# INFORMATION SPILLOVER IN MARKETS WITH HETEROGENEOUS TRADERS

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ABSTRACT. This paper studies the welfare impact of information spillover in divisible good markets with heterogeneous traders and interdependent values. In a setting where two groups of traders trade two distinct but correlated assets, one within each group, the information contained in the price of one asset spills over to the other market. Some "more informed" traders who submit demand schedules may condition their demands on the prices of both assets, while others do not. We prove the existence of a linear equilibrium and examine how information spillover affects trading, information efficiency, and welfare, as the fraction of the more informed traders varies. In the two symmetric benchmarks, full information spillover (all traders are more informed) dominates no information spillover (all traders are less informed) in terms of welfare. However, in markets with heterogeneous traders, information spillover can hurt overall welfare, while still improving information efficiency; we characterize the non-monotonic impact of information spillover on aggregate welfare in large finite markets. Furthermore, information spillover can account for the empirical evidence of excessive price co-movement and volatility transmission in financial markets.

KEYWORDS: information spillover, strategic trading, interdependent values

## 1. INTRODUCTION

It is well-known that markets are interconnected. For instance, in financial markets, trading activities of assets in one market can be informative about the values of assets in other markets. Savvy traders operating in one market will likely use information from other markets to help guide their trading decisions, amplifying underlying correlation among markets and volatility of the impact of economic fundamentals, with potential substantial welfare consequences for all traders. More concretely, during the financial

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crises in the late '90s and 2008, drastic movements in one national stock market had significant impacts on the stock markets across the world (see King et al. (1994), Forbes and Rigobon (2002), Diebold and Yilmaz (2009)). Likewise, several studies have documented the "excess comovement" of asset prices relative to the fundamentals (see Pindyck and Rotemberg (1993), Barberis et al. (2005), Veldkamp (2006)). Similar patterns and issues appear in many markets, such as real estate, petroleum, electricity, etc. Despite the significance of information spillover, there is relatively little study of its impact on strategic trading. How do strategic traders react to information spillover? For traders who do not directly observe or take into account prices in other markets, how are their trading activities indirectly affected? Does information spillover enhance the information efficiency of prices? What determines traders' welfare? Would a policy that encourages more traders to take advantage of information spillover necessarily improve welfare?

This paper develops a theoretical framework to examine these questions. In our model, there are two markets and two assets, one in each market.[1] Traders' values are correlated both within a market and across markets. Traders in each market observe noisy signals about their values and compete in demand schedules of the asset and the equilibrium asset prices are determined by market clearing conditions jointly. Following the strategic trading literature, traders' payoffs are linear in their values net off a quadratic cost, and all traders' asset values and signals are jointly normally distributed. To model information spillover, we assume that some traders in a market, who are called "more informed" traders, can react to the price of the asset in the other market and submit demand schedules of an asset contingent on the prices of both assets and their private signals; the remaining traders, who are called "less informed" traders, can only express their demands of an asset as functions of its own price and their private signals. The fraction of more informed traders in a market, which governs the extent of endogenous information spillover into this market, is the main focus of our exercise.

We first solve for the unique linear Bayes Nash equilibrium in closed form in two benchmarks: either all traders are more informed (Proposition 3.1) or they are all less informed (Proposition 3.2). The differences between these two equilibria illustrate the impact of information spillover, absent of heterogeneity. Notably, all traders in a market benefit from information spillover (Proposition 3.4). Intuitively, because the price in the other market is public, no trader has any informational advantage. Instead, it alleviates adverse selection faced by traders so that they are more willing to trade, which in turn lowers each

[1]The working paper version Huangfu and Liu (2022) extends the analysis to the case with more than two markets and assets.

trader's price impact (i.e., how much a marginal increase in her demand would change the price) and hence increases her expected surplus.

In general, trader heterogeneity, despite a realistic feature in many real-world markets, confounds the information content of prices in the presence of information spillover. In particular, equilibrium prices no longer reveal the average signals of all traders in a market, which is a key property that holds in the symmetric benchmarks. Furthermore, a traders' strategic impact depends on the composition of traders in the markets. Consequently, there are no off-the-shelf results that guarantee the existence of linear Bayes Nash equilibria for the general case. The main contribution of this paper is to establish the existence of an asymmetric linear equilibrium and to characterize the equilibrium and its comparative statics with respect to information spillover in large but finite markets.

For equilibrium existence, we identify four bounded parameters, two in each market, which measure the weighted heterogeneity in bidders' response to their own signals and the asset price in this market, respectively. Theorem 4.4 shows that these parameters uniquely pin down each trader's conditional expectation about her value as a linear function of her signal and prices; moreover, the inference coefficients are bounded under a joint restriction on the number of traders and the noise in the signal, from which we solve for traders' optimal strategies and hence the four parameters in the beginning. Applying Brouwer's fixed point theorem to this mapping of the parameters delivers existence.[2]

Because the equilibria are not in closed forms in general, we first consider the large-market limit as the market size in both markets grows to infinity. Specifically, Proposition 5.1 characterizes the unique limiting equilibrium in closed form, as the numbers of traders in both markets go to infinity, keeping the fraction of more informed traders in each market constant. However, both the price impact of a single trader and her inference from information spillover vanish in the large market limit in our main setting. We also provide numerical examples to illustrate how the equilibrium outcomes vary with the number of traders.

To further examine analytically both the informational and strategic impacts of information spillover, we expand the equilibrium coefficients around their limits and focus on terms that are of order $1/N$ and $1/N^2$, where $N$ is the market size. Remarkably, all these first and second order effects, which also pin down the speed of convergence, are unique and in closed form (Propositions 5.1 and 5.2). Most importantly, these approximations allow us to compare how the more or less informed traders are differentially

[2]The numerical exercises presented after the existence result suggest that the equilibrium is unique for a wide range of primitives, although we do not have a formal proof of uniqueness.

impacted by information spillover (Propositions 5.4 and 5.6). For the more informed, the information content of the asset price in the other market has a first-order effect. This direct "spillover channel" leads to a higher surplus for the more informed than the less informed. Intuitively, the extra information alleviates the more informed traders' adverse selection problem so that their beliefs are less responsive (first-order effect) to changes in their own asset prices and more sensitive (second-order effect) to their own signals than the less informed. Regarding price impacts, because the more informed are more willing to trade with less severe adverse selection, all traders' price impacts in equilibrium are lower with information spillover. Moreover, the price impacts are decreasing in the fraction of more informed, albeit only to a second-order effect.

Based on these characterizations, we obtain three main sets of results on traders' welfare, information efficiency, price co-movement and volatility transmission. First, Proposition 5.8 shows that the fraction of more informed traders has a non-monotonic impact on aggregate welfare. Based on the large-market approximation for the expected surplus, we show that a trader's welfare increases if either (i) she relies more on her own signal ("own signal effect"), or (ii) the two types of traders react more differently to prices so as to create more trade between them ("heterogeneous beliefs effect"), or (iii) her price impact is smaller. Among these three effects, it turns out that only the first two effects, both of which are second-order, vary with respect to the fraction of more informed traders; the term containing price impact in the welfare approximation is independent of the fraction of more informed traders up to the second-order. Therefore, the welfare comparative statics is solely driven by the traders' beliefs and inference in large but finite markets.

For the more informed, whose expected surplus is always larger than the less informed, more information spillover hurts their welfare. As an important step toward this result, we show that when the market sizes are large enough, more informed traders almost perfectly infer the average signals in both markets from the two market prices, independent of the fraction of more informed. Consequently, changes in information spillover do not affect the informativeness of prices nor the informativeness of own signals for the more informed. Thus, only the "heterogeneous beliefs effect" matters for a more informed trader: the gain from trading with the less informed shrinks and so does her expected surplus, as the fraction of the more informed increases. For the less informed, their surplus is U-shaped because of two opposing forces. Suppose the number of more informed traders increases in a market, on the one hand, the price contains more information from the other market, which may crowd out the informativeness of her private signal (a smaller "own signal effect"), which hurts the less informed; on the other hand, competition among the more informed gets stronger (a larger "heterogeneous beliefs effect"), which benefits the less informed. Overall, with a larger number of more informed, the weighted average surplus also turns out to be U-shaped as a function of information spillover.

Second, Proposition 5.9 shows that information spillover improves both information efficiency, measured by variance reduction. When the number of more informed traders increases, the variance of a less informed trader's conditional expectation of her value is always smaller; the variance of the more informed, on the other hand, remains the same up to second-order approximation. That is, information spillover improve overall learning and lower the information advantage of more informed traders. In fact, we can rewrite a trader's expected surplus (up to second-order approximation) as the difference between information efficiency and the variance of her own asset price (Proposition 5.10). Because information spillover leads to larger price variance, this explains the gap between information efficiency and allocation efficiency in our strategic setting.

Finally, our findings are consistent with the empirical puzzles of excessive price co-movements and volatility transmissions mentioned in the beginning: the correlation of prices is larger than the correlation of average signals across markets (Proposition 5.11), and with an exogenous shock to the average signals in one market, the price variance in the other market goes up (Proposition 5.12); both effects are increasing in information spillover.

Our results cover both endogenous and exogenous information spillover. When there are more informed traders in both markets, the equilibrium prices and their information content are endogenously determined through joint market clearing. When only one market has more informed traders, the price in the other market, which is determined by its own market-clearing condition, can be viewed as exogenous and "semi-public" information that is only available to the more informed in the former market. Therefore, in addition to prices from other markets, our analysis applies to broader settings in which a fraction of traders (i.e., insiders) commonly observe some informative signals about their values before trading.[3]

The main policy implication of our results is that an increase in transparency (i.e., the fraction of more informed traders) can have negative welfare consequences in markets with heterogeneously informed traders. While it is well-known that transparency can have detrimental welfare consequences (e.g. Biais et al. (2005)), our analysis highlights

[3]In this exogenous information spillover setting, the same technical issues due to trader asymmetry arise, and our method can be applied directly. We focus on the endogenous information spillover case as it allows us to derive more comparative statics, such as cross market effects.

a new mechanism through the interaction between trader asymmetry and adverse selection, which can be relevant for transparency regulation policies, such as the Trade Reporting and Compliance Engine (TRACE) program in the US.[4] Furthermore, even though higher transparency can make markets more volatile, it also implies that information is more efficiently aggregated in the market prices.

This paper contributes to the literature on divisible-good double auctions with interdependent values and strategic trading in imperfectly competitive markets. Rostek and Yoon (2020) give a contemporary and comprehensive survey of the literature. Starting from the seminal work by Wilson (1979) that considers the symmetric pure common value case,[5] subsequent papers, Vives (2011), Rostek and Weretka (2012), Ausubel et al. (2014), Vives (2014), Du and Zhu (2017), and Lambert et al. (2018) for example, extend the analysis to general interdependent value settings with symmetric equilibria. More recent contributions by Malamud and Rostek (2017), Rostek and Yoon (2021), Rostek and Yoon (2021), Chen and Duffie (2021), Wittwer (2021), and Rostek and Wu (2021) take the market design perspective to analyze the impact of trading technologies (e.g. the arrangement of trading venues, market fragmentation, joint or independent market-clearing across venues, synthetic assets, access to information) in multi-asset markets with either private or interdependent values. An important insight is that, when traders are strategic and can trade in multiple markets, the change in traders' price impact in decentralized markets can outweigh the loss of information or market depth and hence improve overall welfare relative to centralized markets. In comparison, to isolate the cross-market information externality, our main model assumes that each trader only trades in one market so that her payoff does not directly depend on the allocations across markets and analyzes how information spillover, either exogenous or endogenous, affects her price impact and welfare.[6]

Trader heterogeneity in the presence of information spillover is another substantial departure of our paper from the literature. Different from the insider trading models initiated by Kyle (1989) in which some traders are privately informed and the rest has no private information, in our model all traders are privately informed and, in addition, a fraction of traders in each market learns some additional public signal, such as the price in the other market when information spillover is endogenous. A recent contribution

[4]See Asquith et al. (2019) for a recent empirical study of the impact of transparency in the US corporate bond markets.

[5]See Klemperer and Meyer (1989) for seminal study of supply function competition in oligopoly.

[6]Our model is not a special case of Malamud and Rostek (2017) or Rostek and Wu (2021) because traders only observe prices in the other market without being able to trade in the other market. The online appendix contains an analysis of the symmetric case in which all traders trade in both markets.

by Manzano and Vives (2021) also studies asymmetric divisible good auctions with two types of traders, who also provide an extensive and insightful discussion on the relevance of trader heterogeneity in many real-world markets.[7] Manzano and Vives (2021) overcome the potentially complex inference problem under heterogeneous information by studying a setting in which all traders of the same type observe a common type-specific signal so that the market price perfectly aggregates information when there are two types of traders; they also consider trader heterogeneity in other dimensions, such as the marginal cost. In contrast, prices in our model do not perfectly aggregate information and different types of traders hold distinct beliefs; we deal with this non-degenerate adverse selection by exploiting the asymptotic symmetry in large markets as explained in previous paragraphs. In addition, the comparative statics of information spillover is specific to our setting. Our work thus complements Manzano and Vives (2021), in both the economic insights and technical methods, and echoes their advocate for further investigations of markets with heterogeneous traders.

The rest of the paper is organized as follows. Section 2 presents the setting. Section 3 analyzes two symmetric benchmarks. Section 4 establishes equilibrium existence in the general case. Section 5 examines the equilibrium in large finite markets. Section 6 discusses the modeling assumptions and extensions. The proofs are in the appendix. The online appendix contains further details of the extensions.

## 2. Model

Consider a setting with two risky assets, $k \in \{I, II\}$, traded in two separate markets. For each $k \in \{I, II\}$, there are $n_k \in \mathbb{N}_+$ traders who trade asset $k$ in market $k$, and the set of traders in market $k$ is denoted by $\mathcal{N}_k$. For each trader $i \in \mathcal{N}_k$, the per-unit value of asset $k$ to her is $\theta_k^i$. The vector of all traders' values, $(\theta_I^i, \theta_{II}^j)_{i \in \mathcal{N}_I, j \in \mathcal{N}_{II}}$, is jointly normally distributed with a zero mean vector (as a normalization). The variance of $\theta_k^i$ is $\sigma_{\theta_k}^2 > 0$. The covariances satisfy: $\mathrm{Cov}(\theta_k^i, \theta_k^j) = \sigma_{\theta_k}^2 \rho_k > 0$, for all $i, j \in \mathcal{N}_k$, and $\mathrm{Cov}(\theta_k^i, \theta_{-k}^j) = \sigma_{\theta_k} \sigma_{\theta_{-k}} \phi \in \mathbb{R}$, for all $i \in \mathcal{N}_k$, $j \in \mathcal{N}_{-k}$, $k, -k \in \{I, II\}$ with $k \neq -k$. We assume $1 > \rho_k \geq |\phi| > 0$.[8]

For each $k \in \{I, II\}$, trader $i \in \mathcal{N}_k$ privately observes a noisy signal $s_k^i = \theta_k^i + \varepsilon_k^i$ about her value. The noise $\varepsilon_k^i$ is normally distributed with mean zero and variance $\sigma_{\varepsilon_k}^2$ and independent across all traders $i$ and assets $k$. Let $\sigma_k^2 = \sigma_{\varepsilon_k}^2 / \sigma_{\theta_k}^2$ be the variance ratio measuring

[7] Another recent paper by Andreyanov and Sadzik (2021) also studies exchanges with trader heterogeneity from a robust mechanism design perspective. From a broader perspective, our paper is also related to recent work on information spillover in dynamic bargaining, such as Asriyan et al. (2017) and Huangfu and Liu (forthcoming).

[8] The condition $\rho_k \geq |\phi|$ is necessary for the covariance matrix of signals to be positive semi-definite.

the impact of noise relative to the value in market $k$. The noises $(\varepsilon_I^i, \varepsilon_{II}^j)_{i\in\mathcal{N}_I, j\in\mathcal{N}_{II}}$ and the values $(\theta_I^i, \theta_{II}^j)_{i\in\mathcal{N}_I, j\in\mathcal{N}_{II}}$ are independent.

The initial endowment of each trader $i \in \mathcal{N}_k$ is normalized to zero. The payoff of trader $i \in \mathcal{N}_k$ from trading $x_k^i$ units of asset $k$ at a price $p_k \in \mathbb{R}$ is

$$u_k^i(x_k^i, p_k, \theta_k^i) = (\theta_k^i - p_k) \cdot x_k^i - \frac{\gamma}{2}\left(x_k^i\right)^2,$$

which is linear in her value of the asset $\theta_k^i$ net off the asset price $p_k$ and has a quadratic inventory cost, where $\gamma > 0$ is a commonly known constant.

Traders at each market submit net demand schedules for the corresponding asset and the equilibrium prices of the assets are simultaneously determined by the market-clearing conditions at both markets. To examine the impact of information spillover from one market to the other, we assume that there are two types of traders at each market, depending on whether their demand can be contingent on the price of the other asset. Specifically, for each $k \in \{I, II\}$, let $\mathcal{N}_k^1$ be the set of *more informed* traders in market $k$, who can submit demand schedules depending on the prices of the assets in both markets. That is, for each $i \in \mathcal{N}_k^1$, trader $i$ submits a demand function $x_k^i : \mathbb{R}^2 \to \mathbb{R}$ such that $x_k^i(p_k, p_{-k}) \in \mathbb{R}$ specifies the quantity of asset $k$ trader $i$ demands for any price vector $(p_k, p_{-k})$. Let $\mathcal{N}_k^0 = \mathcal{N}_k \setminus \mathcal{N}_k^1$ be the set of *less informed* traders in market $k$, who submit demand schedules only as a function of the price of asset $k$. That is, for each $i' \in \mathcal{N}_k^0$, trader $i'$ submits a demand function $x_k^{i'} : \mathbb{R} \to \mathbb{R}$ specifying the quantity demanded $x_k^{i'}(p_k)$ of asset $k$ for any price $p_k \in \mathbb{R}$. A trader is a buyer if her demand is positive or a seller if her demand is negative. Let $n_k^1 = |\mathcal{N}_k^1|$ and $n_k^0 = |\mathcal{N}_k^0| = n_k - n_k^1$ be the numbers of more and less informed traders, respectively. Denote by $\alpha_k = n_k^1/n_k \in [0,1]$ the fraction of more informed traders in market $k$. Given the submitted demand schedules in both markets, $(x_I^i(p_I, p_{II}), x_I^{i'}(p_I))_{i\in\mathcal{N}_I^1, i'\in\mathcal{N}_I^0}$ and $(x_{II}^j(p_{II}, p_I), x_{II}^{j'}(p_{II}))_{j\in\mathcal{N}_{II}^1, j'\in\mathcal{N}_{II}^0}$, the equilibrium price vector $(p_I^*, p_{II}^*) \in \mathbb{R}^2$ is determined by market clearing:

$$\sum_{i\in\mathcal{N}_I^1} x_I^i(p_I^*, p_{II}^*) + \sum_{i'\in\mathcal{N}_I^0} x_I^{i'}(p_I^*) = 0, \quad \text{and} \quad \sum_{j\in\mathcal{N}_{II}^1} x_{II}^j(p_{II}^*, p_I^*) + \sum_{j'\in\mathcal{N}_{II}^0} x_{II}^{j'}(p_{II}^*) = 0.$$

For each $k \in \{I, II\}$, a more informed trader $i \in \mathcal{N}_k^1$ receives $x_k^i(p_k^*, p_{-k}^*)$ units of asset $k$ and pays $p_k^* x_k^i(p_k^*, p_{-k}^*)$, and a less informed trader $i' \in \mathcal{N}_k^0$ is allocated $x_k^{i'}(p_k^*)$ units of asset $k$ and pays $p_k^* x_k^{i'}(p_k^*)$.

We adopt linear Bayes Nash equilibrium as the solution concept. A strategy of a more informed trader $i \in \mathcal{N}_k^1$ is a mapping $x_k^i(p_k, p_{-k}, s_k^i)$ from her realized signal $s_k^i$ to a demand schedule for asset $k$ contingent on $(p_k, p_{-k})$. A strategy of a less informed trader

$i' \in \mathcal{N}_k^0$ is a mapping $x_k^{i'}(p_k, s_k^{i'})$ from her realized signal $s_k^{i'}$ to a demand schedule for asset $k$ contingent on $p_k$. A *Bayes Nash equilibrium* is a strategy profile $(x_k^i, x_k^{i'})_{k\in\{I,II\}, i\in\mathcal{N}_k^1, i'\in\mathcal{N}_k^0}$ such that for each trader $i \in \mathcal{N}_k^1$ and signal $s_k^i$, the demand schedule maximizes $i$'s expected payoff:

$$\mathbb{E}\left[u_k^i(x_k^i(p_k^*, p_{-k}^*, s_k^i), p_k^*, \theta_k^i)\middle| s_k^i, p_k^*, p_{-k}^*\right] \geq \mathbb{E}\left[u_k^i(\tilde{x}_k^i(\tilde{p}_k, \tilde{p}_{-k}, s_k^i), \tilde{p}_k, \theta_k^i)\middle| s_k^i, \tilde{p}_k, \tilde{p}_{-k}\right],$$

where $(p_k^*, p_{-k}^*)$ is the market-clearing price vector given $x_k^i$ and all other traders' equilibrium strategies and $(\tilde{p}_k, \tilde{p}_{-k})$ is the market-clearing price vector given any strategy $\tilde{x}_k^i$ and others' equilibrium strategies, and for each $i' \in \mathcal{N}_k^0$ and $s_k^{i'}$,

$$\mathbb{E}\left[u_k^{i'}(x_k^{i'}(p_k^*, s_k^{i'}), p_k^*, \theta_k^{i'})\middle| s_k^{i'}, p_k^*\right] \geq \mathbb{E}\left[u_k^{i'}(\tilde{x}_k^{i'}(\tilde{p}_k, s_k^{i'}), \tilde{p}_k, \theta_k^{i'})\middle| s_k^{i'}, \tilde{p}_k\right],$$

where $p_k^*$ is asset $k$'s market-clearing price given $x_k^{i'}$ and all other traders' equilibrium strategies and $\tilde{p}_k$ is asset $k$'s market-clearing price given any strategy $\tilde{x}_k^{i'}$ and all other traders' equilibrium strategies. A Bayes Nash equilibrium is *linear* if all traders' equilibrium strategies are linear functions. Since traders in each of the four subgroups ($\mathcal{N}_I^1$, $\mathcal{N}_I^0$, $\mathcal{N}_{II}^1$, and $\mathcal{N}_{II}^0$) are ex ante symmetric, we further restrict attention to linear Bayes Nash equilibria that are symmetric within each subgroup.

## 3. Benchmarks

To illustrate the spillover effect of asset prices, we first solve for the closed-form equilibria in two benchmark cases. To simplify notations, in this section we assume the two markets are symmetric in the sense that $n_I = n_{II} = n$, $\rho_I = \rho_{II} = \rho$, and $\sigma_I^2 = \sigma_{II}^2 = \sigma^2$. Since the proofs of the results in this section follow standard argument, they are relegated to the online appendix.

### 3.1. **Information Spillover** ($\alpha_I = \alpha_{II} = 1$).

Suppose all traders can condition their demands on the prices of both assets. The equilibrium in market $k \in \{I, II\}$ is given by

$$x_k^i(p_k, s_k^i) = a^1 s_k^i - B^1 p_k + b^1 p_{-k}, \tag{1}$$

where

$$a^1 = \frac{(n-2)F_s^1 - F^1}{\gamma(n-1)},\ B^1 = \frac{(F_s^1 + F^1)a^1}{(F_s^1 + F^1)^2 - (F_-^1)^2},\ b^1 = \frac{F_-^1 a^1}{(F_s^1 + F^1)^2 - (F_-^1)^2}, \tag{2}$$

$$F_s^1 = \frac{1-\rho}{1-\rho+\sigma^2},\ F^1 = \frac{\sigma^2}{1-\rho+\sigma^2} \cdot \frac{\rho^2 + \frac{\rho(1-\rho+\sigma^2)}{n} - \phi^2}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2},\ F_-^1 = \frac{\sigma^2 \cdot \frac{\phi}{n}}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2}.$$

Moreover, trader $i$'s price impact in market $k$ is

$$\lambda^1 \equiv \frac{dp_k}{dx_k^i} = \frac{1}{(n-1)B^1(1-(\frac{b^1}{B^1})^2)}. \tag{3}$$

The above strategy profile is an equilibrium if and only if $a^1 > 0$. A simple sufficient condition for $a^1 > 0$ is $(1-\rho)(n-2) > \sigma^2$. Furthermore, we have the following result regarding the equilibrium coefficients.

**Proposition 3.1.** *If $\alpha_I = \alpha_{II} = 1$ and $(1-\rho)(n-2) > \sigma^2$, the equilibrium is given by (2) and the parameters satisfy:*

*(i) $b^1 > 0$ if and only if $\phi > 0$;*
*(ii) $0 < a^1 < \frac{F_s^1}{\gamma}\frac{n-2}{n-1}$ and $0 < |b^1| < B^1 < \frac{1}{\gamma}\frac{n-2}{n-1}$;*
*(iii) $a^1$, $B^1$, and $|b^1|$ are increasing in $|\phi|$.*

3.2. **No Spillover** $(\alpha_I = \alpha_{II} = 0)$**.** Now suppose no trader can condition her demand on the price of the other asset, thus there is no information spillover and the market-clearing prices are determined separately. The equilibrium in market $k \in \{I, II\}$ is given by

$$x_k^i(p_k, s_k^i) = a^0 s_k^i - B^0 p_k, \tag{4}$$

where

$$a^0 = \frac{(n-2)F_s^0 - F^0}{\gamma(n-1)}, \quad B^0 = \frac{a^0}{F_s^0 + F^0}, \tag{5}$$

$$F_s^0 = \frac{1-\rho}{1-\rho+\sigma^2}, \quad F^0 = \frac{\sigma^2}{1-\rho+\sigma^2} \cdot \frac{\rho}{\rho + \frac{1-\rho+\sigma^2}{n}}. \tag{6}$$

Moreover, trader $i$'s price impact in market $k$ is

$$\lambda^0 \equiv \frac{dp_k}{dx_k^i} = \frac{1}{(n-1)B^0}. \tag{7}$$

The above strategy profile is an equilibrium if and only if $a^0 > 0$. Again, a sufficient condition is $(1-\rho)(n-2) > \sigma^2$. The result is summarized below.

**Proposition 3.2.** *If $\alpha_I = \alpha_{II} = 0$ and $(1-\rho)(n-2) > \sigma^2$, the equilibrium is given by (5) and the parameters satisfy:*

*(i) $0 < a^0 < \frac{F_s^0}{\gamma}\frac{n-2}{n-1}$ and $0 < B^0 < \frac{1}{\gamma}\frac{n-2}{n-1}$;*
*(ii) $a^0$ and $B^0$ are independent of $|\phi|$.*

3.3. **Comparison.** Now we compare the two benchmarks. First, Proposition 3.3 compares traders' demand schedules.

**Proposition 3.3.** *The more informed traders' demand schedules, compared to those of the less informed, are more sensitive to their own price, i.e., $B^1 > B^0$ and more sensitive to their own signals, i.e., $a^1 > a^0$. The price impact under information spillover is lower than that under no spillover: $\lambda^1 < \lambda^0$.*

Intuitively, a more informed trader's inference is less sensitive to the own price, since the price of the other market provides an additional signal for the more informed trader and hence crowds out the informativeness of the own price. As a result, with a lower own price, the more informed trader is less pessimistic about the quality of the asset and thus demands more than a less informed trader. Therefore, the equilibrium market demand under information spillover is more responsive to price ($B^1 > B^0$). Consequently, the equilibrium price is less sensitive to a change of trade, i.e., ($\lambda^1 < \lambda^0$). As an immediate implication, more informed traders are more willing to rely on their own signals ($a^1 > a^0$), since they have smaller price impacts.

Next, we compare traders' equilibrium payoffs. Denote by $W_k^1$ (resp., $W_k^0$) a trader's expected equilibrium payoff in the benchmark with (resp., without) information spillover. Proposition 3.4 establishes that a trader's expected payoff is strictly decreasing in her price impact, and more informed traders' payoffs are higher since information spillover lowers traders' price impacts.

**Proposition 3.4.** *Traders' expected surpluses in the benchmarks are given by*

$$W_k^1 = \frac{\frac{\gamma}{2} + \lambda^1}{(\gamma + \lambda^1)^2}(F_s^1)^2 \mathbb{E}(s_k^i - \bar{s}_k)^2 \quad \textit{and} \quad W_k^0 = \frac{\frac{\gamma}{2} + \lambda^0}{(\gamma + \lambda^0)^2}(F_s^0)^2 \mathbb{E}(s_k^i - \bar{s}_k)^2,$$

*where $W_k^1 > W_k^0$.*

## 4. The General Case

This section establishes the existence and characterization of equilibria in the general setting in which there can be both more and less informed traders in each market. We first characterize both types of traders' equilibrium strategy and inference, and then apply Brouwer's fixed point theorem to prove existence.

For each $k \in \{I, II\}$, we conjecture a linear demand schedule for any less informed trader $i' \in \mathcal{N}_k^0$ in market $k$ as

$$x_{k,i}^0(p_k, s_k^i) = a_k^0 s_k^i - B_k^0 p_k,$$

and another linear demand schedule for any more informed trader $i \in \mathcal{N}_k^1$ in market $k$ as

$$x_{k,i}^1(p_k, p_{-k}, s_k^i) = a_k^1 s_k^i - B_k^1 p_k + b_k^1 p_{-k},$$

where $a_k^0$, $B_k^0$, $a_k^1$, $B_k^1$, and $b_k^1$ are constants. Recall that $\alpha_k = n_k^1/n_k$ is the fraction of more informed traders in market $k$. Let $B_k = \alpha_k B_k^1 + (1-\alpha_k)B_k^0$ and $b_k = \alpha_k b_k^1$ be the average sensitivities of demand in market $k$ to $p_k$ and $p_{-k}$, respectively. Lemma 4.1 characterizes the equilibrium demand schedules.

**Lemma 4.1.** *The equilibrium demand schedule of a more informed trader $i \in \mathcal{N}_k^1$ is*

$$x_{k,i}^1(p_k, p_{-k}, s_k^i) = \frac{1}{\gamma + \lambda_k^1}\left(\mathbb{E}\left[\theta_k^i | s_k^i, p_k, p_{-k}\right] - p_k\right), \tag{8}$$

*where*

$$\lambda_k^1 = \left((\alpha_k n_k - 1)(B_k^1 - b_k^1 \frac{b_{-k}}{B_{-k}}) + (1-\alpha_k) n_k B_k^0\right)^{-1}.$$

*The equilibrium demand schedule of a less informed trader $i \in \mathcal{N}_k^0$ is*

$$x_{k,i}^0(p_k, s_k^i) = \frac{1}{\gamma + \lambda_k^0}\left(\mathbb{E}\left[\theta_k^i | s_k^i, p_k\right] - p_k\right), \tag{9}$$

*where*

$$\lambda_k^0 = \left(\alpha_k n_k (B_k^1 - b_k^1 \frac{b_{-k}}{B_{-k}}) + ((1-\alpha_k) n_k - 1) B_k^0\right)^{-1}.$$

Lemma 4.1 shows that a trader's demand is increasing in the conditional expectation of her value, and is decreasing in the price impact $\lambda_k^1$ (or $\lambda_k^0$). The next Lemma (Lemma 4.2) characterizes the market-clearing prices in terms of traders' signals, given the conjectured strategies of all traders. It establishes that the price $p_k$ is a linear combination of the average signals of traders in different subgroups; as a result, the price vector $(p_I, p_{II})$ is also jointly normally distributed.

**Lemma 4.2.** *The market clearing price of asset $k \in \{I, II\}$ is*

$$p_k = D_k^1 \bar{s}_k^1 + D_k^0 \bar{s}_k^0 + d_k^1 \bar{s}_{-k}^1 + d_k^0 \bar{s}_{-k}^0, \tag{10}$$

*where $\bar{s}_k^1 = \sum_{i \in \mathcal{N}_k^1} s_k^i / n_k^1$, $\bar{s}_k^0 = \sum_{i' \in \mathcal{N}_k^0} s_k^{i'} / n_k^0$, $k, -k \in \{I, II\}, k \neq -k$, and*

$$D_k^1 = \frac{B_{-k}\alpha_k a_k^1}{B_k B_{-k} - b_k b_{-k}},\; d_k^1 = \frac{b_k \alpha_{-k} a_{-k}^1}{B_k B_{-k} - b_k b_{-k}}, D_k^0 = \frac{B_{-k}(1-\alpha_k) a_k^0}{B_k B_{-k} - b_k b_{-k}},\; d_k^0 = \frac{b_k (1-\alpha_{-k}) a_{-k}^0}{B_k B_{-k} - b_k b_{-k}}.$$

Next we examine traders' conditional expectations of their values. Because of trader heterogeneity, there are two types of learning: "cross assets" and "cross subgroups." Unlike the benchmark cases, traders' conditional expectations and equilibrium strategies do not have closed-form solutions due to the asymmetry. To facilitate the economic interpretations of the characterization and to establish equilibrium existence, we introduce the following parameters that capture the impact of signals on market-clearing prices:

- The price impact of own-asset signals, $\zeta_k$:

$$\zeta_k \equiv D_k^1 + D_k^0. \tag{11}$$

- The price impact of cross-asset signals relative to own-asset signals, $\delta_k$:

$$\delta_k \equiv \frac{d_k^1 + d_k^0}{D_k^1 + D_k^0}. \tag{12}$$

- The price impact of cross-subgroup signals within a market, $\pi_k$:

$$\pi_k \equiv \frac{D_k^1}{D_k^1 + D_k^0} = \frac{\alpha_k a_k^1}{\alpha_k a_k^1 + (1-\alpha_k) a_k^0}. \tag{13}$$

The next result (Lemma 4.3) expresses traders' conditional expectations as functions of the parameters $(\zeta_k,\ \delta_k, \pi_k)_{k\in\{I,II\}}$. Besides the associated economic interpretations, these parameters are technically convenient to handle, allowing us to establish tight bounds for the fixed point mapping in the existence proof.

