}]$, holding the other expected charges fixed. If the first-order conditions imply $A_{it}^{rental} + Z_{it}^{rental} > e_{it} + M_{it}$, then firm i chooses $A_{it}^{rental} + Z_{it}^{rental} = e_{it} + M_{it}$ and both A_{it}^{rental} and Z_{it}^{rental} are locally independent of all ψ_{t+s}.

The time t regulator solves:

$$\begin{aligned} V_t^{rental}(M_t, \tilde{g}_t, \tilde{d}_t) = \min_{\psi_t} \Bigg\{ & \int_0^1 C_{it}(A_{it}^{rental}) \, \mathrm{d}i + G_t(Z_t^{rental}; \tilde{g}_t) + D_t(T_t; \tilde{d}_t) \\ & + \frac{1}{1+r} E_t[V_{t+1}^{rental}(M_{t+1}, \tilde{g}_t, \tilde{d}_t)] \Bigg\}. \end{aligned}$$

Using the envelope theorem, the regulator's first-order condition is

$$\begin{aligned} 0 = & \int_0^1 \frac{\partial A_{it}^{rental}}{\partial \psi_t} \left[C'_{it}(A_{it}^{rental}) - \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})] \right] \mathrm{d}i \\ & + \int_0^1 \mathbb{1}_{Z_{it}^{rental} > 0} \frac{\partial Z_{it}^{rental}}{\partial \psi_t} \left[G'_t(Z_t^{rental}) - \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})] \right] \mathrm{d}i. \end{aligned}$$

The partial derivatives are zero for all firms i such that $A_{it}^{rental} + Z_{it}^{rental} \geq e_{it} + M_{it}$. Substituting the other firms' first-order conditions yields

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\psi_{t+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})]. \tag{7}$$

Many sequences of ψ_{t+s} satisfy this condition for given t, but a time-consistent policy satisfies this condition for all t. The following proposition describes the optimal time-consistent policy:

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[16] Firms are small, so they do not account for their infinitesimal effect on M_{t+s} and thus on ψ_{t+s}.

Proposition 2. *The unique time-consistent policy that satisfies* (7) *sets* $\psi_t = \alpha\, D_t'(T_t)$ *at every time* $t \geq 0$.

Proof. A time-consistent policy that satisfies (7) also satisfies:

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t+1}[\psi_{t+1+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t+1}[D'_{t+1+s}(T_{t+1+s})].$$

Taking expectations of both sides with respect to the time t information set and using the law of iterated expectations, we have:

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t}[\psi_{t+1+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t}[D'_{t+1+s}(T_{t+1+s})].$$

Using this, (7) becomes:

$$\psi_t = \alpha\, D_t'(T_t).$$

The proposition follows from observing that the choice of t was arbitrary and that condition (7) holds if $\psi_{t+s} = \alpha\, D'_{t+s}(T_{t+s})$ for all $s \geq 0$.

□

I denote the charge derived in Proposition 2 as $\breve{\psi}_t$.

In Section 2.2, the regulator's desired emission tax $\breve{\tau}_t$ was the present value of the strip of marginal damages incurred by a unit of emissions. Using equation (6), we have:

$$\breve{\tau}_t = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\breve{\psi}_{t+s}]. \tag{8}$$

The optimal rental policy explodes this strip into its constituent pieces, charging firms only as damages are realized and only on the condition that their emissions remain in the atmosphere. Firms' interior solutions are the same whether they face $\breve{\tau}_t$ or the stream of $\breve{\psi}_{t+s}$.[17] Define $\bar{\bar{\tau}}_{it}$ as the smallest τ_t such that

$$A_{it}^{tax} + Z_{it}^{tax} = e_{it} + M_{it}. \tag{9}$$

---

[17] The result relies on forward-looking firms discounting the future at the same rate as the regulator. The next section will show how policy can overcome firms using higher discount rates, there driven by bankruptcy risk. See Barrage (2018) for further analysis of differential social and private discounting.

We immediately have the analogue of the analysis in Section 2.2: the rental policy achieves first-best abatement and emission removal in period t as long as

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\breve{\psi}_{t+s}] \leq \bar{\bar{\tau}}_{it}$$

for all firms i and either the analogous condition holds for all later times at all feasible states or each $D_{t+s}(\cdot)$ is linear for all s sufficiently large.[18] Comparing equations (5) and (9), $\bar{\bar{\tau}}_{it} \geq \bar{\tau}_{it}$: the rental policy can obtain more abatement and emission removal than can the emission tax policy and can therefore achieve first-best in a weakly larger set of cases than can the emission tax policy.

Two points are of special policy relevance. First, note that the optimal allocation would never have $\int_0^1 A_{it}\,\mathrm{d}i + Z_t > M_t + \int_0^1 e_{it}\,\mathrm{d}i$ as $M_t \to 0$ (because $D_t'(0) = 0$). In that case, $\bar{\bar{\tau}}_t \triangleq \sup_i \bar{\bar{\tau}}_{it}$ never binds, so that the rental charge policy can always attain the optimum. It is therefore important to begin implementing the rental policy early, when the preexisting emissions M_0 that escape later charges are still small. Second, $\bar{\tau}_t$ is independent of emission taxes chosen in periods $s < t$ but $\bar{\bar{\tau}}_t$ decreases in rental charges chosen in periods $s < t$. The gains from using a rental policy vanish if those earlier charges were so large as to eliminate earlier emissions (implying $\bar{\bar{\tau}}_t = \bar{\tau}_t$), but the gains potentially become large if those earlier charges were so small that they left substantial emissions in the atmosphere (permitting $\bar{\bar{\tau}}_t \gg \bar{\tau}_t$). If policy must be lax in some early periods (whether due to optimal choices or political constraints), then the gains from using a rental policy are potentially large. Putting these points together, it becomes especially important to immediately begin a rental policy precisely in the case in which policymakers insist on implementing an emission charge that is much smaller than the optimal charge. In such cases, high early emissions make negative emissions more likely to be desirable in later periods. Starting a rental policy earlier provides greater scope for obtaining these negative emissions through decentralized market incentives.

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[18] As in Section 2.2, the regulator obtains more time t abatement and emission removal than in first-best if the analogous condition might not hold at some later time and $D_{t+s}(\cdot)$ is nonlinear at some sufficiently large s. Now, however, that additional abatement and emission removal arises not because the regulator implements a more stringent time t policy but because firms expect the regulator to implement a more stringent policy at the later time $t+s$.

