Endogeneity in Discrete Bayesian Games: U.S. Cellphone Service Deployment[*]

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Abstract

In some Bayesian games, payoff-relevant states are influenced by unobserved player- or game-level heterogeneity that also affect strategic decisions directly. Ignoring such endogeneity in empirical analysis leads to erroneous inference of structural parameters and policy implications. We introduce a control-function approach for estimating discrete Bayesian games with such endogeneity. We apply the method to analyze an entry game of deploying 4G-LTE technology by major U.S. cellphone service providers, taking existing network deployment in the focal market and current deployment in neighboring markets as endogenous. Using lagged demographics of neighboring markets as instruments, we find that a hypothetical T-Mobile and Sprint merger would reduce 4G-LTE deployment across local markets. Moreover, the entry of a fourth national provider, enabled by a partial divestiture of the merger's assets, would not completely offset its negative impact on market entries and the population served. We also find that ignoring endogeneity in network deployment skews the policy implications by under-predicting the merger's negative impacts on market entry.

Keywords: Endogeneity, Discrete Bayesian Games, Control Function, Two-Step Nested Pseudo Likelihood, Entry Game, U.S. Cellphone Service.

JEL Classifications: C31; C35; C57; L13; L41.

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1 Introduction

We propose a control-function approach for estimating discrete Bayesian games when observable payoff-relevant covariates are endogenous. Bayesian games provide a powerful framework for analyzing strategic interaction between individuals or firms with private information (a.k.a., types or signals), and have been studied in a wide range of applied contexts. Examples include choices of effort by students and teachers in classrooms in Todd and Wolpin (2018), choices of fitness exercises by adolescents in Jackson, Lin, and Yu (2020), location choices in video retail industry in Seim (2006), timing of commercials by radio stations in Sweeting (2009), and market entry and exit of grocery stores in Grieco (2014). An important assumption required for inference in these empirical studies is that the covariates, both at the player- and game-level, are exogenous.

When covariates in a Bayesian game are endogenous, identification and estimation require further assumptions on the joint distribution of covariates *and* private types. First of all, this poses challenges that are analogous to endogeneity in single-agent qualitative response models, but aggravated in settings with strategic interaction. More importantly, if such endogeneity is due to unobserved market/game-level heterogeneity that also influences the covariates and types of other players, then the private types are generally correlated even after conditioning on all covariates. This complicates the equilibrium characterization, as well as identification and estimation.[1]

Endogeneity in covariates are common in environments with strategic interaction. For example, consider the decisions by cellphone service providers to deploy a new generation of cellphone technology in local service markets. These providers rely on cellular network infrastructures such as transmission facilities and switching offices to provide cellphone services. As the technology evolved from 3G to 4G-LTE in the last decade, a provider could re-configure and upgrade its 3G network for delivering 4G-LTE services. In addition, the spill-over effects of a provider's 4G-LTE deployment in neighboring markets could reduce the deployment costs in a focal market. Therefore, a provider's 3G deployment in a local

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[1] To see this, consider a binary game with two players i, j, individual-specific covariates Z_i, Z_j, and private types u_i, u_j. Suppose Z_i, Z_j and u_i, u_j are all correlated through some unknown market/game-level factor v_0. In this case, conditioning on (Z_i, Z_j) is not sufficient for attaining independence between (u_i, u_j) in general. Thus i's equilibrium belief about j's decision D_j would be a non-trivial function of its own types $E(D_j|Z_i, Z_j, u_i)$.

market and 4G-LTE deployment in neighboring markets are both important covariates that influence its decision to enter a local 4G-LTE market. Endogeneity in these covariates may result from several sources. For instance, there are unreported demographic or geographic characteristics (e.g., topographic features) that affected both existing 3G deployment and the costs of upgrading facilities for 4G-LTE on a focal market. Besides, a provider's spectrum holdings are not reported in the sample, but are strongly (if not perfectly) correlated between focal and neighboring markets. More broadly speaking, endogeneity is a concern in social-economic settings where payoff-relevant states are influenced by unreported player- or game-level factors. We are not aware of any paper that has allowed for such flexible sources of endogeneity in empirical analysis, or investigated the impact on inference and policy implications when endogeneity is ignored. The goal of our paper is to fill this gap.

We contribute to the econometric and empirical literature on Bayesian games in two ways. First, we introduce a general, feasible control-function method for estimating discrete Bayesian games with endogenous states. We model endogeneity in covariates through a triangular system that is flexible enough to accommodate correlation through both player-level and game-level unobserved heterogeneity. Control function variables are created as residuals from auxiliary regressions using exogenous instruments. (In our application of entry in 4G-LTE markets, we use the lagged demographics of neighboring markets as instruments.) We propose a two-step nested pseudo-likelihood (2SNPL) estimator, and show it is root-n consistent and asymptotically normal. Our monte carlo simulation shows the estimator works well in finite samples with moderate sizes.

Heckman (1978), Newey (1987) and Rivers and Vuong (1988) proposed methods for dealing with endogenous discrete and continuous covariates in single-agent qualitative response models. While there are other solutions for endogeneity in the literature,[2] the control function approach has proliferated due to its simplicity, flexibility and wide applicability.[3] We contribute to this extensive literature by bringing the control function

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[2] Lewbel (2000), Blundell and Powell (2004) and Rothe (2009) dealt with endogeneity in semiparametric binary choice models; Vytlacil and Yildiz (2007) considered nonparametric identification and estimation of average treatment effects of dummy endogenous variables in weakly separable models; Dong and Lewbel (2015) estimated binary choice models with discrete, continuous, or censored endogenous regressors. D'Haultfœuille and Février (2015) and Torgovitsky (2015) showed non-separable models with continuous outcome and endogenous variables can be identified using discrete instruments.

[3] Since its inception by Heckman and Robb (1985), the control function approach has been used in a

approach into a game-theoretic setting with incomplete information.[4] We combine control functions with a nested pseudo likelihood method to handle the simultaneity embedded in the Bayesian game and the endogeneity in regressors at the same time.[5]

Our second contribution is empirical. We apply the 2SNPL estimator to analyze a hypothetical T-Mobile and Sprint merger on the U.S. cellphone service market.[6] In addition to predicting firm entries in post-merger local markets and the population served, we evaluate the impact of adding a fourth national provider, enabled by a government-mandated partial divestiture of assets owned by the merging parties. This is a very meaningful exercise for antitrust and regulatory agencies. For example, while reviewing merger proposals, the Federal Trade Commission (FTC) and the Department of Justice (DOJ) often mandated the merging firms to divest certain assets and facilities to rivaling firms. The goal of such a policy is to strengthen after-merger competition in local markets, and alleviate the loss of consumer welfare due to increased market power of the merged entity.[7] In the case of the 2020 T-Mobile/Sprint merger case, the DOJ required the merging parties to divest parts of Sprint's prepaid businesses, Sprint's 800 MHz spectrum holding, decommissioned cell sites and retail locations to a potential competitor, DISH Network.

A crucial step in our analysis is to allow for endogeneity in providers' network deployments while analyzing their strategic decisions to enter local markets. As noted earlier, two covariates that influence strategic decisions are endogenous (3G deployment in the focal market and 4G-LTE deployment in neighboring markets respectively). Our estimates indicate that unobserved factors in a firm's 4G-LTE deployment decision are negatively correlated with its focal market's 3G deployment and positively correlated

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variety of settings. See, for example, Newey, Powell, and Vella (1999), Chesher (2003), Das, Newey, and Vella (2003), Lee (2007), Florens, Heckman, Meghir, and Vytlacil (2008), Imbens and Newey (2009), Klein and Vella (2010), Petrin and Train (2010), Hahn and Ridder (2011), Kasy (2011) among others.

[4] For econometric analyses of static Bayesian games, see Aradillas-Lopez (2010), Bajari, Hong, Krainer, and Nekipelov (2010), Florens and Sbaï (2010), Tang (2010), De Paula and Tang (2012), Misra (2013), Wan and Xu (2014), Lewbel and Tang (2015), Lin and Xu (2017), Xu (2018), Aguirregabiria and Mira (2019) and Lin, Tang, and Yu (2020).

[5] The fixed point algorithm is typically used to deal with simultaneity of strategic choices in discrete Bayesian games Rust (1987), Aguirregabiria and Mira (2002, 2007) and Kasahara and Shimotsu (2012)).

[6] T-Mobile and Sprint proposed a merger deal in 2019 and were approved to merge in 2020 after lengthy legal battles surrounding antitrust concerns. In our simulations we create a hypothetical merger between these two firms by moving the 2020 merger to the end of 2015.

[7] For example, in 2015 the FTC required Albertsons and Safeway to sell 168 stores in 130 local markets as a condition for approving their $9.2 billion merger case.

with its 4G-LTE deployment in neighboring markets. Both correlations are statistically significant, providing evidence for endogeneity of these two covariates. These covariates are directly impacted by the merger (the new entity owns a union of network facilities of the merging parties). Thus any sound analysis of the merger impact need to start with valid, endogeneity-proof inference of covariate effects.

Using our endogeneity-proof estimates, we find that the hypothetical T-Mobile and Sprint merger would substantially reduce the overall 4G-LTE deployment across local markets. This finding counters a typical pro-merger argument that cost synergies lead to wider cellular coverage and benefit consumers. Moreover, our simulations show that the addition of a fourth national firm, mirroring the DOJ's DISH Network merger remedy through divestiture, would not completely offset the merger's negative impact on market entries and the population served.[8] Lastly, we compare the estimation and simulation results with and without taking into account the endogeneity in network deployment. This comparison shows that ignoring such endogeneity would skew the policy implications for antitrust agencies by under-predicting the merger's negative impacts on entry and over-predicting the merger remedy's compensating effects.

As our work incorporates endogenous assets in oligopolistic firms' strategic choices, we build on the recent empirical literature in industrial organization that evaluates how merger affects product offerings (Fan, 2013, Wollmann, 2018, Li, Mazur, Park, Roberts, Sweeting, and Zhang, 2019, Fan and Yang, 2020) and entry (Berry and Waldfogel, 2001, Sweeting, 2010). Mergers, in the first place, are consolidations of assets and resources, including production facilities, retail outlets, investments, patents and more. Divestitures are the regulators' responses aimed at counteracting the increased concentration in post-merger assets distribution. Empirical work evaluating the role of divestiture practices in merger cases is scarce, partly due to the lack of data and partly due to the lack of a tractable framework to account for the endogeneity of assets and divestiture.[9] To the best of our

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[8] Our results are consistent with findings in Genakos, Valletti, and Verboven (2018), which used mobile operators' prices and accounting information across 33 OECD countries over a decade to show that both prices and investment per operator increased after a merger, and the total industry investment did not change significantly.

[9] Two recent academic papers provide descriptive evidences on the effects of divestitures: Tenn and Yun (2011) compare pre- and post-divestiture performances of divested brands from the 2008 Johnson & Johnson's acquisition of Pfizer' consumer health division; Soetevent, Haan, and Heijnen (2014) evaluate the Dutch government's divestiture requirement when allocating rights to operate highway gasoline stations

knowledge, our work is the first one to evaluate the roles of assets and, more importantly, the role of divestitures in firms' strategic choices using a game-theoretic approach. More broadly, our empirical method provides a very feasible solution to covariate endogeneity in discrete Bayesian games.

The paper unfolds as follows. The next section introduces the discrete Bayesian games with endogeneity and characterizes the Bayesian Nash equilibrium. Section 3 describes the two-step nested pseudo likelihood estimator (2SNPL) and derives its asymptotic properties. Section 4 illustrates the finite-sample performance of the 2SNPL method using two Monte Carlo experiments. Section 5 studies the 4G-LTE entry game of AT&T, Verizon, T-Mobile and Sprint, comparing model estimates and policy implications with and without accounting for endogenous covariates. Section 6 concludes. All proofs, technical details, and robustness checks are rendered in the appendices.