**Lemma 4.3.** *Given the conjectured equilibrium strategies, the conditional expectation of the value of a more informed trader $i \in \mathcal{N}_k^1$ is*

$$\mathbb{E}\left[\theta_k^i | s_k^i, p_k, p_{-k}\right] = C_{ks}^1 s_k^i + C_k^1 p_k + c_k^1 p_{-k}, \tag{14}$$

*and the conditional expectation of the value of a less informed trader $i \in \mathcal{N}_k^0$ is*

$$\mathbb{E}\left[\theta_k^i | s_k^i, p_k\right] = C_{ks}^0 s_k^i + C_k^0 p_k, \tag{15}$$

*where $(C_{ks}^1, C_k^1, c_k^1, C_{ks}^0, C_k^0)_{k\in\{I,II\}}$ depends on $(\delta_k, \zeta_k, \pi_k, \sigma_k^2, \rho_k, n_k, \phi)_{k\in\{I,II\}}$. The exact form is given in equation (22) and (23) in Appendix A.*

Now we state the first main results of the paper (Theorem 4.4) that establishes the existence of a linear equilibrium and demonstrate its properties.

**Theorem 4.4.** *There exists $\bar{\sigma}_k^2 \in (0, (1-\rho_k)(n_k-2))$ such that if $\sigma_k^2 < \bar{\sigma}_k^2$ for $k \in \{I, II\}$,*[9] *there exists a linear Bayes Nash equilibrium such that*

*(1) $a_k^1 > 0$, $B_k^1 > 0$, $|b_k^1| > 0$, $a_k^0 > 0$ and $B_k^0 > 0$.*
*(2) $C_{ks}^1$, $C_{ks}^0 \in (0,1)$, $C_k^1, C_k^0 \in (0, \frac{n_k-2}{n_k-1})$.*
*(3) $b_k^1 > 0$ and $c_k^1 > 0$ if and only if $\phi > 0$.*

[9]The condition $\sigma_k^2 < \bar{\sigma}_k^2$ holds if $n_k$ is large enough or $\sigma_k^2$ is small enough.

Here we provide a sketch of proof for Theorem 4.4. First, we define two parameters to capture the heterogeneity between more and less informed traders:

$$\pi_k = \frac{\alpha_k a_k^1}{\alpha_k a_k^1 + (1-\alpha_k) a_k^0}, \; \Pi_k = \frac{\alpha_k B_k^1}{\alpha_k B_k^1 + (1-\alpha_k) B_k^0}. \tag{16}$$

Then we construct a fixed point mapping as follows:

Step 1: The input of the mapping is $\pi_k \in [0,1]$ and $\Pi_k \in [\underline{\Pi}_k, 1]$ for $k \in \{I, II\}$.[10]

Step 2: Fix any $(\pi_k, \Pi_k)_{k\in\{I,II\}}$, we solve for the unique $(C_{ks}^1, C_k^1, c_k^1, C_{ks}^0, C_k^0)_{k\in\{I,II\}}$, as well as $(\delta_k, \zeta_k)_{k\in\{I,II\}}$.

Step 3: Given $(C_{ks}^1, C_k^1, c_k^1, C_{ks}^0, C_k^0)_{k\in\{I,II\}}$, we get unique $(a_k^1, B_k^1, a_k^0, B_k^0)_{k\in\{I,II\}}$, which are all positive numbers, and also $(b_I^1, b_{II}^1)$.

Step 4: Given $(a_k^1, B_k^1, a_k^0, B_k^0)_{k\in\{I,II\}}$, we obtain the output of the mapping $\pi_k \in [0,1]$ and $\Pi_k \in [\underline{\Pi}_k, 1]$ for $k \in \{I, II\}$.

Along the proof of Theorem 4.4, we also identify the following properties regarding the inference parameters and equilibrium strategies. First, both types of traders respond positively to private signals ($a_k^1 > 0$ and $a_k^0 > 0$) and negatively to the own price ($B_k^1 > 0$ and $B_k^0 > 0$). This is guaranteed by the assumption that the noise of the private signal $\sigma_k^2$ is small enough or there is a large number of trader $n_k$. Intuitively, if a trader's signal is very informative about her value, her demand would mostly rely on her signal as opposed to the prices. If instead it relies more on the information content of prices, then a low price, for example, would imply that the asset is less valuable and thus the trader would lower her demand, violating optimality. Likewise, if there are more traders, the price would be less sensitive to each individual trader's demand, thus the trader is more willing to submit a larger demand when the price is lower. Second, more informed traders react to the price from the other market positively ($b_k^1 > 0$ and $c_k^1 > 0$) if and only if the two assets are positively correlated ($\phi > 0$). Under positive (negative) correlation, a higher price in other market serves a good (bad) news about the value of the asset in a trader's own market. Finally, the sensitivity of the own price for both types of traders is less than $\frac{n_k-2}{n_k-1}$ to prevent the informativeness of the own price from being so precise to crowd out that of private signals and make traders completely abandon their private signals.

4.1. **Numerical Exercises.** For fixed market sizes, the inference problems depend on the composition of traders in both markets, which complicates the characterizations of traders' best replies and thus the equilibrium demand schedules. Here we provide several numerical examples on the impact of information spillover $\alpha_k$. From the mapping constructed

[10] Define $\underline{\Pi}_k \equiv \frac{\alpha_k}{\alpha_k + (1-\alpha_k)(n_k-1)} \in [0,1]$.

in Theorem 4.4, we apply the fixed point iteration to get a numerical solution of $(\pi_k, \Pi_k)$ and hence the equilibrium.[11]

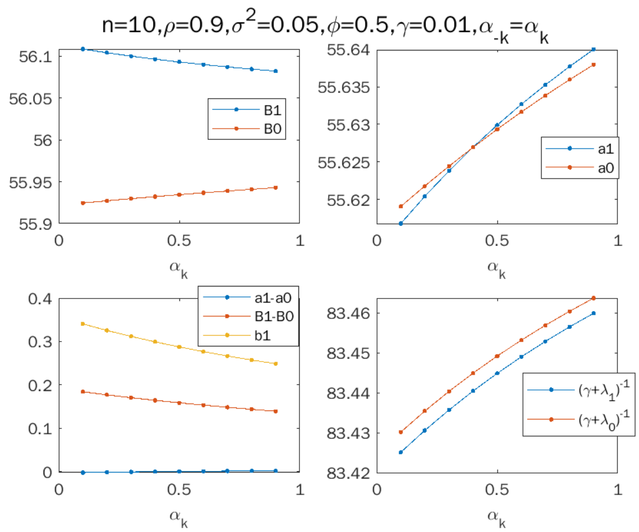


FIGURE 1. Equilibrium Outcome: $n = 10$

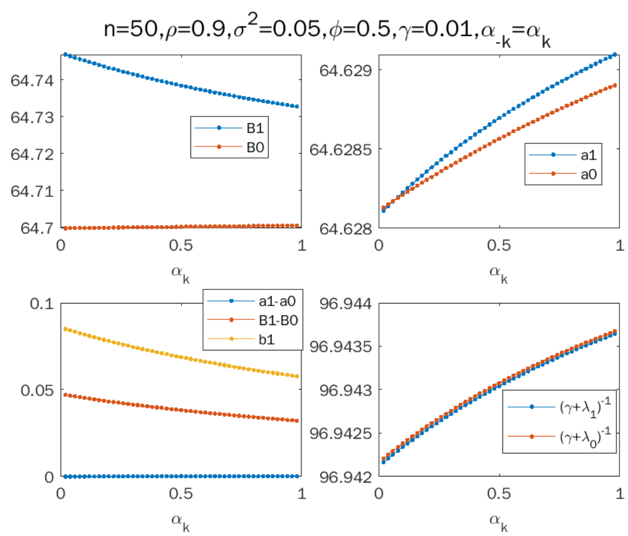


FIGURE 2. Equilibrium Outcome: $n = 50$

First, the more or less informed traders react differently to their own prices $(B_k^1 - B_k^0 \neq 0)$, and their reactions to their own signals also differ but only mildly $(a_k^1 - a_k^0 \neq 0)$. Different from the benchmarks, it is *not* always the case that more informed traders trade more

[11]For simplicity, we focus on the symmetric case where $\alpha_I = \alpha_{II}$ and $n_I = n_{II} \equiv n$, where $n = 10, 50$. The inputs of the fixed point iteration are $\pi = \pi_I = \pi_{II}$ and $\Pi \equiv \Pi_I = \Pi_{II}$. It turns out that $\Pi$ is the unique positive root of a cubic polynomial, hence we can further simplify the input to be $\pi$, which is one-dimensional and significantly simplifies the numerical analysis. See the details in the online appendix Section E.2.

aggressively, i.e., $a_k^1 > a_k^0$ (see Figure 1),[12] while it is still true that more informed traders are more sensitive to prices ($B_k^1 > B_k^0$). When $n_k$ is small, the less informed may trade more aggressively due to a free-riding effect. Since more informed traders are more sensitive to prices, a larger fraction of more informed traders lowers the price impact, which benefits all traders in the market. Compared to the more informed, a less informed trader faces one extra more informed in the market, so she could benefit more from the smaller price impact of her residual demand. However, when the market size is large, the impact of a single trader becomes insignificant; consequently, the informational advantage of a more informed trader dominates: $a_k^1 > a_k^0$ (see Figure 2 and Proposition 5.2).

Second, information spillover ($\alpha_k$) has monotonic impacts on the equilibrium beliefs and strategies. If the fraction of more informed traders increases (a higher $\alpha_k$), then market $k$ becomes more competitive (lower price impacts $\lambda_k^1$ and $\lambda_k^0$); both types of traders trade more aggressively (higher $a_k^1$ and $a_k^0$); and more informed traders are less responsive to both prices, relative to less informed traders (lower $B_k^1 - B_k^0$ and $b_k^1$).

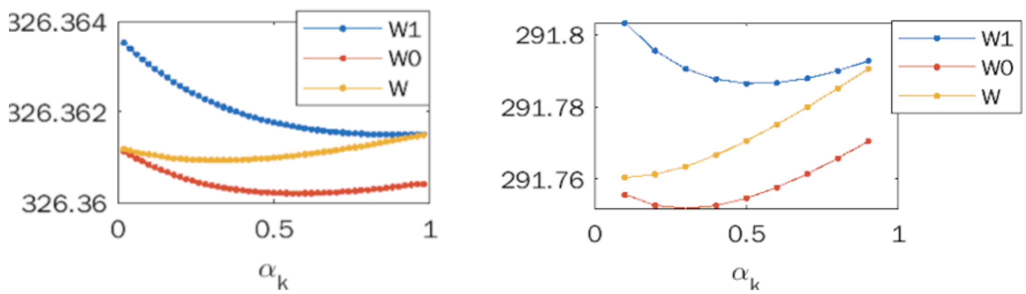


FIGURE 3. Welfare: $n = 50$ and $n = 10$

Finally, the impacts of information spillover ($\alpha_k$) on welfare are non-monotonic (See Figure 3). To be specific, when the number of traders are large enough (see the left figure $n = 50$ in Figure 3), the welfare of more informed traders ($W_k^1$) is decreasing in $\alpha_k$, while the welfare of less informed traders ($W_k^0$) and the aggregate welfare $W_k \equiv \alpha_k W_k^1 + (1 - \alpha_k) W_k^0$ are U-shaped in $\alpha_k$. Note that the welfare in the benchmarks corresponds to the two extreme points in the interval $\alpha_k \in [0, 1]$; thus, the asymmetry between the two types of traders plays has important welfare implications in the general (i.e., interior) case.

## 5. ANALYSIS OF LARGE MARKETS

To further examine the equilibrium properties, this section considers settings in which the numbers of traders at both markets grow large, which allow us to disentangle the

[12]Since we study the symmetric case in the numerical exercises, we drop the dependence of the parameters on $k$ for notational simplicity in Figures 1–3.

information channel of price spillover from its strategic impact on traders' inference, behavior, and welfare. We focus on the generic case in which $\rho_k > |\phi|$ for each $k$, that is, traders' values are more correlated within a market than across markets. Let $N = n_I + n_{II}$ be the total number of traders. Denote by $\chi_k = n_k/N \in (0,1)$ the proportion of traders in market $k \in \{I, II\}$. Recall that $\alpha_k \in (0,1)$ is the fraction of more informed traders in market $k$. We will take $N$ to infinity while holding both $(\chi_k)$ and $(\alpha_k)$ fixed. That is, we study equilibria in large markets keeping the relative sizes of different subgroups the same. In particular, we examine the inference and demand parameters in the orders of $1/N$ and $1/N^2$, which capture most of the direct and indirect spillover effects when $N$ is large.

5.1. **Comparison between two types of traders.** We first compare traders' inferences and strategies in large markets. In particular, the unique limiting equilibrium is symmetric and independent of the fraction of the more informed. This "asymptotic symmetry" lays out the foundation toward the analysis of how information spillover and the composition of traders influence the equilibrium in later sections.

Proposition 5.1 characterizes and compares traders' inferences. Notably, the infinite-market limit as well as the first and second order effects are all unique.

**Proposition 5.1.** *Traders' inference parameters satisfy the following:*

*(1)* $|c_k^1| = \frac{c_k^*}{n_k} + o(\frac{1}{N})$, $|\delta_k| = \frac{\delta_k^*}{n_k} + o(\frac{1}{N})$, $C_k^0 - C_k^1 = \frac{\Delta_k}{n_k} + o(\frac{1}{N})$, *and* $C_{ks}^1 - C_{ks}^0 = \frac{\Delta_{ks}}{n_k^2} + o(\frac{1}{N^2})$, *where* $c_k^* = \frac{\sigma_{\theta_k}(1-\rho_k)|\phi|}{\sigma_{\theta_{-k}}(\rho_k\rho_{-k}-\phi^2)} \frac{1}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}}$.

*(2)* $c_k^* > 0$, $\delta_k^* > 0$, $\Delta_k > 0$, *and* $\Delta_{ks} > 0$.[13]

*(3)* $\lim_{N\to\infty} c_k^1 = \lim_{N\to\infty} \delta_k = 0$, $\lim_{N\to\infty} C_k^1 = \lim_{N\to\infty} C_k^0 = 1 - C_k^*$, *and* $\lim_{N\to\infty} C_{ks}^1 = \lim_{N\to\infty} C_{ks}^0 = C_k^*$, *where* $C_k^* = \frac{1-\rho_k}{1-\rho_k+\sigma_k^2}$.

First, a more informed trader's inference depends on the price in the other market, $|c_k^1| > 0$ when the two assets are correlated ($\phi \neq 0$), as she directly takes into account this information externality. Since $p_k$ is predominantly determined by the average signal $\bar{s}_k$ in market $k$, as the number of traders $n_k$ grows large, $\bar{s}_k$ is almost a perfect signal of her value, i.e., $\frac{\mathrm{Cov}(\theta_k^i, \bar{s}_k)}{\mathrm{Var}(\bar{s}_k)}$ is close to 1. Therefore, the residual explanatory power of $\bar{s}_{-k}$ converges to zero and is proportional to $\frac{1}{n_k}$, i.e., $1 - \frac{\mathrm{Cov}(\theta_k^i, \bar{s}_k)}{\mathrm{Var}(\bar{s}_k)} = \frac{\sigma_{\varepsilon_k}^2}{\mathrm{Var}(\bar{s}_k)} \frac{1}{n_k}$.[14] Furthermore, the cross-asset effect is nontrivial, $|\delta_k| > 0$, which is a direct consequence of information externality.

Second, the less informed are more sensitive to the own price than the more informed: $C_k^0 > C_k^1$. Intuitively, the information disadvantage of the less informed makes them put

[13] All these parameters are solved in closed-form in the proof, see equations (32), (33), (34) and (37).

[14] Note that $\mathrm{Cov}(\theta_k^i, \bar{s}_k) = (\rho_k + \frac{1-\rho_k}{n_k})\sigma_{\theta_k}^2$ and $\mathrm{Var}(\bar{s}_k) = (\rho_k + \frac{1-\rho_k+\sigma_k^2}{n_k})\sigma_{\theta_k}^2$.

more weight on the own price than the more informed. Since this is also a direct implication of information externality, $C_k^0 - C_k^1$ is proportional to $\frac{1}{n_k}$.

Third, the less informed are less sensitive to their own signals than the more informed ($C_{ks}^0 < C_{ks}^1$). Since the less informed put more weight on the own price ($C_k^0 > C_k^1$), which contains information about the other market ($|\delta_k| > 0$), it crowds out their reliance on their own signals than the more informed. Since the difference $C_{ks}^1 - C_{ks}^0$ is jointly determined by both $C_k^0 - C_k^1$ and $|\delta_k|$, it is proportional to $\frac{1}{n_k^2}$.

Finally, one crucial property to establish these approximations is that the higher-order terms in Proposition 5.1 all come from the difference between traders' reactions to their own signals, $a_k^1 - a_k^0$, which has an order higher than $1/N$, as we establish in the next proposition.

**Proposition 5.2.** *Traders' equilibrium strategies satisfy the following:*

(1) $a_k^1 - a_k^0 = \frac{a_k^*}{n_k^2} + o(\frac{1}{N^2})$, $B_k^1 - B_k^0 = \frac{B_k^*}{n_k} + o(\frac{1}{N})$, *and* $|b_k^1| = \frac{b_k^*}{n_k} + o(\frac{1}{N})$.

(2) $a_k^* = \frac{1}{\gamma}\Delta_{ks} > 0$, $B_k^* = \frac{1}{\gamma}\Delta_k > 0$, *and* $b_k^* = \frac{1}{\gamma}c_k^* > 0$.

(3) $\lambda_k^1 = \frac{1}{(n_k-1)B_k} + o(\frac{1}{N^2})$ *and* $\lambda_k^0 = \frac{1}{(n_k-1)B_k} + o(\frac{1}{N^2})$.

(4) $\lim_{N\to\infty} a_k^1 = \lim_{N\to\infty} a_k^0 = \frac{C_k^*}{\gamma}$, $\lim_{N\to\infty} B_k^1 = \lim_{N\to\infty} B_k^0 = \frac{C_k^*}{\gamma}$, $\lim_{N\to\infty} b_k^1 = 0$, *and* $\lim_{N\to\infty} \lambda_k^1 = \lim_{N\to\infty} \lambda_k^0 = 0$.

Proposition 5.2 characterizes traders' equilibrium strategies. In particular, the price impacts for both types of traders are asymptotically the same up to second-order approximations. This is because price impacts are determined by the price elasticity of demand $B_k \equiv \alpha_k B_k^1 + (1-\alpha_k)B_k^0$, so if the market size is large, each trader faces approximately the same set of market participants. Consequently, the behavior differences between the two types of traders are completely due to their information differences. Formally, a more informed trader's demand is more sensitive to the own signal ($a_k^1 > a_k^0$), since her belief is more sensitive to the own signal than the less informed ($C_{ks}^1 > C_{ks}^0$); a more informed trader's demand is also more sensitive to the own price ($B_k^1 > B_k^0$), since her conditional expectation about her value is less sensitive to the own price ($C_k^1 < C_k^0$): with a lower price, she is less pessimistic about the asset value and is willing to buy more assets than the less informed.

5.2. **Impacts of information spillover.** Next we study how information spillover, parameterized by the fraction $\alpha_k$, affects the inference and behavior of traders in market $k$.[15] For inference, the first step is to analyze the key parameters $\delta_k$ and $\zeta_k$, representing cross-asset

[15] The online appendix Section D.1 studies how $\alpha_k$ affects traders in market $-k$.

and own-asset effects, respectively. Let $\bar{s}_k$ be the average signal of all traders in market $k \in \{I, II\}$. From Lemma 4.2, we get

$$p_k = \zeta_k(\bar{s}_k + \delta_k \bar{s}_{-k}) + o(\frac{1}{N}), \quad \text{for } k \in \{I, II\}, \tag{17}$$

where $\zeta_k$ is the impact of $\bar{s}_k$ on $p_k$ and $\delta_k$ is the relative impact of $\bar{s}_{-k}$ on $p_k$. From Proposition 5.1, a larger $\alpha_k$ implies that the total demand in market $k$ is more sensitive to $p_{-k}$, since the fraction of more informed traders reacting to $p_{-k}$ increases. Consequently, the equilibrium price $p_k$ is more sensitive to $\bar{s}_{-k}$ (larger $|\delta_k|$). That is, $p_k$ is less informative of $\bar{s}_{-k}$. Furthermore, since the more informed's inference is less sensitive to $p_k$ than that of the less informed, a larger $\alpha_k$ implies that the total demand in market $k$ is less sensitive to $p_k$, which is predominantly determined by $\bar{s}_k$. As a result, the equilibrium price $p_k$ is less sensitive to $\bar{s}_k$ (lower $\zeta_k$). That is, $p_k$ is more informative of $\bar{s}_k$. Lemma 5.3 below formally establishes these relationships.

**Lemma 5.3.** *If we ignore the terms of order $o(\frac{1}{N})$, then*
- *$|\delta_k|$ and $|\delta_k \zeta_k|$ are increasing in $\alpha_k$.*
- *$\zeta_k$ is decreasing in $\alpha_k$.*

Applying Lemma 5.3, we establish the comparative statics of the inference parameters in Proposition 5.4.

**Proposition 5.4.** *The impacts of information spillover on inference are as follow:*

*(1) First-order effect: if we ignore the terms of order $o(\frac{1}{N})$, then*
- *$C_k^1$ is increasing in $\alpha_k$, and $|c_k^1|$ is decreasing in $\alpha_k$.*
- *$C_k^0$ is independent of $\alpha_k$;*

*(2) Second-order effect: if we ignore the terms of order $o(\frac{1}{N^2})$, then*
- *$C_{ks}^1$ is independent of $\alpha_k$;*
- *$C_{ks}^0$ is decreasing in $\alpha_k$ if and only if $\alpha_k < \frac{1-\rho_k}{\sigma_k^2}$.*

According to Proposition 5.4, information spillover has a first-order effect on the more informed traders' inference from prices, but has no first-order effect on that of the less informed. For the more informed, conditioning on observing $p_{-k}$, the predicting power of $p_k$ on $\bar{s}_{-k}$ is of second-order effect, thus we only need to focus on the direct inference of $\bar{s}_k$ from $p_k$. A larger $\alpha_k$ has two effects: (i) a smaller $\zeta_k$, implying that $p_k$ is more informative about $\bar{s}_k$ (a larger $C_k^1$); (ii) a larger $|\delta_k|$, implying that $p_k$ contains more information about of $\bar{s}_{-k}$, which crowds out the informativeness of $p_{-k}$ in predicting $\bar{s}_{-k}$ (a smaller $|c_k^1|$). For the less informed, the price $p_k$ contains two sources of information: $\bar{s}_k$ and $\bar{s}_{-k}$. Recall that Lemma 5.3 shows that with a larger fraction of more informed traders, $p_k$ is more informative about $\bar{s}_k$ and less informative about $\bar{s}_{-k}$. It turns out that their first-order effects exactly cancel out so that the informativeness of $p_k$ remains unchanged.[16]

Proposition 5.4 also establishes that information spillover has a second-order effect on traders' inference from their own signals. For the more informed, $C^1_{ks}$ is independent of $\alpha_k$. This is because they can almost perfectly infer $\bar{s}_k$ and $\bar{s}_{-k}$ from $p_k$ and $p_{-k}$, independent of the fraction of more informed (see equation (17)). Consequently, changes in information spillover do not affect the informativeness of prices nor the informativeness of own signals for the more informed. For the less informed, $C^0_{ks}$ is decreasing in $\alpha_k$ if and only if $\alpha_k < \frac{1-\rho_k}{\sigma_k^2}$.[17] This non-monotonicity follows from two opposing effects. On the one hand, the correlation between a trader's value and the price is stronger when the fraction of more informed traders increases in the market. As a result, the less informed rely more on the price instead of their signals to predict their values (smaller $C^0_{ks}$). On the other hand, the market price becomes noisier and thus the correlation between a trader's signal and the price is weaker, which implies that the signal contains more information (larger $C^0_{ks}$). In total, these two effects generate the non-monotonity in Proposition 5.4. Moreover, when the fraction of the more informed is small, the former dominates the latter, thus $C^0_{ks}$ is decreasing in $\alpha_k$ for small $\alpha_k$.[18]

Next, we study how information spillover affects traders' equilibrium strategies. The first step (Lemma 5.5) is to analyze the price impact: $\lambda^1_k$ and $\lambda^0_k$. Recall that $B_k \equiv \alpha_k B^1_k + (1-\alpha_k)B^0_k$ is the average sensitivity of demand in market $k$ to the price $p_k$.

**Lemma 5.5.** *The following results hold:*

(1) *First-order effect: if we ignore the terms of order $o(\frac{1}{N})$, then $B_k$ is increasing in $\alpha_k$; $\lambda^1_k$ and $\lambda^0_k$ are independent of $\alpha_k$.*
(2) *Second-order effect: if we ignore the terms of order $o(\frac{1}{N^2})$, then $\lambda^1_k = \lambda^0_k = \frac{1}{(n_k-1)B_k}$ is decreasing in $\alpha_k$.*

Lemma 5.5 states that the price elasticity of the market demand ($B_k$) is increasing in $\alpha_k$. Intuitively, the demand of a more informed trader is more responsive to price, relative to a less informed trader: $B^1_k > B^0_k$ (see Proposition 5.2). Thus, if the fraction of more informed traders increases, the market demand becomes more elastic. Consequently, the price impacts $\lambda^1_k$ and $\lambda^0_k$ are smaller, where the changes are in the second order.

Proposition 5.6 then establishes the comparative statics of the equilibrium.

**Proposition 5.6.** *The impacts of information spillover are the following:*

[16] It is possible that $\alpha_k$ has a second-order effect on $C^0_k$.

[17] If $\frac{1-\rho_k}{\sigma_k^2} > 1$, then $C^0_{ks}$ is decreasing in $\alpha_k$ for all $\alpha_k \in [0,1]$.

[18] See the online appendix Section E.1 for more detailed analysis of this non-monotonicity.

(1) *First-order effect: if we ignore the terms of order $o(\frac{1}{N})$, then $B_k^1$ and $|b_k^1|$ are decreasing in $\alpha_k$; $B_k^0$ is independent of $\alpha_k$.*
(2) *Second-order effect: if we ignore the terms of order $o(\frac{1}{N^2})$, then $a_k^1$ and $a_k^0$ are increasing in $\alpha_k$.*

The first part of Proposition 5.6 follows from the fact that information spillover has a second-order effect on price impacts (Lemma 5.5) and a first-order effect on inference (Proposition 5.4). For the second part, note that even though the inference from signals $C_{ks}^1$ is independent of $\alpha_k$, the price impact of the more informed is smaller when the fraction of more informed traders increases, thus they trade more aggressively with a higher signal, i.e., $a_k^1$ is increasing in $\alpha_k$. For the less informed, their price impact is smaller, yet their inference from signals $C_{ks}^0$ is non-monotone in $\alpha_k$. Overall, the former dominates the latter and hence the less informed trade more aggressively, i.e., $a_k^0$ is increasing in $\alpha_k$.

5.3. **Welfare Analysis.** This section presents the results on traders' welfare. Denote the expected surplus of a more (resp. less) informed trader in market $k$ by $W_k^1$ (resp., $W_k^0$). Recall that $\bar{s}_k$ is the average signal of all traders in market $k$.

Lemma 5.7 characterizes $W_k^1$ and $W_k^0$. Importantly, it decomposes a more (resp. less) informed trader's welfare into three parts: (1) an own signal effect: $\mathbb{E}[C_{ks}^1(s_k^i - \bar{s}_k)]^2$ (resp., $\mathbb{E}[C_{ks}^0(s_k^i - \bar{s}_k)]^2$), respectively; (2) a heterogeneous beliefs effect: $(1-\alpha_k)^2\mathbb{E}[(C_k^1 - C_k^0)\bar{s}_k + c_k^1\bar{s}_{-k}]^2$ (resp., $\alpha_k^2\mathbb{E}[(C_k^1 - C_k^0)\bar{s}_k + c_k^1\bar{s}_{-k}]^2$); and (3) a strategic effect related to the price impacts.

**Lemma 5.7.** *Ignoring the terms of order $o(\frac{1}{N^2})$, we have*

(1) $W_k^1 = \frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2}\left[\mathbb{E}[C_{ks}^1(s_k^i - \bar{s}_k)]^2 + (1-\alpha_k)^2\mathbb{E}[(C_k^1 - C_k^0)\bar{s}_k + c_k^1\bar{s}_{-k}]^2\right]$.
(2) $W_k^0 = \frac{\frac{\gamma}{2}+\lambda_k^0}{(\gamma+\lambda_k^0)^2}\left[\mathbb{E}[C_{ks}^0(s_k^i - \bar{s}_k)]^2 + \alpha_k^2\mathbb{E}[(C_k^1 - C_k^0)\bar{s}_k + c_k^1\bar{s}_{-k}]^2\right]$.

*Furthermore, the strategic effects satisfy $\frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2} = \frac{1}{2\gamma}(1 - (\frac{1}{(n_k-1)C_k^*})^2) + o(\frac{1}{N^2}) = \frac{\frac{\gamma}{2}+\lambda_k^0}{(\gamma+\lambda_k^0)^2}$, which is independent of $\alpha_k$.*

Notably, information spillover does not affect welfare through the strategic effect up to the second order. That is, the welfare impact of information spillover can be almost completely attributed to the information effects. To see this, let us consider the more informed. The same reasoning applies to the less informed. Recall that a larger $\alpha_k$ lowers the price impact $\lambda_k^1$, so the more informed trade more aggressively. However, the marginal benefit of trading equals its marginal cost in equilibrium so that there is no first or second order welfare change when $\alpha_k$ increases. Proposition 5.8 goes on to characterize how the own signal and heterogeneous beliefs effects and thus traders' welfare react to changes in $\alpha_k$.

**Proposition 5.8.** *If we ignore the terms of order $o(\frac{1}{N^2})$, then*

*(1) $W_k^1$ is decreasing in $\alpha_k$.*
*(2) $W_k^0$ is decreasing $\alpha_k$ if and only if $\alpha_k < \alpha_k^* \equiv \frac{1-\rho_k}{1-\rho_k+\sigma_k^2}$.*
*(3) $W_k \equiv \alpha_k W_k^1 + (1-\alpha_k)W_k^0$ is decreasing in $\alpha_k$ if and only if $\alpha_k < \hat{\alpha}_k \equiv \frac{1-\rho_k}{2(1-\rho_k)+\sigma_k^2}$.*
*(4) $W_k$ evaluated at $\alpha_k = 1$ and $\alpha_k = 0$ are the same.*
*(5) $W_k^1 - W_k^0$ is positive and decreasing in $\alpha_k$.*

Proposition 5.8 shows that for any given market composition, the more informed are always better than the less informed ($W_k^1 > W_k^0$), but the welfare advantage $W_k^1 - W_k^0$ shrinks as $\alpha_k$ increases. For the more informed, since $C_{ks}^1$ is independent of $\alpha_k$, the own signal effect is also independent of $\alpha_k$. The heterogeneous beliefs effect is decreasing in $\alpha_k$: a larger $\alpha_k$ means that there are fewer less informed from whom the more informed can take advantage. Therefore, the welfare of the more informed is decreasing in $\alpha_k$. For the less informed, the own signal effect is non-monotonic (Proposition 5.4) and the heterogeneous belief effect is increasing in $\alpha_k$. When the fraction of the more informed is small ($\alpha_k < \alpha_k^*$), the information disadvantage of the less informed dominates, so that the welfare of the uniformed is U-shaped in $\alpha_k$.

