The Climate Risk Premium: How Uncertainty Affects the Social Cost of Carbon*

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I formally analyze the marginal value of reducing greenhouse gas emissions under uncertainty about how much emissions warm the climate, under uncertainty about the consumption loss due to warming, and under stochastic shocks to consumption growth and to the weather. I theoretically demonstrate that each of these sources of uncertainty increases willingness to pay for emission reductions if and only if the coefficient of relative risk aversion is greater than one. In a calibrated numerical application, uncertainty increases the social cost of carbon by 35%. This premium is driven by uncertainty about the consumption loss due to warming, both directly and through its interaction with uncertainty about the magnitude of warming.

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Uncertainty is fundamental to climate change. Today's greenhouse gas emissions will affect the climate for centuries. The optimal emission tax that internalizes the resulting damages depends on the uncertain degree to which emissions generate warming, on the uncertain channels through which warming will impact consumption and the environment, on the uncertain future evolution of greenhouse gas stocks, and on uncertain future growth in total factor productivity and consumption. Nonetheless, the primary tools for analyzing the optimal emission tax have been deterministic climate-economy models, and recently developed recursive dynamic programming versions of these models have analyzed only a single source of uncertainty at a time. I here undertake a more comprehensive theoretical and quantitative investigation of the implications of uncertainty for greenhouse gas emission policies. I analytically disentangle and sign the channels through which uncertainty matters for policy, and I quantitatively demonstrate that deterministic models and models that include only a single source of uncertainty substantially underestimate the value of emission reductions.

I develop a novel theoretical setting with four interacting sources of uncertainty. Consumption evolves stochastically and generates greenhouse gas emissions. Higher temperatures reduce the expected growth rate of consumption. Each instant's temperature is determined by the concentration of greenhouse gases in the atmosphere and by random weather shocks. The policymaker is unsure about the warming that greenhouse gas emissions will generate and about the reduction in consumption growth that warming will impose. The policymaker seeks to value a marginal reduction in today's emissions. This reduction in emissions will produce a stream of payoffs that depend on the realizations of the weather and consumption shocks as well as on the true sensitivities of the climate to emissions and of economic growth to the climate.

I formally demonstrate that each source of uncertainty increases the marginal external benefit of emission reductions (known as the social cost of carbon) under conventional power utility specifications. First, recognizing uncertainty reduces the predictability of future consumption, which induces precautionary savings when the policymaker is prudent. Because reducing emissions is a form of saving, recognizing uncertainty induces precautionary emission reductions. This channel works to increase the social cost of carbon.[1]

Second, introducing uncertainty forces the policymaker to consider the insurance value of emission reductions. In the benchmark consumption-based capital asset pricing model, market agents are willing to accept lower expected returns on assets whose returns covary negatively with consumption, and they require greater expected returns when assets' returns covary positively with consumption (Lucas, 1978; Breeden, 1979). The former type of asset provides insurance against negative consumption shocks, while the latter type of asset tends to pay off in high-consumption states, when additional consumption is less valuable. This same logic applies when pricing the asset defined by a unit of emission reductions: a poli-

[1] This effect is familiar from work describing how consumption volatility lowers the risk-free consumption discount rate employed by prudent agents (Gollier, 2002).

cymaker should be willing to pay more to reduce emissions if emission reductions increase consumption by a large amount when consumption is otherwise low.

I show that this insurance value has two components. A first component works to reduce the social cost of carbon. Under conventional damage specifications, the consumption losses due to climate change increase in the level of consumption. As a result, emission reductions increase future consumption by a larger amount when future consumption is already high. This mechanical correlation between future consumption and the future consumption gains due to emission reductions makes emission reductions seem like an especially risky investment and therefore works to reduce the policymaker's willingness to pay for emission reductions. However, I show that the positive precautionary saving channel dominates this negative "damage scaling" channel when the coefficient of relative risk aversion is greater than 1. Therefore, the precautionary saving channel combines with this first insurance channel to increase the social cost of carbon under the types of preferences typically used in macroeconomic and climate policy analyses.

A second component of the insurance channel considers whether today's emission reductions will increase future consumption growth by a larger amount in states with high future consumption or in states with low future consumption. This "growth insurance" channel most closely corresponds to previous arguments about the insurance value of emission reductions (described below). I show that this channel increases willingness to pay for emission reductions when the coefficient of relative risk aversion is greater than 1.[2]

The intuition depends on the source of uncertainty. Assume that the coefficient of relative risk aversion is greater than 1. Begin by considering uncertainty induced by weather shocks: the weather is variable, and many have suggested that it will become more variable as the earth warms.[3] Now imagine that a sequence of bad weather shocks reduces future consumption to a low state. In this case, any emission reductions undertaken today would end up being especially valuable because the bad weather shocks would have been even more extreme and damaging if the world had been a bit warmer. Reducing greenhouse gas emissions acquires insurance value by protecting against the possibility of future extreme events.

[2] I show that the sign of the growth insurance channel depends on whether an exposure effect or a risk aversion effect dominates. The risk aversion effect dominates when the coefficient of relative risk aversion is greater than one. The exposure effect is similar to the damage scaling channel in reflecting that greater consumption allows for greater consumption losses, whereas the risk aversion effect reflects that marginal utility is greater when consumption is low.

[3] The potential for greater variability is often discussed (e.g., Carney, 2015) but poorly understood. Some scientific studies have detected a recent increase in variability and/or have forecasted a future increase in variability (e.g., Schär et al., 2004; Seager et al., 2011; Hansen et al., 2012). On the other hand, Huntingford et al. (2013) suggest that changes in temperature variability have been regionally heterogeneous and that variability could decrease with future warming. Lemoine and Kapnick (2016) also emphasize regional heterogeneity and find that global warming will increase the variability of unforecasted temperature and precipitation shocks in many parts of the world.

The second and third sources of uncertainty are lack of knowledge about warming per unit of carbon dioxide ("climate sensitivity") and about consumption losses per unit of warming ("damage sensitivity"). These sources of uncertainty again make the growth insurance channel work to increase the social cost of carbon. If the climate is actually very sensitive to carbon dioxide, then each unit of time 0 emissions strongly affects time t temperature and consumption growth. Further, a high climate sensitivity implies relatively low time t consumption because the relatively severe warming at times prior to t reduces economic growth prior to t. Emission reductions are therefore especially effective at increasing time t consumption growth when time t consumption is already low. Similarly, if the economy is actually very sensitive to warming, then each unit of time 0 emissions strongly affects time t consumption growth. A high damage sensitivity implies relatively low time t consumption because the relatively severe damages at times prior to t reduce economic growth prior to t. Once again, emission reductions are especially effective at increasing time t consumption growth when time t consumption is already low. Therefore, under uncertainty about either climate sensitivity or damage sensitivity, reducing greenhouse gas emissions acquires insurance value by smoothing consumption across states of the world.

The fourth source of uncertainty is shocks to business-as-usual consumption growth, as would arise from shocks to technology or productivity. Emissions are an increasing function of consumption, so a positive shock to consumption becomes a positive shock to emissions. Climate science has long established that the first units of carbon dioxide (CO_2) trap more heat than the last units. Therefore, worlds with high consumption and high emissions are also worlds in which the marginal effect of emissions on the climate is small. Reducing greenhouse gas emissions increases the growth rate of future consumption by more when negative consumption shocks have led to a world that is relatively poor and has generated relatively few emissions. Once again, the growth insurance channel works to increase the social cost of carbon.[4]

I quantitatively evaluate the implications of uncertainty in a calibrated numerical application. In the base specification, uncertainty increases the social cost of carbon from \$222 per tCO_2 to \$300 per tCO_2. I find that uncertainty about climate sensitivity and about damage sensitivity each increase the social cost of carbon primarily through the growth insurance channel, whereas uncertainty about business-as-usual consumption growth increases the social cost of carbon primarily through the precautionary saving channel. Uncertainty about damages has over twice as large an effect as uncertainty about business-as-usual consumption growth, which in turn has a 50% larger effect than does uncertainty about warming. Uncertainty about weather shocks has only a tiny effect. Finally, interactions between sources of

[4]The foregoing analysis of shocks to consumption growth assumes that warming reduces the growth rate of consumption, as in the main theoretical setting. We will see that if warming instead reduces the level of consumption, then the growth insurance channel contains an additional component that depends on the rate of increase of marginal damage in temperature. This additional component is theoretically ambiguous but would work to reduce the social cost of carbon in many common numerical implementations.

uncertainty are important. If one were to estimate the total effect of uncertainty by summing the adjustments from models that include only a single source of uncertainty, then one would expect all four sources of uncertainty to increase the social cost of carbon by only \$34 per tCO_2, which is less than half of the correct value of \$78 per tCO_2. I show that the interaction between uncertainty about damages and uncertainty about warming is especially important.

The present paper's analytic focus and calibrated quantitative model both contrast with standard economic analyses of uncertainty and climate change. The primary tools for analyzing the optimal emission tax have been numerical Ramsey-Cass-Koopmans growth models extended to couple the climate and the economy (Nordhaus, 1992). Most of these "integrated assessment models" are deterministic and so are incapable of incorporating uncertainty into the decision problem.[5] A recent literature has developed recursive versions of these models in order to analyze the policy implications of uncertainty about warming (Kelly and Kolstad, 1999; Leach, 2007; Hwang et al., 2014; Kelly and Tan, 2015), about economic growth (Jensen and Traeger, 2014), about damages from climate change (Cai et al., 2013; Crost and Traeger, 2013, 2014; Rudik, 2015), and about tipping points (Lemoine and Traeger, 2014). These models have two advantages over the present setting: they can capture the policy value of anticipated learning and of flexibility to adapt future policies to unexpected outcomes (Lemoine and Rudik, 2017), and by optimizing policy, they go beyond the present paper's focus on the social cost of carbon to calculate the optimal tax on carbon emissions.[6] However, these models also have two disadvantages relative to the present setting: their results are primarily numerical, and the computational demands of dynamic programming have led them to analyze only a single source of uncertainty at a time. In contrast, I will obtain analytic results about how the social cost of carbon changes with volatility and variance parameters, I will quantitatively compare different sources of uncertainty within a single numerical setting, and I will quantitatively demonstrate the importance of including multiple sources of uncertainty when evaluating the social cost of carbon.[7]

Previous theoretical discussions about the implications of uncertainty for the value of

[5] Monte Carlo analyses of deterministic models are not equivalent to models in which the policymaker is aware of uncertainty and can adjust policy accordingly. See Lemoine and Rudik (2017) for an exposition. In fact, Crost and Traeger (2013) show that Monte Carlo analyses may even incorrectly sign the effect of uncertainty on the optimal emission tax.

[6] The social cost of carbon has traditionally been defined as the marginal value of emission reductions along any given path for emissions and consumption, which coincides (in a deterministic setting) with the optimal emission tax along the optimal path for emissions and consumption. The U.S. government's recent calculation of the social cost of carbon emphasizes the value of emission reductions along the no-policy ("business-as-usual") path (Greenstone et al., 2013). The traditional definition matches the present paper's theoretical setting, and the U.S. government's application matches the present paper's numerical calibration.

[7] Golosov et al. (2014) theoretically analyze the social cost of carbon in a dynamic stochastic general equilibrium model of climate change. They assume log utility, as do subsequent closely related papers. We will see that log utility is a knife-edge case in which uncertainty is not interesting.

emission reductions have focused on the growth insurance channel.[8] Several economists have argued that uncertainty about total warming or about damages from warming should increase the value of emission reductions because their greatest payoffs would occur precisely when high damages have reduced consumption (Howarth, 2003; Sandsmark and Vennemo, 2007; Becker et al., 2010).[9] And several economists have argued that uncertainty about future business-as-usual consumption should reduce the value of emission reductions because states with high climate damages tend to correspond to states with high consumption (Litterman, 2013; Weitzman, 2013). Comparing the effects of these multiple uncertainties, two economists have recently argued that the effect of uncertainty about future consumption dominates, so that uncertainty reduces the value of emission reductions: Nordhaus (2008, 2011) shows that consumption and warming are positively correlated in a Monte Carlo analysis of his numerical DICE integrated assessment model, and Gollier (2012) constructs a two-period example in which the correlation between consumption growth and emissions numerically dominates the correlation between emissions and damages. Taking a more formal perspective, the present paper shows that all of these types of uncertainty actually work to increase the value of emission reductions through the growth insurance channel. In particular, previous arguments about the implications of uncertainty about consumption growth have failed to recognize that the physics of climate change imply that worlds with high emissions are also worlds in which marginally reducing emissions avoids relatively less warming.[10] The present paper also demonstrates additional channels through which uncertainty affects the social cost of carbon and quantifies all of these channels in a calibrated application.

The next section highlights the key channels through which uncertainty affects the social cost of carbon. Section 2 describes the full theoretical setting. Section 3 decomposes the social cost of carbon and analyzes how it changes with variance and volatility parameters.

[8]A parallel set of theoretical discussions has recognized that the possibility of shocks to consumption growth can reduce the risk-free consumption discount rate, with many concluding that the adjustment is likely to be small (e.g., Traeger, 2009).

[9]Many have also discussed how the potential for catastrophic climate change can increase the value of emission reductions (e.g., Weitzman, 2007; Becker et al., 2010; Litterman, 2013; Murphy and Topel, 2013; Pindyck, 2013; Weitzman, 2013). The present setting will allow for unexpectedly high damages from climate change, though it will not focus on discrete catastrophes. See Martin and Pindyck (2015) for an analysis of willingness to pay to prevent discrete catastrophes.

[10]The graphical analysis of Nordhaus (2008, 2011) must assume that the marginal effect of emissions on consumption is greatest under high-warming outcomes, which we will see is not always true. The correct graphical analysis would plot consumption against the marginal effect of emissions on consumption (although this analysis would still miss precautionary saving motives). The informal discussions in Litterman (2013) and Weitzman (2013) also do not adopt an explicitly marginal perspective. Gollier (2012) undertakes an explicitly marginal analysis, but in his setting, second-period temperature increases linearly in second-period emissions. In contrast, the present setting recognizes that emissions matter only by increasing the stock of carbon dioxide, which in turn affects temperature nonlinearly and noninstantaneously by trapping heat that would have escaped to space. The nonlinearity turns out to be crucial. Concurrent with the present paper, Dietz et al. (2015) report a positive covariance between consumption and the marginal social cost of emissions in the DICE integrated assessment model. I relate their results to the present paper in Section 5.

Section 4 quantitatively assesses the implications of uncertainty for the social cost of carbon. Section 5 considers the implications of alternate specifications for how climate change affects consumption. It also connects the analysis to the choice of consumption discount rate for use in evaluating climate impacts. The final section concludes. The online appendix contains proofs, derivations, and the description of the numerical calibration. It also analyzes the implications of uncertainty when the policymaker has preferences over environmental quality.

1 Two-period analysis

I begin with a two-period analysis that illustrates the more important forces at play in the full setting. Let time 0 consumption and greenhouse gas emissions be C_0 and e_0, both strictly positive. In the absence of climate change, consumption at time 1 is a random variable C_1 with a strictly positive lower bound. Emissions at time 1 are $e_1(C_1)$, for $e_1(\cdot)$ positive and increasing. Climate change $s\,T(e_0 + e_1) \geq 0$ is driven by the accumulation of emissions. Additional emissions increase climate change ($T'(\cdot) > 0$), and in accord with the scientific understanding described in Section 2, the first units of emissions cause more warming than do the last units ($T''(\cdot) < 0$). The random variable s reflects uncertainty about the strength of warming. It has support in the positive numbers. Warming reduces consumption to $C_1/D(s\,T)$, where damages $D(\cdot) > 0$ are increasing in realized temperature $s\,T$. Welfare is $W = u(C_0) + \beta\, u(C_1/D(s\,T))$, where per-period utility $u(\cdot)$ is increasing and concave and $\beta \in (0, 1]$ is the discount factor.

If the policymaker were to receive ξ units of emission reductions at time 0, the policymaker's expected welfare would increase by approximately

$$-\xi \frac{\mathrm{d}W}{\mathrm{d}e_0} = \xi\, E\left[\beta\, u'\left(\frac{C_1}{D(s\,T(e_0 + e_1(C_1)))}\right) \frac{C_1}{D(s\,T(e_0 + e_1(C_1)))} \frac{D'(s\,T(e_0 + e_1(C_1)))}{D(s\,T(e_0 + e_1(C_1)))}\, s\,T'(e_0 + e_1(C_1))\right],$$

where a prime indicates a derivative. For small ξ, we can ignore higher-order terms. Now imagine that the policymaker has to forgo $x\,\xi$ units of consumption at time 0 to acquire these emission reductions. The disutility from making this payment is approximately $u'(C_0)\, x\,\xi$, again for small ξ. The most consumption that the policymaker would give up to obtain a unit of emission reductions is then

$$scc \triangleq x = E\left[\beta\, \frac{u'\left(\frac{C_1}{D(s\,T(e_0+e_1(C_1)))}\right)}{u'(C_0)} \overbrace{\frac{C_1}{D(s\,T(e_0 + e_1(C_1)))} \frac{D'(s\,T(e_0 + e_1(C_1)))}{D(s\,T(e_0 + e_1(C_1)))}\, s\,T'(e_0 + e_1(C_1))}^{\triangleq G_1}\right]. \tag{1}$$

The term $\beta\, u'\left(\frac{C_1}{D(s\,T(e_0+e_1(C_1)))}\right)/u'(C_0)$ is the stochastic discount factor that prices the time 1 consumption gain G_1 from a marginal reduction in time 0 emissions. The term x is the gross benefit to the policymaker (in terms of time 0 consumption) from marginally reducing time 0 emissions. This gross benefit is also known as the social cost of carbon (scc). The policymaker should undertake projects that provide time 0 emission reductions at a cost less than the social cost of carbon.