2 Discrete Bayesian Games with Endogeneity

Consider a Bayesian game of simultaneous discrete choices among K players, indexed by $k \in \mathcal{K} \equiv \{1, 2, ..., K\}$. Each player k is characterized by a $d_x \times 1$ vector of exogenous covariates X_k, and a $d_z \times 1$ vector of endogenous variables Z_k. Each player k observes a private shock $u_k \in \mathbb{R}$, and makes a simultaneous decision $Y_k \in \{0, 1\}$ based on the public information $\mathbb{I} \equiv \{X_k, Z_k\}_{k \leq K}$ and the private shock u_k. For each player k, let $X_k \equiv (X_{k1}, X_{k2})$ be partitioned into exogenous covariates X_{k1} and instruments X_{k2}. A player k's ex post payoff for $Y_k = 1$ is

$$X_{k1}'\beta_k + Z_k'\gamma_k + \alpha_k \sum_{j \neq k} Y_j + u_k; \tag{1}$$

and that for $Y_k = 0$ is zero. The instruments X_{k2} do not enter the ex post payoffs, but contribute to the endogenous variables as follows:

$$Z_k = \Pi_k' X_k + V_k, \tag{2}$$

where Π_k' is a $d_z \times d_x$ matrix of constant coefficients. Instrument validity requires the coefficients for X_{k2} in the matrix Π_k be non-zero. The regressor Z_k is endogenous whenever the error terms $V_k \in \mathbb{R}^{d_z}$ and $u_k \in \mathbb{R}$ are correlated.

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on prices of divested gasoline stations.

In any pure-strategy Bayesian Nash equilibrium (psBNE), each player k follows a decision rule $Y_k = 1\{Y_k^* > 0\}$, where

$$Y_k^* \equiv X_{k1}'\beta_k + Z_k'\gamma_k + \alpha_k \sum_{j \neq k} \mathbb{E}_k(Y_j|\mathbb{I}, u_k) + u_k, \tag{3}$$

where $\mathbb{E}_k(Y_j|\mathbb{I}, u_k)$ is player k's belief about others' decisions, which is consistent with the common prior of $\{u_j\}_{j\leq K}$ and others' strategies in equilibrium. (We can generalize by letting the strategic interaction term be a weighted sum of other players' choice probabilities, that is, by allowing the weights $\alpha_{k,j}$ to differ across k as well as j.)

Our method for dealing with endogenous covariates in this model applies under intuitive conditions on the unobserved errors, which are formalized as follows. For each k, let η_k denote the error term in the linear projection of u_k on $\{V_j\}_{j\leq K}$. That is,

$$u_k = \sum_{j\leq K} V_j'\lambda_{k,j} + \eta_k,$$

where $\lambda_{k,j}$'s are coefficients in the linear projection.

Assumption 1. *(i) $\{u_k, V_k\}_{k\leq K}$ are independent from $X = \{X_k\}_{k\leq K}$ with zero means. (ii) $\{\eta_k\}_{k\leq K}$ are independent from $\{V_k\}_{k\leq K}$; and η_k's are independent across the players $k = 1, 2, ..., K$.*

This assumption is flexible enough to accommodate different forms of endogeneity in Z_k, including those due to player- or game-level unobserved heterogeneity. For example, consider a data-generating process where V_j is arbitrarily correlated across $j = 1, 2, ..., K$, possibly through some game-level unobserved heterogeneity. Suppose for each player k, u_k is a linear combination of $\{V_j\}_{j\leq K}$ and an idiosyncratic noise ϵ_k, with ϵ_k's being independent across $k = 1, 2, ..., K$, and jointly independent from $\{X_j, V_j\}_{j\leq K}$. Then for all k, the error term from a linear projection of u_k on $\{V_j\}_{j\leq K}$, denoted as η_k, is identical to ϵ_k. Therefore, the conditions in Assumption 1 are satisfied.

Assumption 1 also accommodates situations where endogeneity arises because of unreported individual heterogeneity. For example, suppose there is no unobserved heterogeneity on the game level, and $\{u_k, V_k\}_{k\leq K}$ are independent across all players. For each player k, the vector of individual noises (u_k, V_k') is multivariate normal with non-zero correlation between u_k and the components in V_k due to some unobserved characteristics of player k. Assumption 1 follows from an implication of the multivariate normality. The zero mean restriction in (i) is just a location normalization.

Using Assumption 1, we write the decision rule in (3) as

$$Y_k = 1\{Y_k^* > 0\} = 1\Big\{X_{k1}'\beta_k + Z_k'\gamma_k + \alpha_k \sum_{j\neq k} \mathbb{E}_k(Y_j|\mathbb{I}) + \sum_{j\leq K} V_j'\lambda_{k,j} + \eta_k > 0\Big\}. \tag{4}$$

Note the two conditions in Assumption 1 imply that η_k is independent from $\{X_j, V_j\}_{j\leq K}$, and consequently from $\mathbb{I}$. Besides, the independence of η_k across $k = 1, 2, ..., K$ implies the equilibrium belief $E_k(Y_j|\mathbb{I}, u_k)$ does not depend on $\{\eta_j\}_{j\neq k}$.[10]

Let F_k and f_k denote the marginal distribution and the density function of η_k, respectively. Thus we characterize a psBNE through a vector of conditional choice probabilities (CCPs) $P : \mathbb{I} \mapsto [0, 1]^K$ that solves a fixed-point equation:

$$P = \Gamma(\theta, P), \tag{5}$$

where $\Gamma \equiv (\Gamma_1, ..., \Gamma_K)'$ with

$$\Gamma_k(\theta_k, P) \equiv F_k\Big(X_{k1}'\beta_k + Z_k'\gamma_k + \alpha_k \sum_{j\neq k} P_j + \sum_{j\leq K} V_j'\lambda_{k,j}\Big), \tag{6}$$

and $\theta \equiv \{\theta_k\}_{k\leq K}$ with $\theta_k \equiv (\gamma_k', \beta_k', \alpha_k', \lambda_k')$. The model admits a unique psBNE under the following condition.

Assumption 2. *For each k, the magnitude of α_k is bounded above: $|\alpha_k| < \frac{1}{(K-1)|\sup_t f_k(t)|}$.*

This assumption restricts the strength of interaction between players so that Γ satisfies the contraction mapping property.[11]

Lemma 1. *Under Assumptions 1 and 2, there exists a unique psBNE.*

Proof. See Appendix A. □

Let P^* denote the profile of conditional choice probabilities in Bayesian Nash equilibrium. Identification using Equations (5) and (6) requires the usual rank conditions. That is, the support of the vector $(X_{k1}', Z_k', \sum_{j\neq k} P_j^*, V_1', \cdots, V_K')$ is not contained in a linear subspace. Note that this rules out the cases where the coefficients for X_{k2} in the matrix Π

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[10] To see this, note conditioning on $\mathbb{I}$ and u_k is equivalent to conditioning on $\{X_k, V_k\}_{k\leq K}$ and η_k, and the claim follows from the independence of η_k across k.

[11] Assumption 2 is similar to the Moderate Social Influence (MSI) condition in the interaction game literature (see Glaeser and Scheinkman, 2003, Horst and Scheinkman, 2006). It is used in discrete game literature (Brock and Durlauf, 2001, Lee, Li, and Lin, 2014, Lin and Xu, 2017, Xu, 2018, Jackson, Lin, and Yu, 2020, Lin, Tang, and Yu, 2020) for the uniqueness of Bayesian Nash equilibrium.

are all zeros. This requires a necessary order condition that there are more instruments in X_{k2} than endogenous variables in Z_k.

By Assumption 1-(i), $\{V_k\}_{k\in\mathcal{K}}$ can be recovered directly as the residuals in the regression of Z_k on X_k in Equation (2). Aradillas-Lopez (2010) and Bajari, Hong, Krainer, and Nekipelov (2010) provided two distinct sets of conditions under which the players' ex post utility functions are identified.

3 Estimation

Consider a sample of n independent games $i = 1, 2, ..., n$, each involving K players making simultaneous binary decisions. Throughout this section we use lower-case letters to denote realization of random vectors in the sample. In each game i and for each player k, the sample reports a binary choice $y_{k,i}$, endogenous variables $z_{k,i}$, and exogenous covariates and instruments $x_{k,i} \equiv (x_{k1,i}, x_{k2,i})$. Let $\mathbb{I}_i = \{x_{k,i}, z_{k,i}\}_{k\leq K}$ denote the information set that is common knowledge shared by all players in a game.

Let Θ and $\mathcal{P} \subseteq [0, 1]^{K|\mathcal{X}||\mathcal{Z}|}$ denote the parameter spaces for θ and P respectively, with $\mathcal{X}, \mathcal{Z}$ being marginal support of X_k, Z_k. Let $\theta_0 \in int(\Theta)$ denote the true value of θ in the data-generating process (DGP), and let $P^0 \equiv \{Pr\{Y = y|\mathbb{I} = (x, z)\} : (y, x, z) \in \{0, 1\}^K \times \mathcal{X}^K \times \mathcal{Z}^K\}$ denote the actual equilibrium choice probabilities given θ_0 in the DGP.

Assumption 3. *(i) For any $\theta \neq \theta_0$ and $P(\theta)$ that solves $P = \Gamma(\theta, P)$, $P(\theta) \neq P(\theta_0) \equiv P^0$; (ii) Common knowledge variables X_i and Z_i have finite supports, denoted as $\mathcal{X}$ and $\mathcal{Z}$; (iii) $(Y_i, X_i, Z_i)_{i=1}^n$ are independent across games, and $Pr\{\mathbb{I}_i = (x, z)\} > 0$ for all $(x, z) \in \mathcal{X}^K \times \mathcal{Z}^K$.*

Assumption 3(i) is a standard identification condition for estimating games where the equilibrium is characterized by the solution to a fixed-point problem. See, for example, Assumption 5(C) in Aguirregabiria and Mira (2007) and Assumption 1(e) in Kasahara and Shimotsu (2012). Other papers on asymptotic properties of nested pseudo likelihood estimators in discrete games also assumed finite support of states, e.g., Assumption 4 in Aguirregabiria and Mira (2007) and §2.1 in Kasahara and Shimotsu (2012).

We propose a two-step nested pseudo likelihood (2SNPL) estimator, which builds on a sequential algorithm that combines the nested pseudo likelihood estimator in Aguirregabiria and Mira (2007) with the two-stage conditional maximum likelihood in

Rivers and Vuong (1988). The *pseudo likelihood* is:

$$L_n(\theta, P; \Pi) = \frac{1}{n} \sum_{i=1}^{n} l_i(\theta, P; \Pi),$$

where $l_i(\theta, P; \Pi) \equiv \sum_{k=1}^{K} log f_{k,i}(\theta, P; \Pi)$, with $\Pi \equiv \{\Pi_k\}_{k \leq K}$ and $f_{k,i}(\theta, P; \Pi)$ defined as

$$Pr\{x'_{k1,i}\beta_k + z'_{k,i}\gamma_k + \alpha_k \sum_{j \neq k} P_j + \sum_{j \leq K} (z_{j,i} - \Pi'_j x_{j,i})' \lambda_{k,j} + \eta_{k,i} \geq (<) 0\}$$

if $y_{k,i} = 1$ ($y_{k,i} = 0$). Note in the definition of $f_{k,i}$, the probability measure relates to the marginal distribution of $\eta_{k,i}$, and (x_k, z_k) are fixed realizations.[12]

With a slight abuse of notation, we let $\Gamma(\theta, P; \Pi)$ denote the mapping $\Gamma(\theta, P)$ as defined in Equation (6) when V_j is replaced by its identifiable counterpart $Z_j - \Pi'_j X_j$. This emphasizes how the mapping depends on the first-stage parameter Π.

Our 2SNPL estimator is defined as follows. In the first stage, regress $z_{k,i}$ on $x_{k,i}$ to estimate $\widehat{\Pi}_k$ for each $k \leq K$. In the second stage, plug $\widehat{\Pi} \equiv \{\widehat{\Pi}_k\}_{k \leq K}$ into an iterative algorithm in Aguirregabiria and Mira (2007) to construct a *2SNPL sequence* of estimators as follows:

Step 1. Pick an initial guess $\widehat{P}_0$ for P^0. For example, one can obtain such an initial guess from a reduced-form Probit regression.

Step 2. For each $s \geq 1$, calculate an s-stage estimator for θ as

$$\hat{\theta}_s = \arg\max_{\theta \in \Theta} L_n(\theta, \widehat{P}_{s-1}; \widehat{\Pi}), \tag{7}$$

and update the choice probabilities recursively as

$$\widehat{P}_s = \Gamma(\hat{\theta}_s, \widehat{P}_{s-1}; \widehat{\Pi}). \tag{8}$$

If the initial guess $\widehat{P}_0$ is a consistent estimator for the actual P^0 in the DGP, then all elements in the sequence of estimators are consistent for θ_0. This follows from a similar argument for consistency of two-step pseudo maximum likelihood estimators in Proposition 1 of Aguirregabiria and Mira (2007).