Perhaps more surprisingly, the aggregate (i.e., weighted average) welfare $W_k \equiv \alpha_k W_k^1 + (1-\alpha_k)W_k^0$ is also U-shaped. That is, more information spillover does not always improve the aggregate welfare. To understand this, we consider the aggregate own signal and heterogeneous beliefs effects, respectively. The aggregate own signal effect displays a U-shape, since a larger $\alpha_k$ implies that both the fraction of more informed traders, who are better than the less informed, increases and the less informed may get worse, and the latter dominates when the fraction of more informed is small, i.e., $\alpha_k < \hat{\alpha}_k$. The aggregate heterogeneous beliefs effect displays a reversed U-shape, as intuitively the market becomes more heterogeneous when $\alpha_k$ is in the middle. Overall, the aggregate own asset effect dominates the aggregate heterogeneous belief effect, so $W_k$ is U-shaped in $\alpha_k$.

5.4. **Information Efficiency.** In this section, we first examine the impact of information spillover on information efficiency, captured by an individual trader's uncertainty reduction, formally defined as:

$$\tau_k^1 = \mathrm{Var}(\theta_k^i) - \mathrm{Var}(\theta_k^i | s_k^i, p_k, p_{-k}),\ \tau_k^0 = \mathrm{Var}(\theta_k^i) - \mathrm{Var}(\theta_k^i | s_k^i, p_k).$$

Since a larger fraction of more informed traders $\alpha_k$ improves the informativeness of the price $p_k$, i.e., a higher correlation between the value and the price (see the online appendix Section E.1 for details of this statement), less informed traders in market $k$ get more precise estimation of their values by observing only $p_k$; whereas more informed traders, who observe both prices, can almost perfectly estimate the average signals from both markets, independent of $\alpha_k$. Thus, we have the following proposition.

**Proposition 5.9.** *If we ignore the terms of order $o(\frac{1}{N^2})$, then*

*(1)* $\tau_k^1$ *is independent of* $\alpha_k$.
*(2)* $\tau_k^0$ *is increasing in* $\alpha_k$.
*(3)* $\tau_k^1 - \tau_k^0$ *is positive and decreasing in* $\alpha_k$.

The next result (Proposition 5.10) characterizes the relationship between welfare and information efficiency. The welfare of any trader can be decomposed into two parts: information efficiency and price volatility. Hence, the gap between allocation efficiency and information efficiency comes from the volatility of prices. This gives an alternative explanation about the welfare impact of information spillover: as $\alpha_k$ increases, the welfare deterioration for both types of traders is due to the fact $p_k$ becomes more volatile (higher $\mathrm{Var}(p_k)$) and all traders are risk averse. Another immediate implication is that the advantage of the more informed $W_k^1 - W_k^0$ is completely determined by the information gain $\tau_k^1 - \tau_k^0$, since price variance has the same effect on all traders.

**Proposition 5.10.** *If we ignore the terms of order $o(\frac{1}{N^2})$, then*

*(1)* $W_k^1 = \frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2}\left(\tau_k^1 - \mathrm{Var}(p_k)\right)$ *and* $W_k^0 = \frac{\frac{\gamma}{2}+\lambda_k^0}{(\gamma+\lambda_k^0)^2}\left(\tau_k^0 - \mathrm{Var}(p_k)\right)$.
*(2)* $W_k^1 - W_k^0 = \frac{1}{2\gamma}(\tau_k^1 - \tau_k^0)$.
*(3)* $\mathrm{Var}(p_k)$ *is increasing in* $\alpha_k$.

5.5. **Price co-movement and volatility transmission.** Here we show that information spillover can account for the empirical patterns of market prices mentioned in the introduction. First, it exacerbates the price comovement across markets beyond the correlation in the fundamentals (see Proposition 5.11). Define $r_p$ as the correlation between $p_k$ and $p_{-k}$, and define $r_s$ as the correlation between $\bar{s}_k$ and $\bar{s}_{-k}$, which captures the correlation between fundamentals in two markets. Intuitively, a larger fraction of more informed traders in each market makes the market price contain more information about the other market, thereby increasing the correlation between the two prices.

**Proposition 5.11.** *If we ignore the terms of order $o(\frac{1}{N})$, then*

*(1)* $|r_p|$ *is increasing in both* $\alpha_k$ *and* $\alpha_{-k}$.

*(2)* $|r_p| \geq |r_s|$, *with equality if and only if* $\alpha_k = \alpha_{-k} = 0$.

*(3)* $r_p > 0$ *and* $r_s > 0$ *if and only if* $\phi > 0$.

Furthermore, information spillover amplifies the transmission of price volatility between the two markets (Proposition 5.12). Consider the counterfactual in which the variance of the average signal $\bar{s}_{-k}$ increases, due to higher volatility of true value $\theta^i_{-k}$ or noises $\varepsilon^i_{-k}$. Intuitively, because prices provide information about the average signals of both markets (see equation (17)), a larger $\alpha_k$ (and thus a higher $\delta_k$) implies a higher degree of linkage between two prices. Proposition 5.12 formally establishes that the change in price volatility in market $k$ relative to that in market $-k$, denoted by $T_k$, is increasing in $\alpha_k$.

**Proposition 5.12.** *If we ignore the terms of order* $o(\frac{1}{N})$, *then* $T_k$ *is increasing in* $\alpha_k$.

## 6. Discussion and Extensions

We conclude with a brief discussion of the modeling assumptions and several extensions studied in more detail in the online appendix.

**Trading in both markets.** Our model assumes that each trader only participates in one of the two markets in order to isolate the impact of information externality from prices. Section C of the online appendix contains an analysis of the symmetric case in which all traders trade in both markets and have multidimensional private information. Assuming a trader's demand for each asset can be contingent on both asset prices, we derive the unique symmetric equilibrium. Compared with the full information spillover benchmark in Section 3.1, the equilibrium features two extra incentives, in addition to the information externality from prices: cross-market (and within-market) price impacts and cost linkage of holding different assets; furthermore, traders trade more aggressively based on their signals and prices when they participate in both markets. In the more general case with both traders who trade both assets and those who only trade one asset, the above three types of incentives would confound each other due to this new trader heterogeneity, which is an interesting question that goes beyond the scope of our current analysis.

**Positive information spillover in large markets.** In our main setting, the impact of information spillover vanishes as the market size increases to infinity, which is a consequence of the assumption that a trader's residual uncertainty diminishes when the market size increases, conditional on the average signal of traders in her market (in particular, $s^i_k = \theta^i_k + \varepsilon_k$ and $\rho_k > |\phi|$). There are several ways to enrich the model to reinstall information spillover in the large market limit. We pursue one such extension with systematic risks in Section B of the online appendix.

We model systematic risk by introducing an extra market-specific normally-distributed noise $e_k$ in the signals of all traders in market $k$: $s_k^i = \theta_k^i + e_k + \varepsilon_k$, for all $i \in \mathcal{N}_k$. Even when these noises $e_I$ and $e_{II}$ are independent, in the large market limit, the average signal of traders in market $k$ will not filter out all the noise in $s_k^i$. As in the main setting, we first solve for the unique symmetric equilibrium in the benchmark where all traders are more informed and examine how the systematic noise $e_k$ affects traders' inference. In particular, it muffles the informativeness of the average signal $\bar{s}_k$, but this also means that traders rely more on information spillover (i.e., $\bar{s}_{-k}$) if $e_k$ is noisier. In the large market limit, information spillover matters for traders' inference as long as the variance of the systematic risk is positive.

In the general case with heterogeneously informed traders, the presence of systematic risk implies that the limiting equilibrium remains asymmetric, which further complicates the equilibrium characterization even in large but finite markets. To make progress, we extend the approximation method in the main setting to investigate equilibria around a double limit, first taking the market size to infinity and then taking the variance of the system risk to zero. This again allows us to characterize the comparative statics of information spillover up to second-order approximations. Furthermore, we show that the results in our main setting are robust to systematic risks.

## Appendix A. Proofs of the Results in Section 4

A.1. **Proof of Lemma 4.1.** The first-order condition of a less informed trader $i'$ is

$$\mathbb{E}\left[\theta_k^{i'} | s_k^{i'}, p_k\right] - p_k = (\gamma + \frac{dp_k}{dx_k^{i'}}) x_k^{i'}. \tag{18}$$

From the market-clearing conditions, we derive the price impact of a less informed trader $i'$ as

$$\frac{dp_k}{dx_k^{i'}} = -\left( \sum_{j' \in \mathcal{N}_k^0, j' \neq i'} \frac{\partial x_k^{j'}}{\partial p_k} + \sum_{i \in \mathcal{N}_k^1} \frac{\partial x_k^i}{\partial p_k} - \sum_{i \in \mathcal{N}_k^1} \frac{\partial x_k^i}{\partial p_{-k}} \frac{\sum_{l \in \mathcal{N}_{-k}^1} \frac{\partial x_{-k}^l}{\partial p_k}}{\sum_{l' \in \mathcal{N}_{-k}^0} \frac{\partial x_{-k}^{l'}}{\partial p_{-k}} + \sum_{l \in \mathcal{N}_{-k}^1} \frac{\partial x_{-k}^l}{\partial p_{-k}}} \right)^{-1}. \tag{19}$$

Substituting (19) into (18) and using the conjectured linear strategies, we obtain

$$x_k^{i'}(p_k, s_k^{i'}) = \frac{1}{\gamma + \frac{dp_k}{dx_k^{i'}}} \left( \mathbb{E}\left[\theta_k^{i'} | s_k^{i'}, p_k\right] - p_k \right) = \frac{1}{\gamma + \lambda_k^0} \left( \mathbb{E}\left[\theta_k^{i'} | s_k^{i'}, p_k\right] - p_k \right), \tag{20}$$

where $\lambda_k^0$ is given by (9). Similarly, we obtain the strategy of the more informed.

A.2. **Proof of Lemma 4.2.** The market-clearing conditions can be rewritten as

$$\alpha_k a_k^1 \bar{s}_k^1 + (1-\alpha_k) a_k^0 \bar{s}_k^0 = \left(\alpha_k B_k^1 + (1-\alpha_k) B_k^0\right) p_k - \alpha_k b_k^1 p_{-k} = B_k p_k - b_k p_{-k},$$

for $k, -k \in \{I, II\}$ and $k \neq -k$. Thus, we have $p_k = D_k^1 \bar{s}_k^1 + D_k^0 \bar{s}_k^0 + d_k^1 \bar{s}_{-k}^1 + d_k^0 \bar{s}_{-k}^0$, where the parameters are given in the statement of this lemma.

A.3. **Proof of Lemma 4.3.** We first define some new parameters:

$$C_k^* = \frac{1-\rho_k}{1-\rho_k+\sigma_k^2},\ \kappa_k = 1-\rho_k+\sigma_k^2,\ \eta_k = \frac{\sigma_{\theta_{-k}}}{\sigma_{\theta_k}},$$
$$e_{k1} = \frac{\pi_k}{\alpha_k}\frac{\kappa_k}{n_k},\ e_{k2} = \left(\frac{(1-\pi_k)^2}{1-\alpha_k} + \frac{\pi_k^2}{\alpha_k}\right)\frac{\kappa_k}{n_k},\ e_{k3} = \frac{1-\pi_k}{1-\alpha_k}\frac{\kappa_k}{n_k},$$
$$y_{k1} = \frac{\kappa_k e_{k2} - e_{k1}^2}{\kappa_k - e_{k1}}\frac{\phi}{\eta_k},\ \ y_{k2} = \rho_k(\rho_{-k} + e_{-k2}) - \phi^2,$$
$$y_{k3} = (\rho_k + e_{k2})(\rho_{-k} + e_{-k2}) - \phi^2 - \frac{e_{k1} - e_{k2}}{\kappa_k - e_{k1}}((\rho_k + e_{k1})(\rho_{-k} + e_{-k2}) - \phi^2). \tag{21}$$

**More informed traders:** For a more informed trader $i \in \mathcal{N}_k^1$, since $(\theta_k^i, s_k^i, p_k, p_{-k})$ is jointly normal, we have $\mathbb{E}(\theta_k^i | s_k^i, p_k, p_{-k}) = C_{ks}^1 s_k^i + C_k^1 p_k + c_k^1 p_{-k}$. Let $X = \theta_k^i$ and $Y = (s_k^i, p_k, p_{-k})$. By projection theorem, $\mathbb{E}[X|Y] = \mathbb{E}(X) + \Sigma_{X,Y}\Sigma_{Y,Y}^{-1}(Y - \mathbb{E}(Y))$, where $\Sigma_{X,Y}$ is the covariance matrix of $X$ and $Y$, and $\Sigma_{Y,Y}$ is the variance matrix of $Y$.

Define $\delta_k = \frac{d_k^1 + d_k^0}{D_k^1 + D_k^0}$, $\zeta_k = D_k^1 + D_k^0$, $\pi_k = \frac{D_k^1}{D_k^1 + D_k^0}$. Since $\frac{D_k^1}{D_k^1 + D_k^0} = \frac{d_{-k}^1}{d_{-k}^1 + d_{-k}^0}$ and by $p_k = D_k^1 \bar{s}_k^1 + D_k^0 \bar{s}_k^0 + d_k^1 \bar{s}_{-k}^1 + d_k^0 \bar{s}_{-k}^0$ (see Lemma 4.2), we have

$$p_k = \zeta_k[(\pi_k \bar{s}_k^1 + (1-\pi_k)\bar{s}_k^0) + \delta_k(\pi_{-k}\bar{s}_{-k}^1 + (1-\pi_{-k})\bar{s}_{-k}^0)].$$

In addition, we have $\Sigma_{X,Y}\Sigma_{Y,Y}^{-1} = (C_{ks}^1, C_k^1, c_k^1)$, where

$$C_{ks}^1 = C_k^* - \frac{(1-C_k^*)(e_{k1} - e_{k2})y_{k2}}{(\kappa_k - e_{k1})y_{k3}},\ C_k^1 = \frac{(1-C_k^*)(y_{k2} - y_{k1}\delta_{-k})}{(1-\delta_k\delta_{-k})y_{k3}\zeta_k},$$
$$c_k^1 = \frac{(1-C_k^*)(y_{k1} - y_{k2}\delta_k)}{(1-\delta_k\delta_{-k})y_{k3}\zeta_{-k}}. \tag{22}$$

**Less informed traders:** For a less informed trader $i \in \mathcal{N}_k^0$, since $(\theta_k^i, s_k^i, p_k)$ is jointly normal, we have $\mathbb{E}(\theta_k^i | s_k^i, p_k) = C_{ks}^0 s_k^i + C_k^0 p_k$. Define $X = \theta_k^i$, $Y_0 = (s_k^i, p_k)$. By the projection theorem, we can solve for

$$\mathbb{E}[X|Y_0] = \mathbb{E}(X) + \Sigma_{X,Y_0}\Sigma_{Y_0,Y_0}^{-1}(Y_0 - \mathbb{E}(Y_0)),$$

where $\Sigma_{X,Y_0} = (\mathrm{Cov}(\theta_k^i, s_k^i), \mathrm{Cov}(\theta_k^i, p_k))$, $\Sigma_{Y_0,Y_0} = \begin{bmatrix} \mathrm{Cov}(s_k^i, s_k^i) & \mathrm{Cov}(s_k^i, p_k) \\ \mathrm{Cov}(p_k, s_k^i) & \mathrm{Cov}(p_k, p_k) \end{bmatrix}$.

Therefore, we have $\Sigma_{X,Y_0}\Sigma^{-1}_{Y_0,Y_0} = (C^0_{ks}, C^0_k)$, where

$$C^0_{ks} = C^*_k + \frac{(1-C^*_k)(y_{k2}\eta_k^2\delta_k^2 + \rho_k(e_{k2}-e_{k3}) - e_{k3}\phi\eta_k\delta_k)}{H^0_k}, \tag{23}$$

$$C^0_k = \frac{(1-C^*_k)(\rho_k(\kappa_k - e_{k3}) + \eta_k\delta_k\kappa_k\phi)}{H^0_k\zeta_k},$$

$$H^0_k = \eta_k^2\delta_k^2((\kappa_k+\rho_k)(\rho_{-k}+e_{-k2}) - \phi^2) + (\kappa_k+\rho_k)(\rho_k+e_{k2}) - (\rho_k+e_{k3})^2 + 2\phi(\kappa_k - e_{k3})\eta_k\delta_k.$$

A.4. **Proof of Theorem 4.4.** Define $\pi_k = \frac{\alpha_k a^1_k}{\alpha_k a^1_k + (1-\alpha_k)a^0_k}$, $\Pi_k = \frac{\alpha_k B^1_k}{\alpha_k B^1_k + (1-\alpha_k)B^0_k}$, $\beta_k \equiv \frac{\alpha_k a^1_k + (1-\alpha_k)a^0_k}{\alpha_k B^1_k + (1-\alpha_k)B^0_k}$ and $\bar{C}^1_k = C^1_k + \frac{\delta_{-k}\zeta_{-k}}{\zeta_k}c^1_k$. Simple calculation shows that $\beta_k = (1-\delta_k\delta_{-k})\zeta_k$, then (22) yields

$$C^1_k = \frac{(1-C^*_k)(y_{k2} - y_{k1}\delta_{-k})}{y_{k3}\beta_k},\ c^1_k = \frac{(1-C^*_k)(y_{k1} - y_{k2}\delta_k)}{y_{k3}\beta_{-k}}. \tag{24}$$

We construct a fixed point mapping in Steps 1–4 for the case where $\phi > 0$, which satisfies the conditions in the Brouwer's fixed point theorem (Step 5). Step 6 discusses the case where $\phi < 0$.

**Step 1:** Given $\pi_k \in [0,1]$, $\Pi_k \in [\frac{\alpha_k}{\alpha_k + (1-\alpha_k)(n_k-1)}, 1]$, where $k = I, II$, $C^1_{ks}$, $C^0_{ks}$, $C^1_k$, $C^0_k$, $c^1_k$, and $\bar{C}^1_k$ are pinned down uniquely, which are continuous in $(\pi_k, \Pi_k)_{k=I,II}$.

By (21), we pin down $e_{k1}, e_{k2}, e_{k3}, y_{k1}, y_{k2}, y_{k3}$, which are continuous function of $\pi_k, \pi_{-k}$. By (21), (22), (23) we obtain $C^1_{ks}$, $C^0_{ks}$, $C^1_k$, $C^0_k$ and $c^1_k$, which are continuous functions of $(\pi_k, \delta_k, \zeta_k^{-1})_{k=I,II}$. By $\zeta_k = \frac{\beta_k}{1-\delta_k\delta_{-k}}$ and (24), we obtain $\bar{C}^1_k \equiv C^1_k + \frac{\delta_{-k}\zeta_{-k}}{\zeta_k}c^1_k = C^1_k + \frac{\delta_{-k}\zeta_k^{-1}}{1-\delta_k\delta_{-k}}(\beta_{-k}c^1_k) = C^1_k + \frac{\delta_{-k}\zeta_k^{-1}}{1-\delta_k\delta_{-k}}\frac{(1-C^*_k)(y_{k1}-y_{k2}\delta_k)}{y_{k3}}$, which is continuous in $(\pi_k, \delta_k, \zeta_k^{-1})_{k=I,II}$.

Next, we solve for $\delta_k$ and $\zeta_k^{-1}$. By Lemma 4.2 and (24), we get $\delta_k = \beta_{-k}\frac{\alpha_k b^1_k}{\alpha_k a^1_k + (1-\alpha_k)a^0_k} = \frac{\beta_{-k}\pi_k b^1_k}{a^1_k} = (\beta_{-k}c^1_k)\frac{\pi_k}{C^1_{ks}} = \frac{(1-C^*_k)(y_{k1}-y_{k2}\delta_k)}{y_{k3}}\frac{\pi_k}{C^1_{ks}}$. Consequently,

$$\delta_k = \frac{(1-C^*_k)y_{k1}\pi_k}{(1-C^*_k)y_{k2}\pi_k + C^1_{ks}y_{k3}}. \tag{25}$$

Since $\beta_k = \frac{a^1_k}{B^1_k}\frac{\Pi_k}{\pi_k} = \frac{C^1_{ks}}{1-C^1_k}\frac{\Pi_k}{\pi_k}$ and (24), then we get

$$\beta_k = (1-C^*_k)(\frac{y_{k2}}{y_{k3}} - \frac{y_{k1}}{y_{k3}}\delta_{-k}) + \frac{\Pi_k C^1_{ks}}{\pi_k}. \tag{26}$$

Since $\zeta_k = \frac{\beta_k}{1-\delta_k\delta_{-k}}$, then

$$\zeta_k^{-1} = \frac{\pi_k(1-\delta_k\delta_{-k})}{(1-C^*_k)(\frac{y_{k2}}{y_{k3}} - \frac{y_{k1}}{y_{k3}}\delta_{-k})\pi_k + \Pi_k C^1_{ks}}. \tag{27}$$

By (25) and (27), we obtain $\delta_k$ and $\zeta_k^{-1}$, which are continuous in $(\pi_k, \Pi_k)_{k=I,II}$. Since $C^1_{ks}$, $C^0_{ks}$, $C^1_k$, $C^0_k$, $c^1_k$ and $\bar{C}^1_k$ are continuous in $(\pi_k, \delta_k, \zeta_k^{-1})_{k=I,II}$, hence are continuous in $(\pi_k, \Pi_k)_{k=I,II}$.

**Step 2:** Given $C^1_{ks}, C^0_{ks} \in (0,1)$, $\bar{C}^1_k, C^1_k, C^0_k \in (0, \frac{n_k-2}{n_k-1})$ (see Step 4), there is a unique solution to $a^1_k > 0$, $B^1_k > 0$, $a^0_k > 0$, $B^0_k > 0$, $b^1_k > 0$, $\pi_k \in [0,1]$, $\Pi_k \in [\frac{\alpha_k}{\alpha_k+(1-\alpha_k)(n_k-1)}, 1]$, which are continuous in $C^1_{ks}, C^1_k, C^0_{ks}, \bar{C}^1_k, C^0_k$.

By the definition of $\delta_k$, we have $\delta_k = \frac{\zeta_{-k}}{\zeta_k} \frac{\alpha_k b^1_k}{\alpha_k B^1_k + (1-\alpha_k) B^0_k}$. Thus, $\frac{\alpha_{-k} b^1_{-k}}{\alpha_{-k} B^1_{-k} + (1-\alpha_{-k}) B^0_{-k}} = \frac{\delta_{-k}\zeta_{-k}}{\zeta_k}$. Together with $\frac{b^1_k}{B^1_k} = \frac{c^1_k}{1-C^1_k}$ and $\bar{C}^1_k = C^1_k + \frac{\delta_{-k}\zeta_{-k}}{\zeta_k} c^1_k$, we get

$$B^1_k - \frac{\alpha_{-k} b^1_{-k}}{\alpha_{-k} B^1_{-k} + (1-\alpha_{-k}) B^0_{-k}} b^1_k = B^1_k \left(1 - \frac{b^1_k}{B^1_k} \frac{\delta_{-k}\zeta_{-k}}{\zeta_k}\right) = B^1_k \frac{1-\bar{C}^1_k}{1-C^1_k}.$$

Define $y = \frac{1-\bar{C}^1_k}{1-C^1_k}$. It is apparent that $y > 0$. Substituting the above expression into the definition of $\lambda^1_k$ and $\lambda^0_k$, we get

$$\lambda^1_k = \Big((n\alpha_k - 1) B^1_k y + (1-\alpha_k) n_k B^0_k\Big)^{-1}, \ \lambda^0_k = \Big(n_k \alpha_k B^1_k y + ((1-\alpha_k) n_k - 1) B^0_k\Big)^{-1}.$$

Together with $B^1_k = \frac{1-C^1_k}{\gamma+\lambda^1_k}$ and $B^0_k = \frac{1-C^0_k}{\gamma+\lambda^0_k}$, we get

$$B^0_k = \frac{1-C^0_k}{\gamma} - \frac{x}{\gamma[n_k\alpha_k y + ((1-\alpha_k)n_k - 1)x]} \equiv H_0(x),$$
$$B^1_k = \frac{1-C^1_k}{\gamma} - \frac{1}{\gamma[(n_k\alpha_k - 1)y + (1-\alpha_k)n_k x]} \equiv H_1(x),$$

where $x \equiv \frac{B^0_k}{B^1_k}$. We divide $H_0(x)$ by $H_1(x)$ and get $x = \frac{H_0(x)}{H_1(x)}$. Define

$$G(x) \equiv x H_1(x) - H_0(x).$$

We need to solve $G(x) = 0$ to get the solution $x = \frac{B^0_k}{B^1_k}$. Notice that if $\bar{C}^1_k < 1$ and $C^1_k < 1$, then all the denominators in $H_0(x)$ and $H_1(x)$ are positive, hence $G(x)$ is continuous for $x \geq 0$, Moreover, given $C^1_k$, $C^0_k$ and $\bar{C}^1_k$, it is apparent that $H_1(x)$ is increasing in $x$ and $H_0(x)$ is decreasing in $x$ for any $x \geq 0$.

First, there is a solution $x^* > 0$ such that $G(x^*) = 0$. This holds since $G(0) = -\frac{1-C^0_k}{\gamma} < 0$ and $G(+\infty) = +\infty$.

Second, we prove that the solution $x^* \in (0, \infty)$ is unique. Define $\underline{x}$ such that $H_1(x) > 0$ if and only if $x > \underline{x}$ (it is possible that $\underline{x} = 0$). We prove that $G(x) < 0$ for $x < \underline{x}$. If $x \geq y$, then $H_1(x) \geq H_1(y) = \frac{1-C^1_k}{\gamma} - \frac{1}{\gamma(n_k-1)y} = \frac{1-C^1_k}{\gamma(1-\bar{C}^1_k)}(\frac{n_k-2}{n_k-1} - \bar{C}^1_k) > 0$, where the last inequality holds since $\bar{C}^1_k < \frac{n_k-2}{n_k-1}$. By the definition of $\underline{x}$, we have $x > \underline{x}$. Therefore, $x < \underline{x}$ implies that $x < y$ and hence $H_0(x) > H_0(y) = \frac{1}{\gamma}(\frac{n_k-2}{n_k-1} - C_k^0) > 0$, which holds since $C_k^0 < \frac{n_k-2}{n_k-1}$. Since $xH_1(x) < 0$ and $H_0(x) > 0$ for any $x < \underline{x}$, then we reach the conclusion that $G(x) = xH_1(x) - H_0(x) < 0$ for $x < \underline{x}$. Consequently, any solution $x^*$ of $G(x^*) = 0$ satisfies $x^* > \underline{x}$.

We then prove that $G(x)$ is increasing in $x$ for $x \geq \underline{x}$. Since $H_1(x) > 0$ and $H_1(x)$ is increasing in $x$ for $x \geq \underline{x}$, then $xH_1(x)$ is increasing for $x \geq \underline{x}$, Together with the fact that $H_0(x)$ is decreasing in $x$, we have $G(x) = xH_1(x) - H_0(x)$ is increasing in $x \geq \underline{x}$. Therefore, the solution $x^*$ such that $G(x^*) = 0$ is unique.

Next, we obtain the unique $B_k^1 = H_1(x^*) > 0$ and $B_k^0 = H_0(x^*) > 0$. Moreover, $x^* \in (0, y)$ if and only if $\bar{C}_k^1 < C_k^0$. We know that $G(y) = yH_1(y) - H_0(y) = y\frac{1-C_k^1}{\gamma} - \frac{1-C_k^0}{\gamma} = \frac{1}{\gamma}(C_k^0 - \bar{C}_k^1)$. There are two cases: (1) If $\bar{C}_k^1 > C_k^0$, then $G(y) < 0$, which means that $G(x^*) = 0 > G(y)$. Since $x^* > \underline{x}$ and $y > \underline{x}$ and $G(x)$ is increasing for $x > \underline{x}$, then $x^* > y$. Moreover, $B_k^1 = H_1(x^*) > H_1(y) = \frac{1-C_k^1}{\gamma(1-C_k)}(\frac{n_k-2}{n_k-1} - \bar{C}_k^1) > 0$. Furthermore, $B_k^0 = H_0(x^*) = x^* H_1(x^*) > 0$. (2) If $\bar{C}_k^1 \leq C_k^0$, then $G(y) \geq 0$, which means that $G(x^*) = 0 \leq G(y)$. Consequently, $x^* \leq y$. Moreover, $B_k^0 = H_0(x^*) \geq H_0(y) = \frac{1}{\gamma}(\frac{n_k-2}{n_k-1} - C_k^0) > 0$. Furthermore, $B_k^1 = H_1(x^*) = \frac{1}{x^*}H_0(x^*) > 0$.

Finally, we prove that $x^* \in (0, n_k - 1)$. Since $\bar{C}_k^1 > C_k^1$ (see Step 3), then $y = \frac{1-\bar{C}_k^1}{1-C_k^1} < 1$. If $x^* < y$, then $x^* < y < 1 < n_k - 1$. If $x^* > y$, then $G(x^*) = 0$ implies that $1 - C_k^0 - x^*(1 - C_k^1 + \frac{x^*-y}{(n_k\alpha_k y + ((1-\alpha_k)n_k - 1)x)((n_k\alpha_k - 1)y + (1-\alpha_k)n_k x)}) = 0$, Hence, $x^* < \frac{1-C_k^0}{1-C_k^1} < n_k - 1$, which holds since $C_k^0 \geq 0$ and $C_k^1 < \frac{n_k-2}{n_k-1}$. In all, we prove that $x^* \in (0, n_k - 1)$.

Since we have solved $B_k^1 > 0$ and $B_k^0 > 0$, together with the fact that $C_{ks}^1, C_{ks}^0 \in (0,1)$, $\bar{C}_k^1, C_k^1, C_k^0 \in (0, \frac{n_k-2}{n_k-1})$, we get the unique $a_k^1 = \frac{C_{ks}^1}{1-C_k^1}B_k^1 > 0$, $a_k^0 = \frac{C_{ks}^0}{1-C_k^0}B_k^0 > 0$ and $b_k^1 = \frac{c_k^1}{1-C_k^1}B_k^1 > 0$. Together with $x^* \in (0, n_k - 1)$, we pin down the unique $\pi_k = \frac{\alpha_k a_k^1}{\alpha_k a_k^1 + (1-\alpha_k)a_k^0} \in (0,1)$ and $\Pi_k = \frac{\alpha_k B_k^1}{\alpha_k B_k^1 + (1-\alpha_k)B_k^0} = \frac{\alpha_k}{\alpha_k + (1-\alpha_k)x^*} \in (\frac{\alpha_k}{\alpha_k + (1-\alpha_k)(n_k-1)}, 1)$.

Finally, by implicit function theorem, $x^*$ is continuous in $C_{ks}^1, C_k^1, C_{ks}^0, \bar{C}_k^1, C_k^0$. Hence, $B_k^1$, $B_k^0$, $\pi_k$ and $\Pi_k$ are continuous in $C_{ks}^1, C_k^1, C_{ks}^0, \bar{C}_k^1, C_k^0$.

**Step 3:** We show that (i) $\delta_k \in (0, \frac{y_{k1}}{y_{k2}})$, $\delta_k\delta_{-k} \in (0,1)$; (ii) $C_{ks}^1 < 1$, $C_{ks}^0 < 1$; (iii) if $C_{ks}^1 > 0$, then $\beta_k > 0$, $\zeta_k > 0$, $C_k^1 \in (0,1)$, $C_k^0 > 0$, and $c_k^1 > 0$; (iv) $\bar{C}_k^1 > C_k^1$.

First, we prove that $\delta_k \in (0, \frac{y_{k1}}{y_{k2}})$, $\delta_k\delta_{-k} \in (0,1)$ and $\delta_{-k} < \frac{y_{k2}}{y_{k1}}$. By equation (25) and $\frac{C_{ks}^1}{\pi_k} > 0$, we get $\delta_k < \frac{y_{k1}}{y_{k2}}$. Moreover, by $C_k^* < 1$, $y_{k1} > 0$, $y_{k2} > 0$, $y_{k3} > 0$, we get $\delta_k > 0$. Consequently, $\delta_k\delta_{-k} < \frac{y_{k1}}{y_{k2}}\frac{y_{-k1}}{y_{-k2}}$. Since $\eta_k y_{k1} < y_{-k2}$, $\eta_{-k}y_{-k1} < y_{k2}$ and $\eta_k\eta_{-k} = 1$, then we get $\frac{y_{k1}}{y_{k2}}\frac{y_{-k1}}{y_{-k2}} < 1$. Therefore, $\delta_k\delta_{-k} < \frac{y_{k1}}{y_{k2}}\frac{y_{-k1}}{y_{-k2}} < 1$. Moreover, $\delta_{-k} < \frac{y_{-k1}}{y_{-k2}} < \frac{y_{k2}}{y_{k1}}$.