Using a second-order Taylor expansion of $E[u'(C_1/D)]$ around $E[C_1/D]$, we have

$$scc \approx \frac{\beta}{u'(C_0)}\left\{ \underbrace{u'(E[C_1/D])\, E[G_1]}_{\text{deterministic}} + \underbrace{\frac{1}{2}u'''(E[C_1/D])\, E[G_1]\, Var[C_1/D]}_{\text{precautionary}} + \underbrace{Cov[u'(C_1/D), G_1]}_{\text{insurance}} \right\}. \tag{2}$$

The first term in braces gives the social cost of carbon in a deterministic world. It multiplies the expected time 1 consumption gain from reduced emissions by the marginal utility of consumption at time 1, calculated along the expected consumption trajectory. The second term is a precautionary component, which increases the social cost of carbon as long as $u''' \geq 0$. In this standard case, the agent is prudent in consumption (Leland, 1968; Drèze and Modigliani, 1972; Kimball, 1990). A prudent agent prefers to attach a mean-zero risk to a high-consumption state rather than to a low-consumption state. Making future consumption riskier leads prudent agents to save more today, so that the additional consumption risk is attached to a future with higher baseline consumption. In the present setting, the policymaker saves by reducing emissions. An increase in the variance of future consumption increases a prudent policymaker's willingness to save through emission reductions, and thus increases the social cost of carbon.

The third term is an insurance component. It increases the social cost of carbon if and only if the time 1 consumption gain from additional time 0 emission reductions covaries positively with time 1 marginal utility. In this case, emission reductions become especially valuable because they tend to pay off in states in which additional time 1 consumption is especially valuable. This term is familiar from the consumption-based capital asset pricing model (Lucas, 1978; Breeden, 1979). There, agents require a greater expected return on assets whose returns covary positively with consumption and are willing to accept a lower expected return on assets whose returns covary negatively with consumption. The former type of asset exacerbates the risk in future consumption, whereas the latter type of asset smooths future consumption. The variance of an asset's returns matters only through this covariance. In the present setting, time 0 emission reductions are an asset that generates uncertain consumption payoffs G_1. The variance of these payoffs does not affect the social cost of carbon directly. Instead, their covariance with time 1 consumption (via marginal utility) determines the insurance value of emission reductions.[11,12]

Now assume that utility takes the conventional power form: $u(C) = C^{1-\eta}/(1-\eta)$, with $\eta \geq 0, \neq 1$. In line with recent economic arguments (Pindyck, 2012, 2013; Stern, 2013) and with recent empirical literature (e.g., Bansal and Ochoa, 2011; Dell et al., 2012; Burke et al., 2015; Heal and Park, 2015), let climate change reduce the growth rate of consumption, so that $D(s\,T) = e^{\alpha s T}$, with α a random variable that is positive in expectation. This form is consistent with the common assumption that $D''(s\,T) > 0$. Substituting into equation (1) and using a second-order Taylor expansion of $E[(C_1/D)^{1-\eta}]$ around $C_1/D = E[C_1/D]$, we have:

$$\begin{aligned} scc \approx \frac{\beta}{C_0^{-\eta}} \Big\{ & (E[C_1/D])^{1-\eta}\, E\left[\alpha\, s\, T'\right] && \text{(deterministic)} \\ & + \frac{1}{2}\eta\,(\eta+1)\,(E[C_1/D])^{-\eta-1}\, E[\alpha\, s\, T']\, Var(C_1/D) && \text{(precautionary)} \\ & - \eta\,(E[C_1/D])^{-\eta-1}\, E[\alpha\, s\, T']\, Var(C_1/D) && \text{(damage scaling)} \\ & + Cov\left[(C_1/D)^{1-\eta}, \alpha\, s\, T'\right] \Big\}. && \text{(growth insurance)} \end{aligned} \tag{3}$$

The first line in braces gives the social cost of carbon in a deterministic world. The second line is the precautionary saving channel analyzed previously. The leading coefficient $\frac{1}{2}\eta(\eta+1)$ is familiar from the extended Ramsey rule (Gollier, 2002), in which uncertainty about future consumption reduces the risk-free discount rate (see Section 5). The third and fourth lines divide the insurance channel into a damage scaling channel that captures how the level of consumption responds to a change in its growth rate and a growth insurance channel that captures how the marginal effect of emission reductions on the growth rate of consumption covaries with marginal utility.

The third line is the damage scaling channel. It is negative, working to reduce the social cost of carbon. In most economic models of climate change, damages affect future

[11] Previous literature has focused on whether the "beta" of climate change is positive or negative, where the beta refers to the covariance between G_1 and time 1 consumption and thus to the sign of the insurance channel. However, we here see that the precautionary channel can lead uncertainty to increase the social cost of carbon even if climate change has a positive beta (i.e., even if $Cov[u'(C_1/D), G_1] < 0$ so that the insurance channel is negative). This intuition is different from basic asset pricing models. In the consumption-based capital asset pricing model, the stochastic discount factor is independent of any particular asset's returns. In contrast, when discussing emission reductions that reduce future climate change, it would be a mistake to ignore that greater uncertainty about future climate change makes future baseline consumption less certain and thus increases the value of savings. Therefore uncertainty about the payoffs from the emission "asset" can increase the desire for savings, by affecting the stochastic discount factor itself. See Section 5 for an interpretation in terms of the risk-free discount rate.

[12] See the recent review by Lemoine and Rudik (2017) for a discussion of analogous channels in the context of the optimal carbon tax and recursive integrated assessment models. They also discuss the channels introduced when the policymaker anticipates that he will learn about uncertain parameters over time.

consumption multiplicatively: future consumption is given by C_1/D rather than $C_1 - D$ (see Section 5 for a discussion). In these cases, future climate change reduces consumption by an especially large amount when future consumption would have been especially high due to, for instance, especially rapid technological progress. This positive covariance between future consumption and the future consumption loss from climate change reduces the insurance value of time 0 emission reductions and therefore reduces the time 0 social cost of carbon.[13] Critically, this negative damage scaling channel is dominated by the positive precautionary saving channel when $\eta > 1$, as is typically assumed in economic models of climate change.

The final line is the growth insurance channel: it accounts for uncertainty about the marginal effect of time 0 emissions on the growth rate of consumption. This line is positive (working to increase the social cost of carbon) if the marginal effect of time 0 emissions on consumption growth is large when $(C_1/D)^{1-\eta}$ is large and is negative (working to reduce the social cost of carbon) otherwise. This channel's sign depends on whether $\eta > 1$ or $\eta < 1$. The reason is that the term $(C_1/D)^{1-\eta}$ combines a risk aversion effect and an exposure effect: $(C_1/D)^{-\eta}$ is the marginal utility of time 1 consumption and determines willingness to substitute consumption across states of the world, and C_1/D determines the magnitude of consumption lost from additional climate change, as in the damage scaling channel. The risk aversion effect (controlled by η) tends to make additional climate damages more painful when they occur in states with low C_1/D, but the exposure effect tends to make additional climate damages more painful when they occur in states with high C_1/D. The risk aversion effect dominates the exposure effect if and only if $\eta > 1$.[14] In that standard case, this final line works to increase the time 0 social cost of carbon if and only if time 0 emission reductions tend to increase consumption growth most strongly in states of the world in which time 1 consumption C_1/D happens to be small.

Now consider the sign of the covariance. Begin with the implications of uncertainty about the damage parameter α and the warming parameter s. Large values of α and s imply that D is large, so that net time 1 consumption C_1/D is small. Large values of α and s also imply that $\alpha\, s\, T'$ is large, so that consumption growth is especially sensitive to time 0 emissions. Thus, when $\eta > 1$, uncertainty about α and s works to make the covariance between $(C_1/D)^{1-\eta}$ and $\alpha\, s\, T'$ positive, which works to increase the social cost of carbon through the growth insurance channel.

Now consider the implications of uncertainty about baseline time 1 consumption C_1. Large values of C_1 generate large values of time 1 emissions e_1. The large values of e_1 increase total warming T and thus increase damages D. In any reasonable calibration, the

[13] Note that taking a first-order approximation to C_1/D around $E[C_1/D]$ in $Cov[C_1/D, (C_1/D)^{-\eta}]\, E[\alpha s T']$ yields the damage scaling channel. This derivation illustrates how this channel is one piece of the broader insurance channel analyzed previously.

[14] These two effects exactly cancel in the case of log utility ($\eta = 1$). And it is easy to see that the precautionary saving and damage scaling channels also exactly cancel as $\eta \to 1$. Therefore, in that knife-edge case of log utility, uncertainty does not affect the social cost of carbon.

increase in C_1 outweighs the increase in D, so that large values of C_1 go with large values of C_1/D.[15] Because $T'' < 0$, large values of e_1 go with small values of T': the marginal effect of time 0 emissions on warming becomes small as cumulative emissions become large. Thus, large values of C_1 imply large C_1/D and small $\alpha\, s\, T'$. In other words, if time 1 consumption receives an especially positive shock, then the time 1 stock of CO_2 is especially large and time 0 emission reductions therefore increase consumption growth by an especially small amount. But if time 1 consumption receives an especially negative shock, then the time 1 stock of CO_2 is especially small and time 0 emission reductions therefore increase consumption growth by an especially large amount. The covariance between $(C_1/D)^{1-\eta}$ and $\alpha\, s\, T'$ is again positive for $\eta > 1$, so that uncertainty about baseline consumption growth also works to increase the social cost of carbon through the growth insurance channel.

In sum, we have seen that uncertainty about climate damages, total warming, and baseline consumption growth affects the social cost of carbon through both a precautionary saving channel and an insurance channel, with the latter divided into a damage scaling channel and a growth insurance channel. The precautionary saving channel always works to increase the social cost of carbon, the damage scaling channel always works to reduce the social cost of carbon, and the growth insurance channel works to increase the social cost of carbon if and only if the coefficient of relative risk aversion is greater than unity. The precautionary saving channel dominates the damage scaling channel if and only if the coefficient of relative risk aversion is greater than unity. Standard calibrations of climate-economy integrated assessment models use a coefficient of relative risk aversion that is around 2 (e.g., Nordhaus, 2008). We might therefore expect uncertainty about climate damages, warming, and baseline consumption growth to increase the social cost of carbon. The next section develops the full continuous-time setting. Subsequent sections prove further results in the context of that full setting and quantify the effects of uncertainty in a calibrated numerical implementation.

2 Continuous-Time Setting

I now describe the full, continuous-time setting. This setting will allow for delays in warming, for weather shocks, and for a meaningful numerical calibration.

Let consumption obey a type of geometric Brownian motion:

$$\frac{\mathrm{d}C(t)}{C(t)} = \mu_C\,\mathrm{d}t - \alpha\,T(t)\,\mathrm{d}t + \sigma_C\,\mathrm{d}z_C(t).$$

The drift parameter $\mu_C > \frac{1}{2}\sigma_C^2$ reflects the historical tendency of aggregate consumption $C(t)$ to grow over time in the absence of climate change. The random variable α represents the expected detrimental effect of cumulative global temperature change $T(t)$ on the rate

[15] In particular, $\mathrm{d}[C_1/D]/\mathrm{d}C_1 \geq 0$ if and only if $C_1/D \leq 1/[sD'T'e_1']$, which holds if and only if $C_1 \leq 1/[\alpha s T' e_1']$.

of economic growth. It has finite variance and positive expectation. The Brownian motion $z_C(t)$ reflects the non-climate factors that make consumption volatile, with the variance of consumption controlled by the volatility parameter $\sigma_C \geq 0$.

Climate change is driven by the accumulation of CO_2 in the atmosphere. CO_2 is emitted as a byproduct of consumption. The stock $M(t)$ of atmospheric CO_2 therefore evolves with consumption:

$$\mathrm{d}M(t) = \gamma(t)\, C(t)\, \mathrm{d}t - \delta \left[M(t) - M_{pre} \right] \mathrm{d}t.$$

Time t consumption generates emissions at rate $\gamma(t) > 0$, and CO_2 mean-reverts (or "decays") to the preindustrial level M_{pre} at rate $\delta \geq 0$. The evolution of $\gamma(t)$ reflects exogenous changes in the emission intensity of production technology and in the emission intensity of the consumption bundle.

The accumulation of CO_2 traps heat via the greenhouse effect. The amount of extra heat trapped relative to the heat trapped by preindustrial CO_2 M_{pre} is a metric known as forcing:

$$F(M(t)) = \nu \, \ln \left[M(t)/M_{pre} \right],$$

with $\nu > 0$. Consistent with both the scientific literature (Kondratiev and Niilisk, 1960; Möller, 1963; Rasool and Schneider, 1971; Ramaswamy et al., 2001) and with benchmark integrated climate-economy models (e.g., Nordhaus, 2008, 2014), forcing is logarithmic in CO_2. Additional CO_2 warms the planet by making the atmosphere "optically thick" over a broader range of infrared wavelengths: the upper atmosphere absorbs more of the infrared radiation (i.e., heat) escaping to space, and the surface heats up in order to maintain overall energy balance. Because the atmosphere is already optically thick in the wavelengths over which CO_2 most effectively traps infrared radiation, the primary contribution of additional CO_2 to warming comes from trapping outgoing infrared radiation at wavelengths that are less effectively absorbed by each unit of CO_2. Thus, the contribution of CO_2 to warming is concave in the stock of CO_2 (as assumed in Section 1). An additional unit of CO_2 traps less additional heat when the atmosphere is already holding a lot of CO_2.[16]

Forcing eventually generates warming, but due to the dynamics of the ocean, trapped heat does not immediately translate into surface warming. Temperature responds only gradually to an increase in forcing (Nordhaus, 1991; Lemoine and Rudik, 2014):

$$\mathrm{d}T(t) = \phi \left[s\, F(M(t)) - T(t) \right] \mathrm{d}t + \sigma_T T(t)\, \mathrm{d}z_T(t).$$

This is a mean-reverting process with geometric noise and a mean that evolves with the level of CO_2. If maintained forever, a unit of forcing eventually produces s units of warming, a translation of a parameter commonly known as climate sensitivity. I allow s to be uncertain, with strictly positive expectation and finite variance. The parameter $\phi > 0$ controls

[16] For an accessible explanation of the physics underlying this concave relationship, see `http://www.realclimate.org/index.php/archives/2007/06/a-saturated-gassy-argument-part-ii`.

the degree of inertia in the climate system: as $\phi \to \infty$, changes in forcing pass through to temperature instantaneously (low inertia), and as $\phi \to 0$, changes in forcing pass through to temperature only slowly (high inertia). The Brownian motion $z_T(t)$ and volatility parameter $\sigma_T \geq 0$ reflect stochasticity in the climate system, which determines each instant's weather. I model the weather shocks as geometric rather than additive in order to explore the common assertion that the interannual variability of the climate will increase with global mean temperature (see footnote 3). The Brownian motion that drives temperature is independent of the Brownian motion $z_C(t)$ that drives consumption and emissions.

The policymaker derives utility from consumption. Instantaneous utility $u(\cdot)$ takes the familiar isoelastic form:

$$u\left(\frac{C(t)}{L(t)}\right) = \frac{[C(t)/L(t)]^{1-\eta}}{1-\eta},$$

where $\eta \geq 0, \neq 1$ and where $L(t)$ is the (exogenous) population at time t. I mostly ignore the special case of log utility ($\eta = 1$). The parameter η is the coefficient of relative risk aversion and is also the inverse of the elasticity of intertemporal substitution. Time 0 intertemporal welfare $W(0)$ aggregates instantaneous utility over time and over people, discounting at rate $\rho \geq 0$:

$$W(0) = \int_0^{\tau} e^{-\rho t} L(t) E\left[u(C(t)/L(t))\right] \mathrm{d}t,$$

for $\tau \in (0, \infty)$. All expectations in the text and appendix are with respect to the time 0 information set.

The initial values of the state variables are known and positive: $C(0) > 0$, $M(0) > M_{pre}$, and $T(0) > 0$. The policymaker knows the relations defined in this section and uses them to form expectations about future values of each state variable.

3 The marginal benefit of reducing greenhouse gas emissions

I now consider the marginal gross benefit of CO_2 emission reductions, known as the social cost of carbon. Consider adding m units of time 0 emissions to any realized CO_2 trajectory $M(t)$. These time 0 emissions decay at rate δ, so that the new trajectory is $M(t) + m\, e^{-\delta t}$. Now consider an offer to reduce m. The reduction in greenhouse gas emissions produces a stream of stochastic payoffs. Following Section 1, the policymaker is willing to pay

$$scc \triangleq - E\left[\int_0^{\tau} e^{-\rho t} \frac{u'(C(t)/L(t))}{u'(C(0)/L(0))} \frac{\mathrm{d}C(t)}{\mathrm{d}m} \mathrm{d}t\right] \tag{4}$$

to receive the payoff from reducing m by one unit.[17] The social cost of carbon evaluates the derivative $\mathrm{d}C(t)/\mathrm{d}m$ at $m = 0$. The term

$$e^{-\rho\, t}\, \frac{u'(C(t)/L(t))}{u'(C(0)/L(0))}$$

is the stochastic discount factor that prices a change in time t consumption in terms of time 0 consumption.