More importantly, there exists a neighborhood around P^0 such that, starting from any initial guess $\widehat{P}^0$ in that neighborhood, the NPL sequence constructed above converges almost surely to a root-n consistent and asymptotically normal (CAN) estimator, which

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[12] The term "pseudo likelihood" is used because the argument P in L_n is a generic profile of choice probabilities, rather than the equilibrium choice probabilities P^0.

we refer to as a *2SNPL estimator* and characterize in the next paragraph.

Define a *2SNPL operator* associated with the iterations in (7) and (8):

$$\phi_n(P) \equiv \Gamma(\tilde{\theta}_n(P), P; \widehat{\Pi}), \text{ where } \tilde{\theta}_n(P) \equiv \arg\max_{\theta \in \Theta} L_n(\theta, P; \widehat{\Pi}). \tag{9}$$

The set of *2SNPL fixed points* in a sample is defined as $\Lambda_n \equiv \{(\breve{\theta}, \breve{P}) \in \Theta \times \mathcal{P} : \breve{P} = \phi_n(\breve{P}) \text{ and } \breve{\theta} = \tilde{\theta}_n(\breve{P})\}$. If the maximizer $\tilde{\theta}_n$ is unique for any P and $\widehat{\Pi}$ from a given sample, then the mapping $\tilde{\theta}_n$ is continuous by the theorem of maximum. Thus the 2SNPL operator $\phi_n(\cdot)$ is continuous in the compact and convex set $[0,1]^{K \cdot |\mathcal{X}| \cdot |\mathcal{Z}|} \equiv \mathcal{P}$. It follows from the Brouwer's fixed point theorem that Λ_n is non-empty. We define a *2SNPL estimator* $(\widehat{\theta}_{2SNPL}, \widehat{P}_{2SNPL})$ as the element in Λ_n that leads to the highest value of pseudo likelihood.

3.1 Asymptotic Properties of the 2SNPL Estimator

Let Π_0 denote the true value of Π in the DGP. For simplicity, we also use Π, Π_0 to denote their own vectorization, in which case Π, Π_0 are $K \times d_z \times d_x$ vectors. Define the population counterparts of $L_n, \tilde{\theta}_n, \phi_n$ by

$$L_0(\theta, P) \equiv \mathbb{E}\Big[l_i(\theta, P; \Pi_0)\Big];$$

$$\tilde{\theta}_0(P) \equiv \arg\max_{\theta \in \Theta} L_0(\theta, P)\ ;\ \phi_0(P) \equiv \Gamma(\tilde{\theta}_0(P), P; \Pi_0).$$

The set of 2SNPL fixed points in the population is $\Lambda_0 \equiv \{(\theta, P) \in \Theta \times \mathcal{P} : \theta = \tilde{\theta}_0(P) \text{ and } P = \phi_0(P)\}$. Let $s_{\theta,i} \equiv \nabla_\theta l_i(\theta_0, P^0; \Pi_0)$, and define

$$\begin{aligned}
\Omega_{\theta\theta} &\equiv -E\left[\nabla^2_{\theta\theta} l_i(\theta_0, P^0; \Pi_0)\right] = E\left(s_{\theta,i} s'_{\theta,i}\right); \\
\Omega_{\theta P} &\equiv -E\left[\nabla^2_{\theta P} l_i(\theta_0, P^0; \Pi_0)\right] = E\left(s_{\theta,i} s'_{P,i}\right) \text{ where } s_{P,i} \equiv \nabla_P l_i(\theta_0, P^0; \Pi_0); \\
\Omega_{\theta\Pi} &\equiv -E\left[\nabla^2_{\theta\Pi} l_i(\theta_0, P^0; \Pi_0)\right] = E\left(s_{\theta,i} s'_{\Pi,i}\right) \text{ where } s_{\Pi,i} \equiv \nabla_\Pi l_i(\theta_0, P^0; \Pi_0).
\end{aligned}$$

The equalities following the definition above are due to the information matrix equality with regard to the vector of scores. We denote the Jacobian matrices evaluated at the true value $(\theta_0, P^0; \Pi_0)$ as $\Gamma^0_P \equiv \nabla_{P'} \Gamma(\theta_0, P^0; \Pi_0)$, $\Gamma^0_\theta \equiv \nabla_{\theta'} \Gamma(\theta_0, P^0; \Pi_0)$, and $\Gamma^0_\Pi \equiv \nabla_{\Pi'} \Gamma(\theta_0, P^0; \Pi_0)$. Define $M \equiv \Omega_{\theta\theta} + \Omega_{\theta P}(I - \Gamma^0_P)^{-1} \Gamma^0_\theta$. We establish the asymptotic property of $\widehat{\theta}_{2SNPL}$ under the following regularity conditions.

Assumption 4. *(i)* Θ *is a compact convex subset of a Euclidean space, and* $\mathcal{P}$ *is a compact convex subset of* $(0,1)^{n \cdot |\mathcal{X}| \cdot |\mathcal{Z}|}$*; (ii)* $E\left[\sup_{\theta,P} |l_i(\theta, P; \Pi_0)|\right] < \infty$. *(iii)* (θ_0, P^0) *is an isolated population NPL* *fixed point (i.e., it is unique, or else there is an open ball around it that does not contain any other element of Λ_0); (iv) There exists a closed neighborhood of P^0, denoted by $\mathcal{N}(P^0)$, such that, for all P in $\mathcal{N}(P^0)$, $L_0(\theta, P; \Pi_0)$ is globally concave and its second derivative with respect to θ is a nonsingular matrix; (v) the operator $\phi_0(P) - P$ has a nonsingular Jacobian matrix at P^0; (vi) M is nonsingular.*

Recall that $\widehat{\Pi}$ consists of 1st-stage ordinary least squares (OLS) estimates, and therefore admits a linear, first-order asymptotic representation as

$$\sqrt{n}(\widehat{\Pi} - \Pi_0) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} r_i(\Pi_0) + o_p(1),$$

where $r_i(\Pi_0) \equiv r_{0,i}$ is the influence function characterizing the limit distribution of the OLS estimator.

Theorem 1. *Under Assumptions 1 to 4, $\hat{\theta}_{2SNPL}$ is a consistent estimator and*

$$\sqrt{n}(\widehat{\theta}_{2SNPL} - \theta_0) \xrightarrow{d} N\Big(0, M^{-1} E(\tilde{s}_i \tilde{s}_i') \left(M^{-1}\right)' \Big),$$

where

$$\tilde{s}_i \equiv s_{\theta,i} - [\Omega_{\theta P}(I - \Gamma_P^0)^{-1}\Gamma_\Pi^0 + \Omega_{\theta\Pi}] r_{0,i}.$$

Proof. See Appendix A. □

The proof of the theorem amounts to writing down the first-order conditions and the equilibrium constraints that define the 2SNPL estimator, and then using a first-order expansion to account for the impact of the first-stage estimator $\widehat{\Pi}$ as well as the concurrent iteration over conditional choice probabilities. Similar to Kasahara and Shimotsu (2012), we can establish the following convergence property of the 2SNPL sequence.

Theorem 2. *Suppose Assumptions 1 to 4 hold and $\Omega_{\theta\theta}$ is nonsingular. There exists a neighborhood $\mathcal{N}$ around P^0 such that, starting from any initial value $\widehat{P}_0 \in \mathcal{N}$, $lim_{s\to\infty} \widehat{P}_s = \widehat{P}_{2SNPL}$ almost surely.*

The contraction mapping property in Lemma 1 implies $\rho(\Gamma_P^0) < 1$ where $\rho(\cdot)$ is the spectral radius function. The key condition for convergence in Proposition 1 of Kasahara and Shimotsu (2012) holds.[13] With uniform convergence of $L_n(\cdot; \widehat{\Pi})$ to $L_0(\cdot)$ established the proof of consistency in Theorem 1 (see Appendix A), the proof of Theorem 2 follows from the same steps in Kasahara and Shimotsu (2012), and is therefore omitted for brevity.

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[13] See Section 2.3 of Kasahara and Shimotsu (2012) for more discussions.

4 Monte Carlo Evidences

In this section, we illustrate the finite-sample performance of our 2SNPL estimator by several Monte Carlo experiments. We consider 4 players in the game and each player is associated with X_1 and X_2 which are drawn from the bivariate normal distribution with mean zero, unit variance, and covariances 0.5. A pair of independent standard normal variates (v, η) were drawn. We consider two cases: homogeneous competitive effects and heterogeneous competitive effects

4.1 Homogeneous competitive effects

We generate the error term as $u = \lambda v + \eta$, and the endogenous variable as

$$Z = \pi_0 + \pi_1 X_1 + \pi_2 X_2 + v,$$

Denote $Z = (Z_1, Z_2, Z_3, Z_4)$, $X_1 = (X_{11}, X_{21}, X_{31}, X_{41})$, $X_2 = (X_{12}, X_{22}, X_{32}, X_{42})$, $v = (v_1, v_2, v_3, v_4)$ and $u = (u_1, u_2, u_3, u_4)$ for four players. (X_{k1}, X_{k2})'s are drawn from bivariate normal distribution with mean $(0, 0)$ and variance-covariance matrix $\begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$. We set the true parameter $(\pi_0, \pi_1, \pi_2, \lambda) = (1, 1, 1, 1)$.

The conditional choice probabilities $P^0 = (P_1^*, P_2^*, P_3^*, P_4^*)$ in the BNE are solved by

$$P_k^* = \Phi\Big(\beta_0 + \beta_1 X_{k1} + \gamma Z_k + \alpha \sum_{j \neq k} P_j^* + \lambda v_k\Big), k = 1, 2, 3, 4.$$

The decisions are then generated by

$$Y_k = 1\Big\{\beta_0 + \beta_1 X_{k1} + \gamma Z_k + \alpha \sum_{j \neq k} P_j^* + u_k > 0\Big\}, k = 1, 2, 3, 4.$$

We set the true parameter $(\beta_0, \beta_1, \gamma, \alpha) = (1, 1, 1, -0.5)$. Each simulation was based on a random sample of (200,400,800) observations and was replicated 1000 times. We report the average biases and the mean squared errors for true parameter $(\beta_0, \beta_1, \gamma, \lambda, \alpha) = (1, 1, 1, 1, -0.5)$ in Table 1.

Table 1: Homogemous Competitive Effects

	Average Bias				
n	β		γ	λ	α
200	0.042	0.027	0.016	-0.021	0.019
400	0.010	0.009	0.015	-0.006	0.012
800	0.008	0.010	0.003	-0.002	0.007
	Mean Squared Error				
n	β		γ	λ	α
200	0.102	0.040	0.017	0.021	0.024
400	0.048	0.018	0.009	0.011	0.011
800	0.023	0.010	0.004	0.005	0.006

4.2 Heterogeneous competitive effects

In this section, we consider Monte Carlo designs in which the competition effects differ across "strong" and "weak" players. All other settings are the same as in the homogeneous case except the conditional choice probabilities $P^0 = (P_1^*, P_2^*, P_3^*, P_4^*)$ in BNE are solved by

$$P_k^* = \Phi\Big(\beta_0 + \beta_1 X_{k1} + \gamma Z_k + \alpha_k \sum_{j \neq k} P_j^* + \lambda v_k\Big), k = 1, 2, 3, 4,$$

where $\alpha_1, \alpha_2 = \alpha_S$ and $\alpha_3, \alpha_4 = \alpha_W$.

The decisions are then generated by

$$Y_k = 1\Big\{\beta_0 + \beta_1 X_{k1} + \gamma Z_k + \alpha_k \sum_{j \neq k} P_j^* + u_k > 0\Big\}, k = 1, 2, 3, 4.$$

where we have true parameter $(\alpha_S, \alpha_W) = (-0.5, -1)$. Each simulation is based on a random sample of (200,400,800) observations and is replicated 1000 times. We report the average biases and the mean squared errors for $(\beta_0, \beta_1, \gamma, \lambda, \alpha_S, \alpha_W)$ with true values $(1, 1, 1, 1, -0.5, -1)$ in Table 2.