Second, we prove that $C_{ks}^1 < 1$ and $C_{ks}^0 < 1$. By (22), we get $1 - C_{ks}^1 = (1 - C_k^*)(1 + \frac{(e_{k1}-e_{k2})y_{k2}}{(\kappa_k - e_{k1})y_{k3}}) = \frac{1-C_k^*}{y_{k3}}\Big(\rho_k(\rho_{-k} + e_{-k2}) - \phi^2 + (\rho_{-k} + e_{-k2})(e_{k2} - \frac{e_{k1}-e_{k2}}{\kappa_k - e_{k1}}e_{k1})\Big) > 0$, due to the fact that $\rho_k(\rho_{-k}+e_{-k2})-\phi^2>0$ and $e_{k2}-\frac{e_{k1}-e_{k2}}{\kappa_k-e_{k1}}e_{k1}>0$. In all, $C^1_{ks}<1$. By (23), $1-C^0_{ks}=\frac{1-C^*_k}{H^0_k}[\eta_k^2\delta_k^2\kappa_k(\rho_{-k}+e_{-k2})+\kappa_k e_{k2}-e_{k3}^2+(\rho_k+2\phi\eta_k\delta_k)(\kappa_k-e_{k3})+e_{k3}\phi\eta_k\delta_k]>0$, because $\kappa_k e_{k2}-e_{k3}^2>0$ and $\kappa_k-e_{k3}>0$. Therefore, $C^0_{ks}<1$.

Third, we prove that $\beta_k>0$ and $\zeta_k>0$. Since $\delta_{-k}<\frac{y_{k2}}{y_{k1}}$, $C^1_{ks}>0$, $\pi_k>0$ and $\Pi_k>0$, then (26) implies that $\beta_k>0$. Since $\delta_k\delta_{-k}<1$, then $\zeta_k=\frac{\beta_k}{1-\delta_k\delta_{-k}}>0$.

Next, we prove that $C^1_k\in(0,1)$, $C^0_k>0$ and $c^1_k>0$. By equation (24) and (26), we get

$$C^1_k=\frac{(1-C^*_k)(y_{k2}-y_{k1}\delta_{-k})}{y_{k3}\beta_k}=\frac{(1-C^*_k)(\frac{y_{k2}}{y_{k3}}-\frac{y_{k1}}{y_{k3}}\delta_{-k})}{(1-C^*_k)(\frac{y_{k2}}{y_{k3}}-\frac{y_{k1}}{y_{k3}}\delta_{-k})+C^1_{ks}\frac{\Pi_k}{\pi_k}}.$$

Since $\delta_{-k}<\frac{y_{k2}}{y_{k1}}$ and $\beta_k>0$, then $C^1_k>0$. Furthermore, $C^1_{ks}\frac{\Pi_k}{\pi_k}>0$ implies $C^1_k<1$. Since $H^0_k>0$ and $\kappa_k-e_{k3}>0$, then $C^0_k=\frac{(1-C^*_k)(\rho_k(\kappa_k-e_{k3})+\eta_k\delta_k\kappa_k\phi)}{H^0_k\zeta_k}>0$. By equation (24), $c^1_k>0$ is equivalent to $\delta_k<\frac{y_{k1}}{y_{k2}}$, which holds.

Finally, $\bar{C}^1_k=C^1_k+\frac{\delta_{-k}\zeta_{-k}}{\zeta_k}c^1_k>C^1_k$, since $\delta_{-k},\zeta_{-k},\zeta_k$ and $c^1_k$ are positive.

**Step 4:** If $n_k$ is large or $\sigma_k^2$ is small, then $\bar{C}^1_k,C^1_k,C^0_k\in(0,\frac{n_k-2}{n_k-1})$ and $C^1_{ks},C^0_{ks}\in(0,1)$.

First, we prove that $C^1_k$, $\bar{C}^1_k\in(0,\frac{n_k-2}{n_k-1})$. From (24), we get $C^1_k<\bar{C}^1_k=C^1_k+\frac{\delta_{-k}\zeta_{-k}}{\zeta_k}c^1_k=\frac{1-C^*_k}{\zeta_k}\frac{y_{k2}}{y_{k3}}$. Hence, we need to prove that $\zeta_k>\frac{n_k-1}{n_k-2}\frac{(1-C^*_k)y_{k2}}{y_{k3}}$. By (26) and $\zeta_k=\frac{\beta_k}{1-\delta_k\delta_{-k}}$, we need to prove that

$$\frac{(1-C^*_k)(\frac{y_{k2}}{y_{k3}}-\frac{y_{k1}}{y_{k3}}\delta_{-k})+\frac{\Pi_k C^1_{ks}}{\pi_k}}{1-\delta_k\delta_{-k}}>\frac{n_k-1}{n_k-2}\frac{(1-C^*_k)y_{k2}}{y_{k3}}. \tag{28}$$

We check the case $\delta_{-k}=0$, then (28) is equivalent to

$$n_k-2>\frac{1-C^*_k}{C^1_{ks}}\frac{y_{k2}}{y_{k3}}\frac{\pi_k}{\Pi_k}. \tag{29}$$

As $n_k\to+\infty$ or $\sigma_k^2\to0$, we get $\lim\frac{y_{k2}}{y_{k3}}=1$, $\lim\frac{\pi_k}{\Pi_k}=1$ and $\lim C^1_{ks}=C^*_k$. The right hand side of (29) converges to $\frac{1-C^*_k}{C^*_k}=\frac{\sigma_k^2}{1-\rho_k}$. Moreover, if we ignore the first order term $o(1)$, then (29) is equivalent to and $\sigma_k^2<(1-\rho_k)(n_k-2)$. Since $\lim\delta_{-k}=0$, then (28) also holds.

Next, we prove that $C^0_k<\frac{n_k-2}{n_k-1}$. Since $C^0_k<\frac{1-C^*_k}{\zeta_k}$, then we need to prove that $\zeta_k>\frac{n_k-1}{n_k-2}(1-C^*_k)$. Since $\lim\frac{y_{k2}}{y_{k3}}=1$, then the condition to guarantee $\bar{C}^1_k<\frac{n_k-2}{n_k-1}$ also guarantees that $C^0_k<\frac{n_k-2}{n_k-1}$. Finally, $C^1_{ks}>0$ and $C^0_{ks}>0$ hold since $C^1_{ks}\to C^*_k>0$, $C^0_{ks}\to C^*_k>0$, as $n_k\to+\infty$ or $\sigma_k^2\to0$.

**Step 5:** The existence of a fixed point.

By compositing the two functions defined in Steps 1 and 2, we have a continuous function from $\pi_I\in[0,1],\pi_{II}\in[0,1]$, $\Pi_I\in[\frac{\alpha_I}{\alpha_I+(1-\alpha_I)(n_I-1)},1]$, $\Pi_{II}\in[\frac{\alpha_{II}}{\alpha_{II}+(1-\alpha_{II})(n_{II}-1)},1]$ to the same domain. By Brouwer's fixed point theorem, there exists a fixed point $(\pi_k,\Pi_k)_{k=I,II}$.

By construction of the mapping, we obtain the equilibrium parameters, $a_k^1$, $B_k^1$, $a_k^0$, $B_k^0$, $b_k^1$, $C_{ks}^1, C_k^1, c_k^1, C_{ks}^0$, and $C_k^0$ that satisfy the properties stated in Theorem 4.4.
**Step 6:** The $\phi < 0$ case. If $\phi < 0$, then $y_{k1}$, $\delta_k$, $\phi$, $b_k^1$, and $c_k^1$ are all negative. When we replace $\delta_k$, $\phi$, $b_k^1$, $c_k^1$ with $|\delta_k|$, $|\phi|$, $|b_k^1|$, $|c_k^1|$, the analysis above remains valid.

## Appendix B. Proofs of the Results in Section 5

In the proofs of the results in Section 5, the comparative statics results refer to the estimated version of the equilibrium parameters (i.e., by ignoring the corresponding high order terms).

### B.1. **Proof of Propositions 5.1 and 5.2.** We prove these two results in five steps.

**Step 1:** The limit as $N \to +\infty$.

Theorem 4.4 shows that $\delta_k \leq \frac{y_{k1}}{y_{k2}}$. As $N \to +\infty$, $y_{k1} \to 0$, and hence $\delta_k \to 0$. Consequently, $\lim c_k^1 = 0$, $\lim C_{ks}^1 = C_{ks}^0 = C_k^*$, $\lim C_k^1 = \lim C_k^0 = 1 - C_k^*$, $\lim \lambda_k^1 = \lim \lambda_k^0 = 0$, $\lim a_k^1 = \lim a_k^0 = \lim B_k^1 = \lim B_k^0 = \frac{C_k^*}{\gamma}$. Moreover, $\lim \beta_k = \lim \zeta_k = 1$, $\lim \pi_k = \alpha_k$.
**Step 2:** The coefficients $C_{ks}^1$, $C_k^1$, $c_k^1$, $C_{ks}^0$, and $C_k^0$, ignoring the term $o(\frac{1}{N^2})$.

In equilibrium, $a_k^1 - a_k^0 = o(\frac{1}{N})$ (which is verified in Step 7). Consequently,

$$\pi_k - \alpha_k = \frac{\alpha_k(1-\alpha_k)(a_k^1 - a_k^0)}{\alpha_k a_k^1 + (1-\alpha_k)a_k^0} = o(\frac{1}{N}).$$

Substituting $\pi_k = \alpha_k + o(\frac{1}{N})$ into (21), we get (by ignoring $o(\frac{1}{N^2})$)

$$\begin{aligned} e_{k1} = e_{k2} = e_{k3} &= \frac{\kappa_k}{n_k}, \; y_{k1} = \frac{\kappa_k \phi}{\eta_k n_k}, \\ y_{k2} = \rho_k(\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}}) - \phi^2, \; & y_{k3} = (\rho_k + \frac{\kappa_k}{n_k})(\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}}) - \phi^2. \end{aligned} \tag{30}$$

Substituting (30) into (22) and (23) and ignoring $o(\frac{1}{N^2})$, we get

$$\begin{aligned} & C_{ks}^1 = C_k^*, \; C_k^1 = \frac{(1-C_k^*)(y_{k2} - y_{k1}\delta_{-k})}{y_{k3}\beta_k}, \; c_k^1 = \frac{(1-C_k^*)(y_{k1} - y_{k2}\delta_k)}{y_{k3}\beta_{-k}}, \\ & C_{ks}^0 = C_k^* - \frac{(1-C_k^*)\eta_k^2\delta_k(y_{k1} - y_{k2}\delta_k)}{H_k^0}, \; C_k^0 = \frac{(1-C_k^*)\kappa_k(\frac{n_k-1}{n_k}\rho_k + \phi\eta_k\delta_k)}{H_k^0\zeta_k}, \\ & H_k^0 = \eta_k^2\delta_k^2((\rho_k+\kappa_k)(\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}}) - \phi^2) + \frac{n_k-1}{n_k}\kappa_k(\rho_k + \frac{\kappa_k}{n_k}) + 2\frac{n_k-1}{n_k}\kappa_k\phi\eta_k\delta_k. \end{aligned} \tag{31}$$

**Step 3:** The parameter $|\delta_k|$.

Since $C_{ks}^1 = C_k^* + o(\frac{1}{N^2})$ and $\pi_k = \alpha_k + o(\frac{1}{N})$, then by (25), $y_{k3} = y_{k2} + o(1)$, and $\frac{C_k^*}{1-C_k^*} = \frac{1-\rho_k}{\sigma_k^2}$, we have $\delta_k = \frac{(1-C_k^*)y_{k1}}{(1-C_k^*)y_{k2} + \frac{C_{ks}^1}{\pi_k}y_{k3}} = \frac{y_{k1}}{y_{k2}}\frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} + o(\frac{1}{N})$. Since $\frac{y_{k1}}{y_{k2}} = \frac{\kappa_k\phi}{\eta_k(\rho_k\rho_{-k} - \phi^2)}\frac{1}{n_k} + o(\frac{1}{N})$, we have

$$|\delta_k| = \frac{\delta_k^*}{n_k} + o(\frac{1}{N}), \quad \text{and} \quad \delta_k^* = \frac{\kappa_k|\phi|}{\eta_k(\rho_k\rho_{-k} - \phi^2)} \frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} > 0. \tag{32}$$

**Step 4:** The coefficients $|c_k^1|$, $C_{ks}^1 - C_{ks}^0$, and $C_k^0 - C_k^1$.

By $\beta_{-k} = 1 + o(1)$ and $y_{k3} = y_{k2} + o(1)$, we have $c_k^1 = \frac{(1-C_k^*)(y_{k1}-y_{k2}\delta_k)}{y_{k3}\beta_{-k}} = (1 - C_k^*)(\frac{y_{k1}}{y_{k2}} - \delta_k) + o(\frac{1}{N})$. By (21) and (32) and $y_{k2} = \rho_k\rho_{-k} - \phi^2 + o(1)$,

$$|c_k^1| = \frac{c_k^*}{n_k} + o(\frac{1}{N}) \quad \text{and} \quad c_k^* = \frac{(1-\rho_k)|\phi|}{\eta_k(\rho_k\rho_{-k} - \phi^2)} \frac{1}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} > 0. \tag{33}$$

Since $H_k^0 = \kappa_k\rho_k + o(1)$, then $C_{ks}^1 - C_{ks}^0 = \frac{(1-C_k^*)\eta_k^2\delta_k(y_{k1}-y_{k2}\delta_k)}{H_k^0} = \frac{(1-C_k^*)\eta_k^2\delta_k(y_{k1}-y_{k2}\delta_k)}{\kappa_k\rho_k} + o(\frac{1}{N^2})$. By (21) and (32),

$$C_{ks}^1 - C_{ks}^0 = \frac{\Delta_{ks}}{n_k^2} + o(\frac{1}{N^2}) \quad \text{and} \quad \Delta_{ks} = \frac{(1-\rho_k)\phi^2}{\rho_k(\rho_k\rho_{-k} - \phi^2)} \frac{\alpha_k}{(\alpha_k + \frac{1-\rho_k}{\sigma_k^2})^2} > 0. \tag{34}$$

Substitute $H_k^0 = \frac{n_k-1}{n_k}\kappa_k(\rho_k + \frac{\kappa_k}{n_k}) + 2\frac{n_k-1}{n_k}\kappa_k\phi\eta_k\delta_k + o(\frac{1}{N})$ into $C_k^0$ (see (31)), we get $C_k^0 = \frac{1-C_k^*}{\zeta_k}\frac{\rho_k + \frac{n_k}{n_k-1}\kappa_k\phi\eta_k\delta_k}{\rho_k + \frac{\kappa_k}{n_k} + 2\kappa_k\phi\eta_k\delta_k} + o(\frac{1}{N}) = \frac{1-C_k^*}{\zeta_k}(1 - \frac{\frac{\kappa_k}{n_k} + \frac{n_k}{n_k-1}\phi\eta_k\delta_k}{\rho_k + \frac{\kappa_k}{n_k} + 2\kappa_k\phi\eta_k\delta_k}) + o(\frac{1}{N})$. Together with $\zeta_k = \frac{\beta_k}{1-\delta_k\delta_{-k}} = \beta_k + o(\frac{1}{N})$ and $\rho_k + \frac{\kappa_k}{n_k} + 2\kappa_k\phi\eta_k\delta_k = \rho_k + o(1)$, we obtain $C_k^0$ as follows:

$$C_k^0 = (1 - C_k^*)(1 - \frac{\frac{\kappa_k}{n_k} + \phi\eta_k\delta_k}{\rho_k})\frac{1}{\beta_k} + o(\frac{1}{N}). \tag{35}$$

Since $y_{k1}\delta_{-k} = o(\frac{1}{N})$, then by (31), we obtain $C_k^1$ as follows:

$$C_k^1 = \frac{(1-C_k^*)y_{k2}}{y_{k3}}\frac{1}{\beta_k} + o(\frac{1}{N}) = (1 - C_k^*)(1 - \frac{\frac{\kappa_k}{n_k}(\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}})}{\rho_k(\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}}) - \phi^2})\frac{1}{\beta_k} + o(\frac{1}{N}). \tag{36}$$

By (35), (36) and the definition of $y_{k1}$ and $y_{k2}$ in (30), we obtain

$$C_k^0 - C_k^1 = \frac{(1-C_k^*)\eta_k\phi}{\rho_k\beta_k}(\frac{y_{k1}}{y_{k2}} - \delta_k) + o(\frac{1}{N}).$$

Together with (30) and (32), we get

$$C_k^0 - C_k^1 = \frac{\Delta_k}{n_k} + o(\frac{1}{N}) \quad \text{and} \quad \Delta_k = \frac{(1-\rho_k)\phi^2}{\rho_k(\rho_k\rho_{-k} - \phi^2)} \frac{1}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} > 0. \tag{37}$$

**Step 5:** The parameters $\lambda_k^1$ and $\lambda_k^0$.

Since $b_k^1 = o(1)$ and $b_{-k} = o(1)$ and $B_k - B_k^1 = o(1)$, then $(\lambda_k^1)^{-1} = (\alpha_k n_k - 1)(B_k^1 - b_k^1 \frac{b_{-k}}{B_{-k}}) + (1-\alpha_k) n_k B_k^0 = n_k B_k - B_k^1 + o(1) = (n_k - 1)B_k + o(1)$, which implies that

$$\lambda_k^1 = \frac{1}{(n_k-1)B_k} + o(\frac{1}{N^2}), \;\; \lambda_k^0 = \frac{1}{(n_k-1)B_k} + o(\frac{1}{N^2}) \tag{38}$$

The result for $\lambda_k^0$ follows by the same logic. It is clear that $\lambda_k^1 = o(1)$ and $\lambda_k^0 = o(1)$.

**Step 6:** The coefficients $a_k^1 - a_k^0$, $B_k^1 - B_k^0$, and $|b_k^1|$.

Since $\lambda_k^1 = o(1)$ and $c_k^1 = o(1)$, then $|b_k^1| = \frac{1}{\gamma + \lambda_k^1}|c_k^1| = \frac{1}{\gamma}|c_k^1| + o(\frac{1}{N})$. Similarly, $B_k^1 - B_k^0 = \frac{1}{\gamma}(C_k^0 - C_k^1) + o(\frac{1}{N})$, $a_k^1 - a_k^0 = \frac{1}{\gamma}(C_{ks}^1 - C_{ks}^0) + o(\frac{1}{N^2})$. Consequently, $a_k^1 - a_k^0 = \frac{a_k^*}{n_k} + o(\frac{1}{N^2})$, $B_k^1 - B_k^0 = \frac{B_k^*}{n_k} + o(\frac{1}{N})$, $|b_k^1| = \frac{b_k^*}{n_k} + o(\frac{1}{N})$, where $a_k^* = \frac{1}{\gamma}\Delta_{ks} > 0$, $B_k^* = \frac{1}{\gamma}\Delta_k > 0$, $b_k^* = \frac{1}{\gamma}c_k^* > 0$.

**Step 7:** We verify that $a_k^1 - a_k^0 = o(\frac{1}{N})$.

By Step 6, we have $a_k^1 - a_k^0 = \frac{1}{\gamma}(C_{ks}^1 - C_{ks}^0) + o(\frac{1}{N^2}) = \frac{1}{\gamma}\frac{\Delta_{ks}}{n_k^2} + o(\frac{1}{N^2}) = o(\frac{1}{N})$.

B.2. **Proof of Lemma 5.3.** First, (32) implies that $|\delta_k|$ is increasing in $\alpha_k$.

Next, since $C_k^0 - C_k^1 = \frac{\Delta_k}{n_k} + o(\frac{1}{N})$ and $C_k^1 = 1 - C_k^* + o(1)$, then $\frac{C_{k0}^1 - C_k^1}{1 - C_k^1} = \frac{\Delta_k}{C_k^* n_k} + o(\frac{1}{N})$. Since $B_k^1 - B_k^0 = \frac{1}{\gamma}(C_k^0 - C_k^1) + o(\frac{1}{N})$ and $B_k^1 = \frac{1}{\gamma}(1 - C_k^1) + o(1)$, then $\frac{B_k^1 - B_k^0}{B_k^1} = \frac{C_k^0 - C_k^1}{1 - C_k^1} + o(\frac{1}{N})$. By (37), $\frac{B_k}{B_k^1} = 1 - (1-\alpha_k)\frac{B_k^1 - B_k^0}{B_k^1} = 1 - (1-\alpha_k)\frac{\Delta_k}{C_k^* n_k} + o(\frac{1}{N})$. Since $\Delta_k > 0$ is decreasing in $\alpha_k$, then $\frac{B_k}{B_k^1}$ is increasing in $\alpha_k$.

Next, from (26), we have

$$\beta_k = (1 - C_k^*)(\frac{y_{k2}}{y_{k3}} - \frac{y_{k1}}{y_{k3}}\delta_{-k}) + C_{ks}^1 \frac{\alpha_k}{\pi_k}\frac{B_k^1}{B_k} = \frac{(1-C_k^*)y_{k2}}{y_{k3}} + C_k^* \frac{B_k^1}{B_k} + o(\frac{1}{N}).$$

Since $\frac{B_k}{B_k^1}$ is increasing in $\alpha_k$, then $\beta_k$ is decreasing in $\alpha_k$. Since $\zeta_k = \frac{\beta_k}{1 - \delta_k \delta_{-k}} = \beta_k + o(\frac{1}{N})$, then $\zeta_k$ is also decreasing in $\alpha_k$.

Finally, since $\frac{B_k}{B_k^1} = 1 + o(1)$, then $\zeta_k = \beta_k + o(\frac{1}{N}) = (1 - C_k^*)\frac{y_{k2}}{y_{k3}} + C_k^* + o(1)$. Therefore, $|\delta_k \zeta_k| = |\frac{y_{k1}}{y_{k2}}| \frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}}((1 - C_k^*)\frac{y_{k2}}{y_{k3}} + C_k^*) + o(\frac{1}{N})$, which is increasing in $\alpha_k$.

B.3. **Proof of Proposition 5.4.** Since $\beta_k = \frac{\alpha_k a_k^1 + (1-\alpha_k)a_k^0}{\alpha_k B_k^1 + (1-\alpha_k)B_k^0} = \frac{C_k^*}{1 - (\alpha_k C_k^1 + (1-\alpha_k)C_k^0)} + o(\frac{1}{N^2})$, then $\beta_k = C_k^* + \beta_k(C_k^0 - \alpha_k(C_k^0 - C_k^1))$.

Substituting (35) into this expression, we get

$$\beta_k = 1 - (1 - C_k^*)\frac{\frac{\kappa_k}{n_k} + \phi\eta_k\delta_k}{\rho_k} - \alpha_k \frac{(1-C_k^*)\eta_k\phi}{\rho_k}(\frac{y_{k1}}{y_{k2}} - \delta_k) = 1 - \frac{\frac{\kappa_k}{n_k} + \phi\eta_k\delta_k}{\rho_k} + \frac{\kappa_k C_k^*}{\rho_k n_k} + o(\frac{1}{N}).$$

Substituting the above expression of $\beta_k$ into (35), we obtain

$$C_k^0 = (1 - C_k^*)(1 - \frac{\kappa_k C_k^*}{\rho_k n_k}) + o(\frac{1}{N}). \tag{39}$$

Therefore, $C_k^0$ is independent of $\alpha_k$.

By (37), $C_k^0 - C_k^1$ is decreasing in $\alpha_k$. Since $C_k^0$ is independent of $\alpha_k$, then $C_k^1$ is increasing in $\alpha_k$. By (33), $|c_k^1|$ is decreasing in $\alpha_k$. Since $C_{ks}^1 = C_k^* + o(\frac{1}{N^2})$, then $\frac{\partial C_{ks}^1}{\partial \alpha_k} = 0$. By (34), $C_{ks}^0 - C_{ks}^1 = -\frac{\Delta_{ks}}{n_k^2} + o(\frac{1}{N^2})$. Thus, $\frac{\partial \Delta_{ks}}{\partial \alpha_k} > 0$ if and only if $\alpha_k < \frac{1-\rho_k}{\sigma_k^2}$. Therefore, $\frac{\partial C_{ks}^0}{\partial \alpha_k} < 0$ if and only if $\alpha_k < \frac{1-\rho_k}{\sigma_k^2}$.

B.4. **Proof of Lemma 5.5.** By (37), $\alpha_k(B_k^1 - B_k^0) = \frac{\alpha_k}{\gamma}(C_k^0 - C_k^1) + o(\frac{1}{N}) = \frac{1}{\gamma}\frac{\Delta_k \alpha_k}{n_k} + o(\frac{1}{N})$, which is increasing in $\alpha_k$. Together with $\frac{\partial B_k^0}{\partial \alpha_k} = 0$, we have $B_k = \alpha_k B_k^1 + (1-\alpha_k)B_k^0 = B_k^0 + \alpha_k(B_k^1 - B_k^0)$ is increasing in $\alpha_k$. By (38), we have $\lambda_k^1 = \frac{1}{(n_k-1)B_k} + o(\frac{1}{N^2})$. Since $\frac{\partial B_k}{\partial \alpha_k} > 0$, then $\frac{\partial \lambda_k^1}{\partial \alpha_k} < 0$. Similarly, $\frac{\partial \lambda_k^0}{\partial \alpha_k} < 0$.

B.5. **Proof of Proposition 5.6.** We first prove that $a_k^1$ and $a_k^0$ are increasing in $\alpha_k$. We know that $a_k^1 = \frac{1}{\gamma + \lambda_k^1} C_{ks}^1$. Since $\lambda_k^1$ is decreasing in $\alpha_k$ and $C_{ks}^1$ is independent of $\alpha_k$, then $a_k^1$ is increasing in $\alpha_k$. Since $a_k^0 = \frac{C_{ks}^0}{\gamma + \lambda_k^0}$, then

$$a_k^0 - \frac{C_k^*}{\gamma} = \frac{C_{ks}^0 - C_k^*}{\gamma} + (\frac{1}{\gamma + \lambda_k^0} - \frac{1}{\gamma})C_{ks}^0 = \frac{1}{\gamma}(C_{ks}^0 - C_k^* - \frac{C_k^*}{1 + \gamma(\lambda_k^0)^{-1}}) + o(\frac{1}{N^2}). \tag{40}$$

Since $(\lambda_k^0)^{-1} = (n_k - 1)B_k + o(\frac{1}{N^2})$, $B_k = \alpha_k B_k^1 + (1-\alpha_k)B_k^0$, $\gamma(B_k^1 - B_k^0) = \frac{\Delta_k}{n_k} + o(\frac{1}{N})$, $\gamma B_k^0 = C_k^* + o(1)$, and $\gamma B_k = C_k^* + o(1)$, then

$$\begin{aligned}
\frac{1}{1 + \gamma(\lambda_k^0)^{-1}} &= \frac{1}{1 + \gamma(n_k - 1)B_k} + o(\frac{1}{N^2}) \\
&= \frac{1}{1 + \gamma(n_k - 1)B_k^0} - \frac{\gamma(n_k - 1)\alpha_k(B_k^1 - B_k^0)}{(1 + \gamma(n_k - 1)B_k)(1 + \gamma(n_k - 1)B_k^0)} + o(\frac{1}{N^2}) \\
&= \frac{1}{1 + \gamma(n_k - 1)B_k^0} - \frac{1}{(C_k^*)^2}\frac{\alpha_k \Delta_k}{n_k^2} + o(\frac{1}{N^2}).
\end{aligned}$$

Substituting the above expression and (34) into (40), we obtain

$$a_k^0 = \frac{C_k^*}{\gamma} + \frac{1}{\gamma}\left(-\frac{C_k^*}{1 + \gamma(n_k - 1)B_k^0} + \frac{1}{n_k^2}(\frac{\alpha_k \Delta_k}{C_k^*} - \Delta_{ks})\right) + o(\frac{1}{N^2}) \tag{41}$$

where

$$\frac{\alpha_k \Delta_k}{C_k^*} - \Delta_{ks} = \frac{(1-\rho_k)\phi^2}{\rho_k(\rho_k \rho_{-k} - \phi^2)} \frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} (\frac{1}{C_k^*} - \frac{1}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}}) > 0,$$

which holds by $C_k^* < \frac{1-\rho_k}{\sigma_k^2}$. Since $\frac{1}{C_k^*} - \frac{1}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}}$ and $\frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}}$ are increasing in $\alpha_k$, and $B_k^0$ is independent of $\alpha_k$, then $a_k^0$ is increasing in $\alpha_k$.

Then we prove that $B_k^1$ and $|b_k^1|$ are decreasing in $\alpha_k$ and $B_k^0$ is independent of $\alpha_k$. Since $B_k = \frac{C_k^*}{\gamma} + o(1)$, then $\lambda_k^1 = \frac{1}{(n_k-1)B_k} + o(\frac{1}{N^2}) = \frac{\gamma}{(n_k-1)C_k^*} + o(\frac{1}{N})$. Therefore, $\lambda_k^1$ is independent of $\alpha_k$ if we ignore the term $o(\frac{1}{N})$. By the same logic, $\lambda_k^0$ is independent of $\alpha_k$ if we ignore the term $o(\frac{1}{N})$. Consequently, the comparative statics of $B_k^1$, $|b_k^1|$ and $B_k^0$ are completely determined by $1 - C_k^1$, $|c_k^1|$ and $1 - C_k^0$. Since $1 - C_k^1$ and $|c_k^1|$ are decreasing in $\alpha_k$, so are $B_k^1$ and $|b_k^1|$. Since $C_k^0$ is independent of $\alpha_k$, so is $B_k^0$.

B.6. **Proof of Lemma 5.7.** Define $a_k = \alpha_k a_k^1 + (1-\alpha_k)a_k^0$, $B_k = \alpha_k B_k^1 + (1-\alpha_k)B_k^0$, and $b_k = \alpha_k b_k^1$. We first solve for $\mathbb{E}(x_{k,i}^1)^2$ and $\mathbb{E}(x_{k,i}^0)^2$. Since $a_k^1 - a_k = o(\frac{1}{N})$ and $a_k\bar{s}_k - B_k p_k + b_k p_{-k} = 0$, then $x_{k,i}^1 = a_k^1(s_k^i - \bar{s}_k) + (B_k - B_k^1)p_k - (b_k - b_k^1)p_{-k} + o(\frac{1}{N})$.

Since $p_k = \zeta_k(\bar{s}_k + \delta_k \bar{s}_{-k}) + o(\frac{1}{N}) = \bar{s}_k + o(1)$, $B_k - B_k^1 = -(1-\alpha_k)(B_k^1 - B_k^0) = o(\frac{1}{N})$, and $b_k - b_k^1 = -(1-\alpha_k)b_k^1 = o(1)$, we have

$$x_{k,i}^1 = a_k^1(s_k^i - \bar{s}_k) - (1-\alpha_k)(B_k^1 - B_k^0)\bar{s}_k + (1-\alpha_k)b_k^1\bar{s}_{-k} + o(\frac{1}{N}). \tag{42}$$

Since $\mathbb{E}(s_k^i - \bar{s}_k) = \mathbb{E}(s_k^i - \bar{s}_k)\bar{s}_k = \mathbb{E}(s_k^i - \bar{s}_k)\bar{s}_{-k} = 0$, then (42) implies that

$$\mathbb{E}(x_{k,i}^1)^2 = (a_k^1)^2\mathbb{E}(s_k^i - \bar{s}_k)^2 + (1-\alpha_k^2)\mathbb{E}((B_k^1 - B_k^0)\bar{s}_k - b_k^1\bar{s}_{-k})^2 + o(\frac{1}{N^2}).$$

By $(a_k^1, B_k^1 - B_k^0, b_k^1) = (\gamma + \lambda_k^1)^{-1}(C_{ks}^1, C_k^0 - C_k^1, c_k^1) + o(\frac{1}{N^2})$, we have

$$\mathbb{E}(x_{k,i}^1)^2 = \frac{1}{(\gamma+\lambda_k^1)^2}\Big((C_{ks}^1)^2\mathbb{E}(s_k^i - \bar{s}_k)^2 + (1-\alpha_k)^2 H_k\Big) + o(\frac{1}{N^2}). \tag{43}$$

where $H_k = \mathbb{E}((C_k^1 - C_k^0)\bar{s}_k + c_k^1\bar{s}_{-k})^2$. Similarly, we get

$$\mathbb{E}(x_{k,i}^0)^2 = \frac{1}{(\gamma+\lambda_k^0)^2}\Big((C_{ks}^0)^2\mathbb{E}(s_k^i - \bar{s}_k)^2 + \alpha_k^2 H_k\Big) + o(\frac{1}{N^2}). \tag{44}$$

Next, we solve for $W_k^1$ and $W_k^0$. Since $\mathbb{E}(\theta_k^i | s_k^i, p_k, p_{-k}) - p_k = (\gamma + \lambda_k^1)x_{k,i}^1$, then

$$W_k^1 = \mathbb{E}[x_{k,i}^1(\mathbb{E}(\theta_k^i | s_k^i, p_k, p_{-k}) - p_k) - \frac{\gamma}{2}(x_{k,i}^1)^2] = (\frac{\gamma}{2} + \lambda_k^1)\mathbb{E}(x_{k,i}^1)^2.$$

Substituting (43) into the above expression, we get

$$W_k^1 = \frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2}\Big((C_{ks}^1)^2\mathbb{E}(s_k^i - \bar{s}_k)^2 + (1-\alpha_k)^2\mathbb{E}((C_k^1 - C_k^0)\bar{s}_k + c_k^1\bar{s}_{-k})^2\Big) + o(\frac{1}{N^2}). \tag{45}$$

Similarly,

$$W_k^0 = \frac{\frac{\gamma}{2}+\lambda_k^0}{(\gamma+\lambda_k^0)^2}\Big((C_{ks}^0)^2\mathbb{E}(s_k^i-\bar{s}_k)^2+\alpha_k^2\mathbb{E}((C_k^1-C_k^0)\bar{s}_k+c_k^1\bar{s}_{-k})^2\Big)+o(\frac{1}{N^2}). \tag{46}$$

Finally, we estimate $\frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2}$ and $\frac{\frac{\gamma}{2}+\lambda_k^0}{(\gamma+\lambda_k^0)^2}$. By $\lambda_k^1 = \frac{1}{(n_k-1)B_k}+o(\frac{1}{N^2})$, $\lambda_k^1 = o(1)$ and $(\gamma+\lambda_k^1)B_k = C_k^* + o(1)$, we get $\frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2} = \frac{1}{2\gamma}(1-(\frac{\lambda_k^1}{\gamma+\lambda_k^1})^2) = \frac{1}{2\gamma}(1-(\frac{1}{(n_k-1)B_k(\gamma+\lambda_k^1)})^2)+o(\frac{1}{N^2}) = \frac{1}{2\gamma}(1-(\frac{1}{(n_k-1)C_k^*})^2)+o(\frac{1}{N^2})$. Similarly, $\frac{\frac{\gamma}{2}+\lambda_k^0}{(\gamma+\lambda_k^0)^2} = \frac{1}{2\gamma}(1-(\frac{1}{(n_k-1)C_k^*})^2)+o(\frac{1}{N^2})$.