How does a change in time 0 emissions affect later consumption? Define $\epsilon(t)$ as the semi-elasticity of time t consumption damages with respect to time 0 emissions:

$$\epsilon(t) \triangleq -\frac{\mathrm{d}C(t)}{\mathrm{d}m}\frac{1}{C(t)}.$$

This damage semi-elasticity is a useful analytic device because it separates the damage scaling effect induced by the assumption of a multiplicative damage function from the growth insurance effects induced by the physical relationships between emissions, temperature, and consumption growth. Decomposing the derivative, we have

$$\begin{aligned}
\epsilon(t) =& \frac{-1}{C(t)} \int_0^t \left.\frac{\mathrm{d}C(t)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}} \frac{\mathrm{d}T(i)}{\mathrm{d}m}\mathrm{d}i,\\
\text{where }\ \left.\frac{\mathrm{d}C(t)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}} =& -C(t)\,\alpha \int_i^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[k-i]+\sigma_T\,[z_T(k)-z_T(i)]}\,\mathrm{d}k\\
\text{and }\ \frac{\mathrm{d}T(i)}{\mathrm{d}m} =& \phi\, s \int_0^i e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[i-j]+\sigma_T\,[z_T(i)-z_T(j)]}\,\frac{\mathrm{d}F(M(j)+m\,e^{-\delta j})}{\mathrm{d}m}\,\mathrm{d}j.
\end{aligned}$$

The appendix derives the last two lines. Substituting into equation (4) yields integrals that are easy to evaluate numerically for any given path of the stochastic variables. In $\mathrm{d}T(i)/\mathrm{d}m$, the term $s\,[\mathrm{d}F(M(j)+m\,e^{-\delta j})/\mathrm{d}m]$ describes how time 0 emissions change the steady-state temperature by reducing the time j CO_2 stock, and ϕ determines what fraction of that change in steady-state temperature is realized in a given instant. The term $e^{-\phi(i-j)}$ reflects how changes in temperature at times j earlier than i have decayed by the time we arrive at instant i. As ϕ increases, more of the steady-state warming is realized in a given instant, but the effects of realized warming are less persistent. In $\mathrm{d}C(t)/\mathrm{d}T(i)$, the growth rate of

[17] The other possible definition of the social cost of carbon would vary $M(0)$ rather than varying m. The difference is that the present approach ignores how small changes in time 0 emissions can affect time t CO_2 by changing emissions (via consumption) at times $i \in (0, t)$. The present approach offers substantially greater analytic tractability. Further, the present approach is consistent with a standard definition of the social cost of carbon as resulting from a small change in time 0 emissions along an exogenously specified emission pathway, which is often the business-as-usual pathway (e.g., Greenstone et al., 2013). Any numerical differences between the present approach and that of varying $M(0)$ are likely to be quite small.

consumption is reduced via α at every instant k between times i and t, but the effect of the change in time i temperature on time k temperature decays at rate ϕ.

The magnitude of the damage semi-elasticity $\epsilon(t)$ increases in warming per unit of CO_2 (s) and in damages per unit of warming (α). It depends on CO_2 only through the effect of a change in CO_2 on forcing. From the forcing relationship, we have

$$\frac{\mathrm{d}F(M(j) + m\, e^{-\delta j})}{\mathrm{d}m} = \nu\, \frac{e^{-\delta j}}{M(j) + m\, e^{-\delta j}}.$$

When valuing a unit of emission reductions, the policymaker is concerned with the marginal effect of emissions on the climate. Because of the concave relationship between CO_2 and forcing, emissions have a stronger marginal effect on temperature when there is not much CO_2 in the atmosphere.

Following Section 1, use the assumption of power utility and a second-order approximation to $C(t)^{1-\eta}$ around $(E[C(t)])^{1-\eta}$ to write the social cost of carbon as

$$\begin{aligned}
scc \approx &\int_0^\tau \frac{e^{-\rho t} L(t)^\eta}{[C(0)/L(0)]^{-\eta}} \left(E[C(t)]\right)^{1-\eta} E\left[\epsilon(t)\right]\, \mathrm{d}t && \text{(deterministic)}\\
&+ \frac{1}{2}\eta\,(\eta+1) \int_0^\tau \frac{e^{-\rho t} L(t)^\eta}{[C(0)/L(0)]^{-\eta}}\, Var(C(t))\,(E[C(t)])^{-\eta-1}\, E[\epsilon(t)]\,\mathrm{d}t && \text{(precautionary)}\\
&- \eta \int_0^\tau \frac{e^{-\rho t} L(t)^\eta}{[C(0)/L(0)]^{-\eta}}\, Var(C(t))\,(E[C(t)])^{-\eta-1}\, E[\epsilon(t)]\,\mathrm{d}t && \text{(damage scaling)}\\
&+ \int_0^\tau \frac{e^{-\rho t} L(t)^\eta}{[C(0)/L(0)]^{-\eta}}\, Cov\left[C(t)^{1-\eta}, \epsilon(t)\right]\, \mathrm{d}t. && \text{(growth insurance)}
\end{aligned}$$

We see the same types of channels analyzed in Section 1, except now written with the damage semi-elasticity $\epsilon(t)$. As in the two-period case, the precautionary saving channel works to increase the social cost of carbon and the damage scaling channel works to reduce the social cost of carbon, with the precautionary saving channel dominating if and only if $\eta > 1$. And driven by the same intuition as in the two-period case, we see that the growth insurance channel works to increase the social cost of carbon if and only if the damage semi-elasticity covaries positively with $C(t)^{1-\eta}$, with $\eta > 1$ again indicating that the risk aversion effect dominates the exposure effect.

We now formally analyze how uncertainty affects the social cost of carbon. To start, assume that $\sigma_T = 0$ (i.e., temporarily ignore temperature stochasticity) and that $E[\alpha]\, E[s]$ is sufficiently small that a marginal increase in warming does not end up increasing future consumption by reducing emissions at intervening times. Then:

Proposition 1. *Under the given conditions, $Cov\,[C(t)^{1-\eta}, \epsilon(t)] > 0$ if and only if $\eta > 1$, and the social cost of carbon increases in $Var(s)$, in $Var(\alpha)$, and in σ_C if and only if $\eta > 1$.*

Proof. See appendix. □

Most economic analyses of climate change use $\eta > 1$. The proposition says that, with these preferences, uncertainty about warming, damages, and consumption growth makes the growth insurance channel positive and increases the social cost of carbon. We have already seen that the (positive) precautionary saving channel dominates the (negative) damage scaling channel if and only if $\eta > 1$. The net effect of these two channels grows in $Var(s)$, $Var(\alpha)$, and σ_C because increasing any of these works to increase the variance of future consumption.

It remains to explore the sign of the growth insurance channel. The intuition varies with the source of uncertainty. Begin with uncertainty about the parameter s, which governs warming per unit of emissions. The intuition for the effect of uncertainty about the damage parameter α is analogous. The appendix derives a useful analytic expression for the growth insurance channel induced by uncertainty about s. Higher values of s increase $\epsilon(t)$ in two ways. First, an emission-induced increase in forcing generates more warming at time t when s is large. Second, greater warming occurs prior to time t when s is large, which reduces consumption and emissions prior to t. The resulting reduction in the CO_2 stock increases the marginal effect of time 0 emission reductions on forcing at time t and so increases $\epsilon(t)$. Now consider the effect of s on time t consumption: large values of s reduce time t consumption by increasing warming, but they also increase time t consumption by reducing emissions (via consumption) in earlier periods. The last effect is small when $E[\alpha]\, E[s]$ is small, in which case larger s corresponds to less time t consumption. Uncertainty about the warming parameter s therefore makes the marginal climatic effect of emissions covary negatively with consumption. When $\eta > 1$, the risk aversion effect dominates the exposure effect. In that case, uncertainty about the warming parameter s generates a positive growth insurance channel (increasing the social cost of carbon) by inducing a positive covariance between time t marginal utility and the rate at which time t consumption increases in response to a marginal reduction in time 0 emissions.

Now consider the growth insurance channel induced by uncertainty about consumption growth. The appendix again derives a useful analytic expression. Positive consumption shocks produce emissions and thus increase the future stock of CO_2. The logarithmic forcing relationship means that the marginal effect of time 0 emissions on the time t climate is small when the CO_2 stock is large. Positive consumption shocks therefore imply smaller $\epsilon(t)$. Now consider the effect of consumption shocks on time t consumption. Positive consumption shocks work to increase time t consumption directly, but shocks that arise at instants prior to t also work to decrease time t consumption by engendering additional emissions and warming. This last effect is small when $E[\alpha]\, E[s]$ is small: the direct effect of positive consumption shocks on time t consumption then dominates the indirect effect of positive shocks on time t consumption via intervening climate change. In this case, shocks to consumption growth make states with high consumption correspond to states in which additional emissions have a relatively small effect on the climate.[18] When $\eta > 1$, the risk aversion effect dominates the

[18]The assumption that $E[\alpha]\, E[s]$ is small is the analogue to our assumption in Section 1 that C_1/D

exposure effect. The induced positive correlation between marginal utility and the marginal climatic effect of emissions then generates a positive growth insurance channel.[19]

Having considered the commonly discussed sources of risk that can generate a positive growth insurance channel, I now consider a novel source: the increasing volatility of the climate system as warming progresses. Assume that $\sigma_C = 0$, $Var(\alpha) = 0$, and $Var(s) = 0$ (i.e., temporarily ignore consumption stochasticity and uncertainty about the damage and warming parameters), and assume that the expected value of the damage parameter $E[\alpha]$ is small. Then:

Proposition 2. *Under the given conditions, $Cov\left[C(t)^{1-\eta}, \epsilon(t)\right] > 0$ if and only if $\eta > 1$, and the social cost of carbon increases in σ_T if and only if $\eta > 1$.*

Proof. See appendix. □

Because temperature shocks act multiplicatively on temperature, a sequence of bad weather shocks increases temperature by more than a sequence of good weather shocks reduces temperature. The social cost of carbon is therefore more sensitive to the possibility of a sequence of bad weather shocks. Such shocks directly reduce later consumption, but they indirectly increase later consumption by reducing consumption and emissions in intervening periods. If $E[\alpha]$ is small, then the first effect dominates. Now consider the benefits of having reduced emissions at some earlier time: if earlier emission reductions have limited time t temperature, then they also have limited the warming induced by the sequence of bad weather shocks. Putting these pieces together, the possibility of weather shocks tends to make states with low consumption also be states in which emission reductions have a strong marginal effect on the climate: emission reductions are valuable for limiting the severity of realized extreme events when those realized extreme events have reduced consumption. Therefore, temperature stochasticity induces a positive correlation between time t marginal utility and the marginal effect of time 0 emission reductions on time t consumption growth, which combines with the net effect of the precautionary saving and damage scaling channels to increase the social cost of carbon when $\eta > 1$.

4 Calibrated numerical application

I now numerically simulate the social cost of carbon using equation (4) and the subsequent decomposition in order to learn about the quantitative importance of uncertainty. The ap-

increases in C_1, which we saw held for $\alpha T'$ small.

[19]This positive growth insurance channel arising from uncertainty about future consumption growth contrasts with the results of previous literature because we recognize the concavity of forcing in CO_2 and because we focus on the marginal effect of emissions on the climate. If forcing (and therefore temperature) were linear in CO_2, then this growth insurance channel would disappear altogether: the marginal effect of emissions on the damage semi-elasticity $\epsilon(t)$ would be constant and the covariance would be zero. (In the definition of $\epsilon(t)$, $\mathrm{d}F(M(j))/\mathrm{d}m$ would be independent of $M(j)$, and thus $\mathrm{d}T(j)/\mathrm{d}m$ would be independent of $M(j)$.)

pendix describes how I calibrate the consumption growth, emission, and climate parameters to economic and scientific data. The calibration is methodologically consistent with the U.S. government's definition of the social cost of carbon as the value of emission reductions along a no-policy ("business-as-usual") pathway. The baseline preference parameters come from DICE-2007 (Nordhaus, 2008), which fixes the coefficient of relative risk aversion at $\eta = 2$ and the utility discount rate at 1.5% per year. I take time 0 as the year 2014, and I use a horizon of 200 years.

The warming parameter s becomes $s = \Gamma/(1 - \Delta)$, with $\Gamma > 0$ and $\Delta < 1$. This representation is consistent with the scientific literature (e.g., Roe and Baker, 2007) and with much recent work on the economics of climate change (e.g., Greenstone et al., 2013; Kelly and Tan, 2015; Lemoine and Rudik, 2017). The feedback factor Δ is a random variable drawn from a truncated normal distribution, where the nontruncated distribution has a mean of 0.6 and a standard deviation of 0.13 (Roe and Baker, 2007). At the mean value, doubling the atmospheric concentration of CO_2 would eventually generate 3°C of warming, which is a value for "climate sensitivity" that is consistent with DICE-2007 (Nordhaus, 2008). Following common practice (e.g., Costello et al., 2010; Kelly and Tan, 2015), I truncate the distribution from above. The appendix plots the resulting distribution over climate sensitivity. In experiments in which I vary the standard deviation parameter, I hold the mean value of Δ fixed by simultaneously varying the location parameter.

I calibrate the distribution of the damage parameter α to the survey in Pindyck (2016) of 1,000 leading climate scientists and economists. These experts reported several quantiles of their subjective distributions for the percentage loss in GDP that climate change would cause in fifty years. As described in the appendix, the survey results produce a lognormal distribution for the parameter ϕ_P in the relationship $\phi_P = \alpha \int_0^{50} T(j)\,\mathrm{d}j$. Combining this distribution with a lognormal distribution fit to simulated values of $\int_0^{50} T(j)\,\mathrm{d}j$, I find that α is lognormally distributed with location parameter -6.680 and scale parameter 1.472. I truncate this distribution from above at $\alpha = 0.01$, a point that implies economic losses equal to the largest ones asked about in Pindyck (2016). The appendix plots the resulting distribution for α. This calibration yields an expected value for α of 0.0019 (so that, in expectation, each degree of warming reduces the growth rate of consumption by 0.19 percentage points). In experiments in which I vary the scale parameter, I also vary the location parameter so that the expected value of α remains fixed at 0.0019.

Figure 1 plots the evolution of temperature (top), CO_2 (middle), and global consumption per capita (bottom) over time. The solid lines depict the expected trajectories, and the dashed lines depict the 95% confidence intervals. All three variables increase over time, with total warming exceeding 2°C by the end of the century in the vast majority of simulations. Consumption per capita never falls below its initial value in any of the sampled trajectories. The left column fixes α and Δ at their expected values, so that it depicts the influence of volatility in the temperature and consumption processes. We see that these sources of volatility can induce substantial uncertainty about future consumption per capita, which is almost entirely due to the volatility in the consumption process. This uncertainty about future consumption in turn generates uncertainty about future CO_2. The middle column eliminates these sources of volatility but allows s (via Δ) to be uncertain. Uncertainty about the warming parameter generates substantial uncertainty about the amount of warming that will be experienced by the end of the century, with a long tail towards high-warming outcomes. We also see that the coming century's consumption trajectory is not strongly affected by uncertainty about s because large values of s take time to manifest as temperature and take even longer to manifest as consumption losses. Finally, the right column again fixes Δ at its expected value but now allows α to be uncertain. We see greater uncertainty about future consumption than in the middle column and less uncertainty than in the left column. However, in contrast to the left column, we see much more downside risk than upside risk.

Table 1 reports the year 2014 social cost of carbon as well as each of its components. It does so for cases with only a single source of uncertainty and for the full model with all four sources of uncertainty. The social cost of carbon in the full model is around \$300 per tCO_2, with uncertainty accounting for 26% of the total. As predicted, uncertainty increases the social cost of carbon in every case, with the growth insurance channel always being positive and with the positive precautionary saving channel always dominating the negative damage scaling channel.[20] The deterministic component is around \$197 per tCO_2 when the warming parameter is known and just over \$220 per tCO_2 when it is unknown.[21] Temperature volatility increases the social cost of carbon by only a few cents. Consumption volatility increases the social cost of carbon by almost \$9 per tCO_2, almost entirely because the precautionary saving channel becomes large: the direct consequence of consumption volatility is to make future consumption uncertain, which we have seen raises the value of emission reductions as a means of saving. In contrast, the growth insurance channel becomes especially important when either the damage parameter or the warming parameter is uncertain: when only the warming parameter is uncertain, the growth insurance channel increases the social cost of carbon by just over \$4 per tCO_2, and when only the damage parameter is uncertain, the growth insurance channel increases the social cost of carbon by just over \$17 per tCO_2. The total effect of uncertainty with an uncertain damage parameter is over twice as large as in any other case with only a single source of uncertainty, and the total effect of uncertainty about business-as-usual consumption growth is nearly 60% greater than the total effect of uncertainty about the warming parameter. Finally, the bottom row of

[20]Recall that the damage scaling channel and the growth insurance channel are the two components of the full insurance channel (see equation (2)). Summing these components, we see that temperature volatility and uncertainty about either the damage or warming parameter each make the full insurance channel positive and that consumption volatility makes it negative. The last effect dominates in the setting with all four sources of uncertainty.

[21]Making s uncertain increases the deterministic component because $E[s]$ increases when we hold $E[\Delta]$ fixed while increasing the variance of Δ. Because Δ is the model primitive from a scientific perspective, holding $E[\Delta]$ fixed when changing the variance of Δ is more sensible than is simultaneously adjusting $E[\Delta]$ to hold $E[s]$ fixed.