2.4 The Challenge of Market Churn

Instead of compiling the stream of expected marginal damages into a single emission charge, the rental policy requires firms to pay for marginal damages period by period. However, damages from climate change unfold over a very long time.[19] If firms declare bankruptcy, then they will not be around to pay to remove their old emissions from the atmosphere. Moreover, if they anticipate that they may not be in business at some later time, then they may overemit in the near term because their emissions appear less costly.[20]

Let each firm have probability λ of declaring bankruptcy between any two periods. So as not to conflate issues, imagine that each firm is replaced by a similar firm, leaving aggregate business-as-usual emissions unaffected. The chance of bankruptcy reduces firm i's discount factor to $(1-\lambda)/(1+r)$. The chance of bankruptcy does not affect firms' decisions under the emission tax policy and thus does not affect the optimal emission tax. However, under the rental policy, bankruptcy risk leads firms to undertake less abatement and emission removal for a given sequence of anticipated charges. Moreover, the realization of bankruptcy also reduces M_{it} to 0, as the new firm i does not carry old emission liabilities. The maximum level of abatement plus emission removal that firm i will undertake therefore falls after bankruptcy.

For the regulator, equation (7) becomes:

$$\sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[\psi_{t+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})]. \tag{10}$$

The following proposition describes the optimal time-consistent policy:

Proposition 3. *The unique time-consistent policy that satisfies* (10) *sets* $\psi_t = (1-\lambda)\breve{\psi}_t + \lambda\breve{\tau}_t$ *at every time* $t \geq 0$.

Proof. Rearrange (10):

$$\psi_t = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})] - \frac{1-\lambda}{1+r} \sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[\psi_{t+1+s}]. \tag{11}$$

---

[19] The appendix shows that the rental charges that comprise the currently optimal emission tax remain significant for a century or more.

[20] The atmospheric rental charge policy could have instead been implemented by capping cumulative emissions in each period and requiring current and past emitters to have a permit. This quantity version of the policy is subject to the same concerns about bankruptcy risk.

A time-consistent policy that satisfies (10) also satisfies:

$$\sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_{t+1}[\psi_{t+1+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t+1}[D'_{t+1+s}(T_{t+1+s})].$$

Taking expectations of both sides with respect to the time t information set and using the law of iterated expectations, we have:

$$\sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_{t}[\psi_{t+1+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t}[D'_{t+1+s}(T_{t+1+s})].$$

Using this in (11), simplifying, and adding and subtracting $\lambda\,\alpha\,D'_t(T_t)$, we have:

$$\psi_t = (1-\lambda)\,\alpha\,D'_t(T_t) + \lambda\,\alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})].$$

The proposition follows from observing that the choice of t was arbitrary and using the definitions of $\breve{\tau}_t$ and $\breve{\psi}_t$.

$\square$

The optimal charge is a weighted average of the Pigouvian emission tax and the optimal rental charge in the absence of bankruptcy risk. As $\lambda \to 0$, bankruptcy becomes highly unlikely and we are back to the optimal rental charge analyzed in Section 2.3. As $\lambda \to 1$, firms survive for only a single period. We are back to the Pigouvian emission tax analyzed in Section 2.2, as we must force firms to pay for all future social costs at the time they emit. In between these two extremes, the optimal charge forces firms to immediately pay for the time t slice of marginal social costs associated with time t emissions and also forces them to pay for a share of future marginal social costs that reflects their chance of going bankrupt before paying future charges.

The following corollary establishes that firms' incentives to reduce emissions are as in first-best, as long as firms' solutions are interior:

Corollary 4. *Under the policy from Proposition 3, firms' interior solutions are defined by equations* (1) *and* (2)*.*

Proof. At an interior solution, firm i's first-order conditions imply

$$\begin{aligned} C'_{it}(A_{it}) &= \sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[(1-\lambda)\breve{\psi}_{t+s} + \lambda \breve{\tau}_{t+s}], \\ G'_t(Z_t) &= \sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[(1-\lambda)\breve{\psi}_{t+s} + \lambda \breve{\tau}_{t+s}]. \end{aligned}$$

Using equation (8), the first-order conditions become:

$$\begin{aligned} C'_{it}(A_{it}) &= \sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t \left[(1-\lambda)\breve{\psi}_{t+s} + \lambda \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \breve{\psi}_{t+s+j} \right], \\ G'_t(Z_t) &= \sum_{s=t}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t \left[(1-\lambda)\breve{\psi}_{t+s} + \lambda \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \breve{\psi}_{t+s+j} \right]. \end{aligned}$$

Collecting terms, each right-hand side becomes:

$$\sum_{j=0}^{\infty} E_t[\breve{\psi}_{t+j}] \left[\frac{(1-\lambda)^j}{(1+r)^j}(1-\lambda) + \sum_{s=0}^{j} \frac{(1-\lambda)^s}{(1+r)^s} \frac{1}{(1+r)^{j-s}} \lambda \right],$$

which simplifies to

$$\sum_{j=0}^{\infty} E_t[\breve{\psi}_{t+j}] \left[\frac{(1-\lambda)^j}{(1+r)^j}(1-\lambda) + \sum_{s=0}^{j} \frac{(1-\lambda)^s}{(1+r)^j} \lambda \right].$$

Solving the geometric series in brackets, this becomes:

$$\sum_{j=0}^{\infty} E_t[\breve{\psi}_{t+j}] \left[\frac{(1-\lambda)^j}{(1+r)^j}(1-\lambda) + \frac{\lambda}{(1+r)^j} \frac{1-(1-\lambda)^{j+1)}}{1-(1-\lambda)} \right],$$

which simplifies to

$$\sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t[\breve{\psi}_{t+j}].$$

The corollary follows from substituting for each $\breve{\psi}_{t+j}$ from Proposition 2.

□

The optimal policy successfully forces firms to internalize the social cost of their emissions, despite bankruptcy risk. There is no inefficiency in incentives at the time of emission.[21] The inefficiency arises, as before, from incentives to undertake negative emissions. That inefficiency is less severe than in the case of an emission tax, because firms are charged for past emissions and thus do have an incentive to remove them. But the possibility of bankruptcy reduces the gains relative to an emission tax: if firm i has replaced an older firm that went bankrupt (or, equivalently, if firm i represents a firm that survived but shed its liabilities through bankruptcy), then M_{it} in equation (9) is smaller than it would have been in the absence of bankruptcy. At sufficiently large charges, firm i might pay for less emission removal than if it were accountable for the full history of emissions by firms of type i.