B.7. **Proof of Proposition 5.8.** We first compute two preliminary estimations.

Since $\frac{C_k^1-C_k^0}{c_k^1} = \frac{\phi\eta_k}{\rho_k}+o(\frac{1}{N^2})$, $\lim\mathbb{E}(\bar{s}_k)^2 = \rho_k\sigma_{\theta_k}^2$, and $\lim\mathbb{E}(\bar{s}_{-k})^2 = \rho_{-k}\sigma_{\theta_{-k}}^2$, then

$$\begin{aligned}\mathbb{E}(s_k^i-\bar{s}_k)^2 &= \mathbb{E}(s_k^i)^2-\mathbb{E}(\bar{s}_k)^2 = X_k,\\ E[(C_k^1-C_k^0)\bar{s}_k+c_k^1\bar{s}_{-k}]^2 &= (c_k^1)^2E(\frac{\phi\eta_k}{\rho_k}\bar{s}_k-\bar{s}_{-k})^2 = (c_k^1)^2Y_k+o(\frac{1}{N^2}),\end{aligned} \tag{47}$$

where $X_k \equiv \frac{n_k-1}{n_k}(1-\rho_k+\sigma_k^2)\sigma_{\theta_k}^2$ and $Y_k \equiv \frac{\rho_k\rho_{-k}-\phi^2}{\rho_k}\sigma_{\theta_{-k}}^2$. By (45) and (47), we have

$$W_k^1 = \frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2}\left((C_k^*)^2X_k+\frac{1}{n_k^2}\frac{(1-\rho_k)^2\phi^2\sigma_{\theta_k}^2}{\rho_k(\rho_k\rho_{-k}-\phi^2)}\frac{(1-\alpha_k)^2}{(\alpha_k+\frac{1-\rho_k}{\sigma_k^2})^2}\right)+o(\frac{1}{N^2}), \tag{48}$$

which holds since $C_{ks}^1 = C_k^*+o(\frac{1}{N^2})$ and (33). Thus, we have $\frac{dW_k^1}{d\alpha_k} < 0$.

By (46) and (47), we have

$$W_k^0 = \frac{\frac{\gamma}{2}+\lambda_k^0}{(\gamma+\lambda_k^0)^2}\left((C_k^*)^2X_k-\frac{1}{n_k^2}\frac{(1-\rho_k)^2\phi^2\sigma_{\theta_k}^2}{\rho_k(\rho_k\rho_{-k}-\phi^2)}\frac{2\alpha_k-\alpha_k^2}{(\alpha_k+\frac{1-\rho_k}{\sigma_k^2})^2}\right)+o(\frac{1}{N^2}), \tag{49}$$

which holds because of (33) and (34). Consequently, $\frac{dW_k^0}{d\alpha_k} < 0$ if and only if $\alpha_k < \frac{1-\rho_k}{1-\rho_k+\sigma_k^2}$. Then by (48) and (49), we obtain

$$W_k^1 - W_k^0 = \frac{1}{2\gamma n_k^2}\frac{(1-\rho_k)^2\phi^2\sigma_{\theta_k}^2}{\rho_k(\rho_k\rho_{-k}-\phi^2)}\frac{1}{(\alpha_k+\frac{1-\rho_k}{\sigma_k^2})^2}+o(\frac{1}{N^2}). \tag{50}$$

Thus, $W_k^1 - W_k^0$ is decreasing in $\alpha_k$ and $W_k^1 - W_k^0 > 0$.

By (48), (49), and $\frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2} = \frac{\frac{\gamma}{2}+\lambda_k^0}{(\gamma+\lambda_k^0)^2}+o(\frac{1}{N^2})$, we have

$$W_k = \frac{\frac{\gamma}{2}+\lambda_k^1}{(\gamma+\lambda_k^1)^2}\left((C_k^*)^2X_k-\frac{1}{n_k^2}\frac{(1-\rho_k)^2\phi^2\sigma_{\theta_k}^2}{\rho_k(\rho_k\rho_{-k}-\phi^2)}\frac{\alpha_k(1-\alpha_k)}{(\alpha_k+\frac{1-\rho_k}{\sigma_k^2})^2}\right)+o(\frac{1}{N^2}). \tag{51}$$

Hence, $W_k$ is decreasing in $\alpha_k$ if and only if $\alpha_k < \hat{\alpha}_k = \frac{1-\rho_k}{2(1-\rho_k)+\sigma_k^2}$. Finally, by (51), $W_k$ is the same when $\alpha_k = 1$ or $\alpha_k = 0$.

B.8. **Proof of Proposition 5.9.** Let $\mathrm{Var}_k^1 = \mathrm{Var}(\theta_k^i | s_k^i, p_k, p_{-k})$, $\mathrm{Var}_k^0 = \mathrm{Var}(\theta_k^i | s_k^i, p_k)$. Let $X = \theta_k^i$, $Y = (s_k^i, \bar{s}_k, \bar{s}_{-k})$ and $Y_1 = (s_k^i, p_k, p_{-k})$ and $Y_0 = (s_k^i, p_k)$.

Since $\Sigma_{X,Y}\Sigma_{Y,Y}^{-1} = (C_k^*, (1-C_k^*)\frac{y_{k2}}{y_{k3}}, (1-C_k^*)\frac{y_{k1}}{y_{k3}})$ and $\Sigma_{Y,X} = (1, \rho_k + \frac{1-\rho_k}{n_k}, \phi)\sigma_{\theta_k}^2$, then by the projection theorem,

$$\mathrm{Var}_k^1 = \mathrm{Var}[X|Y] = \mathrm{Var}(X) - \Sigma_{X,Y}\Sigma_{Y,Y}^{-1}\Sigma_{Y,X} = \frac{\sigma_k^2}{\kappa_k}(1-\rho_k^1)\sigma_{\theta_k}^2, \tag{52}$$

where $\rho_k^1 = \rho_k - \frac{\sigma_k^2}{n_k}\frac{y_{k2}}{y_{k3}} = \rho_k - \frac{\sigma_k^2}{n_k} + \frac{\sigma_k^2}{n_k^2}\frac{\rho_{-k}\kappa_k}{\rho_k\rho_{-k}-\phi^2} + o(\frac{1}{N^2})$ is independent of $\alpha_k$. Therefore, $\tau_k^1 = \mathrm{Var}(\theta_k^i) - \mathrm{Var}_k^1$ is independent of $\alpha_k$.

Since $\Sigma_{X,Y_0}\Sigma_{Y_0,Y_0}^{-1} = (C_{ks}^0, C_k^0)$ and $\Sigma_{Y_0,X} = (\mathrm{Cov}(\theta_k^i, s_k^i), \mathrm{Cov}(\theta_k^i, p_k))$ (see Lemma 4.3), then by the projection theorem,

$$\mathrm{Var}_k^0 = \mathrm{Var}[X|Y_0] = \mathrm{Var}(X) - \Sigma_{X,Y_0}\Sigma_{Y_0,Y_0}^{-1}\Sigma_{Y_0,X} = \frac{\sigma_k^2}{\kappa_k}(1-\rho_k^0)\sigma_{\theta_k}^2, \tag{53}$$

where $\rho_k^0 = \rho_k - \frac{\sigma_k^2}{n_k} + \frac{\sigma_k^2}{n_k^2}(\frac{\kappa_k}{\rho_k} + \frac{\kappa_k}{\rho_k}\frac{\phi^2}{\rho_k\rho_{-k}-\phi^2}\frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}}(2 - \frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}})) + o(\frac{1}{N^2})$ is increasing in $\alpha_k$. Since $\tau_k^0 = \mathrm{Var}(\theta_k^i) - \mathrm{Var}_k^0$ is increasing in $\rho_k^0$, then $\tau_k^0$ is increasing in $\alpha_k$.

B.9. **Proof of Proposition 5.10.** Let $E_k^1 = \mathbb{E}(\theta_k^i | s_k^i, p_k, p_{-k})$ and $E_k^0 = \mathbb{E}(\theta_k^i | s_k^i, p_k)$. By (52) and (53), we have

$$\tau_k^1 - \tau_k^0 = \mathrm{Var}_k^0 - \mathrm{Var}_k^1 = \frac{\sigma_k^2}{\kappa_k}(\rho_k^1 - \rho_k^0)\sigma_{\theta_k}^2 = \frac{1}{n_k^2}\frac{(1-\rho_k)\phi^2}{\rho_k(\rho_k\rho_{-k}-\phi^2)}\frac{1}{(\alpha_k + \frac{1-\rho_k}{\sigma_k^2})^2}. \tag{54}$$

Comparing (50) with (54), we get

$$W_k^1 - W_k^0 = \frac{1}{2\gamma}\left(\tau_k^1 - \tau_k^0\right). \tag{55}$$

Next, we deduce $W_k^1 - W_k^0$ in a different way. By Lemma 5.7, we have

$$W_k^1 - W_k^0 = \frac{1}{2\gamma}\left(\mathbb{E}((E_k^1 - p_k)^2 - \mathbb{E}(E_k^0 - p_k)^2\right) + o(\frac{1}{N^2}). \tag{56}$$

By the law of total variance, $\mathrm{Var}(\theta_k^i) = \mathbb{E}(\mathrm{Var}_k^1) + \mathrm{Var}(E_k^1) = \mathrm{Var}_k^1 + \mathbb{E}(E_k^1)^2$, which holds since $\mathrm{Var}_k^1$ is a constant by normality and $\mathbb{E}(E_k^1) = 0$. Therefore,

$$\mathbb{E}(E_k^1)^2 = \mathrm{Var}(\theta_k^i) - \mathrm{Var}_k^1 = \tau_k^1, \;\; \mathbb{E}(E_k^0)^2 = \mathrm{Var}(\theta_k^i) - \mathrm{Var}_k^0 = \tau_k^0. \tag{57}$$

Thus, we have $\mathbb{E}((E_k^1 - p_k)^2 - \mathbb{E}(E_k^0 - p_k)^2 = \tau_k^1 - \tau_k^0 + 2\mathbb{E}(p_k(E_k^0 - E_k^1))$.

We then substitute the above equation to (56) and get

$$W_k^1 - W_k^0 = \frac{1}{2\gamma}\left(\tau_k^1 - \tau_k^0 + 2\mathbb{E}(p_k(E_k^0 - E_k^1))\right) + o(\frac{1}{N^2}). \tag{58}$$

From (55) and (58), we get $\mathbb{E}(p_k(E_k^0 - E_k^1)) = o(\frac{1}{N^2})$. Since $(1-\alpha_k)x_{k,i}^0 + \alpha_k x_{k,i}^1 = 0$, then $(1-\alpha_k)E_k^0 + \alpha_k E_k^1 - p_k = 0$. Therefore, we have

$$\mathbb{E}(p_k E_k^0) = \mathbb{E}(p_k E_k^1) = \mathrm{Var}(p_k). \tag{59}$$

Hence, by (57) and (59), $\mathbb{E}(E_k^1 - p_k)^2 = \mathbb{E}(E_k^1)^2 - \mathbb{E}(p_k)^2 = \tau_k^1 - \mathrm{Var}(p_k)$.

Similarly, $\mathbb{E}(E_k^0 - p_k)^2 = \tau_k^0 - \mathrm{Var}(p_k)$. Therefore, we get the decomposition of $W_k^1$ and $W_k^0$ in Proposition 5.10. In addition, since $\tau_k^1$ is independent of $\alpha_k$ and $W_k^1$ is decreasing in $\alpha_k$, $\mathrm{Var}(p_k)$ is increasing in $\alpha_k$.

B.10. **Proof of Proposition 5.11.** First, we calculate $r_p$ and $r_s$. Since $p_k = \zeta_k(\bar{s}_k + \delta_k \bar{s}_{-k}) + o(\frac{1}{N})$ and $p_{-k} = \zeta_{-k}(\bar{s}_{-k} + \delta_{-k}\bar{s}_k) + o(\frac{1}{N})$, then ignoring the term of order $o(\frac{1}{N})$, we have

$$\mathrm{Cov}(p_k, p_{-k}) = \zeta_k\zeta_{-k}[(1+\delta_k\delta_{-k})\mathrm{Cov}(\bar{s}_k, \bar{s}_{-k}) + \delta_k \mathrm{Var}(\bar{s}_{-k}) + \delta_{-k}\mathrm{Var}(\bar{s}_k)],$$

$$\mathrm{Var}(p_k) = \zeta_k^2[\mathrm{Var}(\bar{s}_k) + \delta_k^2\mathrm{Var}(\bar{s}_{-k}) + 2\delta_k\mathrm{Cov}(\bar{s}_k, \bar{s}_{-k})],$$

$$\mathrm{Var}(p_{-k}) = \zeta_{-k}^2[\mathrm{Var}(\bar{s}_{-k}) + \delta_{-k}^2\mathrm{Var}(\bar{s}_k) + 2\delta_{-k}\mathrm{Cov}(\bar{s}_k, \bar{s}_{-k})].$$

Define $\omega_k = \sqrt{\frac{\mathrm{Var}(\bar{s}_{-k})}{\mathrm{Var}(\bar{s}_k)}}$ and $\omega_{-k} = \sqrt{\frac{\mathrm{Var}(\bar{s}_k)}{\mathrm{Var}(\bar{s}_{-k})}}$. We get

$$r_p = \frac{\mathrm{Cov}(p_k, p_{-k})}{\sqrt{\mathrm{Var}(p_k)\mathrm{Var}(p_{-k})}} = \frac{(1+\delta_k\delta_{-k})r_s + \delta_k\omega_k + \delta_{-k}\omega_{-k}}{\sqrt{1+\delta_k^2\omega_k^2 + 2r_s\omega_k\delta_k}\sqrt{1+\delta_{-k}^2\omega_{-k}^2 + 2r_s\omega_{-k}\delta_{-k}}}.$$

Since $\mathrm{Cov}(\bar{s}_k, \bar{s}_{-k}) = \phi\sigma_{\theta_k}\sigma_{\theta_{-k}}$, $\mathrm{Var}(\bar{s}_k) = (\rho_k + \frac{\kappa_k}{n_k})\sigma_{\theta_k}^2$ and $\mathrm{Var}(\bar{s}_{-k}) = (\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}})\sigma_{\theta_{-k}}^2$, then $r_s = \frac{\phi}{\sqrt{(\rho_k + \frac{\kappa_k}{n_k})(\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}})}}$.

Second, we prove that $r_p > 0$ and $r_s > 0$ if and only if $\phi > 0$. If $\phi > 0$, then $r_s > 0$, $\delta_k > 0$ and $\delta_{-k} > 0$, thus $r_p > 0$. If $\phi < 0$, then $r_s < 0$, $\delta_k < 0$ and $\delta_{-k} < 0$, thus $r_p < 0$.

Next, we prove that $|r_p|$ is increasing in $\alpha_k$ and $\alpha_{-k}$. We first check the case where $\phi > 0$. We have

$$\frac{\partial r_p}{\partial \delta_k} = \frac{\omega_k(1-r^2)(1-\delta_k\delta_{-k})}{(1+\delta_k^2\omega_k^2 + 2r_s\omega_k\delta_k)^{\frac{3}{2}}\sqrt{1+\delta_{-k}^2\omega_{-k}^2 + 2r_s\omega_{-k}\delta_{-k}}} > 0,$$

using the fact that $r_s < 1$, $1 - \delta_k\delta_{-k} > 0$, and $\omega_k\omega_{-k} = 1$. If $\phi < 0$, we replace $\phi$, $r_p$, $r_s$, $\delta_k$ and $\delta_{-k}$ with $|\phi|$, $|r_p|$, $|r_s|$, $|\delta_k|$ and $|\delta_{-k}|$. By definition, $|r_p|$ is increasing in $|\delta_k|$. Together with the fact that $|\delta_k|$ is increasing in $\alpha_k$, we have $|r_p|$ is increasing in $\alpha_k$. By symmetry, $|r_p|$ is also increasing in $\alpha_{-k}$.

Finally, we prove that $|r_p| > |r_s|$. Notice that $|r_p| = |r_s|$ if $\alpha_k = \alpha_{-k} = 0$. Since $|r_p|$ is increasing in $\alpha_k$ and $\alpha_{-k}$, then $|r_p| > |r_s|$.

B.11. **Proof of Proposition 5.12.** Let $p'_k$ and $p'_{-k}$ denote the prices under the counterfactual. Since $p_k = \zeta_k(\bar{s}_k + \delta_k \bar{s}_{-k}) + o(\frac{1}{N})$, $p'_k = \zeta_k(\bar{s}_k + \delta_k \bar{s}'_{-k}) + o(\frac{1}{N})$, $p_{-k} = \zeta_{-k}(\bar{s}_{-k} + \delta_{-k}\bar{s}_k) + o(\frac{1}{N})$ and $p'_{-k} = \zeta_{-k}(\bar{s}'_{-k} + \delta_{-k}\bar{s}_k) + o(\frac{1}{N})$, then

$$T_k \equiv \frac{\mathrm{Var}(p'_k) - \mathrm{Var}(p_k)}{\mathrm{Var}(p'_{-k}) - \mathrm{Var}(p_{-k})} = \frac{\zeta_k^2 \delta_k^2 (\mathrm{Var}(\bar{s}'_{-k}) - \mathrm{Var}(\bar{s}_{-k})) + o(\frac{1}{N})}{\zeta_{-k}^2 (\mathrm{Var}(\bar{s}'_{-k}) - \mathrm{Var}(\bar{s}_{-k})) + o(\frac{1}{N})} = \frac{(\delta_k \zeta_k)^2}{\zeta_{-k}^2} + o(\frac{1}{N}).$$

If we ignore the terms of order $o(\frac{1}{N})$, then $|\delta_k \zeta_k|$ is increasing in $\alpha_k$ (see Lemma 5.3) and $\zeta_{-k}$ is independent of $\alpha_k$ (see online appendix Section D.1), hence $T_k$ is increasing in $\alpha_k$.

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# INFORMATION SPILLOVER IN MARKETS WITH HETEROGENEOUS TRADERS: ONLINE APPENDIX

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In this online Appendix A, we first prove the results in Section 3 of the main paper. In Appendix B, we consider an extension with systematic risk, which restores information spillover even in the large market limit. Appendix C analyzes a setting where all traders trade in both markets. In Appendix D, we provide additional comparative statics results in the main model and some technical results. The proofs of the results in Appendices B–E are relegated to Appendices F and G.

## APPENDIX A. PROOFS OF THE RESULTS IN SECTION 3 OF THE MAIN PAPER

In the full information spillover benchmark, the linear equilibrium is derived as follows.

(i) Trader $i$'s first order condition is

$$\mathbb{E}\left[\theta_k^i | s_k^i, p_k, p_{-k}\right] - p_k = (\gamma + \lambda_k^1) x_k^i, \tag{1}$$

where $\lambda_k^1 = dp_k / dx_k^i$ is trader's price impact in market $k$.

(ii) The price impact $\lambda_k^1$. In the two market-clearing conditions, we take derivative with respect to $x_k^i$ to obtain

$$1 + \sum_{j \neq i, j \in \mathcal{N}_k^1} \left[\frac{\partial x_k^j}{\partial p_k}\frac{dp_k}{dx_k^i} + \frac{\partial x_k^j}{\partial p_{-k}}\frac{dp_{-k}}{dx_k^i}\right] = 0, \quad \sum_{l \in \mathcal{N}_{-k}^1} \left[\frac{\partial x_{-k}^l}{\partial p_{-k}}\frac{dp_{-k}}{dx_k^i} + \frac{\partial x_{-k}^l}{\partial p_k}\frac{dp_k}{dx_k^i}\right] = 0,$$

from which we solve for the price impact:

$$\lambda_k^1 = -\left\{\sum_{j \neq i, j \in \mathcal{N}_k^1} \left[\frac{\partial x_k^j}{\partial p_k} - \frac{\partial x_k^j}{\partial p_{-k}}\left(\frac{\sum_{l \in \mathcal{N}_{-k}^1} \frac{\partial x_{-k}^l}{\partial p_k}}{\sum_{l \in \mathcal{N}_{-k}^1} \frac{\partial x_{-k}^l}{\partial p_{-k}}}\right)\right]\right\}^{-1} = \frac{B_{-k}^1}{(n-1)\left(B_k^1 B_{-k}^1 - b_k^1 b_{-k}^1\right)}, \tag{2}$$

where the second equality follows from the conjectured linear strategy.

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(iii) Trader $i$'s inference: $\mathbb{E}\left[\theta_k^i|s_k^i, p_k, p_{-k}\right]$. The market-clearing condition for asset $k$ implies that

$$\bar{s}_k^1 = \frac{B_k^1 p_k - b_k^1 p_{-k}}{a_k^1}, \tag{3}$$

where $\bar{s}_k^1 = (\sum_{i\in\mathcal{N}_k^1} s_k^i)/n$ is the average signal of all (more informed) traders in market $k$. Therefore, $(p_I, p_{II})$ is a linear combination of $(\bar{s}_I^1, \bar{s}_{II}^1)$ and hence is also jointly normal distributed. The projection theorem then implies that

$$\mathbb{E}\left[\theta_k^i|s_k^i, p_k, p_{-k}\right] = F_s^1 s_k^i + \left(\frac{F^1 B_k^1}{a_k^1} - \frac{F_-^1 b_{-k}^1}{a_{-k}^1}\right) p_k + \left(\frac{F_-^1 B_{-k}^1}{a_{-k}^1} - \frac{F^1 b_k^1}{a_k^1}\right) p_{-k}, \tag{4}$$

where

$$F_s^1 = \frac{1-\rho}{1-\rho+\sigma^2},\ F^1 = \frac{\sigma^2}{1-\rho+\sigma^2}\cdot\frac{\rho^2 + \frac{\rho(1-\rho+\sigma^2)}{n} - \phi^2}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2},\ F_-^1 = \frac{\sigma^2\cdot\frac{\phi}{n}}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2}.$$

(iv) Substituting (4) and (2) into (1) and matching coefficients, we get $a_k^1 \equiv a^1$, $B_k^1 \equiv B^1$, and $b_k^1 \equiv b^1$, for $k = I, II$, where

$$a^1 = \frac{(n-2)F_s^1 - F^1}{\gamma(n-1)},\ B^1 = \frac{(F_s^1 + F^1)a^1}{(F_s^1+F^1)^2 - (F_-^1)^2},\ b^1 = \frac{F_-^1 a^1}{(F_s^1+F^1)^2 - (F_-^1)^2}. \tag{5}$$

The equilibrium in the no information spillover benchmark can be derived similarly.

A.1. **Proof of Proposition 3.1.** We first solve for $\mathbb{E}\left[\theta_k^i|s_k^i, p_k, p_{-k}\right]$. It follows from the market clearing condition (3) that $p_k = a_k^1(B_{-k}^1\bar{s}_k^1 + b_k^1\bar{s}_{-k}^1)/(B_k^1 B_{-k}^1 - b_k^1 b_{-k}^1)$, so it suffices to solve for $\mathbb{E}[\theta_k^i|s_k^i, \bar{s}_k^1, \bar{s}_{-k}^1]$, which is given in equation (4) above.

Next, since $F_-^1 > 0$ if and only if $\phi > 0$ and $b_k^1$ has the same sign as $F_-^1$, we have $b^1 > 0$ if and only if $\phi > 0$. Second, since $\rho^2 + \rho(1-\rho+\sigma^2)/n - \phi^2 > |\phi|(1-\rho+\sigma^2)/n$, we have $F^1 \geq |F_-^1|$ and thus $\frac{|b^1|}{B^1} = \frac{|F_-^1|}{F_s^1+F^1} < \frac{|F_-^1|}{F^1} < 1$. Since $F^1 > 0$, we have $a^1 = \frac{(n-2)F_s^1 - F^1}{\gamma(n-1)} < \frac{F_s^1}{\gamma}\frac{n-2}{n-1}$. Since $F^1 > |F_-^1|$, we also have $B^1 = \frac{a^1}{F_s^1+F^1-|F_-^1|} < \frac{a^1}{F_s^1} < \frac{1}{\gamma}\frac{n-2}{n-1}$. Finally, since $\rho^2 + \rho(1-\rho+\sigma^2)/n < (\rho + (1-\rho+\sigma^2)/n)^2 - \phi^2$, we have $\frac{\partial F^1}{\partial|\phi|} < 0$. Together with $\frac{\partial a^1}{\partial F^1} < 0$, we have $\frac{\partial a^1}{\partial|\phi|} > 0$. In addition, since $\frac{\partial F_-^1}{\partial|\phi|} > 0$, both $\frac{B^1}{a^1}$ and $\frac{|b^1|}{a^1}$ are increasing in $F_-^1$ and decreasing in $F^1$, and thus both are increasing in $|\phi|$. Together with $\frac{\partial a^1}{\partial|\phi|} > 0$, we have that $B^1$ and $|b^1|$ are increasing in $|\phi|$.

A.2. **Proof of Proposition 3.2.** From the market-clearing condition, $p_k = a_k^0 \bar{s}_k^0 / B_k^0$, where $\bar{s}_k^0 = \sum_{i' \in \mathcal{N}_k^0} s_k^{i'} / n$, we have

$$\mathbb{E}\left[\theta_k^{i'} | s_k^{i'}, p_k\right] = F_k^0 s_k^{i'} + F^0 \bar{s}_k^0 = F_k^0 s_k^{i'} + F^0 \frac{B_k^0 p_k}{a_k^0},$$

where

$$F_s^0 = \frac{(1-\rho)}{1-\rho+\sigma^2},\ F^0 = \frac{\sigma^2}{1-\rho+\sigma^2} \frac{\rho}{\rho + \frac{1-\rho+\sigma^2}{n}}.$$

Finally, it is clear that $a^0 > 0$ and $B^0 > 0$ are independent of $\phi$. Since $F^0 > 0$, we have $a^0 = \frac{(n-2)F_s^0 - F^0}{\gamma(n-1)} < \frac{F_s^0}{\gamma} \frac{n-2}{n-1}$ and thus $B^0 = \frac{a^0}{F_s^0 + F^0} < \frac{a^0}{F_s^0} < \frac{1}{\gamma} \frac{n-2}{n-1}$.

A.3. **Proof of Proposition 3.3.** First, $F_s^0 = F_s^1 = (1-\rho)/(1-\rho+\sigma^2)$. Next,

$$F^0 - F^1 = \frac{\sigma^2}{1-\rho+\sigma^2} \cdot \frac{\frac{(1-\rho+\sigma^2)\phi^2}{n}}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)\left(\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2\right)} > 0.$$

Therefore, $a^1 - a^0 = \frac{1}{\gamma(n-1)}(F^0 - F^1) > 0$. Moreover,

$$\frac{B^1}{a^1} = \frac{F_s^1 + F^1}{(F_s^1 + F^1)^2 - (F_-^1)^2} > \frac{1}{F_s^1 + F^1} > \frac{1}{F_s^0 + F^0} = \frac{B^0}{a^0}.$$

Together with $a^1 > a^0$, we have $B^1 > B^0$. Since $a^1 = \frac{F_s^1}{\gamma + \lambda^1}$, $a^0 = \frac{F_s^0}{\gamma + \lambda^0}$, and $F_s^1 = F_s^0$, then $a^1 > a^0$ implies that $\lambda^1 < \lambda^0$.

A.4. **Proof of Proposition 3.4.** With full information spillover, for any trader $i \in \mathcal{N}_k^1$, $x_k^i = \frac{1}{\gamma + \lambda^1}(\mathbb{E}[\theta_k^i | s_k^i, p_k, p_{-k}] - p_k)$, then her expected payoff is

$$W_k^1 = \mathbb{E}((\mathbb{E}[\theta_k^i | s_k^i, p_k, p_{-k}] - p_k)x_k^i - \frac{\gamma (x_k^i)^2}{2}) = (\frac{\gamma}{2} + \lambda^1)\mathbb{E}(x_k^i)^2.$$

By market clearing, the equilibrium demand is $x_k^i = a^1 s_k^i - B^1 p_k + b^1 p_{-k} = a^1(s_k^i - \bar{s}_k)$. Together with $a^1 = \frac{1}{\gamma + \lambda^1} F_s^1$, we have

$$W_k^1 = \frac{\frac{\gamma}{2} + \lambda^1}{(\gamma + \lambda^1)^2}(F_s^1)^2 \mathbb{E}(s_k^i - \bar{s}_k)^2.$$

$W_k^0$ can be solved similarly. Finally, since $\lambda^1 < \lambda^0$ and $F_s^1 = F_s^0$, then $W_k^1 > W_k^0$.

## Appendix B. Systematic Risk

In this section, we study how the equilibrium in market $k$ is influenced by information spillover $\alpha_k$ if there is a systematic risk for each trader in market. Each trader $i \in \mathcal{N}_k$ in market $k \in \{I, II\}$ privately observes a noisy signal

$$s_k^i = \theta_k^i + e_k + \varepsilon_k^i,$$

where the true value $\theta_k^i$, the idiosyncratic noise $\varepsilon_k^i$ and the systematic noise $e_k$ (which is absent in the original model) are normally distributed with mean zero and variance $\sigma_{\theta_k}^2$, $\sigma_{\varepsilon_k}^2$ and $\sigma_{e_k}^2$, respectively. Moreover, $(\varepsilon_I^i, \varepsilon_{II}^j, e_I, e_{II})_{i\in\mathcal{N}_I, j\in\mathcal{N}_{II}}$ are independent. Moreover, the above noises are independent of the true values. Let $\sigma_k^2 = \sigma_{\varepsilon_k}^2/\sigma_{\theta_k}^2$ ($\varsigma_k^2 = \sigma_{e_k}^2/\sigma_{\theta_k}^2$) be the variance ratio measuring the impact of idiosyncratic (systematic) noise relative to the true value in market $k$.

B.1. **Benchmark.** To illustrate the spillover effect of asset prices, we first solve for the closed-form equilibria in two benchmark cases and compare their behavioral and welfare implications. To simplify the algebra, here we assume the two markets are symmetric: $n_I = n_{II} = n$, $\rho_I = \rho_{II} \equiv \rho$, $\sigma_I^2 = \sigma_{II}^2 \equiv \sigma^2$ and $\varsigma_I^2 = \varsigma_{II}^2 \equiv \varsigma^2$.

If all traders are more informed, we conjecture a symmetric linear demand schedule:

$$x_k^i(p_k, s_k^i) = a_k^1 s_k^i - B_k^1 p_k + b_k^1 p_{-k},$$

where $a_k^1$, $B_k^1$ and $b_k^1$ are constants, for each $i \in \mathcal{N}_k^1$ and $k \in \{I, II\}$. If all traders are less informed, we conjecture a symmetric linear demand schedule:

$$x_k^i(p_k, s_k^i) = a_k^0 s_k^i - B_k^0 p_k,$$

where $a_k^0$ and $B_k^0$ are constants, for each $i \in \mathcal{N}_k^0$ and $k \in \{I, II\}$.