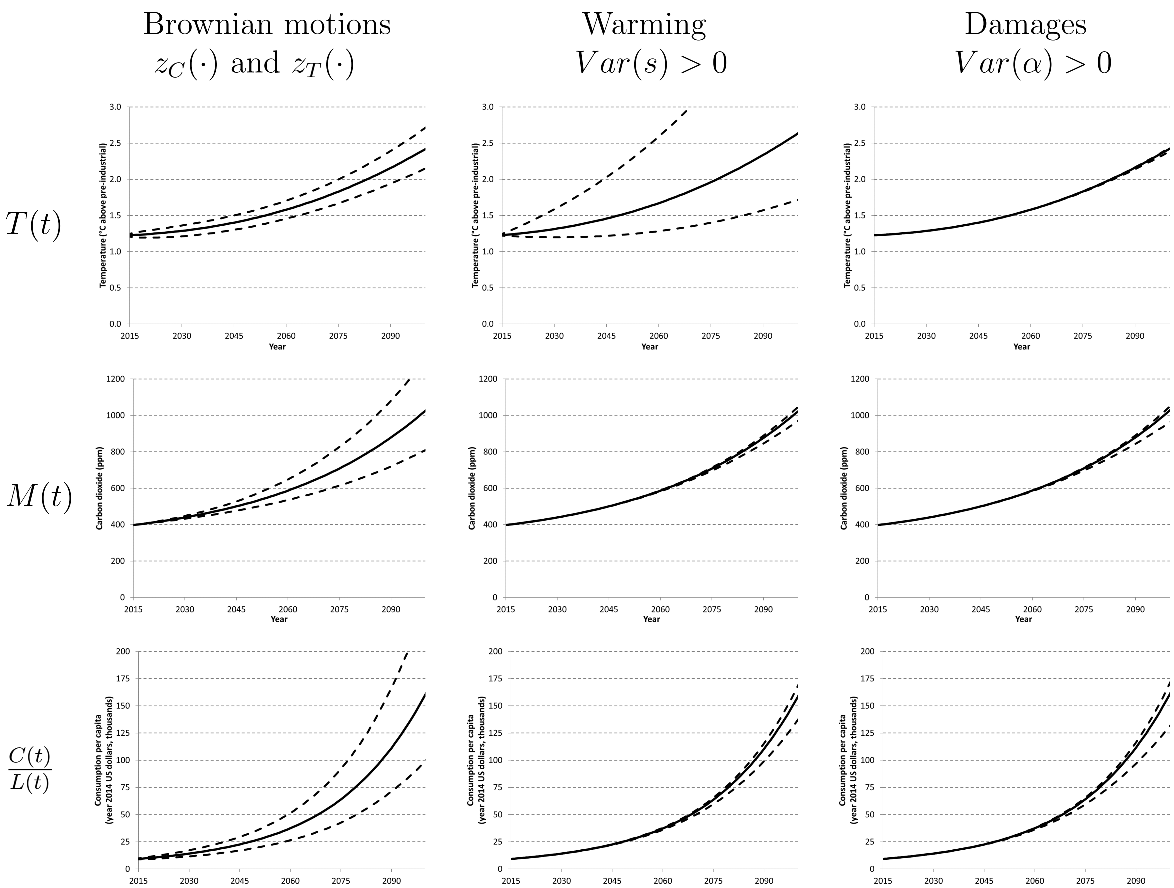


Figure 1: Simulated trajectories for temperature (top), carbon dioxide (middle), and consumption per capita (bottom) in simulations with volatility in consumption and temperature but no uncertainty about s or α (left), with uncertainty about s but no volatility in consumption or temperature or uncertainty about α (middle), and with uncertainty about α but no volatility in consumption or temperature or uncertainty about s (right). The solid lines depict expected values, and the dashed lines depict 95% confidence intervals. Each column simulates 10,000 trajectories.

Table 1: The year 2014 social cost of carbon and its components, all in $\$/tCO_2$.

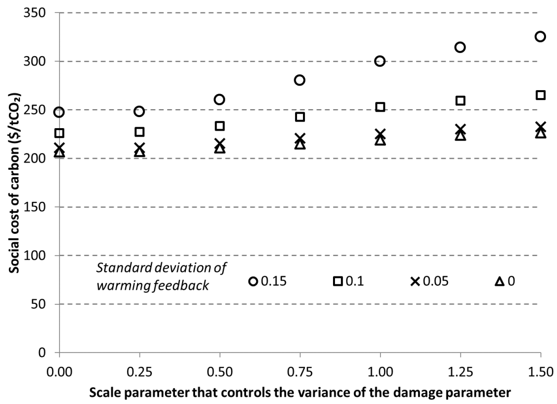
	Channel				
Source of uncertainty	**Deterministic**	**Precautionary**	**Damage Scaling**	**Growth Insurance**	**Total**
Temperature Volatility	197.49	0.02	-0.01	0.04	197.54
Consumption Volatility	197.43	26.14	-17.84	0.24	205.98
Warming Parameter	223.31	3.94	-2.84	4.34	228.73
Damage Parameter	196.34	12.66	-10.06	17.05	215.99
All Four Factors	221.50	488.66	-471.13	60.98	300.01

Table 1 suggests that interactions among sources of uncertainty are important: uncertainty about all four factors increases the social cost of carbon by almost \$80 per tCO_2, whereas summing the individual adjustments would have led one to expect uncertainty to increase the social cost of carbon by only \$34 per tCO_2. In particular, the growth insurance channel increases to \$61 per tCO_2. This large growth insurance channel suggests that the interaction between the uncertain damage parameter and the uncertain warming parameter is especially important.

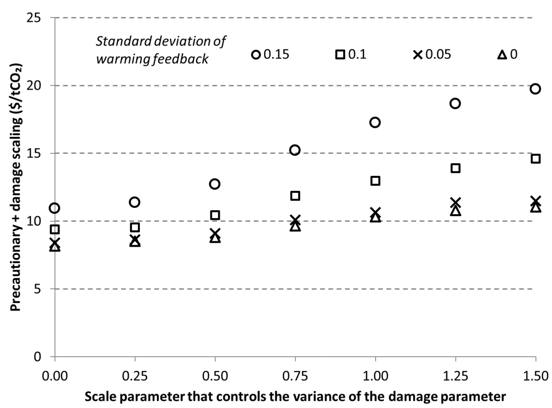
Figure 2 further explores the interaction between uncertainty about the damage and warming parameters. Moving to the right on each panel's horizontal axis increases the variance of the damage parameter, with the different markers corresponding to different standard deviations of the warming feedback Δ. When Δ is fixed at its expected value (triangles), we see that increasing the variance of the damage parameter has only a small effect on the social cost of carbon (top panel), driven almost entirely by the growth insurance channel (bottom panel) rather than by the sum of the precautionary saving and damage scaling channels (middle panel). The social cost of carbon becomes more sensitive to the variance of the damage parameter as the warming parameter becomes more uncertain (e.g., as we move to the right, the circles increase faster than do the triangles), and we also see the social cost of carbon become more sensitive to the variance of the warming parameter as the damage parameter becomes more uncertain (the vertical gaps between different markers grow as we move to the right). Comparing the middle and bottom panels shows that the interaction between the two sources of uncertainty is primarily due to the growth insurance channel. Intuitively, uncertainty about the damage parameter interacts with uncertainty about the warming parameter to make the marginal effect of today's emissions on future consumption growth especially uncertain.[22]

Table 2 shows how the social cost of carbon and its components depend on the utility discount rate ρ (left versus right), on the coefficient of relative risk aversion η (top panel), and

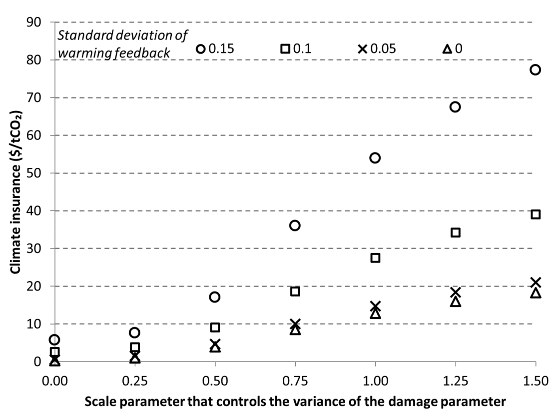
[22] We saw in Section 3 that the growth insurance channel becomes stronger as the variance of the damage semi-elasticity $\epsilon(t)$ increases, and we also saw that $\epsilon(t)$ is a function of $\alpha\, s$. The marginal effect of $Var(\alpha)$ on $Var(\epsilon(t))$ therefore tends to increase in $Var(s)$.



(a) Social cost of carbon



(b) Precautionary + damage scaling channels



(c) Growth insurance channel

Figure 2: The dependence of the social cost of carbon (top) and its components (middle and bottom) on the variance of the damage parameter α (along the horizontal axes) and on the variance of the feedbacks Δ that determine warming (distinguished by the markers). Note that the scale of the vertical axis differs between panels.

on the expected value of the damage parameter $E[\alpha]$ (bottom panel).[23] In all cases, reducing the utility discount rate from its baseline value of 1.5% to the value of 0.01% used in Stern (2007) strongly increases the total social cost of carbon because the policymaker becomes more concerned about the future consumption losses from climate change. Reducing the utility discount rate also increases the influence of uncertainty on the social cost of carbon, in part because many of the consequences of drawing different values of s or α take time to materialize (see Figure 1).

Changing the coefficient of relative risk aversion η from its baseline value of 2 has conflicting effects. First, increasing η makes the policymaker less willing to sacrifice year 2014 consumption for the benefit of a richer future. In models with power utility, η controls both relative risk aversion and the elasticity of intertemporal substitution. Increasing η raises the consumption discount rate by reducing the elasticity of intertemporal substitution. The sensitivity of the deterministic component to η shows that raising the consumption discount rate can strongly reduce the social cost of carbon. Second, when η is less than 1, we see that uncertainty works to reduce the social cost of carbon: as predicted by the theoretical analysis, the growth insurance channel becomes negative and the negative damage scaling channel dominates the positive precautionary saving channel. In particular, uncertainty reduces the social cost of carbon by around 25–30% with $\eta = 0.5$, but the deterministic component becomes so large that this case nonetheless has an exceptionally high social cost of carbon. Third, when η is greater than 1, we see that uncertainty increases the social cost of carbon by 20–120% with the larger utility discount rate and by 40–800% with the smaller utility discount rate. Conventional analyses with $\eta > 1$ that ignore uncertainty may strongly understate the social cost of carbon.

Now consider the effects of changing the expected damage parameter from its baseline value of 0.0019.[24] The bottom panel of Table 2 shows that the deterministic component of the social cost of carbon increases approximately linearly in $E[\alpha]$, but the components introduced by uncertainty increase faster than linearly, with the growth insurance channel growing faster than does the sum of the precautionary and damage scaling channels. As a result, the contribution of uncertainty to the social cost of carbon increases in $E[\alpha]$. Uncertainty becomes especially critical when climate change is expected to cause greater consumption

[23] In unreported simulations, I find that the social cost of carbon is not sensitive to increasing the rate at which the emission intensity of consumption declines. This result suggests that the reported social cost of carbon is close to the open-loop optimal emission tax: emission control policies can be represented as making the emission intensity of consumption decline at a faster rate, so this experiment suggests that the social cost of carbon is not sensitive to the time 0 choice of emission control policy. The social cost of carbon along the open-loop optimal emission path is therefore likely to be close to the social cost of carbon along the no-policy paths considered in this analysis. Note that recursive models calculate a closed-loop optimal emission tax, which allows the policymaker to adjust future policies to shocks and to new information (Lemoine and Rudik, 2017).

[24] These simulations vary the location parameter for the distribution of α in order to match the reported values for $E[\alpha]$. They hold the scale parameter fixed.

Table 2: The dependence of the year 2014 social cost of carbon and its components (all in $/tCO$_2$) on the utility discount rate ρ, the coefficient of relative risk aversion η, and the expected damage parameter $E[\alpha]$. Also, the effect of uncertainty on the social cost of carbon, as a percentage of the deterministic component.

$\boldsymbol{\rho}$:	**1.5% per year**			**0.1% per year**		
$\boldsymbol{\eta}$:	**0.5**	**1.5**	**4**	**0.5**	**1.5**	**4**
Deterministic	51308	819	12	477374	3556	20
Precautionary	8433	863	832	100419	10130	12237
Damage Scaling	-10106	-823	-830	-119336	-9816	-12221
Growth Insurance	-11762	152	12	-127523	1262	143
Total Social Cost of Carbon	37873	1010	26	330933	5133	178
Effect of Uncertainty (%)	-26	23	117	-31	44	790

$\boldsymbol{\rho}$:	**1.5% per year**			**0.1% per year**		
$\boldsymbol{E[\alpha]}$:	**0.0005**	**0.0015**	**0.0025**	**0.0005**	**0.0015**	**0.0025**
Deterministic	56	171	295	158	498	880
Precautionary	17	264	914	153	3261	11685
Damage Scaling	-15	-253	-886	-142	-3196	-11500
Growth Insurance	8	43	87	56	329	689
Total Social Cost of Carbon	66	226	410	225	894	1754
Effect of Uncertainty (%)	18	32	39	42	80	99

losses.

Finally, Figure 3 plots the evolution of the (present value) social cost of carbon and its components over the century, using the year 2014 information set and maintaining a horizon of 200 years from the indicated date. The policymaker should undertake projects that reduce emissions in year t only if the present cost (in time 0 consumption) is less than the plotted social cost of carbon. The social cost of carbon stays approximately constant over the first half of the century. However, it declines over the latter half of the century. The deterministic component is responsible for the eventual decline in the social cost of carbon, in part reflecting the policymaker's impatience at waiting for emission reductions. The contribution of uncertainty to the social cost of carbon increases from 26% in 2014 to 71% in 2100. In particular, the growth insurance channel doubles over the century. This increase occurs because lags in the climate system and in the economic consequences of

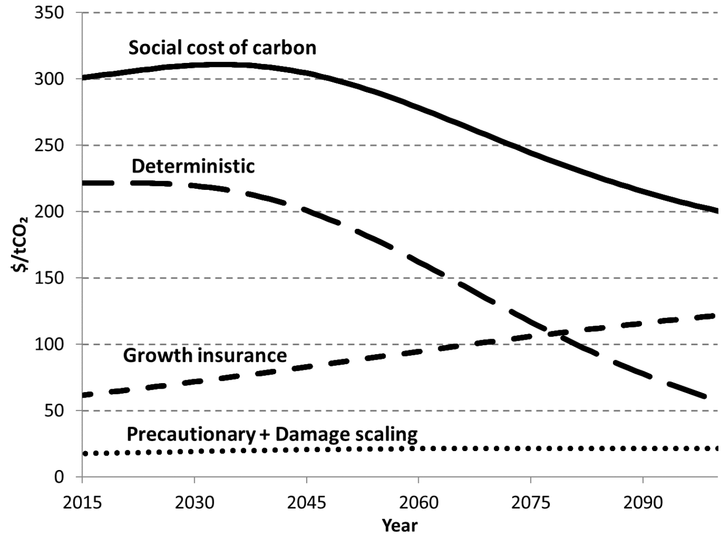


Figure 3: The evolution of the social cost of carbon and its components over time, using year 2014 information and in present value terms.

climate change combine to slow the effect of different possible values of the warming and damage parameters on consumption (see Figure 1).

5 Discussion

I now consider the role of the damage function in the results before demonstrating how the analysis maps into common discussions about the consumption discount rate.

5.1 Alternate damage functions

I have thus far assumed that climate change reduces the growth rate of consumption. This assumption is consistent with recent empirical work that uses cross-country data on climate, output, and asset prices (e.g., Bansal and Ochoa, 2011; Dell et al., 2012; Burke et al., 2015) and with recent empirical work that finds that warming reduces labor productivity (e.g., Heal and Park, 2015). Further, some economists have recently argued for growth rate impacts on the basis that climate change is likely to affect stocks of physical or environmental capital and may divert savings and R&D effort (Pindyck, 2012, 2013; Stern, 2013). However, very little is known about the correct damage specification (e.g., Pindyck, 2013). Many other types of damage specifications are plausible. I here discuss the implications of alternative damage specifications. The appendix analyzes a case in which the policymaker values environmental quality directly.

First, climate change could reduce output multiplicatively without having a growth rate impact. In this more general case, the second argument of the covariance operator in equation (3) includes a $D'(s\,T)/D(s\,T)$ instead of α. $D'(s\,T)/D(s\,T)$ increases in $s\,T$ if and only if $D''(s\,T) \geq [D'(s\,T)]^2/D(s\,T)$. This condition is never satisfied if $D(\cdot)$ is strictly concave, but most economic analyses assume that $D(\cdot)$ is convex. In the benchmark DICE-2007 integrated assessment model, the condition for $D'(s\,T)/D(s\,T)$ to increase in $s\,T$ is satisfied for all policy-relevant temperatures.[25] Assume the condition is satisfied. In that case, large values of s tend to make $D'(s\,T)/D(s\,T)$ large at the same time that they make $s\,T'$ large. This form of damages thus tends to amplify the growth insurance channel induced by uncertainty about the warming induced by emissions.

In contrast, uncertainty about business-as-usual consumption growth makes the growth insurance channel become ambiguous with this alternate damage specification. As discussed in Section 1, states with high consumption growth have small T'. However, these same states have large T and thus, in standard calibrations, large D'/D. It is therefore no longer clear how the second argument of the covariance operator in equation (3) changes with time 1 business-as-usual consumption C_1. If large C_1 increases D'/D by more than it decreases T', then the growth insurance channel could reverse its sign from our analysis.[26]

Second, climate change may reduce output additively rather than multiplicatively (Weitzman, 2010). In that case, climate change reduces consumption to $C_1 - \hat{D}(s\,T)$ in the setting of Section 1, with $\hat{D}(s\,T), \hat{D}'(s\,T) \geq 0$. Equation (3) becomes:

$$\begin{aligned} scc \approx & \frac{\beta}{C_0^{-\eta}} \Big\{ (E[C_1 - \hat{D}])^{-\eta}\, E\left[\hat{D}'\, s\,T'\right] && \text{(deterministic)} \\ & + \frac{1}{2}\eta\,(\eta+1)\,(E[C_1 - \hat{D}])^{-\eta-2}\, E[\hat{D}'\, s\,T']\, Var(C_1 - \hat{D}) && \text{(precautionary)} \\ & + Cov\left[(C_1 - \hat{D})^{-\eta}, \hat{D}'\, s\,T'\right] \Big\}. && \text{(growth insurance)} \end{aligned}$$

The (negative) damage scaling channel has vanished, because the climate-induced loss in consumption is now independent of C_1. Further, the sign of the growth insurance channel is now independent of η, because the exposure effect has also vanished. The growth insurance channel now depends only on the covariance between marginal utility and $\hat{D}'\, s\,T'$. Assume

[25] DICE-2007 uses the following damage specification: $D(s\,T) = 1 + 0.0028\,(s\,T)^2$ (Nordhaus, 2008). The condition for $D'(s\,T)/D(s\,T)$ to increase in $s\,T$ is satisfied if and only if $s\,T \leq 18.9$°C. Temperature remains below 5°C along the optimal policy trajectory in DICE-2007.