In practice, negative emissions may not be optimal until midcentury or later. The risk of any given firm declaring bankruptcy between now and then is not small. These bankruptcies will erode the ability of a system of rental charges to incentivize negative emissions by eroding the base of emission liabilities subject to the charge. I next consider a policy that successfully incentivizes negative emissions even in the presence of arbitrarily severe bankruptcy risk.

2.5 Carbon Shares

We seek a policy that can motivate firms to undertake first-best emission reductions and carbon removal without being vulnerable to the chance that firms go bankrupt before carbon removal becomes optimal. I now propose a new type of policy: carbon shares. This policy will combine the advantage of emission taxes in collecting payment at the time of emission with the advantage of rental charges in preserving incentives to remove old emissions from the atmosphere.

A carbon share policy requires firms to post a bond θ_t per unit of time t emissions. This bond is used to finance a transferable asset that the emitter receives from the regulator. This asset is attached to the unit of carbon emitted. I refer to the asset as a carbon share because it reflects a claim on a part of the carbon in the atmosphere. The face value of the

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[21] In practice the probability of bankruptcy will vary across firms. In that case, an additional inefficiency will arise if each firm knows its own probability of bankruptcy and the regulator is unable to tailor the charge to each type of firm. I here show that even the simplest form of bankruptcy risk creates distortions when negative emissions might be optimal.

carbon share in each period $t+s$ is $B_{t,t+s}$, with $B_{t,t}=\theta_t$. In each period subsequent to emission, shareholders decide whether to leave the unit of carbon in the atmosphere. If they remove that unit of carbon from the atmosphere in time $t+s$, they receive $B_{t,t+s}$; otherwise they receive a dividend $\delta_{t,t+s}$. The policymaker can also charge $\kappa_{t,t+s}$ to the face value of the asset. The face value of the asset evolves as $B_{t,t+s+1}=(1+r)(B_{t,t+s}-\delta_{t,t+s}-\kappa_{t,t+s})$. The policymaker cannot return or deduct any more than the current value of the asset: $\delta_{t,t+s}+\kappa_{t,t+s}\leq B_{t,t+s}$. The policymaker must eventually allocate the entire original bond to either dividends or declared charges: $\lim_{s\to\infty} B_{t,t+s}=0$.

The carbon share is an option to obtain the face value of the bond by spending on carbon removal. The option's holder will not exercise it unless doing so creates value. The option's holder receives the dividends $\delta_{t,t+s}$ whether exercising or holding the option, but the option's holder loses the charges $\kappa_{t,t+s}$ as long as the option is unexercised. The option's value is $\Omega_{t,t+s}$. Clearly, $\Omega_{t,t}\leq\theta_t$ and $\Omega_{t,t+s}\geq 0$. At the time of emission, the firm's net outlays per unit of non-abated emissions are $\theta_t-\Omega_{t,t}\geq 0$. If a firm that held a carbon share were to declare bankruptcy or otherwise liquidate, its creditors would want the carbon share so they could receive its dividends and have the option to eventually reclaim its face value.

The benefit from exercising the option in period $t+s$ is

$$B_{t,t+s}=\sum_{j=0}^{\infty}\frac{1}{(1+r)^j}E_{t+s}[\delta_{t,t+s+j}+\kappa_{t,t+s+j}]. \tag{12}$$

The cost of exercising the option is the cost p_{t+s} of removing the unit of carbon plus the cost $E_{t+s}[\Omega_{t,t+s+1}]/(1+r)$ of losing the option in the future plus the cost of not receiving the dividend $\delta_{t,t+s}$. In a competitive equilibrium with abundant carbon shares, agents exercise their options up to the point at which the cost of removal absorbs the profits from exercise:

$$p_{t+s}=B_{t,t+s}-\frac{1}{1+r}E_{t+s}[\Omega_{t,t+s+1}]-\delta_{t,t+s}. \tag{13}$$

Agents may compete away the entire face value of the carbon share (i.e., $p_{t+s}\leq B_{t,t+s}-\delta_{t,t+s}$) because they must be compensated for forgoing the right to exercise the option in future periods.

The following proposition establishes the equilibrium value of the carbon share:

Proposition 5. *In a competitive equilibrium,*

$$\Omega_{t,t+s} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_{t+s}[\delta_{t,t+s+j}]. \tag{14}$$

Proof. Conjecture that the value of the carbon share depends linearly on each $E_{t+s}[\delta_{t,t+s+j}]$ and $E_{t+s}[\kappa_{t,t+s+j}]$:

$$\Omega_{t,t+s} = \sum_{j=0}^{\infty} \Lambda_{t+s,t+s+j} E_{t+s}[\delta_{t,t+s+j}] + \sum_{j=0}^{\infty} \Phi_{t+s,t+s+j} E_{t+s}[\kappa_{t,t+s+j}],$$

for unknown sequences $\{\Lambda_{t+s,t+s+j}\}_{j=0}^{\infty}$ and $\{\Phi_{t+s,t+s+j}\}_{j=0}^{\infty}$, with the first subscript corresponding to the evaluation period and the second subscript corresponding to the period in which the dividend is received or the charge is incurred. The constant is zero because we know $\Omega_{t,t+s} \to 0$ as $B_{t,t+s} \to 0$. If the option is exercised in period $t+s$, its value is

$$\Omega_{t,t+s} = B_{t,t+s} - p_{t+s}.$$

Using equation (13) and substituting for $\Omega_{t,t+s+1}$, we find

$$\Omega_{t,t+s} = \delta_{t,t+s} + \frac{1}{1+r} \sum_{j=1}^{\infty} \Lambda_{t+s+1,t+s+j} E_{t+s}[\delta_{t,t+s+j}] + \frac{1}{1+r} \sum_{j=1}^{\infty} \Phi_{t+s+1,t+s+j} E_{t+s}[\kappa_{t,t+s+j}].$$