The following proposition illustrates each trader's conditional expectation of the true value, which is the key of the equilibrium behaviors.

**Proposition B.1.** *If all traders are more informed,*

$$\mathbb{E}\left[\theta_k^i | s_k^i, p_k, p_{-k}\right] = C_s^1 s_k^i + \left(\frac{C^1 B_k^1}{a_k^1} - \frac{C_-^1 b_{-k}^1}{a_{-k}^1}\right) p_k + \left(\frac{C_-^1 B_{-k}^1}{a_{-k}^1} - \frac{C^1 b_k^1}{a_k^1}\right) p_{-k}, \tag{6}$$

*where*

$$C_s^1 = \frac{1-\rho}{1-\rho+\sigma^2},\ C^1 = \frac{(\rho + \frac{1-\rho+\sigma^2}{n} + \varsigma^2)(\rho + \frac{1-\rho}{n}) - \phi^2}{(\rho + \frac{1-\rho+\sigma^2}{n} + \varsigma^2)^2 - \phi^2} - C_s^1,$$

$$C_-^1 = \frac{\phi(\frac{\sigma^2}{n} + \varsigma^2)}{(\rho + \frac{1-\rho+\sigma^2}{n} + \varsigma^2)^2 - \phi^2}.$$

*If all traders are less informed,*

$$\mathbb{E}\left[\theta_k^i | s_k^i, p_k\right] = C_s^0 s_k^i + \frac{C^0 B_k^0}{a_k^0} p_k, \tag{7}$$

*where*

$$C_s^0 = \frac{1-\rho}{1-\rho+\sigma^2}, \quad C^0 = \frac{\rho + \frac{1-\rho}{n}}{\rho + \frac{1-\rho+\sigma^2}{n} + \varsigma^2} - \frac{1-\rho}{1-\rho+\sigma^2}.$$

The main finding of Proposition B.1 is that the conditional expectation of the true value with systematic noises is the same as the one in the original model without systematic noises if we replace the term $\frac{\sigma^2}{n}$ by $\frac{\sigma^2}{n} + \varsigma^2$. Intuitively, $\frac{\sigma^2}{n} + \varsigma^2$ represents the variance of $\bar{s}_k$ if there are systematic noises, and $\frac{\sigma^2}{n}$ represents the variance of $\bar{s}_k$ in the original model.

**Lemma B.2.** *The following properties hold:*

(1) $\frac{dC^1}{d\varsigma^2} < 0$ *and* $\frac{dC^0}{d\varsigma^2} < 0$.
(2) $\frac{d|C_-^1|}{d\varsigma^2} > 0$ *if and only if* $\varsigma^2 < \sqrt{(\rho + \frac{1-\rho}{n})^2 - \phi^2} - \frac{\sigma^2}{n}$.

Lemma B.2 studies how the systematic noises influence the equilibrium. If there is higher uncertainty due to systematic risk (larger $\varsigma^2$), then the average signal in the own market $\bar{s}_k$ is less precise to predict the true value of asset $k$. However, the impact of noisier systematic risk on the predictability of the average signal from other market $\bar{s}_{-k}$, which captures information externalities, is mixed. On the one hand, nosier signals hurt the forecast power of both $\bar{s}_k$ and $\bar{s}_{-k}$. On the other hand, there is a crowding-out effect. Less predictability of $\bar{s}_k$ gives more room for information externalities $\bar{s}_k$ to play a role. It turns out that if the systematic risk is small enough, the crowding-out effect dominates.

Lemma B.3 studies the significance of systematic noises in the large market if the number of traders converge to infinity.

**Lemma B.3.** *In the limit as* $n \to +\infty$,

(1) $\lim_{n\to+\infty} C_-^1 = \frac{\phi\varsigma^2}{(\rho+\varsigma^2)^2 - \phi^2}$.
(2) $\lim_{n\to+\infty} b_k^1 = 0$ *if and only if* $\lim_{n\to+\infty} C_-^1 = 0$.

Lemma B.3 shows that the information externalities exist ($b_k^1 \neq 0$) in the limit if and only if there are systematic risks ($\varsigma^2 > 0$). Intuitively, if the own signals only contain idiosyncratic noises ($\varsigma^2 = 0$), then as there are a large number of traders, the own signals get almost perfectly precise in terms of predicting the true value, which leaves no room for other signals such as prices from other markets to play a role, i.e., $b_k^1 = 0$. On the contrary, with systematic noises, the own signal of a trader never becomes perfectly informative of the true value, hence information externalities always have an impact which is actually proportional to the variance of the systematic noises.

B.2. **General Model.** In this section, we study the impact of information spillover on the equilibrium if there are systematic noises and the total number of traders $N = n_I + n_{II}$ converges to infinity.

We first compare two types of traders in terms of inferences and strategies. Proposition B.4 asymptotically compares information inferences between two types of traders. Define $V_s = \varsigma_I^2 + \varsigma_{II}^2$ as the summation of variances of systematic noises in both markets. In order to get analytical solutions, we do Taylor expansion of the equilibrium outcomes in terms of $V_s$, which means that we focus on the limit case where the variances of systematic noises are small enough.[1]

**Proposition B.4.** *The informational comparison of two types of traders is*

*(1)* $|c_k^1| = c_k^* \frac{\varsigma_k^2}{\sigma_k^2} + o(V_s)$, $|\delta_k| = \delta_k^* \frac{\varsigma_k^2}{\sigma_k^2} + o(V_s)$, $C_k^0 - C_k^1 = \Delta_k \frac{\varsigma_k^2}{\sigma_k^2} + o(V_s)$ *and* $C_{ks}^1 - C_{ks}^0 = \Delta_{ks}(\frac{\varsigma_k^2}{\sigma_k^2})^2 + o(V_s^2)$.

*(2)* $c_k^* > 0$, $\delta_k^* > 0$, $\Delta_k > 0$ *and* $\Delta_{ks} > 0$.

*(3) As* $V_s \to 0$, $c_k^1 = 0$, $\delta_k = 0$, $C_k^1 = C_k^0 = 1 - C_k^*$, $C_{ks}^1 = C_{ks}^0 = C_k^*$.

If the variances of systematic noises are small enough, then we have an asymptotic estimation of all the information parameters by doing Taylor expansion in terms of $V_s$, instead of $1/N$. We find that the analytical approximation is the same as that in the original model if we replace $\sigma_k^2/n_k$ by $\varsigma_k^2$. Intuitively, in the model with systematic risk, it is the variance of total noises $\varsigma_k^2 + \sigma_k^2/n_k$ plays the role of information spillover instead of just the variance of idiosyncratic noises $\sigma_k^2/n_k$. Since in the limit $n_k \to +\infty$, idiosyncratic noises do not have an impact due to law of large number, then only the variance of systematic noises $\varsigma_k^2$ determines the information spillover effect.

Since the comparison of information parameters between two types of traders lays the foundation of all the following analysis on how information spillover impacts the equilibrium, an immediate implication of Proposition B.4 is that information spillover $\alpha_k$ has similar impacts on the equilibrium outcome as in the original model, which are summarized as follows.

**Proposition B.5.** *Information spillover impacts traders' inferences as follows:*

*(1) First order effect: if we ignore the term* $o(V_s)$*, then*

[1] Notice that we first take limit $1/N \to 0$ and then take limit $V_s \to 0$. This implies that we implicitly assume that the speed of convergence of $V_s$ is slower than that of $1/N$.

- *$C_k^1$ is increasing in $\alpha_k$, and $|c_k^1|$ is decreasing in $\alpha_k$.*
- *$C_k^0$ is independent of $\alpha_k$;*

*(2) Second order effect: if we ignore the term $o(V_S^2)$, then*
- *$C_{ks}^1$ is independent of $\alpha_k$;*
- *$C_{ks}^0$ is decreasing in $\alpha_k$ if and only if $\alpha_k < \frac{1-\rho_k}{\sigma_k^2}$.*

Since there is no price impact in the limit ($\lambda_k^1 = \lambda_k^0 = 0$), then the results for behavior parameters are derived immediately as follows:

**Proposition B.6.** *The behavioral comparison of two types of traders is as follows:*

*(1) $a_k^1 - a_k^0 = a_k^*(\frac{\varsigma_k^2}{\sigma_k^2})^2 + o(V_s^2)$, $B_k^1 - B_k^0 = B_k^*\frac{\varsigma_k^2}{\sigma_k^2} + o(V_s)$ and $|b_k^1| = b_k^*\frac{\varsigma_k^2}{\sigma_k^2} + o(V_s)$.*

*(2) $a_k^* = \frac{1}{\gamma}\Delta_{ks} > 0$, $B_k^* = \frac{1}{\gamma}\Delta_k > 0$, $b_k^* = \frac{1}{\gamma}c_k^* > 0$.*

*(3) As $V_s \to 0$, $a_k^1 = a_k^0 = B_k^1 = B_k^0 = \frac{C_k^*}{\gamma}$, $b_k^1 = 0$.*

**Proposition B.7.** *Information spillover impacts traders' strategies as follows:*

*(1) First order effect: if we ignore the term $o(V_S)$, then*
- *$B_k^1$ and $|b_k^1|$ are decreasing in $\alpha_k$.*
- *$B_k^0$ is independent of $\alpha_k$.*

*(2) Second order effect: if we ignore the term $o(V_S^2)$, then*
- *$a_k^1$ is independent of $\alpha_k$;*
- *$a_k^0$ is decreasing in $\alpha_k$ if and only if $\alpha_k < \frac{1-\rho_k}{\sigma_k^2}$.*

The welfare implication is also the same as that in the original model. In both settings, it is the information channel that plays a predominant role, instead of price impacts.

**Proposition B.8.** *If we ignore the term $o(V_s^2)$, then*

*(1) $W_k^1$ is decreasing in $\alpha_k$.*

*(2) $W_k^0$ is decreasing $\alpha_k$ if and only if $\alpha_k < \alpha_k^* \equiv \frac{1-\rho_k}{1-\rho_k+\sigma_k^2}$.*

*(3) $W_k \equiv \alpha_k W_k^1 + (1-\alpha_k)W_k^0$ is decreasing in $\alpha_k$ if and only if $\alpha_k < \hat{\alpha}_k \equiv \frac{1-\rho_k}{2(1-\rho_k)+\sigma_k^2}$.*

*(4) $W_k$ is the same if $\alpha_k = 1$ and $\alpha_k = 0$.*

*(5) $W_k^1 - W_k^0$ is positive and decreasing in $\alpha_k$.*

## APPENDIX C. TRADING IN BOTH MARKETS

Consider a setting with two risky assets, $k \in \{I, II\}$, traded in two separate markets. For each $k \in \{I, II\}$, there are $n \in \mathbb{N}_+$ traders who trade both assets in both markets, and the set of traders is denoted by $\mathcal{N}$. For each trader $i \in \mathcal{N}$, the per-unit value of asset $k$ to her is $\theta_k^i$. The vector of all traders' values, $(\theta_I^i, \theta_{II}^i)_{i\in\mathcal{N}}$, is jointly normally distributed with a zero mean vector (as a normalization). The variance of $\theta_k^i$ is $\sigma_{\theta_k}^2 > 0$. The covariances satisfy: $\mathrm{Cov}(\theta_k^i, \theta_k^j) = \sigma_{\theta_k}^2 \rho_k > 0$ and $\mathrm{Cov}(\theta_k^i, \theta_{-k}^j) = \sigma_{\theta_k}\sigma_{\theta_{-k}}\phi \in \mathbb{R}$, for all $i, j \in \mathcal{N}$ and $k, -k \in \{I, II\}$ with $k \neq -k$. We assume $1 > \rho_k \geq |\phi| > 0$ to capture the idea that traders' values are more correlated within a market than across market.[2]

For each $k \in \{I, II\}$, trader $i \in \mathcal{N}$ privately observes two noisy signals $s_I^i = \theta_I^i + \varepsilon_I^i$ and $s_{II}^i = \theta_{II}^i + \varepsilon_{II}^i$ about her assets $I$ and $II$. For $k = I, II$, the noise $\varepsilon_k^i$ is normally distributed with mean zero and variance $\sigma_{\varepsilon_k}^2$ and independent across all traders $i$ and assets $k$. Let $\sigma_k^2 = \sigma_{\varepsilon_k}^2 / \sigma_{\theta_k}^2$ be the variance ratio measuring the impact of noise relative to the true value in market $k$. Assume that the noises $(\varepsilon_I^i, \varepsilon_{II}^i, \varepsilon_I^j, \varepsilon_{II}^j)_{i,j\in\mathcal{N}}$ and the values $(\theta_I^i, \theta_{II}^i, \theta_I^j, \theta_{II}^j)_{i,j\in\mathcal{N}}$ are independent. To simplify the algebra, we assume the two markets are symmetric: $\rho_I = \rho_{II} = \rho$ and $\sigma_I^2 = \sigma_{II}^2 = \sigma^2$.

The initial asset endowment of each trader $i \in \mathcal{N}$ is normalized to zero. The utility of trader $i \in \mathcal{N}$ from trading $x_k^i$ units of asset $k$ at a price $p_k \in \mathbb{R}$ is

$$u^i(x_I^i, x_{II}^i, p_I, p_{II}, \theta_I^i, \theta_{II}^i) = (\theta_I^i - p_I)\cdot x_I^i + (\theta_{II}^i - p_{II})\cdot x_{II}^i - \frac{\gamma}{2}\left((x_I^i)^2 + 2\omega x_I^i x_{II}^i + (x_{II}^i)^2\right),$$

which is linear in her value of the asset $\theta_k^i$ net off the asset price $p_k$ and has an inventory cost that is quadratic in the asset quantity $x_I^i$ and $x_{II}^i$, where $\gamma > 0$ is a commonly known constant representing traders' decreasing marginal values for holding each additional unit of asset and $\omega \in [0,1)$ captures the cost link between two assets.

Traders at each market submit demand schedules for the both assets and the equilibrium prices of the assets are simultaneously determined by the market-clearing conditions at both markets.

Given the submitted demand schedules in both markets $(x_I^i(p_I, p_{II}), x_{II}^i(p_I, p_{II}))_{i\in\mathcal{N}}$, the equilibrium price vector $(p_I^*, p_{II}^*) \in \mathbb{R}^2$ is determined by the two market-clearing conditions: for $k = I, II$,

$$\sum_{i\in\mathcal{N}} x_k^i(p_I^*, p_{II}^*) = 0.$$

For each $k \in \{I, II\}$, a trader $i \in \mathcal{N}$ receives $x_k^i(p_I^*, p_{II}^*)$ units of asset $k$ and pays $p_k^* x_k^i(p_I^*, p_{II}^*)$. We adopt linear Bayes Nash equilibrium as the solution concept. A strategy of a trader $i \in \mathcal{N}$ is a mapping $x_k^i(s_I^i, s_{II}^i, p_I, p_{II})$ from her realized signal $s_I^i$ and $s_{II}^i$ to a demand schedule contingent on $(p_I, p_{II})$. A *Bayes Nash equilibrium* is a strategy profile $(x_k^i)_{k\in\{I,II\}, i\in\mathcal{N}}$ such that for each trader $i \in \mathcal{N}$ and signal realization $s_I^i$ and $s_{II}^i$, the demand schedule

[2] $\rho_k \geq |\phi|$ is necessary for the covariance matrix of all signals to be positive semi-definite.

maximizes trader $i$'s expected payoff:

$$\mathbb{E}\left[u^i(x_I^i, x_{II}^i, p_I^*, p_{II}^*, \theta_I^i, \theta_{II}^i)\middle| s_I^i, s_{II}^i, p_I^*, p_{II}^*\right] \geq \mathbb{E}\left[u^i(\tilde{x}_I^i, \tilde{x}_{II}^i, \tilde{p}_I, \tilde{p}_{II}, \theta_I^i, \theta_{II}^i)\middle| s_I^i, s_{II}^i, \tilde{p}_I, \tilde{p}_{II}\right]$$

where $(p_I^*, p_{II}^*)$ is the market-clearing price vector given $x_I^i$ and $x_{II}^i$ and all other traders' equilibrium strategies and $(\tilde{p}_I, \tilde{p}_{II})$ is the market-clearing price vector given any strategy $\tilde{x}_I^i$ and $\tilde{x}_{II}^i$ and all other traders' equilibrium strategies. A Bayes Nash equilibrium is *linear* if all traders' equilibrium strategies are linear functions. We restrict attention to linear Bayes Nash equilibria that are symmetric.

Suppose all traders can condition their demand on the prices of both assets. We conjecture a symmetric linear demand schedule of a trader $i$ for each asset $k$:

$$x_k^i(p_k, s_k^i) = A s_k^i - a s_{-k}^i - B p_k + b p_{-k}, \tag{8}$$

where $A$, $a$, $B$, and $b$ are constants.

To solve the equilibrium demand schedule, there are four steps as follows:

(i) Trader $i$'s first order condition is:

$$\mathbb{E}\left[\theta_k^i | s_k^i, s_{-k}^i, p_k, p_{-k}\right] - p_k = (\gamma + \lambda_k) x_k^i + (\gamma\omega + L_k) x_{-k}^i. \tag{9}$$

The LHS captures the marginal revenue of trading in market $k$, while the RHS captures that marginal cost of trading in market $k$, which contains two terms $\lambda_k \equiv \frac{dp_k}{dx_k^i}$ and $L_k \equiv \frac{dp_{-k}}{dx_k^i}$, measuring how a change of trade in market $k$ impacts equilibrium price $p_k$ and $p_{-k}$, respectively.

(ii) We solve the price impact $\lambda_k$ and $L_k$. In the two market-clearing conditions, we take the derivative with respect to $x_k^i$ to obtain

$$1 + \sum_{j \neq i, j \in \mathcal{N}_k^1} \left[\frac{\partial x_k^j}{\partial p_k}\frac{dp_k}{dx_k^i} + \frac{\partial x_k^j}{\partial p_{-k}}\frac{dp_{-k}}{dx_k^i}\right] = 0, \sum_{l \in \mathcal{N}_{-k}^1} \left[\frac{\partial x_{-k}^l}{\partial p_{-k}}\frac{dp_{-k}}{dx_k^i} + \frac{\partial x_{-k}^l}{\partial p_k}\frac{dp_k}{dx_k^i}\right] = 0,$$

from which solve for price impacts as follows:

$$\lambda_k = \frac{B}{(n-1)(B^2 - b^2)}, \; L_k = \frac{b}{(n-1)(B^2 - b^2)}. \tag{10}$$

(iii) We solve the trader's belief of the asset value: $\mathbb{E}\left[\theta_k^i | s_k^i, s_{-k}^i, p_k, p_{-k}\right]$. The market-clearing condition for asset $k$ implies that

$$\bar{s}_k = \frac{AB - ab}{A^2 - a^2} p_k - \frac{Ab - aB}{A^2 - a^2} p_{-k}, \tag{11}$$

where $\bar{s}_k = (\sum_{i\in\mathcal{N}} s_k^i)/n$ is the average signal of all traders about asset $k$. Therefore, $(p_I, p_{II})$ is a linear combination of $(\bar{s}_I, \bar{s}_{II})$, and hence $(\theta_k^i, s_k^i, s_{-k}^i, p_k, p_{-k})$ is jointly normal distributed. The projection theorem implies that

(12)

$$\begin{aligned}
&\mathbb{E}\left[\theta_k^i | s_k^i, s_{-k}^i, p_k, p_{-k}\right] = C_s s_k^i + C\bar{s}_k + C_- \bar{s}_{-k} \\
&= C_s s_k^i + \left(\frac{(AB-ab)C}{A^2-a^2} - \frac{(Ab-aB)C_-}{A^2-a^2}\right) p_k + \left(\frac{(AB-ab)C_-}{A^2-a^2} - \frac{(Ab-aB)C}{A^2-a^2}\right) p_{-k},
\end{aligned}$$

where

$$C_s = \frac{1-\rho}{1-\rho+\sigma^2},\ C = \frac{\sigma^2}{1-\rho+\sigma^2} \cdot \frac{\rho^2 + \frac{\rho(1-\rho+\sigma^2)}{n} - \phi^2}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2},\ C_- = \frac{\sigma^2 \cdot \frac{\phi}{n}}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2}.$$

It is worth mentioning that the coefficient of $\bar{s}_{-k}^i$ is zero in (12). Intuitively, $\bar{s}_{-k}$ is a sufficient statistics containing all the information from market $-k$.

(iv) Substituting (12) and (10) into (9) and matching coefficients, we get the unique solution $(A, a, B, b)$:

$$\begin{aligned}
A &= \frac{(n-2)C_s - C + \omega C_-}{\gamma(n-1)(1-\omega^2)},\ a = \frac{C_- + \omega((n-2)C_s - C)}{\gamma(n-1)(1-\omega^2)}, \\
B &= \frac{(C_s+C)A + C_- a}{(C_s+C)^2 - (C_-)^2},\ b = \frac{C_- A + (C_s+C)a}{(C_s+C)^2 - (C_-)^2}.
\end{aligned} \tag{13}$$

The above strategy profile is an equilibrium if and only if $A > 0$. A simple sufficient condition for $A > 0$ is $(1-\rho)(n-2) > \sigma^2$. Furthermore, we establish the following result regarding the equilibrium coefficients.

**Proposition C.1.** *If $(1-\rho)(n-2) > \sigma^2$, there is a linear Bayes Nash equilibrium described by (13) such that $|a| < A < \frac{(n-2)C_s}{\gamma(n-1)(1-\omega^2)}$ and $|b| < B < \frac{n-2}{\gamma(n-1)(1-\omega^2)}$.*

In the equilibrium described above, we distinguish three channels that determine the equilibrium behaviors. First, the parameter $b$ captures the information externalities effect which exists in the main model where traders trade only one asset. Second, the parameter $a$ captures the cross-market price impact effect. Since each trader participates in both markets, then an increase of demand in one market will not only influence the price in the same market, but also the price in the other market, which directly influences the payoff of each trader. Finally, the parameter $\omega$ captures the cost linkage between two assets, which naturally changes the behavior of each trader. Notice that if we let $a = 0$ and $\omega = 0$, then the result is equivalent to the main model in which each trader trades only in one market.

**Proposition C.2.** *If $\phi > 0$, then*

- *$B$ and $b$ are increasing in $a$.*
- *$A$, $a$, $B$ and $b$ are increasing in $\omega$.*

Proposition C.2 establishes that traders react more aggressively to all the signals and prices if they trade in both markets, rather than only in one market.

## APPENDIX D. ADDITIONAL COMPARATIVE STATICS

D.1. **Cross-market impact of information spillover.** In this section, we study how the equilibrium in market $k$ is influenced by information spillover in market $-k$ ($\alpha_{-k}$).

First, we study a benchmark model called *one-sided information spillover* in which $\alpha_k = 1$ and $\alpha_{-k} = 0$. By comparing the equilibrium in market $k$ under one-sided information spillover ($\alpha_k = 1$, $\alpha_{-k} = 0$) and full information spillover ($\alpha_k = \alpha_{-k} = 1$), we get a preliminary result on the impact of cross-market information spillover ($\alpha_{-k}$). Assume that two markets are symmetric: $n_I = n_{II} = n$ and $\rho_I = \rho_{II} = \rho$.

Since all traders in market $-k$ are less informed, the equilibrium strategy is the same as that in the case of no information spillover:

$$x^0_{-k,i}(s^i_{-k}, p_{-k}) = a^0_{-k} s^i_{-k} - B^0_{-k} p_{-k}.$$

Because traders in market $-k$ do not respond to the price change in market $k$, traders in market $k$ cannot influence the market-clearing price $p_{-k}$ of asset $-k$; however, since all traders' signals are correlated, the price $p_{-k}$ affects the inference of traders in market $k$ about their own values. This is the only spillover effect in the one-sided case.

For traders in market $k$, we conjecture a symmetric linear demand schedule:

$$x^1_{k,i}(s^i_k, p_k, p_{-k}) = \hat{a}^1_k s^i_k - \hat{B}^1_k p_k + \hat{b}^1_k p_{-k},$$

where $\hat{a}^1_k$, $\hat{B}^1_k$, and $\hat{b}^1_k$ are constants, for each $i \in \mathcal{N}^1_k$.

From the equilibrium demand schedules of traders of both markets, we derive market-clearing conditions in both markets as follows:

$$\bar{s}_k = \frac{\hat{B}^1_k p_k - \hat{b}^1_k p_{-k}}{\hat{a}^1_k}, \ \bar{s}_{-k} = \frac{B^0_{-k} p_{-k}}{a^0_{-k}}, \tag{14}$$

where $\bar{s}_k = (\sum_{i \in \mathcal{N}^1_k} s^i_k)/n$ is the average signal of all (more informed) traders in market $k$ and $\bar{s}_{-k} = (\sum_{i \in \mathcal{N}^0_{-k}} s^i_{-k})/n$ is the average signal of all (less informed) traders in market $-k$.

Since $E\left[\theta_k^i | s_k^i, \bar{s}_k, \bar{s}_{-k}\right] = C_s^1 s_k^i + C^1 \bar{s}_k + C_-^1 \bar{s}_{-k}$, then by substituting (14) into the above condition expectation, we get

$$\mathbb{E}\left[\theta_k^i | s_k^i, p_k, p_{-k}\right] = C_s^1 s_k^i + \frac{C^1 \hat{B}_k^1}{\hat{a}_k^1} \cdot p_k + \left(\frac{C_-^1 B_{-k}^0}{a_{-k}^0} - \frac{C^1 \hat{b}_k^1}{\hat{a}_k^1}\right) p_{-k}, \tag{15}$$

where

$$C_s^1 = \frac{1-\rho}{1-\rho+\sigma^2}, \quad C^1 = \frac{\sigma^2}{1-\rho+\sigma^2} \cdot \frac{\rho^2 + \frac{\rho(1-\rho+\sigma^2)}{n} - \phi^2}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2}, \; C_-^1 = \frac{\sigma^2 \cdot \frac{\phi}{n}}{\left(\rho + \frac{1-\rho+\sigma^2}{n}\right)^2 - \phi^2}.$$

From trader $i$'s first-order condition and the market-clearing condition, we have

$$x_{k,i}^1(s_k^i, p_k, p_{-k}) = \frac{(n-1)\hat{B}_k^1}{1+(n-1)\gamma \hat{B}_k^1} \cdot \left(\mathbb{E}\left[\theta_k^i | s_k^i, p_k, p_{-k}\right] - p_k\right). \tag{16}$$

Substituting (15) into (16) and matching the coefficients, we obtain

$$\hat{a}_k^1 = \frac{(n-2)C_s^1 - C^1}{\gamma(n-1)}, \quad \hat{B}_k^1 = \frac{\hat{a}_k^1}{C_s^1 + C^1}, \quad \hat{b}_k^1 = \frac{\hat{a}_k^1 C_-^1}{(C_s^1 + C^1)(C_s^0 + C^0)}, \tag{17}$$

where we use the fact that $\frac{B_{-k}^0}{a_{-k}^0} = \frac{1-C^0}{C_s^0}$, $C_s^0 = \frac{1-\rho}{1-\rho+\sigma^2}$, $C^0 = \frac{\sigma^2}{1-\rho+\sigma^2} \cdot \frac{n\rho}{1+(n-1)\rho+\sigma^2}$.

**Proposition D.1.** *Compared to one-sided information spillover ($\alpha_k = 1$, $\alpha_{-k} = 0$),*

*(i) More informed traders' demand schedule in market $k$ under full information spillover ($\alpha_k = \alpha_{-k} = 1$) is*
- *equally sensitive to their own signals, i.e., $a_k^1 = \hat{a}_k^1$;*
- *more sensitive to the price of their own asset, i.e., $B_k^1 > \hat{B}_k^1$;*
- *more sensitive to the other asset's price, i.e, $|b_k^1| > |\hat{b}_k^1|$.*

*(ii) The welfare of trader in market $k$ remains the same under full information spillover ($\alpha_k = \alpha_{-k} = 1$).*

Proposition D.1 analyzes the impact of information spillover $\alpha_{-k}$ on the equilibrium in market $k$. First, a more informed trader is more sensitive to the price of her own asset and the other asset, when traders in the other market become more informed. Intuitively, if traders in market $-k$ becomes more informed, then the price of market $-k$ contains more information, while the price of the market $k$ contains less information. If the correlation $\phi$ is positive, a higher price $p_{-k}$ is a stronger signal of higher quality of asset $k$, and thus traders in market $k$ demand more, while a lower price $p_k$ is a weaker signal of low quality of asset $k$, and thus traders are still willing to demand a lot. In other words, a more informed trader' s demand reacts more to the change of both prices if the traders of other market become more informed.

Second, cross-market information spillover $\alpha_{-k}$ has no impact on the welfare of traders in market $k$. Intuitively, the equilibrium payoffs of more informed traders in market $k$ are completely pinned down by the coefficients of their own signals, regardless of $\alpha_{-k}$.

Next, we study whether Proposition D.1 still holds in the general case where there are both types of traders in each market. The answer is yes if we ignore the terms $o(\frac{1}{N^2})$.

**Proposition D.2.** *If we ignore the terms $o(\frac{1}{N^2})$, then*

$$\frac{\partial C_k^1}{\partial \alpha_{-k}} < 0, \ \frac{\partial |c_k^1|}{\partial \alpha_{-k}} > 0, \ \frac{\partial B_k^1}{\partial \alpha_{-k}} > 0, \ \frac{\partial |b_k^1|}{\partial \alpha_{-k}} > 0.$$

In the general setting, Proposition D.2 reassures the preliminary result in Proposition D.1. There are three observations. (1) $C_k^1$ is decreasing in $\alpha_{-k}$. Higher $\alpha_{-k}$ enhances the cross-asset effect $\delta_{-k}$, which implies that $p_{-k}$ contains more information from market $k$. By observing $p_{-k}$, a more informed trader in market $k$ gets more information about the true value, which crowds out the more informed trader's reliance on $p_k$. (2) $|c_k^1|$ is increasing in $\alpha_{-k}$. Since $|c_k^1|$ is the more informed traders' reaction to $p_{-k}$, then by symmetry, the change of $\alpha_{-k}$ plays the same role as the change of $\alpha_k$ when we study $C_k^1$. Since $C_k^1$ is increasing in $\alpha_k$ in the main model, and thus$|c_k^1|$ is increasing in $\alpha_{-k}$. (3) $B_k^1$ and $|b_k^1|$ are increasing in $\alpha_{-k}$. Intuitively, $B_k^1$ and $|b_k^1|$ are completely determined by $C_k^1$ and $|c_k^1|$, since the strategic channel $\lambda_k^1$ and $\lambda_k^0$ are independent of $\alpha_{-k}$ (see Proposition D.3).

**Proposition D.3.** *If we ignore the term $o(\frac{1}{N^2})$, then $\delta_k$, $\zeta_k$, $C_{ks}^1$, $C_{ks}^0$, $C_k^0$, $a_k^1$, $a_k^0$, $B_k^0$, $\lambda_k^1$, $\lambda_k^0$, $W_k^1$ and $W_k^0$ are independent of $\alpha_{-k}$.*

First, $\delta_k$ and $\zeta_k$ are independent of $\alpha_{-k}$. Intuitively, traders in market $-k$ do not participate the trade in market $k$, hence $\alpha_{-k}$ could only have indirect impacts through information channels: (1) $p_k$ is more sensitive to signals from the other market $-k$ (higher $\delta_k$), since higher $\alpha_{-k}$ increases more informed traders' reaction to $p_{-k}$ (higher $|b_k^1|$, see Proposition D.2); (2) $p_k$ is less sensitive to signals from market $k$ (lower $\delta_k$), since higher $\alpha_{-k}$ decreases more informed traders' reaction to $p_k$ (lower $B_k^1$, see Proposition D.2). However, these indirect impacts are negligible if we only focus on first order and second order effects.

Given the independent result of $\delta_k$ and $\zeta_k$, the rest results hold naturally since all the parameters in Proposition D.3 depend on $\alpha_{-k}$ through $\delta_k$ and $\zeta_k$.

D.2. **Impacts of cross-market correlation.** In this section, we study the impact of cross-market correlation $\phi$ on the equilibrium.

**Proposition D.4.** *The impact of $\phi$ on the equilibrium is as follows:*

(1) *First order effect: if we ignore the term* $o(\frac{1}{N})$, *then*
- $\frac{\partial C_k^1}{\partial|\phi|} < 0$ , $\frac{\partial|c_k^1|}{\partial|\phi|} > 0$, $\frac{\partial B_k^1}{\partial|\phi|} > 0$ , $\frac{\partial|b_k^1|}{\partial|\phi|} > 0$;
- $\frac{\partial C_k^0}{\partial|\phi|} = 0$, $\frac{\partial B_k^0}{\partial|\phi|} = 0$.