Both Table 1 and 2 show our estimator converges to the true parameters values at the parametric root-n rate. In both cases, the variances of the estimators seem to be the dominating component in the mean-squared error (relative to bias).

Table 2: Heterogenous Competitive Effects

	Average Bias					
n	β		γ	λ	α	
200	0.021	0.024	0.028	0.028	-0.010	-0.029
400	0.013	0.016	0.012	0.015	-0.005	-0.015
800	0.000	0.008	0.010	0.006	-0.001	-0.008
	Mean Squared Error					
n	β		γ	λ	α	
200	0.090	0.039	0.017	0.023	0.025	0.024
400	0.041	0.018	0.009	0.011	0.011	0.010
800	0.022	0.009	0.004	0.006	0.006	0.005

5 Empirical Study: An Entry Game of Cellphone Service Providers

In this section, we illustrate the gain from our method, which takes account of the endogenous covariates in a discrete Bayesian game, in a setting featuring oligopolistic firms making strategic 4G-LTE deployment decisions in local markets. The firms are the four national cellphone service providers in the U.S., namely Verizon Wireless, AT&T Mobility, T-Mobile US and Sprint Corporation (often collectively referred to as "Big Four").[14] The time periods we look into are from 2015 to 2018, a few years before the proposal of a T-Mobile and Sprint merger in 2019, which eventually went through in early 2020 after lengthy legal battles over antitrust concerns.

In this industry, firms make capital investment in cellular networks and transmission facilities before providing services to consumers. Such investment was typically made in accordance with the dominant technology of the time. For example, throughout most of the 2000s, the third generation of cellphone technology (3G) was the predominant technology, utilizing the 1850 – 1990 MHz spectrum range. Starting from roughly 2010, it was time for the next generation of technology, 4G-LTE.[15] A firm with 3G deployment in a

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[14] We will refer to them as Verizon, AT&T, T-Mobile and Sprint henceforth. These cellphone service providers are also known as mobile network operators, wireless service providers, wireless carriers, cellular companies, mobile network carriers, etc. In this paper we refer to them as firms, providers, and carriers interchangeably.

[15] 4G-LTE stands for the fourth generation, Long Term Evolution. LTE is the technology to deliver 4G

local market can re-purpose spectrum used by 3G to support 4G-LTE, and utilize existing facilities such as cell towers with upgraded equipment. Such investment also features heavy spatial consideration. For example, extending coverage from central Phoenix to nearby cities and towns would be easier than providing *de novo* services to these markets. We measure a potential entrant's network investment for a local market by the firm's 3G deployment in the focal market and 4G-LTE deployment in nearby markets. These two sets of network investment are the firm-specific, endogenous covariates we focus on in our empirical framework. They are important determinants of a firm's decision to provide a new generation of technology in a local market, while driven by the same source of the unobserved heterogeneity that underlies a firm's entry decision.

In the following subsections, we will describe the background of the U.S. cellphone service industry, the policy relevance of our empirical application, the data we construct, and the empirical specification we use. In particular, we will evaluate a counterfactual experiment in which T-Mobile and Sprint had merged in 2016, which would have led to different 4G-LTE deployment paths in markets these firms had yet to enter in 2016. We will discuss the discrepancies in policy implications and recommendations with and without accounting for the endogeneity in network investment.

5.1 The U.S. Cellphone Service Industry at a Glance

Up until April 2020, Verizon, AT&T, T-Mobile and Sprint were the four major cellphone service providers in the United States. There were also a few regional providers, such as U.S. Cellular and C Spire Wireless, and a fringe of local providers, such as Cricket Wireless and TracFone Wireless, which often offered flexible, more economical prepaid plans. Compared to the Big Four, the other providers' network deployment and market presence are almost negligible.[16]

A consumer (or a household) chooses a plan offered by a provider, considering prices, coverage, speed and customer service. A plan typically ranges from \$30 to \$100. Among

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standards, which is defined as having peak upload and download speeds of at least 100 mbps (mega bits per second). 4G-LTE is still not fully 4G, but is considered the closest to 4G standards by international telecommunications communities.

[16] The Big Four and US Cellular are the only Mobile Network Operators (MNOs) in continental U.S., that is, providers that own and control the spectrum licenses and network infrastructure necessary to provide services to subscribers. All other cellphone providers in the U.S. are Mobile Virtual Network Operators (MVNOs), relying on other firms' network infrastructure to provide services.

the Big Four, Verizon and AT&T were known for best coverage, while T-Mobile and Sprint were considered to offer comparable deals with lower prices and less coverage. The Federal Communications Commission (FCC) is the main regulator of this industry, while the Department of Justice (DOJ) and the Federal Trade Commission (FTC) share the responsibility of evaluating the anti-competitive conducts in this industry.

5.2 Cellular Network Investment

A cellular network is composed of cellphones, base transceiver stations ("cell sites"), mobile telephone switching offices, and the public switched telephone network.[17] When joined together, cellular networks provide radio coverage over a wide geographic area, enabling cellphones to communicate with each other. Globally, major telecommunications providers have deployed cellular networks over most of the inhabited land area of the Earth.

Building a cellular network takes decades of physical and financial investment from a provider. In the past few decades, mobile wireless technologies have experienced multiple generations of evolution, namely from 0G to 5G. In the 2000s, 3G technology was implemented, enabling media streaming with high connection speed. From the start of 2010's, 4G-LTE was rolled out gradually, accounting for more than half of mobile connections for the first time in 2019, hitting 52%.[18] The cellular networks need to be maintained and updated constantly, with substantial cost of sustaining network operation. The Global System for Mobile Communications (GSM) Association projected in 2020 that global network operators would invest more than $1.1 trillion in their networks in the next five years.

The period we are looking into is from 2015 to 2018, during which time 4G-LTE grew to be the dominant network technology. The Big Four have constructed their main 4G-LTE networks, but even extending services to an unserved local market from this main network involves millions to billions of dollars. A potential entrant for a local market needs to first acquire spectrum licenses, depending on the size of the market served.[19] A provider then

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[17] We explain the components and evolution history of cellular networks in Appendix B.

[18] Industry experts predict that 4G will peak at just under 60% by 2023 (The GSM Association Intelligence, "The Mobile Economy 2020.")

[19] A spectrum license gives its holder the exclusive option to use a certain range of frequencies in a well-delineated geographic area. A firm can purchase these licenses in the FCC spectrum auctions, or acquire

needs to build cell sites, purchase radio transmitters and receivers, and acquire access to intermediate links connecting different wired networks ("backhaul"). The firm also must build a distributional network and market its services to retail consumers. To sum up, the biggest hurdle of deploying a new network technology is the substantial costs involved; these costs can become prohibitive in areas with low population density and rugged terrains. Retiring technologies of previous generations can free up spectrum and existing facilities to accommodate the next generation of technology; at the same time, deploying a new technology in a cluster of close-by markets, simultaneously or sequentially, helps a provider to achieve economies of scale. For these reasons, it is essential to incorporate the "network investment" effect in a potential entrant's evaluation of the expected payoff of entering a local market. When we study providers' decisions to enter local markets, not accounting for the network investment factor means ignoring a first-order difference between Verizon, an industry leader, and Cricket Wireless, a fringe player.

5.3 T-Mobile and Sprint Merger: Policy Considerations

T-Mobile and Sprint announced a merger deal of $26 billion on April 29, 2019. The proposed merger would reduce the number of national providers from four to three, leading to antitrust concerns by state governments and regulating agencies.[20] The merging parties claimed a substantial saving of $43.6 billion via cost synergies, which would allow the merged firm to become a stronger competitor against Verizon and AT&T. Proponents for this merger argued that the merger would generate broader coverage, greater capacity, higher service quality and a rapid deployment of a nationwide 5G network (Wallsten, 2019). Opponents argued that the reduction in the number of providers would lead to higher prices, fewer choices, lower quality, and a slow rollout of 5G services.[21]

On July 26, 2019, the DOJ approved the merger deal after T-Mobile and Sprint reached

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them in secondary markets through purchase or renting. Xiao and Yuan (2020) describe the 2008 FCC auction to sell off 700 MHz, mostly used for 4G-LTE deployment.

[20] Internationally, the telecommunication industry has experienced a wave of consolidation activities recently. Most notably, the European Commission allowed four-to-three mergers in the Netherlands, Austria, Ireland, Germany and Italy, but blocked a similar merger in Denmark (Genakos, Valletti, and Verboven, 2018).

[21] Department of Justice Complaint, U.S. et al v. Deutsche Telekom AG, T-Mobile Us, Inc., Softbank Group Corp., and Sprint Corporation, No. 1:19-cv-02232, at 3 (D.D.C. Jul. 26, 2019) Case 1:19-cv-02232, July 26, 2019.

an agreement to sell Sprint's branded prepaid business,[22] Sprint's entire 800 MHz portfolio, and other assets to DISH Network ("DISH" henceforth). The DOJ believed that DISH had the sufficient spectrum holdings and the divestiture from the merger would help DISH become the fourth national provider. The DOJ also prescribed detailed operational instructions for DISH to enter as a facilities-based provider instead of just a reseller.[23] The DOJ argued that this remedy would restore the *ex ante* competitive market conditions before the merger. However, the opponents of the merger deal questioned the effectiveness of this remedy and viewed the remedy as "exceedingly optimistic" (Economides et al., 2019).

On October 18, 2019, the merger received a formal approval by the FCC in a 3-2 commissioner vote, but attorney generals from fourteen states soon filed lawsuits to block the merger. After lengthy negotiations with the states and the DOJ, the merger deal officially closed on April 1, 2020, with the Sprint brand discontinued on August 2, 2020.

Evaluating the overall effects of the merger is beyond the scope of this paper. Instead we focus on evaluating a key claim of this merger's benefit, that the merger would strengthen competition in rural areas and alleviate the divide in cellular infrastructure across the states (Wallsten, 2019). The pre-merger T-Mobile and Sprint did not have sufficient assets and coverage to compete effectively with the industry leaders, especially in rural areas.[24] The merged firm, aided by "the unique combination of spectrum, sites and equipment of T-Mobile and Sprint",[25] would become a comparable rival to AT&T and Verizon. Opponents of the merger, such as the Rural Broadband Association, argued that T-Mobile had shown little incentives to invest in rural areas and, therefore, its incentives were unlikely to change following this merger.

We investigate how a hypothetical T-Mobile and Sprint merger in 2016 would affect the 4G-LTE deployment by national providers. This is an important counterfactual analysis,

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[22] This includes Boost Mobile and Virgin Mobile, representing 9.3 million consumers.

[23] The DOJ imposes on the merging parties an obligation to permit DISH to operate as a reseller on New T-Mobile's wireless network for the entire seven-year term of the settlement. DISH promised to comply with the network build commitments made to the DOJ by 2023. If DISH's own network does not serve 70% of the country by then, it will face penalties up to $2.2 billion.

[24] The FCC reported that in December 2016 more than 98 percent of rural Census blocks had at least one LTE provider, but only 57 percent had at least four providers, compared to 96 percent of non-rural blocks.

[25] T-Mobile and Sprint, "Description of Transaction, Public Interest Statement, and Related Demonstrations," June 18, 2018, page 16.

because no post-merger network deployment data is available as of yet. We also evaluate the remedy proposed by the DOJ, which divests assets from the merger to support DISH as a national provider. We exploit data and a structural model of discrete Bayesian games to analyze the impact of the hypothetical merger and remedy, taking into account the firms' post-merger network consolidation and strategic responses.

5.4 Data We Use

We use three publicly available data sets to construct our sample. The first is the FCC's Mobile Deployment Form 477 Data from 2015 to 2018, which report semi-annually each provider's 2G-4G coverage in every U.S. census block.[26] The FCC requires all facilities-based broadband providers to file Form 477, which discloses where they offer Internet access service at speeds exceeding 200 kbps in at least one direction. In particular, for each mobile network technology deployed in each radio frequency band, facilities-based mobile providers must submit polygons representing their nationwide coverage area of that technology and the advertised data upload and download speeds. Providers' submission of data is mandatory, and they must certify the accuracy of the data submitted.