(Note that this condition is identical to the condition that holds if an option is optimally not exercised in period $t+s$.) Matching coefficients, $\Lambda_{t+s,t+s} = 1$, $\Phi_{t+s,t+s} = 0$, $\Lambda_{t+s,t+s+j} = \Lambda_{t+s+1,t+s+j}/(1+r)$, and $\Phi_{t+s,t+s+j} = \Phi_{t+s+1,t+s+j}/(1+r)$ for $j \geq 1$. Advancing the analysis by one timestep, we find $\Lambda_{t+s+1,t+s+1} = 1$ and $\Phi_{t+s+1,t+s+1} = 0$. Therefore $\Lambda_{t+s,t+s+1} = 1/(1+r)$ and $\Phi_{t+s,t+s+1} = 0$. The proposition follows from repeating these steps for later time periods. □

The equilibrium value of the carbon share is the expected present value of the dividends that it claims. Using equations (14) and (12) in (13), we find

$$p_{t+s} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_{t+s}[\kappa_{t,t+s+j}]. \tag{15}$$

Exercising the carbon share asserts a claim to the present value of expected remaining damage charges, but the equilibrium cost of exercising the share absorbs these benefits. The value of the carbon share in (14) does not reflect future damage charges because their value gets competed away in equilibrium.

Substituting for equilibrium p_{t+s}, equation (15) becomes:

$$G'_{t+s}(Z_{t+s}) = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_{t+s}[\kappa_{t,t+s+j}].$$

Comparing to (2), we find that emission removal decisions are first-best if

$$\kappa_{t,t+s+j} = \alpha[D'_{t+s+j}(T_{t+s+j})] \tag{16}$$

at all possible states.[22] For these charges to be feasible given the initial posted bond, we require

$$B_{t,t+s} \geq \sup_{\{\tilde{g}_{t+s+j},\tilde{d}_{t+s+j}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \alpha[D'_{t+s+j}(T_{t+s+j})]$$

for all $s \geq 0$, which implies

$$\theta_t \geq \sup_{\{\tilde{g}_{t+j},\tilde{d}_{t+j}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \alpha[D'_{t+j}(T_{t+j})]. \tag{17}$$

The cost of emitting in period t is $\theta_t - \Omega_{t,t}$, which from (14) is

$$\theta_t - \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t[\delta_{t,t+j}].$$

Combining this, $\lim_{s\to\infty} B_{t,t+s} = 0$, and (17), we have:

$$\theta_t - \Omega_{t,t} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t[\kappa_{t,t+j}]. \tag{18}$$

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[22] The optimal damage charge is the same for all emission vintages t because damages here depend only on cumulative emissions.

Comparing to (1), we have first-best abatement incentives when $\kappa_{t,t+j}$ is as in (16). We can therefore attain first-best as long as assets remain in the atmosphere, which is true as long as carbon shares remain unexercised. This is exactly the same condition under which the rental charge policy in Section 2.3 attains first-best. And just as in that case, we here can always attain first-best when the policy is begun before too many emissions have accumulated (i.e., when M_0 is small).

This new policy overcomes the challenges posed by the long lifetime of emissions and the potential of bankruptcy by creating a valuable asset that investors want to hold. That asset is financed at the time of emission by the initial bond θ_t. Interpreting (17), that initial bond should be at least as large as the worst-case social cost of carbon. The potentially large bond does not distort firms' abatement incentives because firms receive an asset whose value increases in the size of the bond. Interpreting (16), the regulator should deduct the current period's marginal damage from the face value of any share attached to units of carbon that remain in the atmosphere. The dividends can be structured in any fashion so long as they do not reduce the face value of the share below the current estimate of the worst-case social cost of carbon. The dividend plan that returns the bond to shareholders in the most rapid fashion refunds the change in the worst-case social cost of carbon from period to period.[23] From (14), the value of the carbon share is then the difference between the worst-case social cost of carbon and the expected social cost of carbon.

One might be concerned that the initial bond would challenge firms' liquidity (see Shogren et al., 1993). Note, however, that firms receive a carbon share with value $\Omega_{t,t}$ in exchange for the bond θ_t, and they can sell that asset on. Comparing (18) and (6), their net outlays per unit of emissions are the exact same outlays required by the traditional Pigouvian carbon tax. The carbon share policy therefore need not be any more financially challenging than a conventional carbon emission tax, which is not typically described as challenging liquidity constraints.[24]

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[23] If the worst-case social cost of carbon is calculated correctly, then its value must weakly decline from period to period. If the worst-case social cost of carbon somehow increased over time, then the regulator could require shareholders to increment the posted bond. The regulator in essence marks the face value of the carbon share to the evolving worst-case damage estimate.

[24] Direct outlays are also capped, for two reasons. First, the magnitude of the optimal bond is capped even when marginal damages can become arbitrarily large as long as optimal policy can avoid incurring these damages by removing carbon. Second, any firm could avoid posting the bond by reducing its emissions. The growing number of firms making zero-emission pledges and recent cost projections for removal technologies both suggest that even the maximum gross outlays are limited to a reasonable scale.

3 Quantitative Evaluation

I now quantitatively assess the level of the optimal bond and the gains from enabling negative emissions. I extend the DICE-2016R climate-economy model of Nordhaus (2017) to allow for uncertainty about damages from warming.[25] Prior to 2065, the damage parameter is fixed and known and negative emissions are not allowed. In 2065, a random component of the damage parameter is realized and negative emissions become feasible. I calibrate the variance of damages to the expert survey of year 2066 losses from climate change in Pindyck (2019), following the implementation in Lemoine (2021) that adjusts for uncertainty about warming. In one case ("DICE Damages"), I fix the mean of the distribution to match damages in DICE-2016R, and in the other case ("Expert Damages"), I allow the mean to also be determined by the expert survey. The latter case implies more severe losses from warming. The appendix provides the full equations and parameterization. It also plots optimal trajectories in a deterministic version of each calibration.

Negative emissions are relevant. In the case with DICE damages, emissions are negative in 2065 in 2.6% of cases, emissions are negative at some point after 2065 in 95% of cases, and those negative emissions are substantial enough to eventually remove some pre-2065 emissions in 71% of cases. In the case with expert damages, emissions are negative in 2065 in 48% of cases (including at the mean—see the appendix), are always eventually negative at some point after 2065, and are nearly always substantial enough to eventually remove some pre-2065 emissions.