(2) *Second order effect: if we ignore the term* $o(\frac{1}{N^2})$, *then*
- $\frac{\partial C_{ks}^1}{\partial|\phi|} = 0$, $\frac{\partial a_k^1}{\partial|\phi|} > 0$, $\frac{\partial \lambda_k^1}{\partial|\phi|} < 0$, $\frac{\partial W_k^1}{\partial|\phi|} = 0$;
- $\frac{\partial C_{ks}^0}{\partial|\phi|} < 0$, $\frac{\partial a_k^0}{\partial|\phi|} > 0$, $\frac{\partial \lambda_k^0}{\partial|\phi|} < 0$, $\frac{\partial W_k^0}{\partial|\phi|} < 0$.

We first check the first-order effects. A higher $|\phi|$ implies a strong cross-market signaling effect. Hence, $p_{-k}$ is a stronger signal of asset value in market $k$ (higher $|c_k^1|$) and $p_k$ is a weaker signal of asset value in market $k$ (lower $C_k^1$). Consequently, a more informed trader is more sensitive to the change of both prices (higher $B_k^1$ and $|b_k^1|$). However, for a less informed traders, the above two effects balance each other, and thus the correlation $\phi$ does not have a first-order impact on the informational and behavioral sensitivity to the price ($C_k^0$ and $B_k^0$ are independent of $\phi$.)

Second, we analyze $C_{ks}^1$ and $C_{ks}^0$. Since $\phi$ measures the relative informativeness of two prices, instead of the absolute informativeness of two price, then a change of $\phi$ does not influence the informativeness of the own signal. In other words, $C_{ks}^1$ is independent of $\phi$. However, $|\phi|$ lower the informativeness of the own signal for a less informed trader. Intuitively, higher $|\phi|$ implies a strong information spillover, and thus a strong information disadvantage for less informed traders, i.e., higher $C_{ks}^1 - C_{ks}^0$. Therefore, $C_{ks}^0$ is decreasing in $|\phi|$.

Third, we study the price impacts $\lambda_k^1$ and $\lambda_k^0$. For a higher correlation $|\phi|$, the market demand is more sensitive to the change of own price, due to the fact that the demand of a more informed trader is more sensitive to price change. Therefore, price is less sensitive to a change of market demand (lower $\lambda_k^1$ and $\lambda_k^0$).

Next, we study $a_k^1$ and $a_k^0$. The impact of $|\phi|$ on $a_k^1$ and $a_k^0$ is decomposed into information channel and strategic channel, both are of second-order effects. For an more informed trader, information channel ($C_{ks}^1$) does not play a role, then it is the strategic channel $\lambda_k^1$ that dominates. Hence, $a_k^1$ is increasing in $|\phi|$. For a less informed trader, both channels plays a role, and we find that the strategic channel dominates the information channel, and thus $a_k^0$ is increasing in $|\phi|$.

Finally, we analyze the welfare of more informed traders $W_k^1$ and less informed traders $W_k^0$. In the main model, we have shown that $W_k^1$ ($W_k^0$) is totally determined by the information channel $C_{ks}^1$ ($C_{ks}^0$). Hence, the result holds.

**Proposition D.5.** *Correlation* $|\phi|$ *amplifies the impacts of information spillover:*

(1) *First order effect: if we ignore the term* $o(\frac{1}{N})$*, then*
   - $|\frac{\partial C_k^1}{\partial \alpha_k}|$, $|\frac{\partial |c_k^1|}{\partial \alpha_k}|$, $|\frac{\partial B_k^1}{\partial \alpha_k}|$ *and* $|\frac{\partial |b_k^1|}{\partial \alpha_k}|$ *are increasing in* $|\phi|$.
   - $\frac{\partial C_k^0}{\partial \alpha_k}$ *and* $\frac{\partial B_k^0}{\partial \alpha_k}$ *are independent of* $|\phi|$.

(2) *Second order effect: if we ignore the term* $o(\frac{1}{N^2})$*, then*
   - $\frac{\partial C_{ks}^1}{\partial \alpha_k}$ *and* $\frac{\partial W_k^1}{\partial \alpha_k}$ *are independent of* $|\phi|$; $|\frac{\partial a_k^1}{\partial \alpha_k}|$ *is increasing in* $|\phi|$.
   - $|\frac{\partial C_{ks}^0}{\partial \alpha_k}|$, $|\frac{\partial a_k^0}{\partial \alpha_k}|$, $|\frac{\partial W_k^0}{\partial \alpha_k}|$ *are increasing in* $|\phi|$.

Proposition D.5 discovers that cross-market correlation $|\phi|$ amplifies the impacts of information spillover. Intuitively, the impact of $\alpha_k$ on the equilibrium originates from the cross-market signaling effect, namely $|\phi|$.

## Appendix E. Technical Results

E.1. **Analysis of less informed traders' reaction to own signals.** In this section, we study the less informed traders' reaction to own signals. For less informed traders, $C_{ks}^0$ is decreasing in $\alpha_k$ if and only if $\alpha_k < \frac{1-\rho_k}{\sigma_k^2}$. This non-monotonicity originates from two opposing effects, described by Lemma E.1. Define $r(\theta_k, p_k)$ ($r(\varepsilon_k, p_k)$) as the correlation between $\theta_k^i$ ($\varepsilon_k^i = s_k^i - \theta_k^i$) and $p_k$.

**Lemma E.1.** $C_{ks}^0$ *is a function of* $r(\theta_k, p_k)$ *and* $r(\varepsilon_k, p_k)$ *such that*
- *Crowding-out effect:* $\frac{\partial C_{ks}^0}{\partial r(\theta_k, p_k)} < 0$ *and* $\frac{\partial r(\theta_k, p_k)}{\partial \alpha_k} > 0$.
- *Multicollinearity effect:* $\frac{\partial C_{ks}^0}{\partial r(\varepsilon_k, p_k)} < 0$ *if and only if* $\frac{1-\rho_k}{\sigma_k^2} < 1$; $\frac{\partial r(\varepsilon_k, p_k)}{\partial \alpha_k} < 0$.

On the one hand, there is a crowding-out effect. A larger fraction of more informed traders $\alpha_k$ improves the informativeness of the price $p_k$, which is illustrated by a higher correlation between the true value and the price (higher $r(\theta_k, p_k)$). Therefore, less informed traders rely more on the price instead of own signals to predict the true value (lower $C_{ks}^0$). On the other hand, larger $\alpha_k$ may mitigate the problem of multicollinearity caused by high correlation between independent variables. Intuitively, higher $\alpha_k$ implies that $p_k$ is a noisier signal (higher $\sigma_{p_k}$), which leads to a lower correlation between the own signal and the price (lower $r(\varepsilon_k, p_k)$), and hence a change of the own signal contains more information (higher $C_{ks}^0$). In all, the combination of the above effects creates the non-monotonic relation between $\alpha_k$ and $C_{ks}^0$. If $\alpha_k$ is low enough, the crowding-out effect dominates the effect of mitigated multicollinearity, and thus $C_{ks}^0$ is decreasing in $\alpha_k$.

E.2. **Details of the numerical analysis.** We focus on the symmetric case: $\alpha_I = \alpha_{II} \equiv \alpha$ and $n_I = n_{II} \equiv n$. Hence, the key parameters are $\pi \equiv \pi_I = \pi_{II}$ and $\Pi \equiv \Pi_I = \Pi_{II}$. We apply the following fixed point iteration:

Step 1: The input is $\pi \in [0,1]$ with the initial guess as $\pi = \alpha$.

Step 2: Given $\pi \in [0,1]$, we solve for the unique $\Pi \in [0,1]$ as the solution of a cubic polynomial (see Proposition E.2). Given $\pi$ and $\Pi$, we solve for $\delta_k$, $\zeta_k$, $\beta_k$, $C^1_{ks}$, $C^0_{ks}$, $C^1_k$, $C^0_k$, $c^1_k$, $\bar{C}^1_k$.

Step 3: Given $C^1_{ks}, C^1_k, C^0_{ks}, \bar{C}^1_k, C^0_k$, we get the unique $a^1_k$, $B^1_k$, $a^0_k$, $B^0_k$, $b^1_k$.

Step 4: Given $a^1_k$, $B^1_k$, $a^0_k$, $B^0_k$, we get the output $\pi'$ and then go back to Step 1, until the distance $|\pi' - \pi|$ is less than a tolerance level.

**Proposition E.2.** *If $\pi$ is taken as given, then $\Pi$ is the root of a cubic polynomial.*

## Appendix F. Proofs of the Results in Sections B and C

### F.1. **Proof of Proposition B.1 :**

*Proof.* **Step 1:** We deduce the conditional expectation of a more informed trader in (6).

The market-clearing condition for asset $k$ is $\bar{s}^1_k = \frac{B^1_k p_k - b^1_k p_{-k}}{a^1_k}$, where $\bar{s}^1_k = (\sum_{i \in \mathcal{N}^1_k} s^i_k)/n$. Therefore, $(p_I, p_{II})$ is a linear combination of $(\bar{s}_I, \bar{s}_{II})$, and hence $(\theta^i_k, s^i_k, p_k, p_{-k})$ is jointly normal distributed. Therefore, it suffices to solve for $\mathbb{E}[\theta^i_k | s^i_k, \bar{s}^1_k, \bar{s}^1_{-k}]$. Since $\mathrm{Cov}(s^i_k, s^i_k) = (1 + \sigma^2 + \varsigma^2)\sigma^2_\theta$, $\mathrm{Cov}(\bar{s}^1_k, \theta^i_k) = (\rho + \frac{1-\rho}{n})\sigma^2_\theta$, $\mathrm{Cov}(\bar{s}^1_k, s^i_k) = \mathrm{Var}(\bar{s}^1_k) = (\rho + \frac{1-\rho+\sigma^2}{n} + \varsigma^2)\sigma^2_\theta$, $\mathrm{Cov}(\bar{s}^1_{-k}, \theta^i_k) = \mathrm{Cov}(\bar{s}^1_{-k}, s^i_k) = \mathrm{Cov}(\bar{s}^1_k, \bar{s}^1_{-k}) = \phi\sigma^2_\theta$, then

$$
\begin{aligned}
&\mathbb{E}\left[\theta^i_k | s^i_k, p_k, p_{-k}\right] = \mathbb{E}\left[\theta^i_k | s^i_k, \bar{s}^1_k, \bar{s}^1_{-k}\right] \\
=&\begin{pmatrix} \mathrm{Cov}(\theta^i_k, s^i_k) \\ \mathrm{Cov}(\theta^i_k, \bar{s}^1_k) \\ \mathrm{Cov}(\theta^i_k, \bar{s}^1_{-k}) \end{pmatrix}^T \begin{pmatrix} \mathrm{Cov}(s^i_k, s^i_k) & \mathrm{Cov}(\bar{s}^1_k, s^i_k) & \mathrm{Cov}(\bar{s}^1_{-k}, s^i_k) \\ \mathrm{Cov}(\bar{s}^1_k, s^i_k) & \mathrm{Var}(\bar{s}^1_k) & \mathrm{Cov}(\bar{s}^1_k, \bar{s}^1_{-k}) \\ \mathrm{Cov}(\bar{s}^1_{-k}, s^i_k) & \mathrm{Cov}(\bar{s}^1_k, \bar{s}^1_{-k}) & \mathrm{Var}(\bar{s}^1_{-k}) \end{pmatrix}^{-1} \begin{pmatrix} s^i_k \\ \bar{s}^1_k \\ \bar{s}^1_{-k} \end{pmatrix} \\
=&C^1_s s^i_k + C^1 \bar{s}^1_k + C^1_- \bar{s}^1_{-k} = C^1_s s^i_k + \left(\frac{C^1 B^1_k}{a^1_k} - \frac{C^1_- b^1_{-k}}{a^1_{-k}}\right) p_k + \left(\frac{C^1_- B^1_{-k}}{a^1_{-k}} - \frac{C^1 b^1_k}{a^1_k}\right) p_{-k},
\end{aligned}
$$

where $C^1_s$, $C^1$ and $C^1_-$ satisfy equation (6).

**Step 2:** We deduce the conditional expectation of a less informed trader in (7).

The market-clearing condition for asset $k$ implies that $\bar{s}^0_k = \frac{B^0_k p_k}{a^0_k}$. Therefore, it suffice to solve $\mathbb{E}\left[\theta^{i'}_k | s^{i'}_k, \bar{s}^0_k\right]$. Since $\mathrm{Cov}(s^{i'}_k, s^{i'}_k) = \sigma^2_\theta(1 + \sigma^2 + \varsigma^2)$, $\mathrm{Cov}(\bar{s}^0_k, \theta^{i'}_k) = (\rho + \frac{1-\rho}{n})\sigma^2_\theta$,

$\mathrm{Cov}(\bar{s}_k^0, s_k^{i'}) = \mathrm{Var}(\bar{s}_k^0) = (\rho + \frac{1-\rho+\sigma^2}{n} + \varsigma^2)\sigma_\theta^2$, then

$$\mathbb{E}\left[\theta_k^{i'}|s_k^{i'}, p_k\right] = \mathbb{E}\left[\theta_k^{i'}|s_k^{i'}, \bar{s}_k^0\right] = \begin{pmatrix} \mathrm{Cov}(\theta_k^{i'}, s_k^{i'}) \\ \mathrm{Cov}(\bar{s}_k^0, \theta_k^{i'}) \end{pmatrix}^T \begin{pmatrix} \mathrm{Cov}(s_k^i, s_k^i) & \mathrm{Cov}(\bar{s}_k, s_k^{i'}) \\ \mathrm{Cov}(\bar{s}_k^0, s_k^{i'}) & \mathrm{Var}(\bar{s}_k^0) \end{pmatrix}^{-1} \begin{pmatrix} s_k^{i'} \\ \bar{s}_k^0 \end{pmatrix}$$
$$= C_s^0 s_k^{i'} + C^0 \bar{s}_k^0 = C_s^0 s_k^{i'} + C^0 \frac{B_k^0}{a_k^0} p_k.$$

where $C_s^0$ and $C^0$ satisfy equation (7).

□

F.2. **Proof of Lemma B.2 :**

*Proof.* Define $x = \frac{\sigma^2}{n} + \varsigma^2$. First, we check $C^1$. By taking derivative of $C^1$ with respect to $\varsigma^2$, w find that $\frac{dC^1}{d\varsigma^2} < 0$ is equivalent to

$$F(x) \equiv (\rho + \frac{1-\rho}{n})((\rho + \frac{1-\rho}{n} + x)^2 - \phi^2(\rho + \frac{1-\rho}{n} + 2x) > 0.$$

It is apparent that $F(0) = (\rho + \frac{1-\rho}{n})((\rho + \frac{1-\rho}{n})^2 - \phi^2) > 0$. Moreover, $F'(x) = 2((\rho + \frac{1-\rho}{n})(\rho + \frac{1-\rho}{n} + x) - \phi^2) > 0$. Therefore, $F(x) > 0$ for any $x > 0$. Next, we check $C_-^1$:

$$\frac{dC_-^1}{d\varsigma^2} = \frac{dC_-^1}{dx} = \frac{\phi((\rho + \frac{1-\rho}{n})^2 - \phi^2 - x^2)}{[(\rho + \frac{1-\rho}{n} + x)^2 - \phi^2]^2}.$$

Therefore, $\frac{d|C_-^1|}{d\varsigma^2} > 0$ if and only if $x^2 < (\rho + \frac{1-\rho}{n})^2 - \phi^2$, which is equivalent to $\varsigma^2 < \sqrt{(\rho + \frac{1-\rho}{n})^2 - \phi^2} - \frac{\sigma^2}{n}$. Finally, by calculation, $C_S^0 + C^0 \in (0,1)$ and $\frac{dC^0}{d\varsigma^2} < 0$.

□

F.3. **Proof of Proposition B.4 :**

*Proof.* **Step 1.** We define new parameters which will be used in Steps 2-3.

$$C_k^* = \frac{1-\rho_k}{1-\rho_k+\sigma_k^2},\ \kappa_k = 1-\rho_k+\sigma_k^2,\ \eta_k = \frac{\sigma_{\theta_{-k}}}{\sigma_{\theta_k}}, \tag{18}$$
$$e_{k1} = \frac{\pi_k}{\alpha_k}(\frac{\kappa_k}{n_k} + \varsigma_k^2),\ e_{k2} = (\frac{(1-\pi_k)^2}{1-\alpha_k} + \frac{\pi_k^2}{\alpha_k})(\frac{\kappa_k}{n_k} + \varsigma_k^2),\ e_{k3} = \frac{1-\pi_k}{1-\alpha_k}(\frac{\kappa_k}{n_k} + \varsigma_k^2),$$

$$M_{k0} = (\rho_k + \frac{\pi_k}{\alpha_k}\frac{1-\rho_k}{n_k})(\rho_{-k} + e_{-k2}) - \phi^2,$$
$$M_{k1} = (\rho_k + e_{k1})(\rho_{-k} + e_{-k2}) - \phi^2,\ M_{k2} = (\rho_k + e_{k2})(\rho_{-k} + e_{-k2}) - \phi^2$$

**Step 2. (More informed traders)** We solve $C_{ks}^1$, $C_k^1$, $c_k^1$ and $\delta_k$.

For a more informed trader $i \in \mathcal{N}_k^1$, since $(\theta_k^i, s_k^i, p_k, p_{-k})$ is jointly normal, we have

$$E(\theta_k^i | s_k^i, p_k, p_{-k}) = C_{ks}^1 s_k^i + C_k^1 p_k + c_k^1 p_{-k}.$$

Define $X = \theta_k^i$, $Y = (s_k^i, p_k, p_{-k})$. By projection theorem,

$$E[X|Y] = E(X) + \Sigma_{X,Y}\Sigma_{Y,Y}^{-1}(Y - E(Y)),$$

where $\Sigma_{X,Y}$ ( $\Sigma_{Y,Y}$) is the covariance matrix between $X$ and $Y$ (between $Y$ and $Y$).

Define $\delta_k = \frac{d_k^1 + d_k^0}{D_k^1 + D_k^0}$, $\zeta_k = D_k^1 + D_k^0$, $\pi_k = \frac{D_k^1}{D_k^1 + D_k^0} = \frac{d_{-k}^1}{d_{-k}^1 + d_{-k}^0}$. By $p_k = D_k^1 \bar{s}_k^1 + D_k^0 \bar{s}_k^0 + d_k^1 \bar{s}_{-k}^1 + d_k^0 \bar{s}_{-k}^0$,

$$p_k = \zeta_k[(\pi_k \bar{s}_k^1 + (1 - \pi_k)\bar{s}_k^0) + \delta_k(\pi_{-k}\bar{s}_{-k}^1 + (1 - \pi_{-k})\bar{s}_{-k}^0)].$$

Since we know the covariance of $(\theta_k^i, s_k^i, \bar{s}_k^1, \bar{s}_k^0, \bar{s}_{-k}^1, \bar{s}_{-k}^0)$, then we calculate $\Sigma_{X,Y}\Sigma_{Y,Y}^{-1} = (C_{ks}^1, C_k^1, c_k^1)$ as follows:

$$C_{ks}^1 = \frac{(1 - \frac{\pi_k}{\alpha_k n_k})(1 - \rho_k)M_{k2} - (e_{k1} - e_{k2})M_{k0}}{Y_{k3}}, \tag{19}$$

$$C_k^1 = \frac{Y_{k2} - Y_{k1}\delta_{-k}}{(1 - \delta_k\delta_{-k})Y_{k3}\zeta_k},\ c_k^1 = \frac{Y_{k1} - Y_{k2}\delta_k}{(1 - \delta_k\delta_{-k})Y_{k3}\zeta_{-k}}.$$

where $Y_{k1} = \frac{\phi}{\eta_k}\Big((\kappa_k + \varsigma_k^2 - e_{k1})(e_{k1} - \frac{\pi_k}{\alpha_k}\frac{1-\rho_k}{n_k}) - (\sigma_k^2 + \varsigma_k^2)(e_{k1} - e_{k2})\Big)$, $Y_{k2} = \sigma_k^2(1 - \frac{\pi_k}{\alpha_k n_n})(\rho_k(\rho_{-k} + e_{-k2}) - \phi^2) + \varsigma_k^2((\rho_k + \frac{\pi_k}{\alpha_k}\frac{1-\rho_k}{n_k} - \frac{\pi_k}{\alpha_k})(\rho_{-k} + e_{-k2}) - (1 - \frac{\pi_k}{\alpha_k})\phi^2)$ and $Y_{k3} = (\kappa_k + \varsigma_k^2 - e_{k1})M_{k2} - (e_{k1} - e_{k2})M_{k1}$.

Define $\beta_k = \frac{\alpha_k a_k + (1-\alpha_k)a_{k0}}{\alpha_k B_k + (1-\alpha_k)B_{k0}}$. Then, $\beta_k = (1 - \delta_k\delta_{-k})\zeta_k$. Substituting this into (19),

$$C_k^1 = \frac{Y_{k2} - Y_{k1}\delta_{-k}}{Y_{k3}\beta_k},\ c_k^1 = \frac{Y_{k1} - Y_{k2}\delta_k}{Y_{k3}\beta_{-k}}. \tag{20}$$

We solve for $\delta_k$:

$$\delta_k = \beta_{-k}\frac{\alpha_k b_k^1}{\alpha_k a_k^1 + (1 - \alpha_k)a_k^0} = \frac{\beta_{-k}\pi_k b_k^1}{a_k^1} = \frac{\pi_k \beta_{-k} c_k^1}{C_{ks}^1}. \tag{21}$$

Substituting $c_k^1$ of (20) into (21), we get $\delta_k = (Y_{k1} - Y_{k2}\delta_k)\frac{\pi_k}{Y_{k3}C_{ks}^1}$, which implies that

$$\delta_k = \frac{Y_{k1}}{Y_{k2} + \frac{C_{ks}^1}{\pi_k}Y_{k3}}. \tag{22}$$

**Step 3. (Less informed traders)** We solve $C_{ks}^0$ and $C_k^0$.

For a less informed trader $i \in \mathcal{N}_k^0$, since $(\theta_k^i, s_k^i, p_k)$ is jointly normal, we have

$$E(\theta_k^i | s_k^i, p_k) = C_{ks}^0 s_k^i + C_k^0 p_k.$$

Define $X = \theta_k^i$, $Y_0 = (s_k^i, p_k)$. By projection theorem, we can solve

$$E[X|Y_0] = E(X) + \Sigma_{X,Y_0}\Sigma_{Y_0,Y_0}^{-1}(Y_0 - E(Y_0)),$$

where $\Sigma_{X,Y_0}$ ( $\Sigma_{Y_0,Y_0}$) is the covariance matrix between $X$ and $Y_0$ (between $Y_0$ and $Y_0$).

Therefore, $(C_{ks}^0, C_k^0) \equiv \Sigma_{X,Y_0}\Sigma_{Y_0,Y_0}^{-1}$, where

$$C_{ks}^0 = \frac{\Upsilon_{k4}}{H_k^0},\ C_k^0 = \frac{(\sigma_k^2 + \varsigma_k^2)(\rho_k + \phi\eta_k\delta_k) - e_{k3} + (1 + \sigma_k^2 + \varsigma_k^2)\frac{1-\pi_k}{1-\alpha_k}\frac{1-\rho_k}{n_k}}{H_k^0\zeta_k}, \tag{23}$$

where $\Upsilon_{k4} = (1 - \frac{1-\pi_k}{(1-\alpha_k)n_k})(1-\rho_k)(\rho_k + e_{k3}) + (e_{k2} - e_{k3}) + \eta_k^2\delta_k^2(\rho_{-k} + e_{-k2} - \phi^2) + \phi((2 - \frac{1-\pi_k}{(1-\alpha_k)n_k})(1-\rho_k) - e_{k3})\eta_k\delta_k$, and $H_k^0 = (\kappa_k - e_{k3} + \varsigma_k^2)(\rho_k + e_{k3}) + (\kappa_k + \rho_k + \varsigma_k^2)(e_{k2} - e_{k3}) + \eta_k^2\delta_k^2((\kappa_k + \rho_k + \varsigma_k^2)(\rho_{-k} + e_{-k2}) - \phi^2) + 2\phi(\kappa_k + \varsigma_k^2 - e_{k3})\eta_k\delta_k$.

**Step 4:** We study the limit result if $V_s \to 0$ and $N \to +\infty$.

It is clear that $\delta_k \leq \frac{\Upsilon_{k1}}{\Upsilon_{k2}}$. As $V_s \to +\infty$, and $N \to +\infty$, $\Upsilon_{k1} \to 0$, and hence $\delta_k \to 0$. Consequently, $\lim c_k^1 = 0$, $\lim C_{ks}^1 = C_{ks}^0 = C_k^*$, $\lim C_k^1 = \lim C_k^0 = 1 - C_k^*$, $\lim \lambda_k^1 = \lim \lambda_k^0 = 0$, $\lim a_k^1 = \lim a_k^0 = \lim B_k^1 = \lim B_k^0 = \frac{C_k^*}{\gamma}$. Moreover, $\lim \beta_k = \lim \zeta_k = 1$, $\lim \pi_k = \alpha_k$.

**Step 5:** We solve the limits of $C_{ks}^1$, $C_k^1$, $c_k^1$, $C_{ks}^0$, $C_k^0$ by ignoring the term $o(V_s^2)$.

In the equilibrium, $a_k^1 - a_k^0 = o(V_s)$ (which is verified in Step 8). Consequently,

$$\pi_k - \alpha_k = \frac{\alpha_k(1-\alpha_k)(a_k^1 - a_k^0)}{\alpha_k a_k^1 + (1-\alpha_k)a_k^0} = o(V_s).$$

Therefore, by the definition of $e_{k1}$, $e_{k2}$ and $e_{k3}$,we get

$$e_{k1} = e_{k2} = e_{k3} = \varsigma_k^2 + o(V_s^2). \tag{24}$$

Substituting (24) into (19) and (23), we get (by ignoring $o(V_s^2)$)

$$C_{ks}^1 = C_k^*,\ C_k^1 = \frac{\Upsilon_{k2} - \Upsilon_{k1}\delta_{-k}}{(1-\delta_k\delta_{-k})\Upsilon_{k3}\zeta_k},\ c_k^1 = \frac{\Upsilon_{k1} - \Upsilon_{k2}\delta_k}{(1-\delta_k\delta_{-k})\Upsilon_{k3}\zeta_{-k}}. \tag{25}$$

$$C_{ks}^0 = C_k^* - \frac{\eta_k^2\delta_k(\Upsilon_{k1} - \Upsilon_{k2}\delta_k)}{\kappa_k H_k^0},\ C_k^0 = \frac{(\sigma_k^2 + \varsigma_k^2)(\rho_k + \phi\eta_k\delta_k) - \varsigma_k^2}{H_k^0\zeta_k},$$

where $\Upsilon_{k1} = \frac{\phi}{\eta_k}\kappa_k\varsigma_k^2$, $\Upsilon_{k2} = \sigma_k^2(\rho_k(\rho_{-k} + \varsigma_{-k}^2) - \phi^2) - \varsigma_k^2(1-\rho_k)(\rho_{-k} + \varsigma_{-k}^2)$, $\Upsilon_{k3} = \kappa_k\left((\rho_k + \varsigma_k^2)(\rho_{-k} + \varsigma_{-k}^2)\right.$ and $H_k^0 = \kappa_k(\rho_k + \varsigma_k^2) + \eta_k^2\delta_k^2((\kappa_k + \rho_k + \varsigma_k^2)(\rho_{-k} + \varsigma_{-k}^2) - \phi^2) + 2\phi\kappa_k\eta_k\delta_k$.

**Step 6:** We study $|\delta_k|$, $|c_k^1|$, $C_{ks}^1 - C_{ks}^0$ and $C_k^0 - C_k^1$ in the limit.

Since $C_{ks}^1 = C_k^* + o(V_s^2)$ , $\Upsilon_{k3} = \frac{\kappa_k}{\sigma_k^2}\Upsilon_{k2} + O(V_s)$ and $\pi_k = \alpha_k + o(V_s)$, then by (22),

$$\delta_k = \frac{\Upsilon_{k1}}{\Upsilon_{k2}}\frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} + o(V_s). \tag{26}$$

Since $\frac{Y_{k1}}{Y_{k2}} = \frac{\kappa_k \phi \varsigma_k^2}{\eta_k \sigma_k^2(\rho_k\rho_{-k}-\phi^2)} + o(V_s)$,

$$|\delta_k| = \delta_k^* \frac{\varsigma_k^2}{\sigma_k^2} + o(V_s),\text{ where } \delta_k^* = \frac{\kappa_k|\phi|}{\eta_k(\rho_k\rho_{-k}-\phi^2)} \frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} > 0. \tag{27}$$

By $\beta_{-k} = 1 + O(V_s)$ and $Y_{k3} = \frac{\kappa_k}{\sigma_k^2}Y_{k2} + O(V_s)$, we have $c_k^1 = \frac{Y_{k1}-Y_{k2}\delta_k}{Y_{k3}\beta_{-k}} = \frac{\sigma_k^2}{\kappa_k}(\frac{Y_{k1}}{Y_{k2}} - \delta_k) + o(\varsigma_k^2)$. By (18) and (27) and $y_{k2} = \rho_k\rho_{-k} - \phi^2 + o(1)$,

$$|c_k^1| = c_k^* \frac{\varsigma_k^2}{\sigma_k^2} + o(V_s),\text{ where } c_k^* = \frac{(1-\rho_k)|\phi|}{\eta_k(\rho_k\rho_{-k}-\phi^2)} \frac{1}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} > 0. \tag{28}$$

Since $H_k^0 = \kappa_k\rho_k + O(V_s)$, then $C_{ks}^1 - C_{ks}^0 = \frac{\eta_k^2\delta_k(Y_{k1}-Y_{k2}\delta_k)}{\kappa_k H_k^0} = \frac{\eta_k^2\delta_k(Y_{k1}-Y_{k2}\delta_k)}{\kappa_k^2\rho_k} + o(V_s^2)$. By (18) and (27), we have $C_{ks}^1 - C_{ks}^0 = \Delta_{ks}(\frac{\varsigma_k^2}{\sigma_k^2})^2 + o(V_s^2)$, where

$$C_{ks}^1 - C_{ks}^0 = \Delta_{ks}(\frac{\varsigma_k^2}{\sigma_k^2})^2 + o(V_s^2),\text{ where } \Delta_{ks} = \frac{(1-\rho_k)\phi^2}{\rho_k(\rho_k\rho_{-k}-\phi^2)} \frac{\alpha_k}{(\alpha_k + \frac{1-\rho_k}{\sigma_k^2})^2} > 0. \tag{29}$$

Since $H_k^0 = \kappa_k(\rho_k + \varsigma_k^2) + 2\phi\kappa_k\eta_k\delta_k + o(V_s)$, then

$$C_k^0 = \frac{(\sigma_k^2+\varsigma_k^2)(\rho_k + \phi\eta_k\delta_k) - \varsigma_k^2}{\zeta_k H_k^0} = (1-C_k^*)(1 - \frac{\kappa_k\frac{\varsigma_k^2}{\sigma_k^2} + \phi\eta_k\delta_k}{\rho_k})\frac{1}{\beta_k} + o(V_s^2).$$

Hence, $C_k^0 - C_k^1 = \frac{(1-C_k^*)\eta_k\phi}{\rho_k}(\frac{y_{k1}}{y_{k2}} - \delta_k) + o(V_s)$. Together with (18) and (27),

$$C_k^0 - C_k^1 = \Delta_k \frac{\varsigma_k^2}{\sigma_k^2} + o(V_s),\text{ where } \Delta_k = \frac{(1-\rho_k)\phi^2}{\rho_k(\rho_k\rho_{-k}-\phi^2)} \frac{1}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} > 0. \tag{30}$$

**Step 7:** We study $a_k^1 - a_k^0$, $B_k^1 - B_k^0$ and $|b_k^1|$ in the limit.

It is clear that $a_k^1 - a_k^0 = a_k^*\frac{\varsigma_k^2}{\sigma_k^2} + o(V_s^2)$, $B_k^1 - B_k^0 = B_k^*\frac{\varsigma_k^2}{\sigma_k^2} + o(V_s)$ and $|b_k^1| = b_k^*\frac{\varsigma_k^2}{\sigma_k^2} + o(V_s)$, where $a_k^* = \frac{1}{\gamma}\Delta_{ks} > 0$, $B_k^* = \frac{1}{\gamma}\Delta_k > 0$, $b_k^* = \frac{1}{\gamma}c_k^* > 0$.

**Step 8:** We verify that $a_k^1 - a_k^0 = o(V_s)$.

$a_k^1 - a_k^0 = \frac{1}{\gamma}(C_{ks}^1 - C_{ks}^0) + o(V_s^2) = \frac{1}{\gamma}\Delta_{ks}(\frac{\varsigma_k^2}{\sigma_k^2})^2 + o(V_s^2) = o(V_s)$.

□

## F.4. **Proof of Propositions C.1 and C.2 :**

*Proof.* By (12), we get $C_s > 0$, $C > |C_-|$ and $C_s + C + |C_-| < 1$. Since $(1-\rho)(n-2) > \sigma^2$, then $C_s > \frac{1}{n-1}$.