[26] Dietz et al. (2015) report a positive covariance between consumption and the marginal benefit of emission reductions in the DICE integrated assessment model. Their results suggest that the insurance channel in equation (2) reduces the social cost of carbon, which means that the growth insurance channel and damage scaling channel in equation (3) are jointly negative. We find the same result in the setting with all four sources of uncertainty (see Table 1). Their analysis does not tell us about the precautionary saving channel, about the net impact of uncertainty on the social cost of carbon, or about the sign of the growth insurance channel.

that $\hat{D}(s\,T)$ is convex. In that case, large values of s increase marginal utility by reducing $C_1 - \hat{D}$ and increase $\hat{D}'\,s\,T'$ by increasing both $\hat{D}'$ and $s\,T'$. The growth insurance channel is thus positive. And because the precautionary saving channel is also positive, uncertainty about s increases the social cost of carbon for all $\eta \geq 0$. Now consider the implications of uncertainty about business-as-usual consumption growth. States with high C_1 have low marginal utility and, by increasing both e_1 and T, they also have large values of $\hat{D}'$ and small values of T'. If raising C_1 increases $\hat{D}'$ by more than it decreases T', then uncertainty about business-as-usual economic growth makes the growth insurance negative for all $\eta \geq 0$ (and thus makes it conflict with the precautionary saving channel), but if $\hat{D}$ is approximately linear, then uncertainty about business-as-usual consumption growth increases the social cost of carbon through both the growth insurance and precautionary saving channels for all $\eta \geq 0$.

5.2 A discount rate interpretation

I have expressed all results in terms of how uncertainty affects willingness to pay for emission reductions. However, discussions in environmental economics have historically focused on the choice of discount rate for pricing known payoffs rather than on the specification of a stochastic discount factor for pricing uncertain payoffs. The present section formally demonstrates the connection between the two perspectives.

Imagine that we know $E[C(t)]E[\epsilon(t)]$ at some future time t, and we seek the consumption discount rate r_t to use when valuing the portion of the benefit from time 0 emission reductions that is realized in time t. Within the formalism of Section 3, the consumption discount rate r_t that properly prices this additional future consumption must satisfy:

$$e^{-r_t\,t}\,E\left[C(t)\right]\;E\left[\epsilon(t)\right]\;u'\left(\frac{C(0)}{L(0)}\right) = e^{-\rho t}E\left[u'\left(\frac{C(t)}{L(t)}\right)C(t)\,\epsilon(t)\right].$$

Applying our power utility specification, we seek the discount rate r_t such that

$$e^{-r_t\,t}E\left[C(t)\right]\;E\left[\epsilon(t)\right] = e^{-\rho t}\frac{L(t)^\eta}{L(0)^\eta}\frac{1}{C(0)^{-\eta}}\left\{E\left[C(t)\right]^{1-\eta}E\left[\epsilon(t)\right] + Cov\left[C(t)^{1-\eta},\epsilon(t)\right]\right\}.$$

Taking logs and using a second-order Taylor series expansion of $E[C(t)^{1-\eta}]$ around $E[C(t)]$, we have

$$\begin{aligned} r_t\,t \approx& \rho\,t - \eta\ln\frac{L(t)}{L(0)} + \ln\{E[C(t)]E[\epsilon(t)]\}\\ &- \ln\left\{\frac{E[C(t)]^{1-\eta}E[\epsilon(t)]}{C(0)^{-\eta}} + \frac{1}{2}\eta(\eta+1)Var(C(t))\,E[C(t)]^{-\eta-1}E[\epsilon(t)]\frac{1}{C(0)^{-\eta}}\right.\\ &\left.\qquad - \eta\,Var(C(t))E[C(t)]^{-\eta-1}E[\epsilon(t)]\frac{1}{C(0)^{-\eta}} + Cov\left[C(t)^{1-\eta},\epsilon(t)\right]\frac{1}{C(0)^{-\eta}}\right\}. \end{aligned}$$

The terms in braces on the second and third lines are familiar from our decomposition of the social cost of carbon in Section 3. The term with $Var(C(t))$ on the second line reduces the risk-free discount rate in order to generate precautionary savings, as is familiar from the extended Ramsey rule (Gollier, 2002). The terms on the third line adjust the consumption discount rate for risk, as determined by the insurance component of the social cost of carbon. If uncertainty increases the social cost of carbon, then the terms in braces with $Var(C(t))$ and $Cov[C(t)^{1-\eta}, \epsilon(t)]$ are positive in aggregate: these terms are proportional to the sum of the time t contribution to the precautionary saving, damage scaling, and growth insurance channels, which we have seen is positive as long as $\eta > 1$ and α is not too large. In this case,

$$\begin{aligned} r_t <& \rho - \frac{1}{t}\eta \ln \frac{L(t)}{L(0)} + \frac{1}{t}\ln\{E[C(t)]E[\epsilon(t)]\} - \frac{1}{t}\ln \frac{E[C(t)]^{1-\eta}E[\epsilon(t)]}{C(0)^{-\eta}} \\ =& \rho + \eta\,[g_C - g_L] \\ =& \rho + \eta\, g_{C/L}, \end{aligned}$$

where g_C, g_L, and $g_{C/L}$ are the annual growth rates of expected consumption, population, and expected consumption per capita, respectively, between times 0 and t. The last line is the familiar Ramsey discount rate formula. We see that uncertainty increases the social cost of carbon if and only if it reduces the discount rate that should be applied when evaluating emission reductions at the expected damage semi-elasticity and expected consumption.[27]

6 Conclusions

Conventional dynamic general equilibrium models of climate policy have been limited to numerically exploring only a single type of uncertainty at a time. This paper instead adopts an asset pricing perspective to shed new theoretical and quantitative light on the determinants of the social cost of carbon. I show that uncertainty about future consumption affects the social cost of carbon through precautionary saving motives, through the tendency of damages to scale with consumption, and through the covariance between future consumption and the effect of today's emission reductions on future consumption growth. Further, I show that if climate change reduces the growth rate of consumption and per-period utility takes the conventional power form, then uncertainty about consumption growth, uncertainty about each year's weather shocks, uncertainty about the warming generated by emissions, and uncertainty about the consumption losses due to warming all increase the social cost of carbon if and only if the coefficient of relative risk aversion is greater than 1. Calibrated numerical simulations suggest that uncertainty about the damage parameter is especially

[27] Our numerical results in Section 4 show that the risk adjustment raises the consumption discount rate when all four sources of uncertainty are considered at once (see footnote 20). However, it also shows that the precautionary reduction in the risk-free rate dominates the risk adjustment, so that uncertainty acts on net to reduce the consumption discount rate.

important. These simulations also show that uncertainty about future climate damages interacts strongly with uncertainty about the warming generated by emissions. It is therefore important to simultaneously evaluate multiple sources of uncertainty within a single setting, in contrast to the approach taken by recently developed recursive integrated assessment models.

The results suggest several guideposts for future work. First, I have adopted conventional reduced-form relationships for the effect of warming on consumption growth and for the effect of consumption growth on emissions. However, understanding the structural drivers of consumption growth and of emissions could be critically important for signing the covariance between future consumption and the future payoffs to current emission reductions. For instance, technological change could increase or decrease emissions, and the shocks that generate uncertainty about consumption and emissions could be shocks to energy-augmenting technologies, to other factors' productivity, or to energy supply. Future work should explicitly model the structural channels through which consumption growth arises and fluctuates, through which warming affects the level and growth rate of output, and through which consumption generates emissions.

Second, Epstein-Zin-Weil preferences ("recursive utility") have become a critical piece of several leading explanations for prominent macrofinance puzzles. Compared to common power utility specifications, calibrations of Epstein-Zin-Weil preferences tend to simultaneously support greater relative risk aversion and a greater elasticity of intertemporal substitution. Increasing the elasticity of intertemporal substitution should work directly to increase the social cost of carbon, while increasing relative risk aversion should work to amplify all of the channels through which uncertainty affects the social cost of carbon. Future work should extend the present setting to investigate the implications of these preferences and also of preferences that display aversion to ambiguity about, for instance, the potential damages from climate change.

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Appendix

Section A derives several relationships used in the main text and in the proofs of the propositions. Section B describes the numerical calibration and computational procedure. Section C analyzes a case in which the policymaker values environmental quality directly. Section D contains proofs.

A Formal Analysis

Solving the differential equation for the pollution stock $M(t)$ yields

$$M(t) = e^{-\delta t} M(0) + \int_0^t \gamma(w)\, C(w)\, e^{-\delta(t-w)}\, \mathrm{d}w + (1 - e^{-\delta t}) M_{pre}.$$

Solving the stochastic differential equation for temperature $T(t)$ yields

$$T(t) = e^{\left(-\phi - \frac{1}{2}\sigma_T^2\right)t + \sigma_T\, z_T(t)} \left[T(0) + \int_0^t e^{\left(\phi + \frac{1}{2}\sigma_T^2\right)j - \sigma_T\, z_T(j)}\, \phi\, s\, F(M(j))\, \mathrm{d}j \right]. \tag{A-1}$$

And solving the stochastic differential equation for consumption $C(t)$ yields:

$$C(t) = C(0)\, e^{-\alpha \int_0^t T(j)\, \mathrm{d}j + \left(\mu_C - \frac{1}{2}\sigma_C^2\right)t + \sigma_C\, z_C(t)}. \tag{A-2}$$

These relationships can be verified by applying Itô's Lemma. Note that the solutions to each stochastic differential equation retain endogenous variables on the right-hand side; we do not analytically solve the whole system of stochastic differential equations.

As described in the main text, we consider changing any given realized CO_2 trajectory $M(t)$ to $M(t) + m\, e^{-\delta t}$ and value the offer to marginally reduce m. The derivative of time t consumption with respect to m is

$$\frac{\mathrm{d}C(t)}{\mathrm{d}m} = \int_0^t \left.\frac{\mathrm{d}C(t)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}} \frac{\mathrm{d}T(i)}{\mathrm{d}m} \mathrm{d}i. \tag{A-3}$$

Note that

$$\frac{\mathrm{d}F(M(t) + m\, e^{-\delta t})}{\mathrm{d}m} = \nu\, \frac{e^{-\delta t}}{M(t) + m\, e^{-\delta t}}$$

and

$$\frac{\mathrm{d}T(t)}{\mathrm{d}m} = \phi\, s\, e^{\left(-\phi - \frac{1}{2}\sigma_T^2\right)t + \sigma_T\, z_T(t)} \int_0^t e^{\left(\phi + \frac{1}{2}\sigma_T^2\right)j - \sigma_T\, z_T(j)} \frac{\mathrm{d}F(M(j) + m\, e^{-\delta j})}{\mathrm{d}m}\, \mathrm{d}j. \tag{A-4}$$

Both equations recognize that we are only changing the CO_2 trajectory through the direct addition of $m\,e^{-\delta t}$ to any time t realization of CO_2, so that we abstract from how adding $m\,e^{-\delta i}$ units of CO_2 at time i might affect CO_2 at times $j > i$ by affecting emissions (via consumption) at intermediate times. We are considering the offer to exchange one CO_2 trajectory for another, where the initial trajectory is not known ex ante but the change in the trajectory is known ex ante.

From equation (A-1), we have:

$$T(t) = e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)(t-i)+\sigma_T\,(z_T(t)-z_T(i))}\left[T(i)+\int_0^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)(i-j)+\sigma_T\,(z_T(i)-z_T(j))}\,\phi\,s\,F(M(j)+m\,e^{-\delta j})\,\mathrm{d}j\right].$$

Using this equation and recognizing that we are allowing CO_2 to change only with $m\,e^{-\delta t}$, we have, for $i \leq t$,

$$\left.\frac{\mathrm{d}T(t)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}} = e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[t-i]+\sigma_T\,[z_T(t)-z_T(i)]}.$$

Now differentiate $C(t)$ with respect to $T(i)$, for $i \leq t$:

$$\begin{aligned}\left.\frac{\mathrm{d}C(t)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}} &= -\,C(t)\,\alpha\int_i^t \left.\frac{\mathrm{d}T(k)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}}\mathrm{d}k\\ &= -\,C(t)\,\alpha\int_i^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[k-i]+\sigma_T\,[z_T(k)-z_T(i)]}\mathrm{d}k. \qquad \text{(A-5)}\end{aligned}$$

Substituting into equation (A-3) and rearranging, we find that

$$\frac{\mathrm{d}C(t)}{\mathrm{d}m} = -\,C(t)\,\alpha\,\phi\,s\,\nu\int_0^t\int_0^i\int_i^t \frac{e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[h-j]+\sigma_T[z_T(h)-z_T(j)]}}{M(0)+\int_0^j\gamma(w)\,C(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}+m}\,\mathrm{d}h\,\mathrm{d}j\,\mathrm{d}i.$$

We directly have the semi-elasticity of consumption with respect to a reduction in time 0 emissions:

$$\begin{aligned}\epsilon(t) &\triangleq -\frac{1}{C(t)}\frac{\mathrm{d}C(t)}{\mathrm{d}m}\\ &=\alpha\,\phi\,s\,\nu\int_0^t\int_0^i\int_i^t \frac{e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[h-j]+\sigma_T[z_T(h)-z_T(j)]}}{M(0)+\int_0^j\gamma(w)\,C(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}+m}\,\mathrm{d}h\,\mathrm{d}j\,\mathrm{d}i. \qquad \text{(A-6)}\end{aligned}$$

Henceforth, we evaluate these expressions around $m = 0$.

One way to sign $Cov[C(t)^{1-\eta}, \epsilon(t)]$ is to sign the partial derivative of each argument with respect to the random variables α, s, $z_C(x)$, and $z_T(x)$, for $x \in (0, t]$. From Theorem 236 in Hardy et al. (1952) (which extends the single-variable case of Tchebychef), we know that the covariance is positive if all of these partial derivatives have the same sign. However, we want to know not just the sign of $Cov[C(t)^{1-\eta}, \epsilon(t)]$ but also how this covariance changes in $Var(\alpha)$, $Var(s)$, σ_C, and σ_T. To that end, we analyze the covariance directly by taking first-order Taylor series approximations to each argument around the random variables' expected values. When it comes to signing the covariance, this approach ends up being similar to applying Theorem 236 in Hardy et al. (1952).

Analyze the covariance between $\epsilon(t)$ and $C(t)^{1-\eta}$:

$$
\begin{aligned}
&Cov\left[C(t)^{1-\eta}, \epsilon(t)\right] \\
&=Cov\Bigg[C(0)^{1-\eta}\, e^{-(1-\eta)\alpha \int_0^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)j+\sigma_T\, z_T(j)} T(0)\, \mathrm{d}j} \\
&\qquad e^{-(1-\eta)\alpha \int_0^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)j+\sigma_T\, z_T(j)} \left[\int_0^j e^{\left(\phi+\frac{1}{2}\sigma_T^2\right)k-\sigma_T\, z_T(k)}\, \phi\, s\, \nu\, \ln\left[\frac{e^{-\delta k} M(0)+\int_0^k \gamma(w)\, C(w)\, e^{-\delta(k-w)}\, \mathrm{d}w+(1-e^{-\delta k})M_{pre}}{M_{pre}}\right] \mathrm{d}k\right] \mathrm{d}j} \\
&\qquad e^{(1-\eta)\left(\mu_C-\frac{1}{2}\sigma_C^2\right)t+(1-\eta)\sigma_C\, z_C(t)}, \\
&\quad \alpha\, \phi\, s\, \nu \int_0^t \int_i^t \int_0^i \frac{e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[h-j]+\sigma_T[z_T(h)-z_T(j)]}}{M(0)+\int_0^j \gamma(w)\, C(w)\, e^{\delta w}\, \mathrm{d}w+(e^{\delta j}-1)M_{pre}}\, \mathrm{d}j\, \mathrm{d}h\, \mathrm{d}i\Bigg].
\end{aligned}
$$

In order to derive the expressions described in the main text, I now impose the assumption that $\sigma_T = 0$:

$$
\begin{aligned}
&Cov\left[C(t)^{1-\eta}, \epsilon(t)\right] \\
&=\phi\, \nu\, C(0)^{1-\eta}\; Cov\Bigg[\alpha\, s \int_0^t \int_i^t \int_0^i \frac{e^{-\phi[h-j]}}{M(0)+\int_0^j \gamma(w)\, C(w)\, e^{\delta w}\, \mathrm{d}w+(e^{\delta j}-1)M_{pre}}\, \mathrm{d}j\, \mathrm{d}h\, \mathrm{d}i, \\
&\qquad e^{(1-\eta)\left(\mu_C-\frac{1}{2}\sigma_C^2\right)t+(1-\eta)\sigma_C\, z_C(t)}\, e^{-(1-\eta)\alpha \int_0^t e^{-\phi j} T(0)\, \mathrm{d}j} \\
&\qquad e^{-(1-\eta)\alpha \int_0^t e^{-\phi j}\left[\int_0^j e^{\phi k}\, \phi\, s\, \nu\, \ln\left[\frac{e^{-\delta k} M(0)+\int_0^k \gamma(w)\, C(w)\, e^{-\delta(k-w)}\, \mathrm{d}w+(1-e^{-\delta k})M_{pre}}{M_{pre}}\right] \mathrm{d}k\right] \mathrm{d}j}\Bigg].
\end{aligned}
$$