Figure 1 plots the percentage of cases in which bonds of varying sizes end up being large enough to fund the ex-post optimal sequence of per-period charges. The left panel shows that the case with expert damages requires much larger bonds, reflecting its much larger emission charges. A bond of 300 $\$/tCO_2$ covers 93% of outcomes under expert damages, whereas a bond of 50 $\$/tCO_2$ covers 96% of outcomes in the case with DICE damages. The right panel plots these same bonds as a percentage of the optimal year 2015 optimal emission tax, which is 192 $\$/tCO_2$ in the case of expert damages and 23 $\$/tCO_2$ in the case

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[25] Because it allows abatement to exceed 100%, the abatement cost function in DICE-2016R implicitly accounts for carbon dioxide removal technologies. I maintain this cost function and focus on uncertainty about damages. Allowing for the possibility of cheaper carbon removal would increase the benefits of the carbon share policy. A full analysis would incorporate uncertainty about these costs and about other parameters, including those controlling economic growth and the sensitivity of the climate to emissions. This first analysis builds on evidence that uncertainty about damages is especially important (Lemoine, 2021).

Table 1: Balanced growth equivalent gain from optimal and constrained-optimal policy.

	Expert Damages		DICE Damages	
	BGE (%)	Loss (% of BGE)	BGE (%)	Loss (% of BGE)
Optimal	40.3	-	1.51	-
No Negative Emissions	37.6	6.8	1.37	9.1

Balanced growth equivalent gain (BGE) is relative to a case with abatement fixed at zero (but savings optimized). The BGE translates changes in welfare into the constant relative difference in consumption between two counterfactual consumption trajectories that grow at the same constant rate (Mirrlees and Stern, 1972).

of DICE damages.[26] The two curves track each other remarkably closely until we get to the very highest damage realizations. Requiring that firms post a bond equal to twice what the initial emission tax would have been has more than a 95% chance of covering the stream of optimal damage charges.[27]

Table 1 reports the balanced growth equivalent (BGE) increase in consumption from implementing policy (Mirrlees and Stern, 1972). Policy is far more valuable in the calibration to expert damages, providing expected benefits equivalent to a permanent 40% increase in consumption as opposed to a permanent 1.5% increase in consumption. The second row constrains the policymaker from obtaining negative emissions in any period, as when implementing policy through an emission tax.[28] This constraint imposes expected losses of 7–9%. Adopting a carbon share policy can thus substantially increase the potential value from controlling emissions.

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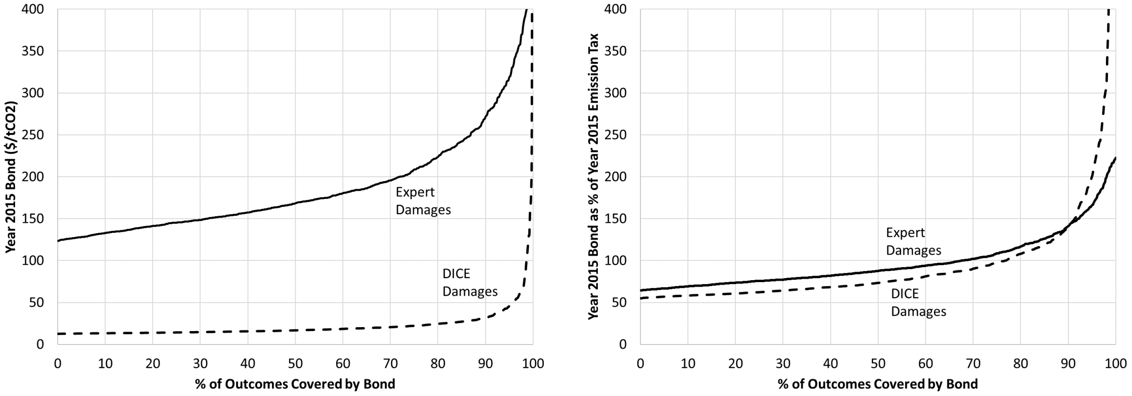
[26] The optimal tax with DICE damages is slightly below the optimal tax of 31 $\$/\mathrm{tCO}_2$ from DICE-2016R, primarily because I update the carbon cycle and climate system in accord with recommendations in Dietz et al. (2020). With either damage model, the initial period's optimal tax under uncertainty is very close to the optimal tax without uncertainty.

[27] Experiments with DICE damages and a lower utility discount rate of 0.1% per year (as in Stern, 2007) suggest that both this result and the expected loss from using an emission tax (see below) are robust to that discount rate.

[28] The regulator chooses policy in full knowledge of this constraint. To correct for the chance that the negative emission constraint will bind, the regulator increases the initial period's emission tax to 226 (24) $\$/\mathrm{tCO}_2$ with expert (DICE) damages.



(a) Bond ($\$/tCO_2$) (b) Bond (as % of initial tax))

Figure 1: The percentage of cases in which a given bond is large enough to cover the ex-post optimal damage charges, with the bond measured in $\$/tCO_2$ (left) and as a percentage of the year 2015 emission tax that would be optimal in the absence of the bond (right).

4 Discussion

I have described a new climate change policy that replaces an emission tax with a bond used to fund an asset called a "carbon share". The bond should be set equal to the worst-case social cost of carbon, and the share's face value should be reduced as climate change damages are realized. The share's remaining face value is refunded upon removing its underlying unit of carbon from the atmosphere. This new policy improves on commonly proposed emission tax and cap-and-trade policies by optimally incentivizing both emission reductions and emission removal.

There is an additional benefit to the carbon share policy that I have not explored formally. By establishing a larger market for carbon removal technologies, this policy should accelerate those technologies' development. Further, if climate damages do end up warranting negative emissions, then innovators should receive a strong signal in the form of high per-period charges in advance of those technologies being needed. By directing innovation, the carbon share policy offers additional insurance against worst-case outcomes. These effects on technical change are difficult to quantify but could be especially important.

A few objections may arise. First, one may wonder how the regulator is to develop an estimate of either the period-by-period charge or the worst-case social cost of carbon. This problem also bedevils Pigouvian emission pricing. In fact, the informational challenge is smaller under the carbon share policy than under conventional emission tax or cap-and-trade policies: specifying the worst-case social cost of carbon is less informationally demanding than specifying the expected social cost of carbon, the current period's charge is but one piece of the current period's expected social cost of carbon, and we no longer have to adjust current policy for the chance that a negative emission constraint will bind in the future.