With providers' submitted data on coverage polygons, the FCC reports the percentage of area in a census block covered with each technology (including 2G, 3G, 4G-non-LTE,[27] and 4G-LTE) by each provider, using a computationally intensive process.[28] In addition, the FCC reports the percentage of a census block covered by "any" technology. From December 2015 to December 2018, the FCC data provides seven snapshots of each firm's granular-level network deployment information. Each snapshot of data has about 45 million observations at the firm-census block level.

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[26] The FCC started to report the Mobile Deployment (including both voice and broadband) data from December 2014, but 2015 was the first year the FCC reported the actual area coverage within a census block by each provider. Much of the information presented on data description is based on the FCC's Public Notice (DA 16-1107), released on September 30, 2016.

[27] 4G-non-LTE refers to technologies that do not reach 4G standards but was marketed as 4G by cellphone providers. 4G-non-LTE will be ultimately replaced by 4G-LTE. Sprint and Clearwire, for example, invested in WiMax rather than LTE and had to rebuild their 4G networks.

[28] The FCC first removes the spectrum and speed information from each shapefile filed by a provider, and then consolidates different polygons for a particular technology for a particular provider into a single, unique polygon. The FCC then determines how much of a census block is covered by this unique polygon. The FCC has not calculated how much coverage reported for one technology does or does not overlap with coverage of another technology, e.g. 2G and 3G overlap within a census block.

The second data set is the 2016 American Community Survey. We obtain demographic variables such as population size, age, gender and ethnicity profiles, income, and commuting patterns that are potential determinants of a consumer's cellphone use. The third data set is the 2000 Population Census. We construct exactly the same set of variables as the ones we obtain from the 2016 American Community Survey, to be used to construct our instrument variables – the lagged demographics of neighboring markets – for endogenous network investment variables.

5.5 Variable Construction and Sample Selection

With the raw data, we define open markets for 4G-LTE deployment by the four major national providers, and then merge in demographic variables at the census tract level.

5.5.1 Aggregation to census tracts

We use the December editions of the FCC's Mobile Deployment Form 477 data from 2015 to 2018, which yield a four-year snapshot of mobile network deployment for the universe of U.S. census blocks. A census block is a smallest geographic unit in the U.S. Census, amounting to more than 11 millions observations in 2010 Census. A census block is typically a very small geographic area, for example, it is often a city block bounded on all sides by streets, and we do not think that deployment decisions are made on such a fine-grained geographic basis. We, therefore, aggregate these census blocks to the universe of 73,057 census tracts. A census tract is designed to be relatively homogeneous units with respect to population characteristics, economic status and living conditions, generally encompassing a population between 2,500 to 8,000 people.[29] A census tract usually covers a contiguous area; however, the spatial size of census tracts varies widely depending on the density of settlement. A rough estimate of the radius of a typical census tract is 6.5 kilometers.[30] Although cell towers have a maximum range of 50 to 70 kilometers, they are typically spaced two to three kilometers apart to adequately handle cellphone traffic.[31] Based on the above comparison, we define census tracts as geographic markets based on

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[29] Due to its size and internal homogeneity, Seim (2006) uses census tracts as location choices for video retail stores.

[30] The total area of the U.S. is 9.857 million square kilometers, covering 73,057 census tracts. A census tract covers 134.9 square kilometers on average, with roughly 6.5 kilometers as the radius.

[31] In urban areas, cell towers may be 400 to 800 meters apart to accommodate dense population.

which cellphone providers make investment and network deployment decisions.

For every firm in every census tract, we calculate the percentage of census blocks covered within the census tract by a given technology. The FCC-reported census block coverage has a bipolar distribution, with a small peak between 0% and 10% coverage and a major peak at 100%.[32] We think some census blocks may experience low, spillover coverage from a nearby cell site in another census block, and this is not an actual entry. Therefore, we define a provider's coverage of a census block under a given technology as a dummy variable that equals 0 if the FCC-reported coverage falls below 10%, and 1 otherwise. When we aggregate to census tracts, we use the same reasoning and define the entry dummy for 4G-LTE as 0 if the percentage of census blocks covered by 4G-LTE within a census tract falls below 10%, and 1 otherwise.

5.5.2 Sample selection

By December 2015, 4G-LTE has been well deployed by the Big Four across the U.S., although the fringe competitors (about 80 of them in total) lagged distantly behind. Verizon has entered 98.8% of the 73,057 census tracts, AT&T followed closely with 98.1%, and T-Mobile and Sprint trailed behind by 95.3% and 92.4% respectively. The non-big-four firms have much smaller coverage in comparison. Even the largest one, U.S. Cellular, only entered 13.9% of the 55,644 census tracts to which it can be considered as a potential entrant.[33] Other fringe competitors' entry rates are typically below 5% in any given year, and they often only consider urban markets or urban clusters in rural areas.

For the Big Four, we can safely argue the national 4G-LTE network has been mostly laid out by the end of 2015, and the remaining task is about the leftover, often isolated open markets. We focus on their decisions to enter these local, isolated markets. For each provider, a census tract is defined as an open market for 4G-LTE deployment (an entry decision) if the deployment dummy was 0 in December 2015. We then use the 2018 data to measure 4G entry into the open markets as well as deployment of earlier technologies,

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[32] For example, for AT&T and Verizon, the 4G-LTE coverage of a census block is already 100% at the 10% percentile for most of our data period; for T-Mobile and Sprint, this number is at high 90% at the 10% percentile.

[33] A firm usually needs to obtain approval from a state before entry (Fan and Xiao, 2015). If a firm has not operated in a single census tract in a state, we do not consider this firm as a potential entrant to any census tracts of the state. Verizon and AT&T has operated in all states (including the District of Columbia); T-Mobile has entered 50 states and Sprint 49 states.

treating the time between 2016 and 2018 as a single period in cross-section data.

To summarize, during 2006 and 2018, the Big Four are the main competitors in the U.S cellphone industry, and they are strategically considering whether to enter the remaining scattered, few markets left open to 4G-LTE deployment. We define a potential entrant on a market as a Big Four provider who had no 4G-LTE deployment on the market by the end of 2015. A potential entrant is observed to enter a market if it makes 4G-LTE deployment by the end of 2018.[34] We drop all census tracks with only one potential entrants in order to better focus on the game theoretic aspects in entry decisions.

5.5.3 Summary statistics: the Big Four's cellphone deployment

In Table 3, we present summary statistics of the Big Four′ cellphone technology deployment in their open 4G-LTE markets by the end of 2015. Of the 2,582 census tracts in our sample, Verizon had not entered 645 by the end of 2015 (i.e, no 4G-LTE deployment by the end of 2015), AT&T 1,132, T-Mobile 2,185, and Sprint 2,182. Table 3 shows how the Big Four differed in their technology mix of 2G, 3G, 4G-non-LTE, and 4G-LTE. From 2016 to 2018, Verizon focused almost completely on 4G-LTE; AT&T retired 2G, and pushed for 3G, 4G-non-LTE, and 4G-LTE, with 4G-LTE leading the growth; T-Mobile grew all four technologies, again with 4G-LTE making the largest stride; Sprint never deployed 4G-non-LTE and made relatively small steps compared to its rivals. Of the four technologies, 4G-LTE is the one that experienced the most growth from 2015 to 2018 across the board. The 4G-LTE growth is also reflected by the percentage of 4G-LTE coverage in other tracts of the same county (referred to as "neighboring tracts" henceforth) and the number of incumbents offering 4G-LTE in the focal markets.

We use two network deployment variables to capture a potential entrant's existing facilities in the focal market and nearby areas. The first is the firm's 3G deployment in the focal market by the end of 2015 (we call it Z_{k1}). The second is the firm's 4G-LTE deployment in neighboring tracts by the end of 2018(we call it Z_{k2}). As we discussed in Section 5.2, different generations of cellphone technologies can share some basic facilities(e.g. cell towers), and close-by cell sites reduce the cost of extending the network to extra miles (e.g. nearby conduits can be extended to bordering neighborhoods). Therefore, Z_{k1} and Z_{k2} can be viewed as cost shifters for a provider k's entry decision into the focal market.

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[34] We keep the other fringe competitors when counting the number of incumbents in a census tract.

A potential entrant's network investment can be measured in different dimensions. For robustness, we use a potential entrant k's deployment via *any* previous generation to 4G-LTE in the focal market by the end of 2015 as Z_{k1}, and its 4G-LTE deployment in neighboring tracts by the end of 2015 as Z_{k2}. We discuss the robustness of our results under different measurement of Z_{k1} and Z_{k2} in Appendix B.

5.5.4 Summary statistics: market attributes

In Table 4, we compare the market attributes of the census tracts in the sample for our entry game and those of the remaining parts of the country. The most important determinant of entry is the population size. Demand for cellphone services depends on market demographics such as gender, age, ethnicity profiles, education, labor force participation, income and household. Workers' commuting patterns also contribute to the intensity of cellphone use. Lastly, population density, ruralness and the presence of large water areas can be considered as cost shifters for network deployment.

As shown in Table 4, the 2,582 census tracts, which have at least two Big-four potential entrants, are notably different from the rest of the country in all dimensions. They have much smaller population and very different demographic compositions. They are more rural, sparsely-populated, poorer and less educated. They spend more time working from home and less time commuting to work. In short, these markets seem to belong to the bottom side of the "digital divide," which refers to the significant disparity in Internet access across different demographic groups and geographic areas in the country.

5.6 Instrumental Variables

To clearly specify our equations (1) and (2) in this cellphone 4G-LTE entry game application, we reiterate our notation:

- Y_k: potential entrant k's 4G-LTE entry decision;
- Z_k: include potential entrant k's 3G deployment in the focal census tract, Z_{k1}, and its 4G-LTE deployment in neighboring tracts, Z_{k2};
- X_{k1}: tract attributes from 2016 ACS + the number of 4G-LTE incumbents in the focal

Table 3: Cellphone Service Coverage (2015-2018), by the Big Four

	2015		2018	
Variable	Mean	S.D.	Mean	S.D.
Verizon: potential entrant to 645 tracts				
% blocks with 2G	0.007	0.052	0.019	0.106
% blocks with 3G	0.005	0.043	0.016	0.094
% blocks with 4G-non-LTE	0	0	0	0
% blocks with 4G-LTE	-	-	0.122	0.233
% blocks 4G-LTE, neighbor tracts average	0.447	0.301	0.534	0.365
# incumbents with 4G-LTE	1.297	1.108	1.964	1.460
Entry with 4G-LTE	-	-	0.267	0.443
AT&T: potential entrant to 1,132 tracts				
% blocks with 2G	0.233	0.357	0	0
% blocks with 3G	0.384	0.414	0.468	0.433
% blocks with 4G-non-LTE	0.357	0.409	0.403	0.424
% blocks with 4G-LTE	-	-	0.362	0.396
% blocks 4G-LTE, neighbor tracts average	0.336	0.372	0.545	0.334
# incumbents with 4G-LTE	1.479	0.877	2.511	1.329
Entry with 4G-LTE	-	-	0.542	0.498
T-Mobile: potential entrant to 2,185 tracts				
% blocks with 2G	0.046	0.163	0.133	0.312
% blocks with 3G	0.011	0.083	0.164	0.315
% blocks with 4G-non-LTE	0.003	0.032	0.243	0.366
% blocks with 4G-LTE	-	-	0.496	0.423
% blocks 4G-LTE, neighbor tracts average	0.206	0.326	0.534	0.326
# incumbents with 4G-LTE	1.876	0.821	3.011	1.195
Entry with 4G-LTE	-	-	0.648	0.478
Sprint: potential entrant to 2,182 tracts				
% blocks with 2G	0.154	0.314	0.195	0.345
% blocks with 3G	0.147	0.309	0.173	0.326
% blocks with 4G-non-LTE	0	0	0	0
% blocks with 4G-LTE	-	-	0.208	0.358
% blocks 4G-LTE, neighbor tracts average	0.107	0.231	0.259	0.317
# incumbents with 4G-LTE	2.055	0.787	3.286	1.015
Entry with 4G-LTE	-	-	0.293	0.455

Notes: This table is based on 6,244 tract-firm observations (2,582 Census Tracts, two to four potential entracts in each tract). This table reports Big Four's coverage of census blocks by each generation of technology, summarized over Census tracts each firm has yet to enter with 4G-LTE by the end of 2015.