Take a first-order approximation to the first argument in the covariance operator around $E[\alpha]$, $E[s]$, and $E[z_C(k)]$ for $k \leq t$, dropping the zero-order terms because they are not random:

$$
\begin{aligned}
&Cov\left[C(t)^{1-\eta}, \epsilon(t)\right] \\
&\quad \approx \phi\,\nu\,C(0)^{1-\eta} \\
&\quad\quad Cov\Bigg[(s-E[s])E[\alpha]\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}}{M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i \\
&\quad\quad\quad -(s-E[s])E[\alpha]E[s]\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}\int_0^j \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}s}\,e^{\delta w}\mathrm{d}w}{\left(M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i \\
&\quad\quad\quad +(\alpha-E[\alpha])E[s]\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}}{M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i \\
&\quad\quad\quad -(\alpha-E[\alpha])E[\alpha]E[s]\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}\int_0^j \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}\alpha}\,e^{\delta w}\mathrm{d}w}{\left(M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i \\
&\quad\quad\quad -\int_0^t z_C(x)\,E[\alpha]\,E[s]\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}\int_0^j \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_C(x)}\,e^{\delta w}\mathrm{d}w}{\left(M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i, \\
&\quad\quad\quad\quad e^{(1-\eta)\left(\mu_C-\frac{1}{2}\sigma_C^2\right)t+(1-\eta)\sigma_C\,z_C(t)}e^{-(1-\eta)\alpha\int_0^t e^{-\phi j}T(0)\,\mathrm{d}j} \\
&\quad\quad\quad\quad e^{-(1-\eta)\alpha\int_0^t e^{-\phi j}\left[\int_0^j e^{\phi k}\,\phi\,s\,\nu\,\ln\left[\frac{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}}{M_{pre}}\right]\mathrm{d}k\right]\mathrm{d}j}\Bigg],
\end{aligned}
$$

where I write $\bar{C}(w)$ to indicate $C(w)$ evaluated at $\alpha = E[\alpha]$, $s = E[s]$, and $z_C(\cdot) = 0$. In an abuse of notation, writing a derivative of $\bar{C}(w)$ with respect to α, s, or $z_C(x)$ implies differentiating $C(w)$ and then evaluating at $\alpha = E[\alpha]$, $s = E[s]$, and $z_C(\cdot) = 0$. Now take a first-order approximation to the second argument of the covariance operator around its exponent evaluated at $\alpha = E[\alpha]$, $s = E[s]$, and $z_C(\cdot) = 0$:

$$
\begin{aligned}
&Cov\left[C(t)^{1-\eta}, \epsilon(t)\right]\\
&\quad\approx E[\alpha]\,\phi\,\nu\,\bar{C}(t)^{1-\eta}\\
&\qquad\left\{\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}}{M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i\right.\\
&\qquad\quad\left. - E[s]\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}\int_0^j \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}s}\,e^{\delta w}\mathrm{d}w}{\left(M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i\right\}\\
&\qquad Cov\Bigg[s,\quad -(1-\eta)\alpha\int_0^t e^{-\phi j}\Bigg(\int_0^j e^{\phi k}\,\phi\,s\,\nu\\
&\qquad\qquad\qquad \ln\left[\frac{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w + (1-e^{-\delta k})M_{pre}}{M_{pre}}\right]\mathrm{d}k\Bigg)\,\mathrm{d}j\Bigg]\\
&\quad + E[s]\,\phi\,\nu\,\bar{C}(t)^{1-\eta}\\
&\qquad\left\{\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}}{M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i\right.\\
&\qquad\quad\left. - E[\alpha]\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}\int_0^j \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}\alpha}\,e^{\delta w}\mathrm{d}w}{\left(M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i\right\}\\
&\qquad Cov\Bigg[\alpha,\quad -(1-\eta)\alpha\int_0^t e^{-\phi j}\Bigg(\int_0^j e^{\phi k}\,\phi\,s\,\nu\\
&\qquad\qquad\qquad \ln\left[\frac{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w + (1-e^{-\delta k})M_{pre}}{M_{pre}}\right]\mathrm{d}k\Bigg)\,\mathrm{d}j\Bigg]\\
&\quad - E[\alpha]\,E[s]\,\phi\,\nu\,\bar{C}(t)^{1-\eta}\\
&\qquad\int_0^t\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}\int_0^j \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_C(x)}\,e^{\delta w}\mathrm{d}w}{\left(M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i\\
&\qquad Cov\Bigg[z_C(x),\quad (1-\eta)\sigma_C\,z_C(t) - (1-\eta)\alpha\int_0^t e^{-\phi j}\Bigg(\int_0^j e^{\phi k}\,\phi\,s\,\nu\\
&\qquad\qquad\qquad \ln\left[\frac{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w + (1-e^{-\delta k})M_{pre}}{M_{pre}}\right]\mathrm{d}k\Bigg)\,\mathrm{d}j\Bigg]\,\mathrm{d}x,
\end{aligned}
\tag{A-7}
$$

where I drop all nonrandom terms inside the covariance operator and recognize that s, α, and $z_C(\cdot)$ are independent. Label the first five lines on the right-hand side A_1, the next five lines A_2, and the final four lines A_3.

Begin by analyzing A_1, the first five lines in equation (A-7). Simplify the covariance operator's second argument to obtain

$$
\begin{aligned}
A_1 =& -(1-\eta)E[\alpha]\phi\,\nu\,\bar{C}(t)^{1-\eta}\\
&\left\{\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}}{M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i\right.\\
&\quad\left. -E[s]\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}\int_0^j \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}s}\,e^{\delta w}\mathrm{d}w}{\left(M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i\right\}\\
&\int_0^t e^{-\phi j}\int_0^j e^{\phi k}\,\phi\,\nu\,Cov\left(s,\ \alpha\,s\,\ln\left[\frac{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}}{M_{pre}}\right]\right)\mathrm{d}k\,\mathrm{d}j.
\end{aligned}
$$

Taking a first-order approximation of the covariance operator's second argument around $E[s]$, $E[\alpha]$, and $z_C(\cdot)=0$, we have

$$
\begin{aligned}
&Cov\left(s,\ \alpha\,s\,\ln\left[e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}\right]\right)\\
\approx& Var(s)\,E[\alpha]\ \ln\left[\frac{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,\bar{C}(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}}{M_{pre}}\right]\\
&+Var(s)\,E[\alpha]\,E[s]\,\frac{\int_0^k \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}s}\,e^{-\delta(k-w)}\,\mathrm{d}w}{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,\bar{C}(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}}.
\end{aligned}
$$

Substituting into A_1 and using the solution for $M(t)$ yields:

$$
\begin{aligned}
A_1 \approx& -(1-\eta)\,Var(s)\,E[\alpha]^2\phi\,\nu\,\bar{C}(t)^{1-\eta}\\
&\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}}{e^{\delta j}\bar{M}(j)}\left(1-E[s]\frac{\mathrm{d}\ln(\bar{M}(k))}{\mathrm{d}s}\right)\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i\\
&\int_0^t e^{-\phi j}\int_0^j e^{\phi k}\,\phi\,\nu\left[\ln\left(\frac{\bar{M}(k)}{M_{pre}}\right)+E[s]\frac{\mathrm{d}\ln(\bar{M}(k))}{\mathrm{d}s}\right]\mathrm{d}k\,\mathrm{d}j,
\end{aligned}
\tag{A-8}
$$

where I write $\bar{M}(j)$ to indicate $M(j)$ evaluated at $\alpha=E[\alpha]$, $s=E[s]$, and $z_C(\cdot)=0$. This is the expression that the main text uses to discuss how uncertainty about s generates a growth insurance premium. The second line describes how a large value of s affects the damage semi-elasticity $\epsilon(t)$. The first term in parentheses on the second line captures how larger s makes any given increase in forcing generate more warming. The second term in parentheses on the second line reflects how larger s induces more warming prior to time

t, which reduces consumption prior to time t and so reduces time t CO_2.[1] The reduction in time t CO_2 increases the damage semi-elasticity because of the concavity of the forcing operator. The final line describes how larger s affects time t consumption. The first term in brackets reflects how larger s reduces time t consumption by magnifying the warming effect of each earlier instant's CO_2. The second term in brackets reflects how larger s increases time t consumption by reducing consumption (and so emissions) in earlier instants.

By an analogous derivation, we also have

$$\begin{aligned} A_2 \approx &- (1-\eta)\, Var(\alpha)\, E[s]^2 \phi\, \nu\, \bar{C}(t)^{1-\eta} \\ &\int_0^t \int_i^t \int_0^i \frac{e^{-\phi[h-j]}}{e^{\delta j}\bar{M}(j)} \left(1 - E[s]\frac{\mathrm{d}\ln(\bar{M}(k))}{\mathrm{d}\alpha}\right) \mathrm{d}j\, \mathrm{d}h\, \mathrm{d}i \\ &\int_0^t e^{-\phi j} \int_0^j e^{\phi k}\, \phi\, \nu \left[\ln\left(\frac{\bar{M}(k)}{M_{pre}}\right) + E[\alpha]\frac{\mathrm{d}\ln(\bar{M}(k))}{\mathrm{d}\alpha}\right] \mathrm{d}k\, \mathrm{d}j. \end{aligned} \tag{A-9}$$

The following lemma will be useful in proving the propositions:

Lemma 1. *Assume $\sigma_T = 0$. If $M(0)$ is sufficiently large, τ is sufficiently small, or $E[\alpha]\, E[s]$ is sufficiently small, then $\mathrm{d}\bar{C}(t)/\mathrm{d}s \leq 0$, $\mathrm{d}\bar{M}(t)/\mathrm{d}s \leq 0$, $\mathrm{d}\bar{C}(t)/\mathrm{d}\alpha \leq 0$, and $\mathrm{d}\bar{M}(t)/\mathrm{d}\alpha \leq 0$ for all $t \in [0, \tau]$.*

Proof. From equation (A-2), we have

$$\frac{\mathrm{d}\bar{C}(t)}{\mathrm{d}s} = - E[\alpha]\, \bar{C}(t) \int_0^t \frac{\mathrm{d}\bar{T}}{\mathrm{d}s}\, \mathrm{d}j, \tag{A-10}$$

where I write $\bar{T}$ by analogy to $\bar{C}$ and $\bar{M}$. From equation (A-1), we have (with $\sigma_T = 0$):

$$\begin{aligned} \frac{\mathrm{d}\bar{T}}{\mathrm{d}s} =& \phi \int_0^j e^{-\phi\,(j-i)} F(\bar{M}(i))\, \mathrm{d}i \\ &+ E[s]\, \phi\, \nu \int_0^j e^{-\phi\,(j-i)} \frac{1}{\bar{M}(i)} \int_0^i \gamma(w)\, \frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}s}\, e^{-\delta(i-w)} \mathrm{d}w\, \mathrm{d}i. \end{aligned} \tag{A-11}$$

The first line is positive and increasing in $M(0)$. The second line is of ambiguous sign and is small when $E[\alpha]\, E[s]$ is small (using equation (A-10) for $\mathrm{d}\bar{C}(w)/\mathrm{d}s$). The sign of $\mathrm{d}\bar{C}(t)/\mathrm{d}s$ is opposite to that of $\mathrm{d}\bar{T}/\mathrm{d}s$. Therefore, if $M(0)$ is sufficiently large or $E[\alpha]\, E[s]$ is sufficiently small, then $\mathrm{d}\bar{C}(t)/\mathrm{d}s$ is negative.

[1] This interpretation is true as long as $\mathrm{d}\bar{C}(w)/\mathrm{d}s$ is negative, which Lemma 1 shows is the case for $E[\alpha]\, E[s]$ sufficiently small. A marginal increase in expected warming per unit of emissions tends to reduce consumption, but by reducing consumption, it also tends to reduce emissions and so increase later consumption. The derivative is negative as long as a marginal increase in expected warming per unit of emissions does not decrease future consumption and emissions so strongly that it eventually begins increasing consumption at some date prior to w.

It remains to show that $\mathrm{d}\bar{C}(t)/\mathrm{d}s$ is negative for t sufficiently small. Assume that there exists $i > 0$ such that $\mathrm{d}\bar{C}(t)/\mathrm{d}s \geq 0$ for all $t \in [0, i)$. Then the second line in equation (A-11) is positive for $j \leq i$, in which case $\mathrm{d}\bar{C}(i)/\mathrm{d}s \leq 0$ from equation (A-10). Note that $\mathrm{d}\bar{C}(0)/\mathrm{d}s = 0$ as $t \to 0$. Thus at times k infinitesimally close to zero, we satisfy the assumption and can conclude that $\mathrm{d}\bar{C}(k)/\mathrm{d}s \leq 0$. Therefore there is always some $\tau > 0$ such that $\mathrm{d}\bar{C}(t)/\mathrm{d}s \leq 0$ for $t < \tau$.

Finally, note that $\mathrm{d}\bar{M}(t)/\mathrm{d}s \leq 0$ if $\mathrm{d}\bar{C}(j)/\mathrm{d}s \leq 0$ for $j \in [0, t]$.

The proof for the derivatives with respect to α is directly analogous.

□

Now analyze A_3, the final four lines in equation (A-7). Recognizing that $Cov(z_C(x), z_C(t)) = \min\{x, t\}$, we have

$$
\begin{aligned}
A_3 =& \\
&-(1-\eta)E[\alpha]\,E[s]\,\phi\,\nu\,\bar{C}(t)^{1-\eta}\int_0^t\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}\int_0^j \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_C(x)}\,e^{\delta w}\mathrm{d}w}{\left(M(0)+\int_0^j \gamma(w)\,\bar{C}(w)\,e^{\delta w}\,\mathrm{d}w+(e^{\delta j}-1)M_{pre}\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i \\
&\Bigg[\sigma_C\,x - E[\alpha]\,E[s]\int_0^t e^{-\phi j}\int_0^j e^{\phi k}\,\phi\,\nu \\
&\qquad Cov\Bigg(z_C(x),\ \ln\left[e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}\right]\Bigg)\,\mathrm{d}k\,\mathrm{d}j\Bigg]\,\mathrm{d}x.
\end{aligned}
$$

Taking a first-order approximation of the covariance operator's second argument around $E[s]$, $E[\alpha]$, and $z_C(\cdot) = 0$, we have

$$
\begin{aligned}
&Cov\Bigg(z_C(x),\ \ln\left[e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}\right]\Bigg) \\
\approx& \frac{\int_0^k\int_0^w \min\{x,v\}\,\gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_C(v)}\,e^{-\delta(k-w)}\,\mathrm{d}v\,\mathrm{d}w}{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,\bar{C}(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}}.
\end{aligned}
$$

Substituting into A_3 and using the solution for $M(t)$, we have:

$$
\begin{aligned}
A_3 \approx& -(1-\eta)E[\alpha]\,E[s]\,\phi\,\nu\,\bar{C}(t)^{1-\eta}\int_0^t\int_0^t\int_i^t\int_0^i \frac{e^{-\phi[h-j]}e^{\delta j}\,\frac{\mathrm{d}\bar{M}(j)}{\mathrm{d}z_C(x)}}{\left(e^{\delta j}\bar{M}(j)\right)^2}\,\mathrm{d}j\,\mathrm{d}h\,\mathrm{d}i \\
&\left[\sigma_C\,x - E[\alpha]\int_0^t e^{-\phi j}\int_0^j e^{\phi k}\,\phi\,E[s]\,\nu\frac{\int_0^k\int_0^w \min\{x,v\}\,\gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_C(v)}\,e^{-\delta(k-w)}\,\mathrm{d}v\,\mathrm{d}w}{\bar{M}(k)}\,\mathrm{d}k\,\mathrm{d}j\right]\mathrm{d}x.
\end{aligned}
\tag{A-12}
$$

This is the expression that the main text uses to discuss how uncertainty about consumption growth generates a growth insurance premium. The integrals on the first line account for how a sequence of positive consumption shocks affects the damage semi-elasticity $\epsilon(t)$. They indicate that worlds in which consumption has had positive shocks are also worlds in which the CO_2 stock is large, and so the marginal effect of emissions on the climate is small.[2] The final line reflects how positive shocks to consumption affect time t consumption. The positive first term accounts for how positive time x shocks directly induce greater time t consumption. The negative second term accounts for how positive time x consumption shocks tend to reduce time t consumption by engendering additional emissions and warming.