Second, one may be concerned about the regulator's incentive to confiscate the bonds (see Shogren et al., 1993) or to set high per-period charges that raise revenue from inelastic, prior emission decisions. Such concerns are valid, but they should be counterpoised against concerns that shareholders would lobby for small per-period charges and against the possibility that prior emissions may not in fact be inelastic. Further work should consider the design of institutions in more detail, for this policy as well as for traditional emission taxes and caps.

Finally, I have described the carbon share as a price instrument, but it could be implemented as a quantity instrument. The shares would be funded by the upfront bond as already described, but instead of announcing charges period by period, the regulator would announce a cap on cumulative emissions. Each shareholder would bid in the damage charge above which they will remove their underlying unit of carbon from the atmosphere. The market-clearing charge would be deducted from the face value of each outstanding share.[29] Whether the regulator sets damage charges directly or discovers them after setting caps on cumulative emissions, the key is that carbon shareholders will not pay out of pocket after the time of emission and will trade off the cost of carbon removal against expected future charges. The regulator thereby divorces cleanup from emission decisions and optimally incentivizes each, enabling announced climate goals to be achieved through market-based policies.

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[29] Future work could explore how to discover the value of the bond through a quantity instrument.

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Appendix to "Incentivizing Negative Emissions Through Carbon Shares"

This appendix gives the full equations for the model, which follows DICE-2016R (Nordhaus, 2017). The only modifications are to change the horizon, to allow uncertainty about a damage parameter, to allow negative emissions to begin as soon as that uncertainty is resolved, and to update the carbon cycle and climate system. Table A-1 reports the values of the model parameters. A Matlab implementation of DICE-2016R (including various extensions) can be found at `https://github.com/dlemoine1/DICE-2016R-Matlab`.

The DICE model is a Ramsey growth model coupled to a climate module. An infinitely lived representative agent aims to maximize the sum of the stream of discounted utility from consuming output. The timestep is Δ years and the horizon is here 400 years, or $\bar{t} = 400/\Delta$ periods.[30] I follow DICE-2016R in setting $\Delta = 5$. The initial year is 2015, denoted here as time 0. At time 0, the policymaker chooses the abatement rate μ_t and savings rate s_t to maximize a utilitarian expected welfare function of consumption C_t and population L_t:

$$\max_{\{\mu_t, s_t\}_{t=0}^{\bar{t}-1}} E_0 \left[\sum_{t=0}^{\bar{t}-1} \frac{1}{(1+\rho)^{\Delta\, t}} L_t\, u(C_t; L_t) \right], \tag{Welfare}$$

where expectations are taken at the time 0 information set. Per-period utility is:

$$u(C_t; L_t) = \frac{(C_t/L_t)^{1-\eta}}{1-\eta}, \tag{Utility}$$

with $\eta \geq 0, \neq 1$ is the inverse of the elasticity of intertemporal substitution and also the coefficient of relative risk aversion. Utility is discounted at annual rate ρ. As described below, the policymaker chooses abatement and savings rates as functions of information about damages (i.e., as closed-loop policies), not as functions of time.

To produce time t gross output Y_t^g, the agent combines capital K_t with labor L_t and technology A_t in a Cobb-Douglas production function:

$$Y_t^g = A_t\, (L_t/1000)^{1-\kappa}\, K_t^{\kappa}. \tag{Gross output}$$

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[30] The horizon in DICE-2016R is 500 years. Shortening the horizon to 400 years does not sacrifice much but helps when optimizing under uncertainty because the number of controls becomes large.

Some of this output is lost to damages caused by surface warming T_t, so that output net of damages is given by

$$Y_t^n = Y_t^g \left[1 - \min\{0.95, d_t \, [T_t]^2\}\right]. \qquad \text{(Net output)}$$

The parameter d_t is constant and known prior to 2065, with value d. It is also constant from 2065 on, with value $\tilde{d}$. The policymaker does not know $\tilde{d}$ until 2065. In the DICE damage specification, $d = 0.00236$ and the distribution of $\tilde{d}$ is lognormal with mean 0.00236. The standard deviation of $\ln \tilde{d}$ is 1.286, from Appendix C.1 of Lemoine (2021). That calibration fits a distribution to the Pindyck (2019) expert survey of losses from climate change in fifty years after adjusting for uncertainty about warming. The expert damage specification increases both d and the mean of $\tilde{d}$ to 0.0228 in order to match the survey results and truncates the distribution from above at 0.1132 (see Lemoine, 2021). I cap the losses in any one period at 95%.

The policymaker allocates net output to consumption C_t, investment I_t, or spending Ψ_t on emission abatement. Industrial emissions (net of abatement) per timestep are:

$$E_t^I = \Delta\, \sigma_t (1 - \mu_t) Y_t^g, \qquad \text{(Industrial emissions)}$$

where σ_t is the emission intensity of production at time t. Emissions E_t (net of abatement) per timestep are

$$E_t = E_t^I + \Delta\, E_t^{\sim I}, \qquad \text{(Emissions)}$$

where $E_t^{\sim I}$ gives (exogenous) annual emissions from deforestation. Cumulative industrial emissions up to each time t are constrained by the stock of available carbon:

$$\sum_{t=0}^{\tau} \left[400 + \max\{0, E_t^I\}\right] \leq 6000 \quad \text{for all } \tau \in [0, T-1], \qquad \text{(Cumulative fossil constraint)}$$

where E_t^I is measured in Gt C. The cost of abating fraction μ_t of industrial emissions is

$$\Psi_t = \psi_t Y_t^g [\mu_t]^{a_2}. \qquad \text{(Abatement cost)}$$

The carbon tax is equal to marginal abatement cost. I constrain $\mu_t \leq 1$ prior to 2065.[31]

The economy's resource constraint is:

$$C_t + I_t + \Psi_t \leq Y_t^n. \tag{Resource constraint}$$

Capital depreciates at annual rate δ_K:

$$K_{t+1} = K_t\,(1-\delta_k)^{\Delta} + \Delta\, I_t. \tag{Capital}$$

Annual investment is determined by the savings rate s_t:

$$I_t = s_t[Y_t^n - \Psi_t]. \tag{Investment}$$

The final fifty years' savings rate is fixed at 0.2583.