Table 4: Census Tract Attributes

		Markets to enter		Other markets	
Variable	Definition	Mean	S.D.	Mean	S.D.
Pop(in 000's)	Population in thousands	2.901	1.758	4.414	2.171
% Female	% female in population	0.495	0.045	0.508	0.050
% Senior	% 65 and older in population	0.285	0.266	0.153	0.092
% White	% White in population	0.862	0.209	0.724	0.253
% Black	% Black in population	0.038	0.107	0.142	0.222
% Native	% Native in population	0.044	0.161	0.008	0.035
% Asian	% Asian in population	0.012	0.037	0.049	0.091
% Hispanic	% Hispanic in population	0.072	0.129	0.163	0.215
% College	% above 25, with college degree	0.193	0.092	0.294	0.190
% Labor force	% above 16, in labor force	0.573	0.109	0.631	0.103
% Work home	% above 16 & employed, working at home	0.056	0.047	0.045	0.040
% Long comm.	% above 16 & employed, commuting for 40+ minutes	0.167	0.106	0.201	0.129
HH income	Median household income in 2016 $, 000's	46.127	14.499	59.641	29.860
HH size	Household size	4.651	5.688	2.915	1.835
Pop density	Population/land area	0.0002	0.0007	0.002	0.005
% Rural	% population in rural area	0.683	0.404	0.190	0.348
Mostly water	If water area ≥ 90%	0.113	0.317	0.0007	0.027

Notes: This table is based on 73,057 Census tracts, which include 2,582 tracts for the final sample we use for estimation and 70,475 tracts for the rest of the data.

census tract by the end of 2015;[35]

- X_{k2}: instrumental variables for Z_{k1} and Z_{k2} (all variables summarized in Table 4);
- u_k: unobserved errors in the ex post payoffs (equation (1));
- V_k: unobserved errors that determine Z_{k1}, Z_{k2} (equation (2)).

In the above specification, we focus on 4G-LTE competition. For example, we do not consider an incumbent that offers 3G only as a legitimate competitor to a 4G-LTE provider. We have two endogenous covariates in a potential entrant's expected payoff function: Z_{k1} and Z_{k2}. In this subsection, we discuss our choice of X_{k2}, which serve as instrumental variables for Z_{k1} and Z_{k2}.

In the ex post payoff function, the unobserved error u_k is a potential entrant's private information. The potential entrant observes u_k before deploying different generations of technology in the focal and nearby markets. A good example of this unobserved term is each firm's spectrum holdings for the focal market. This unobservable enters both the firm's expected payoff function (equation (1)) and the technology deployment function (equation (2)), causing the correlation between u_k and V_k. Valid instruments for Zs need to be excluded from the entry payoff function, to be orthogonal to u_k and V_k, and to enter the technology deployment equation.

For each focal census tract, we use demographics of its "neighbors" (i.e., other tracts in the same county) in 2000 as instruments for Z_{k1} and Z_{k2}. The 2000 demographic variables of neighboring tracts determine the 3G and 4G-LTE deployment in these neighboring markets, but do not enter the 4G-LTE deployment of the focal market directly conditioning on the focal market's observables. Furthermore, it is plausible to assume that these neighboring demographics are orthogonal to the unobserved factors determining deployments in focal and neighboring markets (u_k and V_k), once conditional on market-level observables.

These instruments affect the 4G-LTE deployment decision of a firm in the focal market indirectly through the “spillover” effect. The 3G deployment in neighboring markets

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[35] We treat the number of incumbents as predetermined and uncorrelated with the unobserved u_k in the entry payoff equation. An incumbent's entry decision was made earlier, before the realization of a potential entrant's time-varying private shocks.

could lower the cost of 3G deployment in the focal market (Z_{k1}), which, in turn, lowers the cost of 4G-LTE deployment in the focal market. Similarly, the 4G-LTE deployment in neighboring markets (Z_2) could lower the cost of 4G-LTE deployment in the focal market.

One may worry that a potential entrant decides entry on a much larger scale than a census tract, so these instruments will enter the payoff function of the focal market. However, the remaining tracts to enter in 2016 are typically isolated spots with the surrounding tracts well served before our sampling period starts, as attested by Table 3's summary statistics on the percentage of census blocks covered with 4G-LTE in the neighboring tracts. Therefore, modeling the Big-four providers' post-2015 4G-LTE deployment decisions on the level of local markets serves as a first-order approximation that captures the firms' main strategic concerns.

We choose neighboring markets' attributes in 2000 (instead of in 2016) as our instruments for several reasons. First, 3G technology was actively deployed between year 2000 to year 2010; therefore, the 2000 Census' market attributes are more relevant for 3G deployment. Second, the correlation between 2000 Census' market attributes and 2016's market-level unobserved heterogeneity are weakened with time, giving us better justification for the orthogonality of the instruments. Lastly, as we can reasonably argue that the detailed market-level attributes we include in X_{k1} capture the spatial correlation across census tracts, we abstract away spatial correlation in the unobservables. That is, conditional on X_{k1}, the error term u_k, which captures firm- and market-specific heterogeneity, is not spatially correlated.

5.7 Estimation Results

Among the Big Four, AT&T and Verizon lead in terms of spectrum holdings, network built and customer base. Our baseline specification categorizes AT&T and Verizon as "strong" competitors and T-Mobile and Sprint as "weak". We estimate heterogeneous competitive effects based on whether the potential entrant is strong or weak. We present results treating all four firms as equal competitors in Appendix C. We adopt a specification where firms share the same coefficients for all covariates (other than aforementioned heterogeneous competition effects) in ex post payoffs. Thus, for simplicity, we suppress the generic index k in $Z_{k1}, Z_{k2}, V_{k1}, V_{k2}$ while reporting our estimation and simulation results.

Table 5 presents estimation results from two models with and without accounting for endogeneity in Z respectively. In the latter case, all covariates in X1 and Z are considered as exogenous in MLE estimation. Using our estimator in Section 3 to allow for endogenous Z, we get estimates that mostly conform to our expectations. The "expected competition" effects (α_k) are significantly negative, with a stronger negative effect for weak potential entrants. The incumbent effect is also significantly negative. Population size contributes to 4G-LTE entry positively, but the percentages of seniors and Natives as well as water coverage act in the opposite direction. The percentage of labor force participation and population density, surprisingly, contribute to 4G-LTE entry negatively. We suspect that the markets that had not been entered by some of the Big Four providers by 2016 may have inherent differences from other markets, in terms of how population density and labor participation affect profitability.

Allowing for potential endogenous Z's turns out to have a big impact on the estimates of network investment effects. Both models produce significantly positive estimates for the coefficients of Z's, but ignoring the endogeneity in Z underestimates the effect of Z_1 while over-estimating that of Z_2. The reason for such discrepancies can be attributed to the roles of structural errors (Vs) in the expected entry payoff. These Vs are firm- or market-level heterogeneity, which may contribute to 3G/4G-LTE deployment in the focal market and 4G-LTE deployment in the neighboring markets in different directions. For instance, V_1 may have a positive effect on 3G deployment before 2015 but a negative impact on 4G-LTE deployment after 2015, while V_2 is the other way around.

A good example example of V's that can lead to these patterns is each firm's spectrum holdings for different generations of cellphone technology. Spectrum of a certain frequency often best serves a particular generation of cellphone technology and has different suitability for urban, suburban and rural deployment. For example, 700 MHz is considered the right band for 4G-LTE and 2.5GHz right for 5G. A firm may have a rich stock of 3G spectrum, but a poor stock of 4G-LTE spectrum simply due to budget constraints.[36] The negative correlation between a firm's 3G and 4G-LTE spectrum holdings on a focal market (which are captured in V_1 and u respectively) is consistent with a negative coefficient for V_1 in our estimates which account for endogenous Z_1.

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[36] For example, T-Mobile did not (and still does not) have enough low-band spectrum (600 MHz), which has wider reach and is better suited for rural deployment; instead, it relies on 1,700MHz and 1900MHz for 4G-LTE deployment, which is better suited for urban and suburban areas.

In addition, this negative correlation also explains the negative bias in the estimated coefficient for Z_1 when its endogeneity is ignored (i.e., 2.024<3.809). At the same time, note that if a firm owns a 4G-LTE spectrum license for the focal census tract, this license covers at least the entire county due to its indivisible nature.[37] Hence there is positive correlation between the 4G-LTE spectrum holdings on the focal and neighboring markets (captured by u and V_2 respectively). This is consistent with a positive coefficient for V_2 in our estimates accounting for endogenous Z, and leads to positive bias in the estimated coefficient for Z_2 when endogeneity is ignored (i.e., 3.042>1.284).

5.8 Counterfactual Results: Evaluating the Merger and the Merger Remedy

In this section, we investigate the impact of a hypothetical merger between T-Mobile and Sprint in 2016. In the first scenario, we use the structural estimates from Table 5 to simulate market outcomes under a baseline scenario with no mergers. In the second scenario for simulation, T-Mobile and Sprint are merged into a "strong" competitor with integrated T-Mobile and Sprint network (henceforth referred to as a “New T-Mobile”).[38] In the third scenario we introduce DISH as a new potential entrant. It is modeled as a "weak competitor" that takes over the decommissioned network originally owned by Sprint. That is, in this scenario, the T-Mobile-and-Sprint merger is mandated to divest assets to the new competitor DISH, enabling DISH’s entry as a facilities-based provider. This scenario corresponds to the DOJ’s proposed remedy out of anti-trust concerns. We keep all 2,582 open market in the baseline simulation, which has a combined total population of 17,209,450.[39]

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[37] The FCC’s smallest coverage for a spectrum license is the Cellular Market Area, which typically covers three to four counties. Even if firms divide spectrum licenses for reselling and lease in secondary markets, they do not break down counties (Kavalar, 2014).

[38] After the merger, T-Mobile will bridge the two network cores together by routing Sprint traffic to the T-Mobile anchor network. An estimated 11,000 Sprint cell sites will be retained to improve capacity and/or coverage on the new network. We implement the after-merger network integration in data by taking the union or the maximum of T-Mobile and Sprint’s coverage at the census block level. These two methods yield almost identical results because in census tracts where both firms serve, they mostly serve at 100% coverage. We report the maximum result in the paper.

[39] In the case of one of T-Mobile and Sprint was a 4G-LTE incumbent in a census tract in 2015 and the other was a potential entrant, we assume that after merger New T-Mobile will re-evaluate the profitability

Table 5: Estimation Results of the 4G-LTE Entry Game: Structural Coefficients

	Endogenous Z		No endogenous Z	
	(1)		(2)	
Variable	Estimate	Std. Error	Estimate	Std. Error
Pop (in 000's)	0.076***	0.021	0.081***	0.019
% Female	0.513	0.730	0.417	0.743
% Senior	-1.033**	0.466	-1.110**	0.434
% White	-0.367	0.563	-0.170	0.502
% Black	-0.502	0.589	-0.367	0.527
% Native	-1.493***	0.573	-1.611***	0.514
% Asian	0.593	1.562	1.290	1.302
% Hispanic	-0.425	0.398	-0.185	0.313
% College	-0.089	0.445	-0.063	0.404
% Labor force	-1.020***	0.328	-0.627**	0.308
% Work home	0.177	0.576	0.768	0.606
% Long comm.	0.213	0.346	-0.508*	0.307
HH income	-0.002	0.003	-0.006*	0.003
HH size	-0.037	0.028	-0.059**	0.025
Pop density	-0.156*	0.093	-0.229***	0.088
% Rural	0.184	0.135	0.188**	0.094
Mostly water	-1.769***	0.585	-2.418***	0.532
# Incumbents	-0.139***	0.050	-0.225***	0.041
Intercept	1.109	0.795	0.871	0.757
Network Investment Effects				
Z_1	3.809***	0.855	2.024***	0.176
Z_2	1.284***	0.390	3.042***	0.096
V_1	-1.791**	0.853	–	–
V_2	1.837***	0.407	–	–
Expected Competition Effects				
Strong potential entrant	-1.053***	0.086	-1.084***	0.085
Weak potential entrant	-1.105***	0.062	-1.133***	0.062

Notes: The results are based on 6,244 observations, corresponding to two to four potential entrants for 2,582 Census tracts. Standard errors are obtained from resampling of markets with replacement 1,000 times. Asterisks indicate the statistical significance at the 1% (***), 5% (**), and 10% (*) levels.

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of the market and re-decide on entry again. In this case and in the case of Sprint was a 4G-LTE incumbent in a census tract in 2015, the number of incumbents in 2016 will be reduced by one after the merger.