The following lemma will be useful in proving the propositions:

Lemma 2. *Assume $\sigma_T = 0$. If $E[\alpha]\, E[s]\, \phi\, \nu$ is sufficiently small or τ is sufficiently small, then $\mathrm{d}\bar{C}(t)/\mathrm{d}z_C(x) \geq 0$ and $\mathrm{d}\bar{M}(t)/\mathrm{d}z_C(x) \geq 0$ for all $t \in [0, \tau]$ and all $x \geq 0$.*

Proof. From equation (A-2), we have

$$\begin{aligned}
\frac{\mathrm{d}\bar{C}(t)}{\mathrm{d}z_C(t)} =& \sigma_C\, \bar{C}(t),\\
\frac{\mathrm{d}\bar{C}(t)}{\mathrm{d}z_C(x)} =& \sigma_C\, \bar{C}(t) - E[\alpha]\, \bar{C}(t) \int_x^t \frac{\mathrm{d}\bar{T}}{\mathrm{d}z_C(x)} \mathrm{d}j \quad \text{for } x \in [0, t),\\
\frac{\mathrm{d}\bar{C}(t)}{\mathrm{d}z_C(x)} =& 0 \quad \text{for } x > t.
\end{aligned} \tag{A-13}$$

Positive shocks increase contemporary consumption directly but decrease future consumption by raising future temperatures. From equation (A-1), we have (with $\sigma_T = 0$):

$$\begin{aligned}
\frac{\mathrm{d}\bar{T}}{\mathrm{d}z_C(x)} =& E[s]\, \phi\, \nu \int_0^j e^{-\phi\,(j-i)} \frac{1}{\bar{M}(i)} \int_0^i \gamma(w)\, \frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_C(x)}\, e^{-\delta(i-w)} \mathrm{d}w\, \mathrm{d}i\\
=& E[s]\, \phi\, \nu \int_{\min\{x,j\}}^j e^{-\phi\,(j-i)} \frac{1}{\bar{M}(i)} \int_x^i \gamma(w)\, \frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_C(x)}\, e^{-\delta(i-w)} \mathrm{d}w\, \mathrm{d}i.
\end{aligned} \tag{A-14}$$

If $\mathrm{d}\bar{T}(j)/\mathrm{d}z_C(x) < 0$ for all $x \in [0, t), j \in (x, t)$, then $\mathrm{d}\bar{C}(w)/\mathrm{d}z_C(x) > 0$ for all $w \in (0, t]$, by equation (A-13). But then equation (A-14) implies that $\mathrm{d}\bar{T}(j)/\mathrm{d}z_C(x) > 0$ for all $x \in [0, t), j \in (x, t)$, which generates a contradiction. Therefore it must be the case that for some $k > 0$ sufficiently small, $\mathrm{d}\bar{T}(j)/\mathrm{d}z_C(x) \geq 0$ for all $x \in [0, k), j \in (x, k)$. Now consider $\mathrm{d}\bar{C}(t)/\mathrm{d}z_C(x)$. By equation (A-13) and the result that some $\mathrm{d}\bar{T}(j)/\mathrm{d}z_C(x) \geq 0$, this expression is always ambiguously signed. However, for $E[\alpha]\, E[s]\, \phi\, \nu$ sufficiently small (using equation (A-13) to substitute for $\mathrm{d}\bar{C}(w)/\mathrm{d}z_C(x)$), the positive term dominates. The positive term also dominates for t sufficiently small.

[2] In the interpretation here and immediately below, I assume that $\mathrm{d}\bar{C}(w)/\mathrm{d}z_C(x) \geq 0$ and $\mathrm{d}\bar{M}(j)/\mathrm{d}z_C(x) \geq 0$. Lemma 2 shows that these derivatives will be positive as long as the direct effect of past consumption shocks on time w consumption dominates the indirect effects via induced warming. The direct effect dominates as long as $E[\alpha]\, E[s]$ is sufficiently small. This assumption is the analogue to the assumption used in the interpretation of the main text's two-period example.

Finally, note that $\mathrm{d}\bar{M}(t)/\mathrm{d}z_C(x) \geq 0$ if $\mathrm{d}\bar{C}(j)/\mathrm{d}z_C(x) \geq 0$ for $j \in [0, t]$.

□

B Numerical calibration and computation

This section describes the numerical setting's calibration to historical data and to a benchmark climate-economy model. It also describes the computational method.

The numerical setting measures time in years, with the initial period (time 0) being 2014 and the horizon extending out for 200 years ($\tau = 200$). The annual utility discount rate of 1.5% matches the value in the benchmark climate-economy model DICE-2007 (Nordhaus, 2008), which implies $\rho = 0.015$.

In World Bank data for GDP since 1960, global output grew at an average of 3.9% per year with a realized standard deviation of 2.6% per year. I therefore take $\mu_C = 0.039$ and $\sigma_C = 0.026$. Global GDP in the year 2013 was 55 trillion dollars (in year 2005 U.S. dollars). The 2014 U.S. GDP deflator with respect to year 2005 dollars is 117.72. I therefore fix $C(0) = 55 \times 10^{12} \times 1.1772$, or \$64.75 trillion.

Figure B1 depicts the evolution of annual CO_2 emissions and world output per capita since 1960. In accord with these data, I allow emission intensity $\gamma(t)$ to decline over time:

$$\gamma(t) = \gamma_0 e^{-\gamma_1 (t+2014-1960)}/10^{12},$$

where t is measured in years from 2014 and emission intensity is measured in Gt C per trillion dollars of output, in year 2014 dollars. I calibrate the parameters γ_0 and γ_1 via the following regression with the emission and global GDP time series:

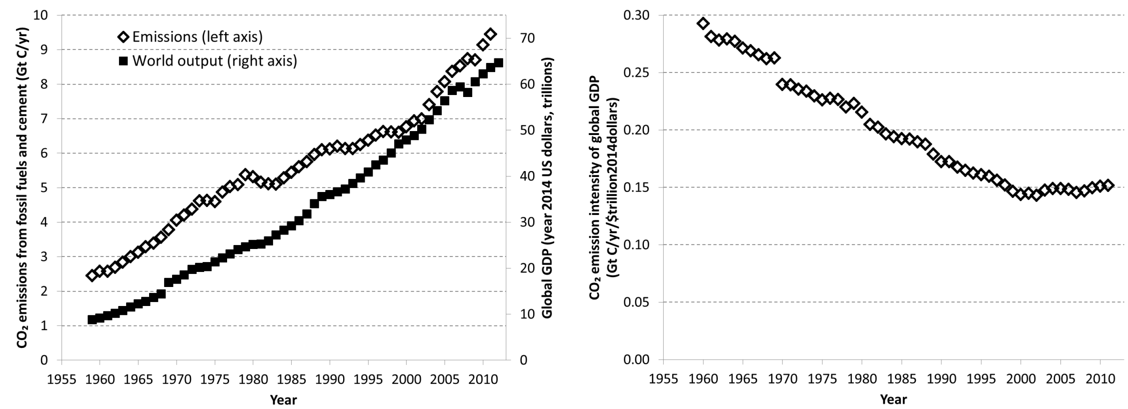
$$\ln\left(E(t)/C(t)\right) = \ln(\gamma_0) - \gamma_1 t,$$

where $E(t)$ is emissions in year t and t is here measured in years since 1960. Both estimated coefficients are significantly different from zero at the 1% level. This regression yields $\gamma_0 = 0.29$ and $\gamma_1 = 0.015$.

The exogenous population $L(t)$ is

$$L(t) = L(0)\, e^{\sum_{s=1}^{t} g_L(s)\, \mathrm{d}s},$$

where t is measured in years from 2014 and $g_L(t)$ is the time t growth rate of population. I allow this growth rate to vary over time: $g_L(t) = \max\{0, g_{L0} + g_{L1}[t + 2014 - 1960]\}$. Using World Bank data on global population to regress population growth rates against time, we find that $g_{L0} = 0.021$ and $g_{L1} = -0.00018$ (both estimated coefficients are significantly different from zero at the 1% level). Using data from 2013, I fix initial population at $L(0) = 7.1 \times 10^9$ people.



(a) Emissions and world output (b) Emission intensity of world output

Figure B1: The evolution of emission intensity over the past 50 years. Emissions data are from Boden et al. (2015), and world output is from the World Bank.

Following the calibration of Lemoine and Rudik (2014) to DICE-2007, CO_2 in excess of its pre-industrial level decays at rate $\delta = 0.0138$. Blasing (2014) reports the year 2013 average concentration as 395.4 parts per million (ppm). Expressing this in Gt C, we have $M(0) = 854$.

For the forcing relationship, Blasing (2014) reports the pre-1750 tropospheric CO_2 concentration as 280 ppm, which implies $M_{pre} = 605$ Gt C. Following Ramaswamy et al. (2001, Table 6.2), I take $\nu = 5.35$ W m^{-2}, which is approximately equal to the parameter used in DICE-2007 (Nordhaus, 2008).

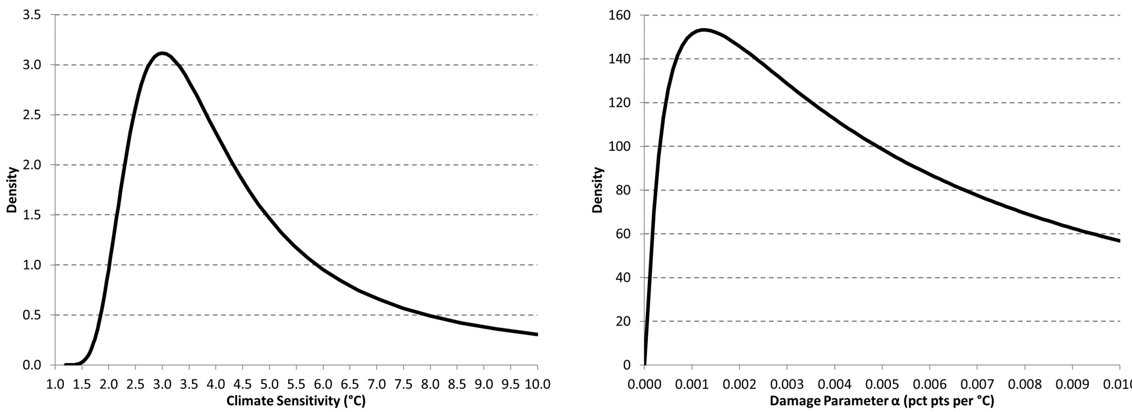
"Climate sensitivity" (S) is a standard metric that described the equilibrium warming that results from doubling the atmospheric concentration of carbon dioxide. Following Lemoine and Rudik (2014), climate sensitivity S relates to the present setting's warming parameter s as follows:

$$s = \frac{S}{5.35 \ln 2}.$$

The scientific literature has produced many distributions for climate sensitivity. Roe and Baker (2007) theoretically derive the standard shape of these distributions from linear feedback analysis, and Greenstone et al. (2013) find that the resulting distribution matches the scientific literature better than does fitting various conventional distributions directly over S. Linear feedback analysis derives climate sensitivity S as

$$S = \frac{\lambda_0}{1 - \Delta},$$

where $\lambda_0 = 1.2$°C is the climate sensitivity for a reference system (lacking earth system feedbacks) and $\Delta < 1$ is the climate system's (dimensionless) feedback factor. See Roe (2009)



(a) Climate sensitivity S (b) Damage parameter α

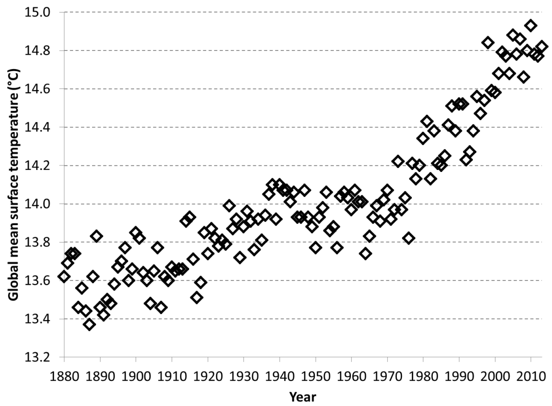
Figure B2: The calibrated distributions for climate sensitivity S (left) and the damage parameter α (right).

and Lemoine (2010) for discussions. In Roe and Baker (2007), Δ is normally distributed with a standard deviation of 0.13 and a mean of 0.6. This mean generates a climate sensitivity of 3°C, which is consistent with the value in DICE-2007 (Nordhaus, 2008). I use a version of this distribution truncated at 0.88. The truncation point corresponds to $S = 10$°C, which is a much higher climate sensitivity than is plausible over the timescale of a century or two. See Costello et al. (2010) and Kelly and Tan (2015) for more on truncation of the distribution of climate sensitivity. The left panel of Figure B2 plots the distribution of S implied by the distribution of Δ.

The inertia parameter ϕ follows Lemoine and Rudik (2014): $\phi = 0.0091$. This value was originally calibrated to DICE-2007 (Nordhaus, 2008) via Lemoine and Traeger (2014). I assume that ϕ is fixed independently of S, but the potential correlation between these parameters is worthy of future study.

In the Geophysical Fluid Dynamics Laboratory (GFDL) global mean temperature series (Figure B3), the standard deviation of the year-on-year changes in annual temperature over 1880–2013 was 1.02%, and temperature in 2013 was 1.2245°C above the 1880–1899 average, implying $T(0) = 1.2245$. Assume that the realized standard deviation for temperature holds around 1°C and would double at 2°C. (Note that temperatures below 1°C are irrelevant for our analysis.) This ad hoc assumption yields $\sigma_T = 0.0102$.

The damages from climate change are deeply uncertain. Pindyck (2016) reports the results of a recent survey of around 1,000 climate scientists and economists. He asked these experts to report their subjective percentiles for the percentage reduction in GDP that climate change would cause in fifty years, assuming that no emission controls are enacted before then. He fit four distributions to the results and found that a lognormal distribution pro-



(a) Annual level

(b) Year-on-year change

Figure B3: Global mean surface temperature since 1880.

duced the highest corrected R^2. The location parameter for his fitted lognormal distribution is -2.446 and the scale parameter is 1.476. His distribution describes a parameter ϕ_P that maps to our setting as

$$\phi_P = \alpha \int_0^{50} T(j)\,\mathrm{d}j,$$

where the subscript P distinguishes Pindyck's parameter from our inertia parameter. The distribution of his parameter ϕ_P captures uncertainty about warming over the fifty year interval and also about economic damages from that warming. Simulating $T(j)$ 10,000 times with $\sigma_C = \sigma_T = 0$ and $\alpha = 0$, we find that a normal distribution fit to $\ln\left[\int_0^{50} T(j)\,\mathrm{d}j\right]$ has mean 4.234 and standard deviation 0.108. Assume that α and $\int_0^{50} T(j)\,\mathrm{d}j$ are independent. Then, by properties of the lognormal distribution, we have that α is lognormally distributed with location parameter $-2.446 - 4.234 = -6.680$ and scale parameter $(1.476^2 - 0.108^2)^{0.5} = 1.472$. I truncate the distribution of α at 0.01. This truncation point would imply a consumption loss of 50% in fifty years at the mean of $\int_0^{50} T(j)\,\mathrm{d}j$, which is also the largest value asked about in the survey for Pindyck (2016). This truncated lognormal distribution implies $E[\alpha] = 0.0019$. Because some effects of past climate change are wrapped up in the calibration of consumption growth, I assume that the damage parameter α acts on the difference between time t temperature $T(t)$ and initial temperature $T(0)$.[3] The right panel of Figure B2 plots the distribution of α.

The baseline preference specification uses a coefficient of relative risk aversion (η) equal to 2. This value is consistent with preferences revealed via income tax rates (Stern, 1976),

[3]However, earlier in this paragraph I assumed that the damage parameter acts on $T(t)$ for the purposes of calibrating to the survey results reported in Pindyck (2016) (which could be interpreted as asking about the future losses from total climate change, not just from future climate change).

with the climate-economy model DICE-2007 (Nordhaus, 2008), and with recommendations in Weitzman (2007) and Dasgupta (2008).

In order to calculate the social cost of carbon, I take 10,000 random draws of each Brownian motion's trajectory, approximate the distribution over Δ using quadrature with 8 nodes, and approximate the distribution over α by using quadrature with 8 nodes.[4] For each quadrature node and each set of Brownian motion trajectories, I calculate the semi-elasticity $\epsilon(t)$ using equations (A-3), (A-4), and (A-5). Combining this semi-elasticity with directly calculated trajectories for consumption, I obtain the social cost of carbon by taking expected values (using quadrature weights for α and s and averaging over the Monte Carlo draws for the Brownian motions). I calculate the growth insurance channel as the difference between the actual social cost of carbon and a version calculated using the expected damage semi-elasticity. I calculate the damage scaling channel by multiplying the expected damage semi-elasticity and the covariance between consumption and marginal utility. The deterministic social cost of carbon uses expected consumption and the expected damage semi-elasticity. The precautionary channel is the remainder after subtracting the growth insurance, damage scaling, and deterministic social cost of carbon from the full social cost of carbon. All differential equations are simulated using the Euler-Maruyama method with a timestep of 1 year. Time integrals are calculated using the trapezoidal rule, with spacing of 1 year.

C The environmental risk premium

I have heretofore considered how today's emissions affect future consumption, but many welfare costs of climate change arise through impacts on environmental quality, aside from any impacts on consumption. In this section, I extend the theoretical framework to allow for direct preferences over environmental quality.

Numerical integrated assessment models have typically monetized climate change impacts via effects on consumption or output, but the most serious welfare consequences of climate change potentially arise through direct preferences for various dimensions of environmental quality. If we expect the environment to become scarcer relative to consumption goods, then our increasing willingness to sacrifice present consumption to obtain future environmental quality acts like applying a lower risk-free discount rate when valuing environmental impacts than when valuing consumption impacts (Hoel and Sterner, 2007; Gollier, 2010), which can strongly increase the value of emission reductions in conventional integrated assessment settings (Sterner and Persson, 2008).

Let there be two types of goods: a consumption good and an environmental good. Climate change induces a downward drift in each good. As before, the policymaker seeks to

[4]Experiments with $\eta = 4$ use 9 nodes in each dimension. Some experiments with a very small scale parameter for either Δ or α use less than 8 nodes, due to numerical difficulties in defining a large number of nodes over a small domain.

value a marginal unit of greenhouse gas emissions via its effect on future welfare.

The processes driving consumption, CO_2, and temperature are as before, except we now take the damage parameter α as known, for purposes of exposition. The new pieces are the process driving environmental quality and the policymaker's preferences over environmental quality. Environmental quality $Q(t)$ evolves geometrically:

$$\frac{\mathrm{d}Q(t)}{Q(t)} = \mu_Q\,\mathrm{d}t - \alpha_Q\,T(t)\,\mathrm{d}t + \sigma_Q\,\mathrm{d}z_Q(t).$$

Environmental quality may drift upward or downward, according to the sign of μ_Q. Global warming tends to reduce environmental quality through the environmental damage parameter $\alpha_Q > 0$. The Brownian motion $z_Q(t)$ captures all non-temperature shocks to environmental quality, with volatility parameter $\sigma_Q \geq 0$. This Brownian motion is independent of the Brownian motions driving temperature and consumption.[5]

The policymaker now cares about both environmental quality and consumption. For analytic tractability, let the policymaker's preferences be additively separable in these two types of goods and have the conventional isoelastic (power) representation in each dimension. Instantaneous utility therefore takes the form

$$u(C(t)/L(t), Q(t)) = \frac{[C(t)/L(t)]^{1-\eta}}{1-\eta} + \frac{Q(t)^{1-\eta_Q}}{1-\eta_Q}.$$

The parameter $\eta_Q \geq 0, \neq 1$ is the inverse of the (constant) elasticity of intertemporal substitution (and so also is the coefficient of relative risk aversion) for the environmental good. Intertemporal welfare is defined as before, except using this two-dimensional instantaneous utility function.