By (13), $A - a = \frac{(n-1)C_s-(C_s+C+C_-)}{\gamma(n-1)(1+\omega)} > \frac{1-(C_s+C+C_-)}{\gamma(n-1)(1+\omega)} > 0$ and $B - b = \frac{A-a}{C_s+C+C_-} > 0$. Moreover, $A + a = \frac{(n-1)C_s-(C_s+C-C_-)}{\gamma(n-1)(1-\omega)} > \frac{1-(C_s+C-C_-)}{\gamma(n-1)(1+\omega)} > 0$ and $B + b = \frac{A+a}{C_s+C-C_-} > 0$. In all, $A > |a|$ and $B > |b|$.

By (13) , we get $A = \frac{(n-2)C_s-C+\omega C_-}{\gamma(n-1)(1-\omega^2)} < \frac{(n-2)C_s}{\gamma(n-1)(1-\omega^2)}$, due to $C - wC_- > |C_-| - wC_- > 0$. By (13), we get $B = \frac{(C_s+C)A+C_-a}{(C_s+C)^2-(C_-)^2} \leq \frac{(C_s+C)A+|C_-||a|}{(C_s+C)^2-(C_-)^2} = \frac{(C_s+C+|C_-|)A-|C_-|(A-|a|)}{(C_s+C)^2-(C_-)^2} < \frac{A}{C_s+C-|C_-|} < \frac{A}{C_s} < \frac{(n-2)}{\gamma(n-1)(1-\omega^2)}$.

If $\phi > 0$, then $C_- > 0$. Consequently, $\frac{\partial B}{\partial a} = \frac{C_-}{(C_s+C)^2-(C_-)^2} > 0$ and $\frac{\partial b}{\partial a} = \frac{C_s+C}{(C_s+C)^2-(C_-)^2} > 0$. Moreover, $C_- > 0$ also imply that $A$ and $a$ are increasing in $\omega$. As a result, $B$ and $b$ are also increasing in $\omega$.

□

## Appendix G. Proofs of the Results in Sections D and E

### G.1. Proof of Proposition D.1 :

*Proof.* We know from the main model that

$$a_k^1 = \frac{(n-2)C_s^1 - C^1}{\gamma(n-1)}, \quad B_k^1 = \frac{(C_s^1 + C^1)a^1}{(C_s^1 + C^1)^2 - (C_-^1)^2}, \quad b_k^1 = \frac{C_-^1 a^1}{(C_s^1 + C^1)^2 - (C_-^1)^2}.$$

It is clear that $\hat{a}_k^1 = a_k^1 = \frac{(n-2)C_s^1 - C^1}{\gamma(n-1)}$. Since $C^1 > |C_-^1|$, then

$$B_k^1 = \frac{a_k^1(C_s^1 + C^1)}{(C_s^1 + C^1)^2 - (C_-^1)^2} = \frac{\hat{a}_k^1(C_s^1 + C^1)}{(C_s^1 + C^1)^2 - (C_-^1)^2} > \frac{\hat{a}_k^1}{C_s^1 + C^1} = \hat{B}_k^1.$$

Finally, since $C^0 > C^1$ and $(C_-^1)^2 > 0$, we have

$$|b_k^1| = \frac{a_k^1|C_-^1|}{(C_s^1 + C^1)^2 - (C_-^1)^2} = \frac{\hat{a}_k^1|C_-^1|}{(C_s^1 + C^1)^2 - (C_-^1)^2} > \frac{\hat{a}_k^1|C_-^1|}{(C_s^1 + C^1)(C_s^0 + C^0)} = |\hat{b}_k^1|.$$

Under full (one-sided) information spillover, the expected payoff of trader $i$ in market $k$ is $\left(C_s a_k^1 - \frac{\gamma(a_k^1)^2}{2}\right)\mathbb{E}\left[(s_k^i - \bar{s}_k)^2\right]$ and $\left(C_s \hat{a}_k^1 - \frac{\gamma(\hat{a}_k^1)^2}{2}\right)\mathbb{E}\left[(s_k^i - \bar{s}_k)^2\right]$, respectively. Since $\hat{a}_k^1 = a_k^1$, then the welfare is the same under both cases.

□

### G.2. Proof of Proposition D.2: 

First, we analyze $C_k^1$ and $B_k^1$. We know

$$C_k^1 = \frac{(1 - C_k^*)(y_{k2} - y_{k1}\delta_{-k})}{y_{k3}\beta_k} = \frac{(1 - C_k^*)(y_{k2} - y_{k1}\delta_{-k})}{y_{k3}(1 - \delta_k\delta_{-k})}\frac{1}{\zeta_k} + o(\frac{1}{N^2}). \tag{31}$$

By $\delta_k < \frac{y_{k1}}{y_{k2}}$, we get $C_k^1$ is decreasing in $\delta_{-k}$. Since $\delta_k$ and $\zeta_k$ are independent of $\alpha_{-k}$ (see Proposition D.3) and $\delta_{-k}$ is increasing in $\alpha_{-k}$, then $C_k^1$ is decreasing in $\alpha_{-k}$. Since $\lambda_k^1$ is independent of $\alpha_{-k}$ (see Proposition D.3), then $B_k^1 = \frac{1}{\gamma+\lambda_k^1}(1 - C_k^1)$ is increasing in $\alpha_{-k}$.

Next, we analyze $|c_k^1|$ and $|b_k^1|$. We know that $c_k^1 = \frac{(1-C_k^*)(\frac{y_{k1}}{y_{k3}} - \frac{y_{2k}}{y_{k3}}\delta_k)}{\beta_{-k}} + o(\frac{1}{N^2}) = \frac{(1-C_k^*)(\frac{y_{k1}}{y_{k3}} - \frac{y_{2k}}{y_{k3}}\delta_k)}{\zeta_{-k}} + o(\frac{1}{N^2})$. Since $\zeta_{-k}$ is decreasing in $\alpha_{-k}$ and $\delta_k$ is independent of $\alpha_{-k}$ (see Proposition D.3), then $|c_k^1|$ is increasing in $\alpha_{-k}$. Since $\lambda_k^1$ is independent of $\alpha_{-k}$ (see Proposition D.3), then $|b_k^1| = \frac{1}{\gamma+\lambda_k^1}|c_k^1|$ is increasing in $\alpha_{-k}$.

G.3. **Proof of Proposition D.3:** First, we prove that $\delta_k$ is independent of $\alpha_{-k}$.

$$\delta_k = \frac{(1 - C_k^*)y_{k1}}{(1 - C_k^*)y_{k2} + \frac{C_{ks}^1}{\pi_k}y_{k3}} = \frac{y_{k1}}{y_{k2} + \frac{C_k^*}{\alpha_k(1-C_k^*)}y_{k3}} + o(\frac{1}{N^2}), \tag{32}$$

which is independent of $\alpha_{-k}$.

Second, we prove that $\zeta_k$ is independent of $\alpha_{-k}$.

$$\zeta_k = \frac{\beta_k}{1 - \delta_k\delta_{-k}} = \frac{\alpha_k a_k^1 + (1-\alpha_k)a_k^0}{\alpha_k B_k^1 + (1-\alpha_k)B_k^0}\frac{1}{1 - \delta_k\delta_{-k}} = \frac{\alpha_k C_{ks}^1 + (1-\alpha_k)C_{ks}^0}{1 - \alpha_k C_k^1 - (1-\alpha_k)C_k^0}\frac{1}{1 - \delta_k\delta_{-k}} + o(\frac{1}{N^2}).$$

$$= \alpha_k\zeta_k C_k^1 + \frac{\alpha_k C_{ks}^1 + (1-\alpha_k)C_{ks}^0}{1 - \delta_k\delta_{-k}} + (1-\alpha_k)\zeta_k C_k^0 + o(\frac{1}{N^2}).$$

By (31) and (32),

$$\zeta_k = \alpha_k\frac{(1 - C_k^*)(\frac{y_{k2}}{y_{k3}} - \frac{y_{k1}}{y_{k3}}\delta_{-k}) + \frac{\alpha_k C_{ks}^1 + (1-\alpha_k)C_{ks}^0}{\alpha_k}}{1 - \delta_k\delta_{-k}} + (1-\alpha_k)\zeta_k C_k^0 + o(\frac{1}{N^2}).$$

$$= \frac{\alpha_k}{1 - \delta_k\delta_{-k}}(\frac{\alpha_k C_{ks}^1 + (1-\alpha_k)C_{ks}^0}{\alpha_k} - \frac{C_{ks}^1}{\pi_k}) + \alpha_k((1 - C_k^*)\frac{y_{k2}}{y_{k3}} + \frac{C_{ks}^1}{\pi_k}) + (1-\alpha_k)\zeta_k C_k^0 + o(\frac{1}{N^2}).$$

Since $\zeta_k C_{k0}^1 = \frac{(1-C_k^*)(\rho_k(\kappa_k - e_{k3}) + \eta_k\delta_k\kappa_k\phi)}{H_k^0}$ is not related to $\delta_{-k}$, then the only term that could be related to $\delta_{-k}$ is the first term. Since $C_{ks}^0 = C_{ks}^1 + o(\frac{1}{N})$ and $\pi_k = \alpha_k + o(\frac{1}{N})$, then $\frac{\alpha_k C_{ks}^1 + (1-\alpha_k)C_{ks}^0}{\alpha_k} - \frac{C_{ks}^1}{\pi_k} = o(\frac{1}{N})$. Therefore, the first term can be approximated by

$$\frac{\alpha_k}{1 - \delta_k\delta_{-k}}(\frac{\alpha_k C_{ks}^1 + (1-\alpha_k)C_{ks}^0}{\alpha_k} - \frac{C_{ks}^1}{\pi_k}) = \alpha_k(\frac{\alpha_k C_{ks}^1 + (1-\alpha_k)C_{ks}^0}{\alpha_k} - \frac{C_{ks}^1}{\pi_k}) + o(\frac{1}{N^2}).$$

which is independent of $\delta_{-k}$. In all, $\zeta_k$ is independent of $\delta_{-k}$ (hence $\alpha_{-k}$).

Third, we study $C_{ks}^1$, $C_{ks}^0$ and $C_k^0$. Since $C_{ks}^1 = C_k^* + o(\frac{1}{N^2})$, then $C_{ks}^1$ is independent of $\alpha_{-k}$. Since $C_{ks}^0 = C_k^* - \frac{(1-C_k^*)\eta_k^2\delta_k(y_{k1} - y_{k2}\delta_k)}{H_k^0} + o(\frac{1}{N^2})$ and $\delta_k$ is independent of $\alpha_{-k}$, then $C_{ks}^0$ is independent of $\alpha_{-k}$. We know that $C_k^0 = \frac{(1-C_k^*)\kappa_k(\frac{n_k-1}{n_k}\rho_k + \phi\eta_k\delta_k)}{H_k^0\zeta_k} + o(\frac{1}{N^2})$. Since $H_k^0$ depends only on $\delta_k$, and $\delta_k$ is independent of $\alpha_{-k}$, then $H_k^0$ is independent of $\alpha_{-k}$. Together with the fact that $\zeta_k$ is independent of $\alpha_{-k}$, we have $C_k^0$ is independent of $\alpha_{-k}$.

Next, $\lambda_k^1$ and $\lambda_k^0$ are independent of $\alpha_{-k}$ if we ignore $o(\frac{1}{N^2})$. We know that $\lambda_k^1 = ((n_k - 1)B_k)^{-1} + o(\frac{1}{N^2}) = \lambda_k^0$. In order to prove that $\lambda_k^1$ and $\lambda_k^0$ are independent of $\alpha_{-k}$ if we ignore $o(\frac{1}{N^2})$, we only need to prove that both $B_k^1$ and $B_k^0$ are independent of $\alpha_{-k}$ if we ignore the term $o(\frac{1}{N})$. Since $\zeta_k$ is independent of $\alpha_{-k}$, then $C_k^1 = \frac{(1-C_k^*)(y_{k2}-y_{k1}\delta_{-k})}{y_{k3}\beta_k} = \frac{(1-C_k^*)y_{k2}}{y_{k3}\zeta_k} + o(\frac{1}{N})$ is independent of $\alpha_{-k}$ if we ignore the term $o(\frac{1}{N})$. Together with $C_k^0$, $\frac{1}{\gamma+\lambda_k^1}$ and $\frac{1}{\gamma+\lambda_k^0}$ are independent of $\alpha_{-k}$ if we ignore $o(\frac{1}{N})$, then $B_k^1$ and $B_k^0$ are independent of $\alpha_{-k}$ if we ignore $o(\frac{1}{N})$.

Next, $a_k^1$, $a_k^0$ and $B_k^0$ are independent of $\alpha_{-k}$ if we ignore $o(\frac{1}{N^2})$. This holds since $\lambda_k^1$, $\lambda_k^0$, $C_{ks}^1$, $C_{ks}^0$ and $C_k^0$ are independent of $\alpha_{-k}$ if we ignore $o(\frac{1}{N^2})$.

Finally, in the main model, we prove that $W_k^1$ and $W_k^0$ are independent of $\alpha_{-k}$.

G.4. **Proof of Proposition D.4:** First, by definition, $\Delta_{ks}$, $\Delta_k$, $c_k^*$ and $\delta_k^*$ are increasing in $|\phi|$. Second, we analyze $C_k^0$, $C_k^1$, $c_k^1$, $B_k^0$, $B_k^1$ and $b_k^1$. In the main model,

$$C_k^0 = (1 - C_k^*)(1 - \frac{\kappa_k C_k^*}{\rho_k n_k}) + o(\frac{1}{N}). \tag{33}$$

Hence, $\frac{\partial C_k^0}{\partial|\phi|} = 0$. Since $C_k^1 = C_k^0 - \frac{\Delta_k}{n_k} + o(\frac{1}{N})$, $|c_k^1| = \frac{c_k^*}{n_k} + o(\frac{1}{N})$, then $\frac{\partial C_k^1}{\partial|\phi|} < 0$ and $\frac{\partial|c_k^1|}{\partial|\phi|} > 0$. Since $\lambda_k^1 = ((n_k-1)B_k)^{-1} + o(\frac{1}{N^2}) = ((n_k-1)\frac{C_k^*}{\gamma})^{-1} + o(\frac{1}{N})$, then $\frac{1}{\gamma+\lambda_k^1} = \frac{1}{\gamma(1+((n_k-1)C_k^*)^{-1})} + o(\frac{1}{N})$. Similarly, $\frac{1}{\gamma+\lambda_k^0} = \frac{1}{\gamma(1+((n_k-1)C_k^*)^{-1})} + o(\frac{1}{N})$. By $(B_k^1, b_k^1) = \frac{1}{\gamma+\lambda_k^1}(1 - C_k^1, c_k^1)$ and $B_k^0 = \frac{1}{\gamma+\lambda_k^0}(1 - C_k^0)$, then we get $\frac{\partial B_k^1}{\partial|\phi|} > 0$ , $\frac{\partial|b_k^1|}{\partial|\phi|} > 0$ and $\frac{\partial B_k^0}{\partial|\phi|} = 0$.

Third, we study $C_{ks}^1$, $C_{ks}^0$, $W_k^1$ and $W_k^0$. Since $C_{ks}^1 = C_k^* + o(\frac{1}{N^2})$, then $\frac{\partial C_{ks}^1}{\partial|\phi|} = 0$. Since $C_{ks}^0 = C_{ks}^1 - \frac{\Delta_{ks}}{n_k} + o(\frac{1}{N^2})$ and $\Delta_{ks}$ is increasing in $|\phi|$, then $\frac{\partial C_{ks}^0}{\partial|\phi|} < 0$. Since $W_k^1$ ($W_k^0$) is determined by $C_{ks}^1$ ($C_{ks}^0$), then $\frac{\partial W_k^1}{\partial|\phi|} = 0$, $\frac{\partial W_k^0}{\partial|\phi|} < 0$.

Next, we study $\lambda_k^1$ and $\lambda_k^0$. We know that $\lambda_k^1 = ((n_k-1)B_k)^{-1} + o(\frac{1}{N^2})$. Since $\frac{\partial B_k^1}{\partial|\phi|} > 0$ and $\frac{\partial B_k^0}{\partial|\phi|} = 0$, then $\frac{\partial \lambda_k^1}{\partial|\phi|} < 0$. By the same logic, $\frac{\partial \lambda_k^0}{\partial|\phi|} < 0$.

Finally, we study $a_k^1$ and $a_k^0$. Since $\frac{\partial \lambda_k^1}{\partial|\phi|} < 0$ and $\frac{\partial C_{ks}^1}{\partial|\phi|} = 0$, then $a_k^1 = \frac{1}{\gamma+\lambda_k^1}C_{ks}^1$ is increasing in $|\phi|$. From the main model, we obtain that

$$a_k^0 = \frac{C_k^*}{\gamma} + \frac{1}{\gamma}\left(-\frac{C_k^*}{1+\gamma(n_k-1)B_k^0} + \frac{1}{n_k^2}(\frac{\alpha_k\Delta_k}{C_k^*} - \Delta_{ks})\right) + o(\frac{1}{N^2}) \tag{34}$$

where $\frac{\alpha_k \Delta_k}{C_k^*} - \Delta_{ks} = \frac{(1-\rho_k)\phi^2}{\rho_k(\rho_k\rho_{-k}-\phi^2)} \frac{\alpha_k}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}} (\frac{1}{C_k^*} - \frac{1}{\alpha_k + \frac{1-\rho_k}{\sigma_k^2}})$ is increasing in $|\phi|$. Together with $\frac{\partial B_k^0}{\partial|\phi|} = 0$, we get $\frac{\partial a_k^0}{\partial|\phi|} > 0$.

G.5. **Proof of Proposition D.5 :** First, $|\frac{\partial \Delta_{ks}}{\partial \alpha}|$, $|\frac{\partial \Delta_k}{\partial \alpha}|$, $|\frac{\partial c_k^*}{\partial \alpha}|$ and $|\frac{\partial \delta_k^*}{\partial \alpha}|$ are increasing in $|\phi|$, by definition. Second, we analyze $C_k^0$, $C_k^1$, $c_k^1$, $B_k^0$, $B_k^1$ and $b_k^1$. By (33), $\frac{\partial C_k^0}{\partial \alpha}$ is independent of $|\phi|$. Since $C_k^1 = C_k^0 - \frac{\Delta_k}{n_k} + o(\frac{1}{N})$, then $|\frac{\partial C_k^1}{\partial \alpha|}| = |\frac{\partial \Delta_k}{\partial \alpha}|$ is increasing in $|\phi|$. Since $|c_k^1| = \frac{c_k^*}{n_k} + o(\frac{1}{N})$, then $|\frac{\partial c_k^1}{\partial \alpha}| = \frac{1}{n_k}|\frac{\partial c_k^*}{\partial \alpha}|$ is increasing in $|\phi|$. We know that $\frac{1}{\gamma+\lambda_k^1} = \Gamma_k + o(\frac{1}{N}) = \frac{1}{\gamma+\lambda_k^0}$, where $\Gamma_k \equiv \frac{1}{\gamma(1+((n_k-1)C_k^*)^{-1})}$. Therefore, $|\frac{\partial B_k^1}{\partial \alpha}| = \Gamma_k |\frac{\partial C_k^1}{\partial \alpha}| + o(\frac{1}{N})$ and $|\frac{\partial b_k^1}{\partial \alpha}| = \Gamma_k |\frac{\partial c_k^1}{\partial \alpha}| + o(\frac{1}{N})$ are increasing in $|\phi|$. Moreover, $|\frac{\partial B_k^0}{\partial \alpha}| = \Gamma_k |\frac{\partial C_k^0}{\partial \alpha}| + o(\frac{1}{N})$ is independent of $|\phi|$.

Third, we study $C_{ks}^1$, $C_{ks}^0$, $W_k^1$ and $W_k^0$. Since $C_{ks}^1 = C_k^* + o(\frac{1}{N^2})$, then $\frac{\partial C_{ks}^1}{\partial \alpha}$ is independent of $|\phi|$. Since $C_{ks}^0 = C_{ks}^1 - \frac{\Delta_{ks}}{n_k} + o(\frac{1}{N^2})$, then $|\frac{\partial C_{ks}^0}{\partial \alpha}| = |\frac{\partial \Delta_{ks}}{\partial \alpha}|$ is increasing in $|\phi|$. Since $W_k^1$ ($W_k^0$) is determined by $C_{ks}^1$ ($C_{ks}^0$), then $\frac{\partial W_k^1}{\partial \alpha}$ is independent of $|\phi|$ and $|\frac{\partial W_k^0}{\partial \alpha}|$ is increasing in $|\phi|$.

Next, we study $(\gamma + \lambda_k^1)^{-1}$ and $(\gamma + \lambda_k^0)^{-1}$. Since $\lambda_k^1 = ((n_k-1)B_k)^{-1}$, then $(\gamma + \lambda_k^1)^{-1} = \frac{B_k}{B_k + \frac{\gamma}{n_k - 1}}$. Since $|\frac{\partial B_k}{\partial \alpha}|$ is increasing in $|\phi|$, then $|\frac{\partial(\gamma+\lambda_k^1)^{-1}}{\partial \alpha}|$ is increasing in $|\phi|$. Similarly, $|\frac{\partial(\gamma+\lambda_k^0)^{-1}}{\partial \alpha}|$ is increasing in $|\phi|$.

Finally, we study $a_k^1$ and $a_k^0$. We know that $a_k^1 = (\gamma + \lambda_k^1)^{-1} C_{ks}^1$. Since $|\frac{\partial(\gamma+\lambda_k^1)^{-1}}{\partial \alpha}|$ is increasing in $|\phi|$ and $\frac{\partial C_{ks}^1}{\partial \alpha}$ is independent of $|\phi|$, then $|\frac{\partial a_k^1}{\partial \alpha}|$ is increasing in $|\phi|$. By (34), $|\frac{\partial a_k^0}{\partial \alpha}|$ is increasing in $|\phi|$.

G.6. **Proof of Lemma E.1:** Define $r_1 \equiv r(\theta_k, p_k)$ and $r_2 \equiv r(\varepsilon_k, p_k)$.

$$(C_{ks}^0, C_k^0) = (\text{Cov}(\theta_k^i, s_k^i),\ \text{Cov}(\theta_k^i, p_k)) \begin{bmatrix} \text{Var}(s_k^i) & \text{Cov}(s_k^i, p_k) \\ \text{Cov}(p_k, s_k^i) & \text{Var}(p_k) \end{bmatrix}^{-1}.$$

$$C_{ks}^0 = \frac{\text{Cov}(\theta_k^i, s_k^i)\text{Var}(p_k) - \text{Cov}(\theta_k^i, p_k)\text{Cov}(s_k^i, p_k)}{\text{Var}(s_k^i)\text{Var}(p_k) - \text{Cov}(s_k^i, p_k)^2} = \frac{1 - r_1^2 - r_1 r_2 \sigma_k^2}{1 + \sigma_k^2 - (r_1 + \sigma_k^2 r_2)^2}.$$

Therefore, by the fact that $r_2 = 0 + o(1)$ and $r_1^2 = \rho_k + o(1)$, we get

$$\frac{\partial C_{ks}^0}{\partial r_1} = \frac{-2r_1\sigma_k^2 + r_2\sigma_k^2(1 - \sigma_k^2 + (r_1 + r_2\sigma_k^2)^2)}{(1 + \sigma_k^2 - (r_1 + \sigma_k^2 r_2)^2)^2} = \frac{-2r_1\sigma_k^2}{(1 + \sigma_k^2 - r_1^2)^2} + o(1) < 0.$$

$$\frac{\partial C_{ks}^0}{\partial r_2} = \frac{2\sigma_k^2(r_1 + \sigma_k^2 r_2) - r_1\sigma_k^2(1 + \sigma_k^2) - r_1\sigma_k^2(r_1 + \sigma_k^2 r_2)^2}{(1 + \sigma_k^2 - (r_1 + \sigma_k^2 r_2)^2)^2} = \frac{2r_1\sigma_k^2(1 - \sigma_k^2 - \rho_k)}{(1 + \sigma_k^2 - r_1^2)^2} + o(1).$$

Therefore, $C_{ks}^0$ is decreasing in $r_2$ if and only if $\frac{1-\rho_k}{\sigma_k^2} < 1$.

Next, we prove $\frac{dr_1}{d\alpha_k} > 0$. Since $p_k = \zeta_k(\bar{s}_k + \delta_k \bar{s}_{-k})$, then $\mathrm{Cov}(\theta_k^i, p_k) = \zeta_k(\rho_k + \frac{1-\rho_k}{n_k} + \eta_k\delta_k\phi)\sigma^2_{\theta_k}$ and $\mathrm{Var}(p_k) = \zeta_k^2[\mathrm{Var}(\bar{s}_k) + \delta_k^2 \mathrm{Var}(\bar{s}_{-k}) + 2\delta_k \mathrm{Cov}(\bar{s}_k, \bar{s}_{-k})]$. We know that $\mathrm{Cov}(\bar{s}_k, \bar{s}_{-k}) = \phi\sigma_{\theta_k}\sigma_{\theta_{-k}}$, $\mathrm{Var}(\bar{s}_k) = (\rho_k + \frac{\kappa_k}{n_k})\sigma^2_{\theta_k}$ and $\mathrm{Var}(\bar{s}_{-k}) = (\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}})\sigma^2_{\theta_{-k}}$, then $\mathrm{Var}(p_k) = \zeta_k^2\sigma^2_{\theta_k}(\rho_k + \frac{\kappa_k}{n_k} + (\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}})(\eta_k\delta_k)^2 + 2\phi\eta_k\delta_k)$. Define $\delta_k^* = \eta_k\delta_k$, we get

$$r_1^2 = \frac{\mathrm{Cov}(\theta_k^i, p_k)^2}{\sigma^2_{\theta_k}\mathrm{Var}(p_k)} = \frac{(\rho_k + \frac{1-\rho_k}{n_k} + \delta_k^*\phi)^2}{\rho_k + \frac{\kappa_k}{n_k} + (\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}})(\delta_k^*)^2 + 2\phi\delta_k^*},$$

$$\frac{dr_1^2}{d\delta_k^*} = \frac{2(\rho_k + \frac{1-\rho_k}{n_k} + \delta_k^*\phi)(\frac{\sigma_k^2\phi}{n_k} - ((\rho_k + \frac{1-\rho_k}{n_k})(\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}}) - \phi^2)\delta_k^*)}{(\rho_k + \frac{\kappa_k}{n_k} + (\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}})(\delta_k^*)^2 + 2\phi\delta_k^*)^2} > 0,$$

since $\frac{\sigma_k^2\phi}{n_k} - ((\rho_k + \frac{1-\rho_k}{n_k})(\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}}) - \phi^2)\delta_k^* = \eta_k y_{k2}(\frac{\sigma_k^2}{\kappa_k}\frac{y_{k1}}{y_{k2}} - \delta_k) + o(\frac{1}{N}) > 0$, which holds since $\alpha_k \le 1$ and $\delta_k = \frac{1}{1 + \frac{1-\rho_k}{\sigma_k^2}\frac{1}{\alpha_k}}\frac{y_{k1}}{y_{k2}} + o(\frac{1}{N})$. Since $\delta_k^* = \eta_k\delta_k$ is increasing in $\alpha_k$, then $r_1$ is increasing in $\alpha_k$.

Since $r_2^2 = \frac{\mathrm{Cov}(\varepsilon_k^i, p_k)^2}{\sigma^2_{\varepsilon_k}\mathrm{Var}(p_k)} = \sigma_k^2 n_k^{-2}\left(\rho_k + \frac{\kappa_k}{n_k} + (\rho_{-k} + \frac{\kappa_{-k}}{n_{-k}})(\delta_k^*)^2 + 2\phi\delta_k^*\right)^{-1}$ and $\delta_k^*$ is increasing in $\alpha_k$, then $r_2$ is decreasing in $\alpha_k$.

## G.7. **Proof of Proposition E.2.**

*Proof.* Define $\beta_k \equiv \frac{\alpha_k a_k^1 + (1-\alpha_k)a_k^0}{\alpha_k B_k^1 + (1-\alpha_k)B_k^0}$, $\pi_k \equiv \frac{\alpha_k a_k^1}{\alpha_k a_k^1 + (1-\alpha_k)a_k^0}$, $\Pi_k \equiv \frac{\alpha_k B_k^1}{\alpha_k B_k^1 + (1-\alpha_k)B_k^0}$ and $x \equiv \frac{B_k^0}{B_k^1}$. Since $\Pi_k = \frac{\alpha_k}{\alpha_k + (1-\alpha_k)x}$, we need to prove that $x$ is the root of a cubic polynomial. In the existence result, $x$ is the root of $G(x) = 0$, which is equivalent to

$$C_k^0 - C_k^1 - (1 - C_k^1)(1-x) - \frac{x(E-x)}{f(x)} = 0. \tag{35}$$

where $E \equiv \frac{1-\bar{C}_k^1}{1-C_k^1}$, $f(x) = [n_k\alpha_k E + ((1-\alpha_k)n_k - 1)x][(n_k\alpha_k - 1)E + (1-\alpha_k)n_k x]$.

Next, we solve $E$ and $f(x)$. By the definition of $C_k^1$ and $\bar{C}_k^1$, $\beta_k$ and $\Pi_k = \frac{\alpha_k}{\alpha_k + (1-\alpha_k)x}$,

$$E = \frac{1-\bar{C}_k^1}{1-C_k^1} = \frac{\beta_k - (1-C_k^*)(1-\delta_k\delta_{-k})\frac{y_{k2}}{y_{k3}}}{\beta_k - \frac{(1-C_k^*)(y_{k2} - y_{k1}\delta_{-k})}{y_{k3}}} = E_0 + E_1 x, \tag{36}$$

where

$$E_0 = 1 + \delta_{-k}(1-C_k^*)(\frac{y_{k2}\delta_k}{y_{k3}} - \frac{y_{k1}}{y_{k3}})\frac{\pi_k}{C_{ks}^1},\ E_1 = \delta_{-k}(1-C_k^*)(\frac{y_{k2}\delta_k}{y_{k3}} - \frac{y_{k1}}{y_{k3}})\frac{\pi_k}{C_{ks}^1}\frac{1-\alpha_k}{\alpha_k}.$$

Substituting (36) into the definition of $f(x)$ (see (35)), we get

$$f(x) = f_2 x^2 + f_1 x + f_0, \tag{37}$$

$$f_2 = ((1-\alpha_k)n_k + n_k\alpha_k E_1 - 1)((1-\alpha_k)n_k + (n_k\alpha_k - 1)E_1),$$
$$f_1 = ((1-\alpha_k)n_k + n_k\alpha_k E_1 - 1)(n_k\alpha_k - 1)E_0 + n_k\alpha_k E_0((1-\alpha_k)n_k + (n_k\alpha_k - 1)E_1),$$
$$f_0 = n_k\alpha_k(n_k\alpha_k - 1)E_0^2.$$

Substituting (36), (37) and the definition of $C_k^0$ and $C_k^1$ into (35), we get $x$ as the root of the following cubic polynomial (38):

$$A_3x^3 + A_2x^2 + A_1x + A_0 = 0. \tag{38}$$

where

$$A_3 = \left(\frac{D(1-\alpha_k)}{\alpha_k} + \frac{C_{ks}^1}{\pi_k}\right)f_2 + g_1(1-E_1),$$
$$A_2 = \left(D - \frac{C_{ks}^1}{\pi_k}\right)f_2 + \left(\frac{D(1-\alpha_k)}{\alpha_k} + \frac{C_{ks}^1}{\pi_k}\right)f_1 + g_0(1-E_1) - g_1E_0,$$
$$A_1 = \left(D - \frac{C_{ks}^1}{\pi_k}\right)f_1 + \left(\frac{D(1-\alpha_k)}{\alpha_k} + \frac{C_{ks}^1}{\pi_k}\right)f_0 - g_0E_0, \;\; A_0 = \left(D - \frac{C_{ks}^1}{\pi_k}\right)f_0,$$
$$g_1 = (1-C_k^*)\left(\frac{y_{k2}}{y_{k3}} - \frac{y_{k1}}{y_{k3}}\delta_{-k}\right)\frac{1-\alpha_k}{\alpha_k}, \;\; g_0 = (1-C_k^*)\left(\frac{y_{k2}}{y_{k3}} - \frac{y_{k1}}{y_{k3}}\delta_{-k}\right) + \frac{C_{ks}^1}{\pi_k},$$
$$D = \frac{(1-C_k^*)(\rho_k(\kappa_k - e_{k3}) + \eta_k\delta_k\kappa_k\phi)(1-\delta_k\delta_{-k})}{H_k^0} - (1-C_k^*)\left(\frac{y_{k2}}{y_{k3}} - \frac{y_{k1}}{y_{k3}}\delta_{-k}\right).$$

Notice that $A_3$, $A_2$, $A_1$ and $A_0$ are coefficients that depend on $\pi$.

□