We use the estimated coefficients in Table 5 to simulate local market entry decisions of Verizon, AT&T and New T-Mobile (and DISH in the third scenario). For comparison, in each scenario, we simulate two sets of outcomes, with and without accounting for endogenous Z. Panel A in Table 6 presents the simulated market entries across different scenarios; Panel B in Table 6 reports population still under-served (that is, population with the number of providers less than or equal to one) by the end 2018 across these scenarios.

Table 6: Counterfactual Results under Alternative Models

Panel A: Entry outcomes						
	Allow endogenous Z			No endogenous Z		
	(1)	(2)	(3)	(4)	(5)	(6)
# markets with	Baseline	4 to 3	DISH	Baseline	4 to 3	DISH
n entrants = 0	520	659	540	526	588	535
n entrants = 1	1,275	1,606	1,227	1,270	1,662	1220
n entrants = 2	733	310	765	735	323	774
n entrants = 3	53	7	49	49	9	47
n entrants = 4	1	–	1	2	–	6
Total # entry occurrences	2,904	2,247	2,908	2,895	2,335	2,933
by AT&T	723	724	708	722	745	715
by Verizon	175	176	148	173	192	153
by T-Mobile/New T-Mobile	1,195	1,347	1,257	1,185	1,398	1,255
by Sprint/DISH Network	811	–	795	815	–	810
Panel B: Population (in 1000's) under-served (# incumbents in 2018 ≤ 1)						
	Allow endogenous Z			No endogenous Z		
	(1)	(2)	(3)	(4)	(5)	(6)
Under-served population	Baseline	4 to 3	DISH	Baseline	4 to 3	DISH
Total population	203	260	250	209	255	239
Minority population	123	133	128	121	129	124
Rural population	155	204	191	162	201	183

Notes: The above results are based on 2,582 census tracts, each with one to four potential entrants. In Panel A, the number of entrants does not include the number of incumbents prior to 2016; in Panel B, the number of incumbents in 2018 includes both the incumbents prior to 2016 and the entrants between 2016 and 2018.

Columns (1) to (3) of Table 6 are simulation results under the three scenarios, using structural estimates that account for endogeneity (column (1) of Table 5). Comparing column (1) to column (2), we can see T-Mobile and Sprint merger reduces the number of total entry occurrences from 2,904 to 2,247, corresponding to a 23% reduction rate. This leads to a large increase (28%) in underserved population, especially in rural population under-served (32%). There are two explanations for such reduction: First, there are fewer potential entrants on the markets after the merger. Second, the New T-mobile emerging from the merger is a strong competitor with integrated deployment from Sprint and T-mobile, and therefore is more likely to deter entry by the other competitors. As for each firm's entry occurrences after the merger, New T-Mobile would gain sizable grounds after the merger (compared to the pre-merger T-Mobile) while AT&T and Verizon would stay roughly the same. Overall, the reduction of total instances of market entry after the merger is mostly due to the fact there would be fewer potential entrants.

Now consider the scenario where Dish is introduced after the merger as a fourth competitor, enabled by divestiture from the New T-mobile as mandated by the DOJ (assuming that DISH is able to achieve Sprint's deployment in 2015). In this case, our simulation suggest the number of entry occurrences would be 2,908. This is substantially higher than the scenario without the divestiture required by the DOJ, and practically restores the level of market entries before the merger. However, the big gap in population under-served remains — the population under-served would increase by 23% from column (1) to column (3). This suggests that the New T-Mobile and DISH would be inclined to choose to enter different census tracts after the merger, leading to a change in the composition of markets served. Analyzing each firm's entry pattern after the merger with the DOJ-mandated divestiture, we can see that New T-Mobile would increase its market presence at the cost of the other three firms. In this simulation, we have DISH assuming exactly the same network deployment as Sprint in 2015, and AT&T and Verizon have stayed as the same firms as before, so these three firms' reduction in entry can only be a reaction to T-Mobile's advances: T-Mobile not only has a stronger network but also becomes a "strong" potential entrant after the merger, discouraging other potential entrants to enter. The different responses of these three firms (notably, Verizon retracted the most from entry) can only be a strategic, equilibrium response due to the different configuration of strong and weak competitors on the markets.

Columns (4) to (6) of Table 6 report simulation results using estimates that do not account for endogeneity in 3G and neighboring 4G-LTE deployment (the first column in panel (2) of Table 5). Comparing column (1) to column (4), we can see a slight underestimation of entry occurrences and a slight overestimation of population under-served, creating the impression that the impact of ignoring endogeneity in Z in the counterfactual simulation is negligible. The predicted changes in market entry under the second (merger) and the third (merger + DOJ-mandated divestiture) scenario in columns (5) and (6), however, are very different from those predicted in columns (2) and (3). Specifically, if we ignore endogeneity in Z, we will underestimate the effect of the T-Mobile-and-Sprint merger on the reduction of entry (19% reduction from column (4) to column (5) using the model with no endogenous Z) and on population affected by the reduced entry (22% increase in population under-served and 24% in rural population under-served from column (4) to column (5) using the model with no endogenous Z). Moreover, we will overestimate the effect under the third scenario (merger + DOJ-mandated divestiture). Using the model with no endogenous Z, with DISH entry the number of entry occurrences would increase slightly and the population under-served would only increase by 14% from column (4) to column (6). The key message is that researchers would paint a much more rosy picture of the consequences due to the merger and the remedy of proposed divestiture to alleviate the negative merger effect if they use the model without considering Z's endogeneity. Relying on such a biased prediction, policy makers would lean more toward approving the proposed merger.

6 Conclusion

We propose a new method for estimating discrete Bayesian games with endogenous covariates. The approach is flexible enough to accommodate endogeneity due to player- or game-level unobserved heterogeneity. We apply the method to estimate an entry game of 4G LTE deployments between major wireless service providers in the U.S.. In this setting, existing 3G network deployment and neighboring 4G-LTE deployment are endogenous covariates. Our results show that whether to incorporate the endogeneity of network investment affects our estimates of economic primitives, the counterfactual simulations under a hypothetical merger between T-Mobile and Sprint in 2016, and the policy implications. We find that this merger would reduce the 4G-LTE deployment significantly, and the divestiture remedy would not completely reverse the negative outcomes of the merger. Based on our results, we recommend that competition and regulatory authorities fully consider the multi-dimensional trade-offs between market power effects and efficiency gains from drastic changes in market structure.

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Appendix A Proofs

Proof of Lemma 1. The existence of BNEs follows from the continuity of the mapping Γ and an application of the Schauder Fixed-point Theorem. We show the uniqueness of BNE by contradiction.

Suppose there are two distinct BNEs, $P^1 \equiv (P_1^1, \cdots, P_K^1) \neq (P_1^2, \cdots, P_K^2) \equiv P^2$, From Equation (5), we have

$$\begin{aligned} |P_k^1 - P_k^2| &= \left| f_k\Big(X_{k1}'\beta_k + Z_k'\gamma_k + \alpha_k \sum_{j\neq k} \tilde{P}_j + \sum_{j\in\mathcal{K}} V_j'\lambda_{k,j}\Big) \cdot \alpha_k \cdot \sum_{j\neq k} (P_j^1 - P_j^2) \right| \\ &\leq \sup_t f_k(t) \cdot |\alpha_k| \cdot (K-1) \max_{j\in\mathcal{K}} |P_j^1 - P_j^2| < \max_{j\in\mathcal{K}} |P_j^1 - P_j^2|, \end{aligned}$$

where $\tilde{P} = (\tilde{P}_1, \cdots, \tilde{P}_K)$ is a vector of probabilities between P^1 and P^2. The first equality is due to the Mean Value theorem and the second inequality due to Assumption 2. Taking maximization of the left-hand side over $\mathcal{K}$ leads to a contradiction: $\max_{k\in\mathcal{K}} |P_k^1 - P_k^2| < \max_{j\in\mathcal{K}} |P_j^1 - P_j^2|$. □

Proof of Theorem 1. (Consistency of $\widehat{\theta}_{2SNPL}$) First, we show that $L_n(\cdot,\cdot;\widehat{\Pi}) \xrightarrow{p} L_0(\cdot,\cdot)$ uniformly over $\Theta \times P$. By the mean value theorem, for any $\theta \in \Theta$, $P \in \mathcal{P}$,

$$L_n(\theta, P; \widehat{\Pi}) - L_n(\theta, P; \Pi_0) = \nabla_\Pi L_n(\theta, P; \Pi^+)(\widehat{\Pi} - \Pi_0), \tag{10}$$

where Π^+ denotes an intermediate value betweeen $\widehat{\Pi}$ and Π_0. By (10) and the triangular inequality,

$$\begin{aligned} &\sup\nolimits_{\theta,P} \left| L_n(\theta, P; \widehat{\Pi}) - L_0(\theta, P) \right| \\ \leq\ & \sup\nolimits_{\theta,P} |\nabla_\Pi L_n(\theta, P; \Pi^+)| \left| \widehat{\Pi} - \Pi_0 \right| + \sup\nolimits_{\theta,P} |L_n(\theta, P; \Pi_0) - L_0(\theta, P)|. \end{aligned}$$

Under our maintained conditions, $\sup_{\theta,P} |\nabla_\Pi L_n(\theta, P; \Pi^+)| = O_p(1)$. Because $\widehat{\Pi} \xrightarrow{p} \Pi_0$, the first term on the righthand side of the inequality is $o_p(1)$. By Assumption 4-(ii) and the fact that $l_i(\theta, P; \Pi_0)$ is continuous at each θ, P with probability one, the second term on the righthand side of the inequality is $o_p(1)$. This establishes the uniform convergence of L_n to $L_0(\cdot)$ over $\Theta \times \mathcal{P}$.

Note that by Assumption 3-(ii) and the Kullback-Leibler information inequality, (θ_0, P^0) uniquely maximizes $L_0(\theta, P)$ in the set Λ_0. Define

$$T(\theta, P; \Pi_0) \equiv \max_{c\in\Theta} \Big\{ L_0(c, P; \Pi_0) \Big\} - L_0(\theta, P; \Pi_0),$$

where we write out dependence of L_0 on Π_0 explicitly. Because $L_0(\theta, P; \Pi_0)$ is continuous and $\Theta\times\mathcal{P}$ is compact, Berge's maximum theorem establishes that $T(\theta, P; \Pi_0)$ is a continuous function. By construction, $T(\theta, P; \Pi_0) \geq 0$ for any (θ, P). Define

$$\mathcal{E} \equiv \Big\{(\theta, P) \in \Theta \times \mathcal{P} : P = \Gamma(\theta, P; \Pi_0)\Big\}.$$

Since $\Theta \times \mathcal{P}$ is compact and Γ is continuous, $\mathcal{E}$ is a compact set. By definition, Λ_0 is a subset of $\mathcal{E}$. For each element in Λ_0, consider an arbitrarily small open ball that contains it. Let $B_\epsilon(\theta_0, P^0)$ denote the union of such open balls containing elements of Λ_0. Let B_ϵ^c denote the complement of B_ϵ. We then see that $B_\epsilon^c(\theta_0, P^0) \cap \mathcal{E}$ is also compact. Define the constant

$$\tau \equiv \min_{(\theta,P) \in B_\epsilon^c(\theta_0, P^0) \cap \mathcal{E}} T(\theta, P; \Pi_0) > 0. \tag{11}$$

Define the event

$$A_n \equiv \Big\{\Big|L_n(\theta, P; \widehat{\Pi}) - L_0(\theta, P; \Pi_0)\Big| < \tau/2 \text{ for all } (\theta, P) \in \Theta \times \mathcal{P}\Big\}.$$

Let $\left(\theta_n^*, P_n^*\right)$ be an element of Λ_n. Then A_n implies

$$\begin{aligned} &L_0\Big(\theta_n^*, P_n^*; \Pi_0\Big) > L_n\Big(\theta_n^*, P_n^*; \widehat{\Pi}\Big) - \frac{\tau}{2}; \text{ and} \\ &L_n\Big(\theta, P_n^*; \widehat{\Pi}\Big) > L_0\Big(\theta, P_n^*; \Pi_0\Big) - \frac{\tau}{2} \text{ for any } \theta \in \Theta. \end{aligned}$$