The model's exogenous economic processes are

$$\begin{aligned}
L_{t+1} =& L_t \left(\frac{L_\infty}{L_t}\right)^{g_L\,\Delta/5}, && \text{(Population)}\\
A_{t+1} =& A_t/(1-g_{A,t})^{\Delta/5}, && \text{(Production technology)}\\
g_{A,t+1} =& g_{A,0}\,e^{-\Delta\,(t+1)\,\delta_A}. && \text{(Production technology growth rate)}
\end{aligned}$$

The model's exogenous climate-related processes are

$$\begin{aligned}
\sigma_{t+1} &= \sigma_t e^{\Delta\, g_{\sigma,t}}, && \text{(Gross emissions per unit of output)}\\
g_{\sigma,t+1} &= g_{\sigma,t}(1+\delta_\sigma)^{\Delta}, && \text{(Growth rate of gross emissions per unit of output)}\\
\psi_{t+1} &= \frac{a_1(1-g_\psi)^{t\,\Delta/5}\,\sigma_{t+1}}{1000\,a_2}, && \text{(Abatement cost coefficient)}\\
E^{\sim I}_{t+1} &= E^{\sim I}_0\,(1-g_E)^{(t+1)\,\Delta/5}, && \text{(Emissions from deforestation)}\\
EF_{t+1} &= EF_0 + (EF_{100} - EF_0)\min\{\Delta\, t/(5*17), 1\}. && \text{(Non-CO}_2\text{ forcing)}
\end{aligned}$$

I now describe the carbon cycle and climate model, both of which deviate from DICE-2016R. The carbon cycle follows Joos et al. (2013, Table 5), as recommended and compiled

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[31] In DICE-2016R, $\mu_t \leq 1$ for the first 145 years and $\mu_t \leq 1.2$ afterward.

by Dietz et al. (2020).[32] That carbon cycle has

$$\boldsymbol{M}_{t+1} = \boldsymbol{\Lambda}^{\Delta} M_t + \boldsymbol{b} E_t \tag{Carbon reservoirs}$$

where $\boldsymbol{M}$ is a 4×1 vector of atmospheric carbon reservoirs. The coefficient matrices are:

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9975 & 0 & 0 \\ 0 & 0 & 0.9730 & 0 \\ 0 & 0 & 0 & 0.7927 \end{bmatrix}$$

and

$$\boldsymbol{b} = \begin{bmatrix} 0.2173 \\ 0.2240 \\ 0.2824 \\ 0.2763 \end{bmatrix}.$$

The year 2015 values (in Gt C) are

$$\boldsymbol{M}_0 = \begin{bmatrix} 588 + 139.1 \\ 90.2 \\ 29.2 \\ 4.2 \end{bmatrix}, \tag{Carbon starting value}$$

where 588 Gt C is the stock of preindustrial carbon.

The parameters of the climate model come from Geoffroy et al. (2013), as compiled by Dietz et al. (2020). Additional atmospheric carbon dioxide (CO_2) increases radiative forcing $F_t(\boldsymbol{M}_t)$, which measures additional energy at the earth's surface due to CO_2 in the atmosphere. Forcing is

$$F_t(\boldsymbol{M}_t) = f_{2x} \frac{\ln\left(\sum_{i=1}^{4} M_t^i / 588\right)}{\ln(2)} + EF_t, \tag{Forcing}$$

where i indicates element i of $\boldsymbol{M}_t$, EF_t is exogenous forcing from non-CO_2 greenhouse gases

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[32] Dietz et al. (2020) additionally recommend using the FAIR model to capture carbon cycle feedbacks, but doing so would further increase the complexity of an already nontrivial optimization problem.

(defined above), and f_{2x} is forcing induced by doubling CO_2. Surface temperature evolves as

$$T^s_{t+1} = T^s_t + \frac{\Delta}{5}\phi_1 \left[F_{t+1}(\boldsymbol{M}_{t+1}) - \lambda T^s_t - \phi_3 \left(T^s_t - T^o_t\right)\right]. \qquad \text{(Surface temperature)}$$

Ocean temperature evolves as

$$T^o_{t+1} = T^o_t + \frac{\Delta}{5}\phi_4 \left[T^s_t - T^o_t\right]. \qquad \text{(Ocean temperature)}$$

Steady-state warming from doubled carbon dioxide ("climate sensitivity") is $f_{2x}/\lambda = 3.1$°C.

I solve the model by searching over contingent trajectories for μ_t, s_t, K_t, $\boldsymbol{M}_t$, T^s_t, and T^o_t, treating the transition equations as constraints. With this form, I can supply an analytic gradient for the objective and an analytic Jacobian for the constraints. I approximate the distribution over $\tilde{d}$ using quadrature with 5 nodes.[33] The trajectories are contingent because they vary by quadrature node. I solve the model in Matlab. When optimizing the full model, I search over 2,880 values. When simulating the distribution of future outcomes, I use the year 2065 state reached along the optimal trajectory (defined by policy optimized under uncertainty) and take 1,000 draws from the damage distribution.

I calculate the bond required by each draw from the damage distribution by combining the optimal year 2065 emission tax (as chosen upon learning the value of $\tilde{d}$ with the strip of pre-2065 charges. I calculate the pre-2065 charges by perturbing year 2015 emissions and calculating the change in each period's welfare.

Figure A-1 reports the abatement, emission tax, and temperature trajectories in a deterministic model in which the damage parameter is fixed to its mean at all times (i.e., $\tilde{d} = d$). Negative emissions occur in midcentury in the case with expert damages and occur early in the next century in the case with DICE damages.[34] Near-term abatement is greater in the case with expert damages, but long-term abatement is reduced because early abatement leaves a smaller stock of atmospheric carbon. The emission tax trajectory reveals corre-