Besides, $L_n\Big(\theta_n^*, P_n^*; \widehat{\Pi}\Big) \geq L_n\Big(\theta, P_n^*; \widehat{\Pi}\Big)$ by definition of Λ_n. Thus

$$L_0\Big(\theta_n^*, P_n^*; \Pi_0\Big) > L_n\Big(\theta_n^*, P_n^*; \widehat{\Pi}\Big) - \frac{\tau}{2} \geq L_n\Big(\theta, P_n^*; \widehat{\Pi}\Big) - \frac{\tau}{2} > L_0\Big(\theta, P_n^*; \Pi_0\Big) - \tau$$

for any $\theta \in \Theta$. Therefore

$$\begin{aligned} A_n \;&\Rightarrow\; \Big\{L_0\Big(\theta_n^*, P_n^*; \Pi_0\Big) > L_0\Big(\theta, P_n^*; \Pi_0\Big) - \tau, \forall \theta \in \Theta\Big\}, \\ &\Rightarrow\; \Big\{L_0\Big(\theta_n^*, P_n^*; \Pi_0\Big) > \max_{\theta \in \Theta} L_0\Big(\theta, P_n^*; \Pi_0\Big) - \tau\Big\}, \\ &\Rightarrow\; \Big\{\tau > T\Big(\theta_n^*, P_n^*; \Pi_0\Big)\Big\}, \\ &\Rightarrow\; \Big\{\min_{(\theta,P) \in B_\epsilon^c(\theta_0; P^0) \cap \mathcal{E}} T(\theta, P; \Pi_0) > T\Big(\theta_n^*, P_n^*; \Pi_0\Big)\Big\} \text{ by (11)}, \\ &\Rightarrow\; \Big\{(\theta_n^*; P_n^*) \in B_\epsilon(\theta_0; P^0)\Big\}. \end{aligned}$$

The last induction uses the fact that $(\theta_n^*, P_n^*) \in \mathcal{E}$. Therefore, $\Pr(A_n) \leq \Pr\Big((\theta_n^*, P_n^*) \in B_\epsilon(\theta_0; P^0)\Big)$. By the uniform convergence of $L_n(\cdot; \widehat{\Pi})$ to $L_0(\cdot)$, $\Pr(A_n) \to 1$ as $n \to \infty$. Thus

$$\Pr\Big((\theta_n^*, P_n^*) \in B_\epsilon(\theta_0; P^0)\Big) \to 1. \tag{12}$$

For the case where Λ_0 is a singleton, this suffices for consistency of $\widehat{\theta}_{2SNPL}$.

In the general case where Λ_0 has multiple elements, the proof follows from the same arguments in Aguirregabiria and Mira (2007), which proceed by showing the following results sequentially: (1)ϕ_n converges to ϕ_0 in probability uniformly in a neighborhood around P^0; (2) with probability approaching 1, there exists an element (θ_n^*, P_n^*) of Λ_n in any open ball around (θ_0, P^0); and (3) with probability approaching 1, the 2SNPL estimator is the element of Λ_n that belongs to an open ball around (θ_0, P^0).

(Asymptotic Normality of $\widehat{\theta}_{2SNPL}$) We now derive the limit distribution of $\widehat{\theta}_{2SNPL}$. To simplify notation, we drop the subscript $_{2SNPL}$ from the notation of this estimator in the proof below. By definition,

$$\frac{1}{n}\sum\nolimits_{i=1}^{n} \nabla_\theta l_i(\hat{\theta}, \hat{P}; \widehat{\Pi}) = 0 \text{ and } \hat{P} - \Gamma(\hat{\theta}, \hat{P}; \widehat{\Pi}) = 0.$$

A stochastic mean value theorem between $(\theta_0, P^0; \Pi_0)$ and $(\hat{\theta}, \hat{P}; \widehat{\Pi})$, together with consistency of $(\hat{\theta}, \hat{P})$ and $\widehat{\Pi}$, imply that

$$\begin{aligned}\frac{1}{\sqrt{n}}\sum_{i=1}^{n} s_{\theta,i} - \Omega_{\theta\theta}\sqrt{n}\left(\hat{\theta} - \theta_0\right) - \Omega_{\theta P}\sqrt{n}\left(\hat{P} - P^0\right) - \Omega_{\theta\Pi}\sqrt{n}\left(\widehat{\Pi} - \Pi_0\right) &= o_p(1), \\ \left(I - \Gamma_P^0\right)\sqrt{n}(\hat{P} - P^0) - \Gamma_\theta^0\sqrt{n}(\hat{\theta} - \theta_0) - \Gamma_\Pi^0\sqrt{n}(\widehat{\Pi} - \Pi_0) &= o_p(1).\end{aligned}$$

Solving for $\sqrt{n}(\hat{P} - P^0)$ from the second set of equations and substituting into the first set, we get

$$\begin{aligned} &\underbrace{[\Omega_{\theta\theta} + \Omega_{\theta P}(I - \Gamma_P^0)^{-1}\Gamma_\theta^0]}_{\equiv M}\sqrt{n}(\hat{\theta} - \theta_0) \\ = &\ \frac{1}{\sqrt{n}}\sum_{i=1}^{n} s_{\theta,i} - [\Omega_{\theta P}(I - \Gamma_P^0)^{-1}\Gamma_\Pi^0 + \Omega_{\theta\Pi}]\sqrt{n}(\hat{\Pi} - \Pi_0) + o_p(1) \\ = &\ \frac{1}{\sqrt{n}}\sum_{i=1}^{n} \underbrace{\left\{s_{\theta,i} - [\Omega_{\theta P}(I - \Gamma_P^0)^{-1}\Gamma_\Pi^0 + \Omega_{\theta\Pi}]r_{0,i}\right\}}_{\equiv\tilde{s}_i} + o_p(1),\end{aligned}$$

where the second equality uses the asymptotic linear representation of $\sqrt{n}(\widehat{\Pi} - \Pi_0)$ and its influence function $r_{0,i} \equiv r_i(\Pi_0)$. The asymptotic distribution of $\hat{\theta}$then follows from the continuous mapping theorem,

□

Appendix B Cellular Network Explained

A cellphone is a portable telephone that can make and receive calls over a radio frequency (“spectrum”) while the user is moving within a service area. When a user makes a phone call or sends a message, her cellphone converts her voice or message into electrical signals, which is transmitted from her location to the nearest cell tower via radio waves.

The network of cell towers then relays the radio waves to the receiver's cellphone, which converts it to electrical signals and then back to sound, text, or image again. In this process, data travels in a "cellular network," which is composed of cellphones, base transceiver stations ("cell sites"), mobile telephone switching offices, and the public switched telephone network. A cellphone is a type of Mobile Subscriber Units, which consists of a control unit and a transceiver that transmits and receives radio transmissions to and from a cell site. The term cell site refers to the physical location of radio equipment that provides coverage within a cell. A list of hardware located at a cell site includes power sources, interface equipment, radio frequency transmitters and receivers, and antenna systems. A mobile telephone switching office is the central office for mobile switching. It houses the mobile switching center, field monitoring, and relay stations for switching calls from cell sites to wire-line central offices. The public switched telephone network is made up of local networks, the exchange area networks, and the long-haul network that interconnect telephones and other communication devices on a worldwide basis. Boccuzzi(2019) describes the basics of cellular communications.

A new generation of network technology arrived almost every decade since the inception of such technology. The first two generations (0G and 1G) were before the widespread use of cellphones.[40] In the 1990's, 2G started the use of digital transmission instead of analog transmission, marking the start of widespread use of cellphones in our lives. In the 2000's, 3G was the predominant technology and in the 2010's 4G-LTE. Now we are facing the transformation from 4G-LTE to 5G, the newest generation of network technology.

Appendix C Robustness of Table 5

We check the robustness of our Table 5's results by: 1) restricting to homogeneous competition effects; 2) using any generation of technology (2G, 3G, and 4G non-LTE combined) instead of 3G to measure a firm's previous network investment in the focal market; 3) using a firm's 4G deployment in neighboring markets in 2015, instead of that in 2018, to measure the firm's network investment in neighboring markets.

Table 7 reports the results from these alternative specifications. In specification (1), the Big Four are treated as equal competitors. Results in this specification are very close to specification (1) in Table 5, suggesting only small differences in how AT&T/Verizon and T-Mobile/Sprint reacted to expected competition. In specification (2), for Z_1 we expand from 3G to include any previous generation of technology deployment in the focal market. Compared with specification (1) in Table 5, the biggest change is that the estimate of the coefficient for $V1$ loses its statistical significance. This can be attributed to the multiple

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[40] 0G refers to pre-cellphone mobile technology, such as radio telephones that were placed in cars before the advent of cellphones. 1G refers to Analog Cellular Networks, which employ multiple cell sites to transfer calls from one site to the next as the user travels between cell sites during a conversation.

dimensions of firm- and market-level unobserved heterogeneity, each contributing to the deployment of different generations of cellphone technology in different directions.[41] In specification (3), we do not consider the concurrent deployment of 4G-LTE in the neighboring census tracts; instead, we restrict to 4G-LTE deployment in the neighboring census tracts to the status quo before the start of the entry game. In this specification, Z_2 would stay an endogenous variable, but how the unobserved heterogeneity in the Z_2 equation contributes to 4G-LTE entry in the focal market will have a confounding direction. A firm selectively chose to enter the neighboring markets by 2015, and this selection may lead to negative correlation between u and V_2. Results support our conjecture, with an insignificant Z_2 effect and a much smaller V_2 effect.

In summary, these alternative specifications often produce different magnitudes in estimates, but all point to the importance of the network investment effect as well as a consistently negative expected competition effect.

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[41] As discussed earlier, each generation of technology is deployed in a different era with different demand- and supply- side factors as well as under different regulatory regimes.

Table 7: Robustness of Table 5

	Homogeneous Competition		Use any G for Z_1		Use 2015’s 4G for Z_2	
	(1)		(2)		(3)	
Variable	Est.	S.E.	Est.	S.E.	Est.	S.E.
Pop (in 000’s)	0.075***	0.021	0.071***	0.022	0.044***	0.017
% Female	0.515	0.729	0.646	0.756	0.329	0.571
% Senior	-1.041**	0.465	-0.981**	0.502	-0.746*	0.388
% White	-0.357	0.564	-0.365	0.571	-0.506	0.466
% Black	-0.503	0.590	-0.516	0.583	-0.734	0.483
% Native	-1.492***	0.573	-1.642***	0.608	-1.012**	0.457
% Asian	0.613	1.566	0.806	1.417	0.200	1.298
% Hispanic	-0.414	0.398	-0.188	0.445	-0.561*	0.319
% College	-0.094	0.444	0.002	0.415	0.145	0.357
% Labor force	-1.022***	0.328	-0.855***	0.318	-1.110***	0.282
% Work home	0.168	0.574	0.351	0.571	-0.054	0.455
% Long commute	0.205	0.346	0.030	0.352	0.576**	0.245
HH income	-0.002	0.003	-0.005	0.003	0.002	0.003
HH size	-0.037	0.028	-0.039	0.029	-0.011	0.023
Pop density	-0.157*	0.093	-0.169*	0.088	-0.061	0.074
% Rural	0.177	0.135	0.077	0.130	0.108	0.115
Mostly Water	-1.792***	0.583	-2.131***	0.592	-1.324***	0.487
# Incumbents	-0.146***	0.049	-0.162***	0.050	0.008	0.037
Intercept	1.140	0.797	1.254	0.832	0.684	0.603
Network Investment Effects						
Z_1	3.745***	0.853	2.176***	0.841	3.931***	0.685
Z_2	1.286***	0.390	1.589***	0.393	0.102	0.306
V_1	-1.731**	0.852	-0.213	0.823	-1.339**	0.679
V_2	1.823***	0.407	1.493***	0.407	0.645*	0.339
Expected Competition Effects						
Any P.E.	-1.097***	0.061	-	-	-	-
Strong P.E.	-	-	-1.008***	0.081	-0.453***	0.083
Weak P.E.	-	-	-1.081***	0.063	-0.365***	0.062

Notes: The results are based on 6,244 observations, corresponding to two to four potential entrants for 2,582 Census tracts. Standard errors are obtained from resampling of markets with replacement 1,000 times). Asterisks indicate the statistical significance at the 1% (***), 5% (**), and 10% (*) levels.