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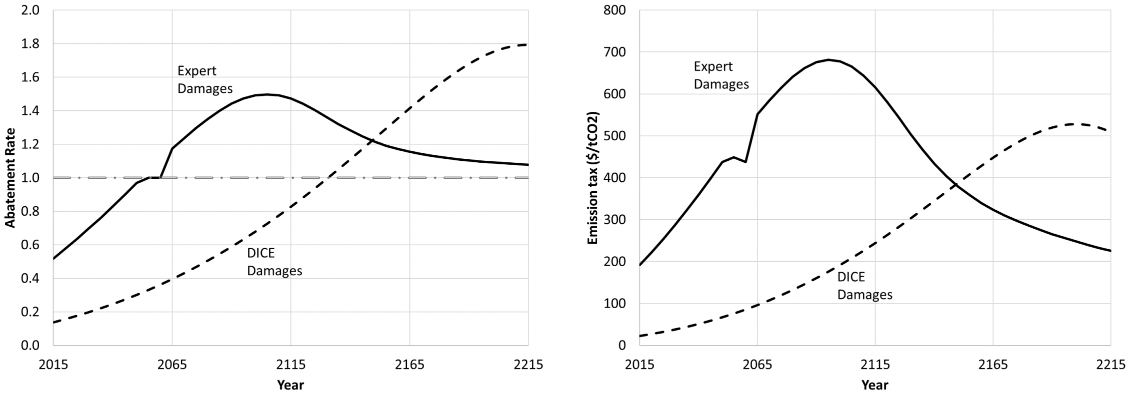
[33] I use the compecon toolbox to obtain Gaussian quadrature nodes for non-truncated distributions (Miranda and Fackler, 2002) and use the Fortran90 version of truncated_normal_rule (available at `http://people.math.sc.edu/Burkardt/c_src/truncated_normal_rule/truncated_normal_rule.html`) to obtain quadrature nodes for truncated distributions.

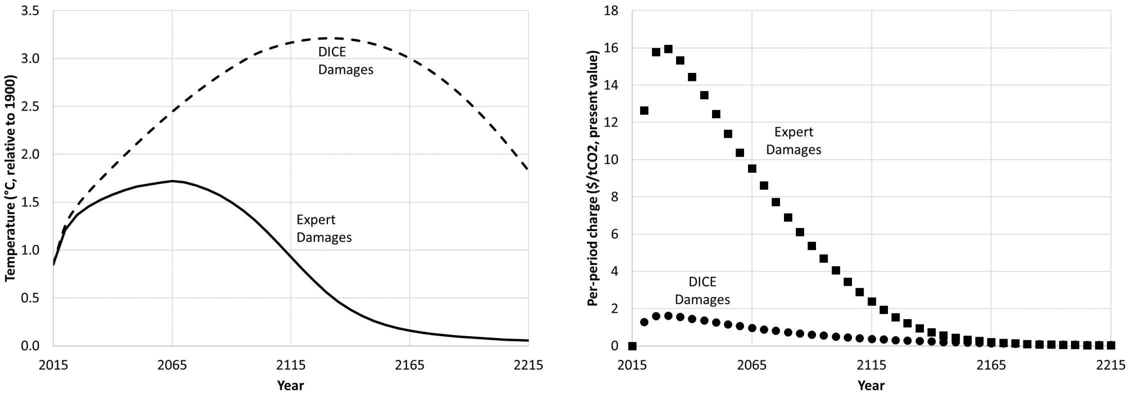
[34] The kink in the case with expert damages arises because the pre-2065 constraint that abatement be weakly less than 100% briefly binds. The tax declines over this interval because exogenously improving technology gradually reduces the tax needed to obtain 100% abatement.

sponding effects on the implied emission price. Negative emissions eventually undo some warming in both calibrations, with the expert damage calibration never allowing warming to exceed 2 degrees C.

The lower right panel explodes the tax into the strip of per-period marginal damages. These are the optimal rental charges per 5-year timestep and are also the optimal sequence of per-period damage charges that would be implemented under the carbon share policy. The sum of each set of points equals the optimal emission tax. The charges increase over the next decades as current emissions translate into warming and as higher temperatures interact with convex damages. The charges eventually decline due to the effect of discounting, the eventual decline in temperature, and the decay of initial emissions. The charges spread the emission tax's upfront payment over more than a century, with the peak charges comprising only a small fraction of the optimal emission tax.



(a) Abatement Rate (b) Emission Tax



(c) Temperature (d) Rental Charge

Figure A-1: Optimal trajectories in deterministic versions of each damage calibration.

Table A-1: Parameters

Parameter	Value	Description
Δ	5	Timestep (years)
$\bar{t}$	80	Horizon (periods)
A_0	5.115	Initial production technology
$g_{A,0}$	0.076	Initial growth rate of production technology, per five years
δ_A	0.005	Annual decline in growth rate of production technology
L_0	7403	Year 2015 population (millions)
L_∞	11500	Asymptotic population (millions)
g_L	0.134	Rate of approach to asymptotic population level, per five years
σ_0	0.0955	Initial emission intensity of output (Gt C per trillion 2010$)
$g_{\sigma,0}$	-0.0152	Initial annual growth rate of emission intensity
δ_σ	-0.001	Annual change in growth rate of emission intensity
a_1	2016.7	Cost of backstop technology in 2015 (2010$ per ton of C)
a_2	2.6	Abatement cost function exponent
g_ψ	0.025	Decline rate of backstop cost, per five years
$E_0^{\sim I}$	0.71	Initial emissions from deforestation (Gt C per year)
g_E	0.115	Decline rate of deforestation emissions, per five years
EF_0	0.5	Year 2015 non-CO_2 forcing (W/m^2)
EF_{100}	1	Year 2100 non-CO_2 forcing (W/m^2)
κ	0.3	Capital share in production
δ_K	0.1	Annual capital depreciation rate
ρ	0.015	Annual utility discount rate
η	1.45	Inverse of elasticity of intertemporal substitution; also RRA
ϕ_1	0.386	Warming delay parameter
ϕ_3	0.73	Parameter governing transfer of heat from ocean to surface
ϕ_4	0.034	Parameter governing transfer of heat from surface to ocean
f_{2x}	3.503	Forcing from doubling CO_2 (W/m^2)
λ	1.13	Forcing per degree warming ($[W/m^2]/°C$)
$d, \tilde{d}$	see text	Damage parameters

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Table A-1 – continued from previous page

Parameter	Value	Description
K_0	223	Year 2015 capital (trillion 2010$)
$\boldsymbol{M}_0$	see text	Year 2015 carbon reservoirs (Gt C)
T_0^s	0.85	Year 2015 surface temperature (°C, wrt 1900)
T_0^o	0.0068	Year 2015 lower ocean temperature (°C, wrt 1900)

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