By previous reasoning, the social cost of carbon is

$$\begin{aligned} scc = &\underbrace{\int_0^\tau \frac{e^{-\rho t} L(t)^\eta}{[C(0)/L(0)]^{-\eta}} \left[E[C(t)^{1-\eta}]\, E[\epsilon(t)] + Cov[C(t)^{1-\eta}, \epsilon(t)] \right] \mathrm{d}t}_{scc_{cons}} \\ &+ \underbrace{\int_0^\tau \frac{e^{-\rho t} L(t)}{[C(0)/L(0)]^{-\eta}} \left[E[Q(t)^{1-\eta_Q}]\, E[\epsilon_Q(t)] + Cov[Q(t)^{1-\eta_Q}, \epsilon_Q(t)] \right] \mathrm{d}t}_{scc_{env}}, \end{aligned} \tag{A-15}$$

where $\epsilon(t)$ is the semi-elasticity of time t consumption with respect to time 0 emissions and is exactly the same as in previous sections. The new term $\epsilon_Q(t)$ is the semi-elasticity of time

[5] One could imagine a setting in which shocks to consumption and environmental quality are negatively correlated, but such a setting should also make the drift in environmental quality depend on consumption. This type of setting is beyond the scope of the present paper. I leave its exploration to future work.

t environmental quality with respect to time 0 emissions:

$$\epsilon_Q(t) \triangleq -\frac{\mathrm{d}Q(t)}{\mathrm{d}m}\frac{1}{Q(t)} = \frac{\alpha_Q}{\alpha}\,\epsilon(t), \tag{A-16}$$

where the second equality is derived in the proof of Proposition 3.

These separable preferences enable us to cleanly separate the social cost of carbon into a consumption component and an environmental component, as in equation (A-15). The consumption component scc_{cons} has been analyzed above. We here focus on the environmental component scc_{env}, which includes the same channels analyzed previously. Because the damage scaling premium works exactly as before, I focus on the growth insurance channel $Cov[Q(t)^{1-\eta_Q}, \epsilon_Q(t)]$. The following proposition describes how each source of risk affects this covariance:

Proposition 3.

1. *Assume* $\sigma_C\, Var(s) > 0$ *and* $\sigma_T = 0$*. Also assume that* $\alpha\, E[s]$ *is small, as in Proposition 1. If* $Var(s)/\sigma_C$ *is sufficiently large, then* $Cov[Q(t)^{1-\eta_Q}, \epsilon_Q(t)] > 0$ *if and only if* $\eta_Q > 1$*. If* $Var(s)/\sigma_C$ *is sufficiently small, then* $Cov[Q(t)^{1-\eta_Q}, \epsilon_Q(t)] < 0$ *if and only if* $\eta_Q > 1$*.*

2. *Assume* $\sigma_C = 0$*,* $Var(s) = 0$*, and* $\sigma_T > 0$*. If* α_Q *is sufficiently small, then* $Cov[Q(t)^{1-\eta_Q}, \epsilon_Q(t)] > 0$ *if and only if* $\eta_Q > 1$*.*

3. *Assume* $\sigma_C = 0$*,* $Var(s) = 0$*, and* $\sigma_T = 0$*. Then* $Cov[Q(t)^{1-\eta_Q}, \epsilon_Q(t)] = 0$*.*

Proof. See Section D. □

This proposition contains several results about the growth insurance channel in scc_{env}. Assume $\eta_Q > 1$, in line with conventional preferences over consumption. First, uncertainty about the warming parameter s increases the environmental component of the social cost of carbon (as with the consumption component), but consumption volatility reduces the environmental component of the social cost of carbon (in contrast to the consumption component). Second, volatility in the climate system increases the social cost of carbon. And third, non-climatic volatility in environmental quality has no effect on the social cost of carbon.

The intuition for how temperature volatility and uncertainty about warming induce a positive covariance is the same as in the case of the consumption good. The intuition for why purely environmental volatility does not induce any covariance is that the environmental damage semi-elasticity $\epsilon_Q(t)$ is independent of the level of environmental quality, as seen in (A-16). This environmental damage semi-elasticity is therefore also independent of direct shocks to environmental quality.

The most interesting result is that, in contrast to the case of scc_{cons}, consumption volatility induces a negative growth insurance channel in scc_{env} when $\eta_Q > 1$. We have already seen that consumption volatility tends to make states in which emissions only weakly affect the climate (i.e., in which $\epsilon(t)$ is small) accompany states in which consumption is high. States with high consumption are states with high CO_2, high warming, and low environmental quality. Therefore, positive shocks to consumption tend to reduce the marginal effect of emissions on the climate while also reducing environmental quality. The potential for consumption shocks induces a positive covariance between $\epsilon_Q(t)$ and $Q(t)$ and so generates a negative growth insurance channel in scc_{env}.

D Proofs

This section contains the proofs of the propositions contained in the main text.

D.1 Proof of Proposition 1

Assume that $\sigma_T = 0$ and $\sigma_C \, Var(s) > 0$. $Cov[C(t)^{1-\eta}, \epsilon(t)]$ is (to a first-order approximation) equal to the sum of equations (A-8), (A-9), and (A-12).

By Lemma 1, $\mathrm{d}\bar{M}(k)/\mathrm{d}s \leq 0$ if $E[\alpha]\,E[s]$ is sufficiently small. And if $\mathrm{d}\bar{M}(k)/\mathrm{d}s \leq 0$, then equations (A-8) and (A-9) are positive if $\eta > 1$ and $E[\alpha]\,E[s]$ is sufficiently small, because the first two lines are unambiguously positive and the negative term in each equation's third line is small if $E[\alpha]\,E[s]$ is small. By analogous logic, if $\mathrm{d}\bar{M}(k)/\mathrm{d}s \leq 0$, then equations (A-8) and (A-9) are negative if $\eta < 1$ and $E[\alpha]\,E[s]$ is sufficiently small. Therefore, if $E[\alpha]\,E[s]$ is small, then equations (A-8) and (A-9) are positive if and only if $\eta > 1$.

By Lemma 2, $\mathrm{d}\bar{C}(w)/\mathrm{d}z_C(x) \geq 0$ and $\mathrm{d}\bar{M}(w)/\mathrm{d}z_C(x) \geq 0$ if $E[\alpha]\,E[s]$ is sufficiently small. And if $\mathrm{d}\bar{C}(w)/\mathrm{d}z_C(x) \geq 0$ and $\mathrm{d}\bar{M}(w)/\mathrm{d}z_C(x) \geq 0$, then equation (A-12) is positive if $\eta > 1$ and $E[\alpha]\,E[s]$ is sufficiently small, because the first line is unambiguously positive and the negative term in the second line is small if $E[\alpha]\,E[s]$ is small. By analogous logic, if $\mathrm{d}\bar{C}(w)/\mathrm{d}z_C(x) \geq 0$ and $\mathrm{d}\bar{M}(w)/\mathrm{d}z_C(x) \geq 0$, then equation (A-12) is negative if $\eta < 1$ and $E[\alpha]\,E[s]$ is sufficiently small. Therefore, if $E[\alpha]\,E[s]$ is small, then equation (A-12) is positive if and only if $\eta > 1$.

Therefore, if $\sigma_T = 0$ and $E[\alpha]\,E[s]$ is small, then $Cov[C(t)^{1-\eta}, \epsilon(t)]$ is positive if and only if $\eta > 1$.

Now consider the main text's decomposition of the social cost of carbon into the deterministic social cost of carbon, the precautionary channel, the damage scaling channel, and the growth insurance channel. Using the expression for $\epsilon(t)$ in equation (A-6) and a second-order expansion of $E[\epsilon(t)]$ around $E[C(w)]$, we see that $E[\epsilon(t)]$ increases in $Var[C(w)]$, which increases in $Var(\alpha)$, $Var(s)$, and σ_C. The sum of the precautionary and damage scaling channels increases in each of $Var(\alpha)$, $Var(s)$, and σ_C if and only if these two channels sum to a positive value, and they sum to a positive value if and only if $\eta > 1$. Finally, under the assumptions of the proposition, equation (A-8) increases in $Var(s)$ if and only if $\eta > 1$, equation (A-9) increases in $Var(\alpha)$ if and only if $\eta > 1$, and equation (A-12) increases in σ_C if and only if $\eta > 1$.

D.2 Proof of Proposition 2

Assume that $\sigma_T > 0$, $\sigma_C = 0$, $Var(\alpha) = 0$, and $Var(s) = 0$. We have

$$
\begin{aligned}
Cov(\epsilon(t), C(t)^{1-\eta}) =& Cov\Bigg(\alpha\,\phi\,s\,\nu \int_0^t \int_0^i \int_i^t \frac{e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[h-j]+\sigma_T[z_T(h)-z_T(j)]}}{M(0)+\int_0^j \gamma(w)\,C(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}}\,\mathrm{d}h\,\mathrm{d}j\,\mathrm{d}i,\\
& C(0)^{1-\eta}\,e^{-(1-\eta)\alpha\int_0^t T(j)\,\mathrm{d}j+(1-\eta)\left(\mu_C-\frac{1}{2}\sigma_C^2\right)t+(1-\eta)\sigma_C\,z_C(t)}\Bigg)\\
=& C(0)^{1-\eta}e^{(1-\eta)\mu_C t}\\
& Cov\Bigg(\alpha\,\phi\,s\,\nu \int_0^t \int_0^i \int_i^t \frac{e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[h-j]+\sigma_T[z_T(h)-z_T(j)]}}{M(0)+\int_0^j \gamma(w)\,C(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}}\,\mathrm{d}h\,\mathrm{d}j\,\mathrm{d}i,\;\; e^{-(1-\eta)}\\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (\text{using } \sigma_C = 0)\\
=& C(0)^{1-\eta}e^{(1-\eta)\mu_C t}\alpha\,\phi\,s\,\nu\\
& Cov\Bigg(\int_0^t \int_0^i \int_i^t \frac{e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[h-j]+\sigma_T[z_T(h)-z_T(j)]}}{M(0)+\int_0^j \gamma(w)\,C(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}}\,\mathrm{d}h\,\mathrm{d}j\,\mathrm{d}i,\\
& \quad e^{-(1-\eta)\,\alpha\int_0^t\int_0^j e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[j-k]+\sigma_T\,[z_T(j)-z_T(k)]}\,\phi\,s\,\nu\,\ln\left[\frac{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}}{M_{pre}}\right.}\\
& \qquad e^{-(1-\eta)\,\alpha\int_0^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)j+\sigma_T\,z_T(j)}T(0)\,\mathrm{d}j}\Bigg).
\end{aligned}
$$

We have that $Cov(\epsilon(t), C(t)^{1-\eta})$ is positive (negative) if the following is positive (negative) for all (h, j) such that $h \in (0, t)$ and $j \in (0, h)$:

$$
\begin{aligned}
Cov\Bigg[&\frac{e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[h-j]+\sigma_T[z_T(h)-z_T(j)]}}{M(0)+\int_0^j \gamma(w)\,C(w)\,e^{\delta w}\,\mathrm{d}w + (e^{\delta j}-1)M_{pre}},\\
& e^{-(1-\eta)\,\alpha\int_0^t\int_0^j e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[j-k]+\sigma_T\,[z_T(j)-z_T(k)]}\,\phi\,s\,\nu\,\ln\left[\frac{e^{-\delta k}M(0)+\int_0^k \gamma(w)\,C(w)\,e^{-\delta(k-w)}\,\mathrm{d}w+(1-e^{-\delta k})M_{pre}}{M_{pre}}\right]\mathrm{d}k\,\mathrm{d}j}\\
& e^{-(1-\eta)\,\alpha\int_0^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)j+\sigma_T\,z_T(j)}T(0)\,\mathrm{d}j}\Bigg].
\end{aligned}
\tag{A-17}
$$

Take a first-order approximation of each argument of the covariance operator with respect to $z_T(\cdot)$ around $z_T(\cdot) = 0$, use $\bar{C}(t)$ and $\bar{M}(t)$ to indicate time t consumption and CO_2

evaluated at $z_T(\cdot) = 0$, recognize that the covariance of a constant term with anything else is zero, and factor out the common (positive) constant parts of the first-order terms in the approximations to obtain that the covariance in (A-17) is proportional to the following expression:

$$
\begin{aligned}
Cov\Bigg[& \int_j^h \sigma_T \, z_T(x) \, \mathrm{d}x - \frac{1}{e^{\delta j} \bar{M}(j)} \int_0^j \int_0^w \gamma(w) \, \frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_T(x)} \, e^{\delta w} z_T(x) \, \mathrm{d}x \, \mathrm{d}w, \\
& - (1-\eta)\, \alpha \int_0^t \int_0^m e^{\left(-\phi - \frac{1}{2}\sigma_T^2\right)[m-k]} \, \phi \, s \, \nu \, \ln\left[\frac{\bar{M}(k)}{M_{pre}}\right] \\
& \qquad \int_k^m \sigma_T \, z_T(x) \, \mathrm{d}x \, \mathrm{d}k \, \mathrm{d}m \\
& - (1-\eta)\, \alpha \int_0^t \int_0^m e^{\left(-\phi - \frac{1}{2}\sigma_T^2\right)[m-k]} \, \phi \, s \, \nu \frac{1}{\bar{M}(k)} \int_0^k \int_0^w \gamma(w) \, \frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_T(x)} \, e^{-\delta(k-w)} z_T(x) \, \mathrm{d}x \, \mathrm{d}w \, \mathrm{d}k \, \mathrm{d}m \\
& - (1-\eta)\, \alpha \int_0^t e^{\left(-\phi - \frac{1}{2}\sigma_T^2\right)m} \int_0^m \sigma_T \, z_T(x) \, \mathrm{d}x \; T(0) \, \mathrm{d}m \Bigg],
\end{aligned}
$$

where, as before, I abuse notation by using $\mathrm{d}\bar{C}(w)/\mathrm{d}z_T(x)$ to indicate the derivative of $C(w)$ with respect to $z_T(x)$ evaluated at $z_T(\cdot) = 0$. Breaking up the covariance operator and factoring out α yields:

$$
\begin{aligned}
&-(1-\eta)\int_0^t \int_0^m e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[m-k]}\,\phi\, s\, \nu\, \ln\left[\frac{\bar{M}(k)}{M_{pre}}\right] \\
&\qquad \int_k^m \int_j^h \sigma_T^2\, \min\{x,i\}\,\mathrm{d}i\,\mathrm{d}x\,\mathrm{d}k\,\mathrm{d}m \\
&-(1-\eta)\int_0^t \int_0^m e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[m-k]}\phi\, s\, \nu \frac{1}{\bar{M}(k)}\int_0^k \int_0^w \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_T(x)} e^{-\delta(k-w)} \int_j^h \sigma_T\, \min\{x,i\}\,\mathrm{d}i\,\mathrm{d}x\,\mathrm{d}w\,\mathrm{d}k\,\mathrm{d}m \\
&-(1-\eta)\int_0^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)m} \int_0^m \int_j^h \sigma_T^2\, \min\{x,i\}\,\mathrm{d}i\,\mathrm{d}x\; T(0)\,\mathrm{d}m \\
&+(1-\eta)\int_0^t \int_0^m e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[m-k]}\,\phi\, s\, \nu\, \ln\left[\frac{\bar{M}(k)}{M_{pre}}\right] \\
&\qquad \int_k^m \sigma_T \frac{1}{e^{\delta j}\bar{M}(j)}\int_0^j \int_0^w \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_T(i)}\, e^{\delta w}\min\{x,i\}\,\mathrm{d}i\,\mathrm{d}w\,\mathrm{d}x\,\mathrm{d}k\,\mathrm{d}m \\
&+(1-\eta)\int_0^t \int_0^m e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)[m-k]}\,\phi\, s\, \nu \int_0^k \int_0^w \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_T(x)} \\
&\qquad \frac{1}{e^{\delta j}\bar{M}(j)}\int_0^j \int_0^v \gamma(v)\,\frac{\mathrm{d}\bar{C}(v)}{\mathrm{d}z_T(i)}\, e^{\delta v}\min\{x,i\}\,\mathrm{d}i\,\mathrm{d}v\,\mathrm{d}x\,\mathrm{d}w\,\mathrm{d}k\,\mathrm{d}m \\
&+(1-\eta)\int_0^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right)m} \int_0^m \sigma_T \frac{1}{e^{\delta j}\bar{M}(j)}\int_0^j \int_0^w \gamma(w)\,\frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_T(i)}\, e^{\delta w}\min\{x,i\}\,\mathrm{d}i\,\mathrm{d}w\,\mathrm{d}x\; T(0)\,\mathrm{d}m.
\end{aligned}
\tag{A-18}
$$

The first two lines and the fourth line are positive if and only if $\eta > 1$. All other lines are proportional to $\mathrm{d}\bar{C}(w)/\mathrm{d}z_T(x)$. From equation (A-2), $\mathrm{d}\bar{C}(w)/\mathrm{d}z_T(x)$ goes to zero as α goes to zero, while $\bar{C}(w)$ increases as α goes to zero. Therefore, for α sufficiently small, the first two lines and the fourth line dominate the rest, making the overall expression positive if and only if $\eta > 1$. Therefore, if α is sufficiently small and $\eta > 1$, then (A-17) is positive for all (h, j) such that $h \in (0, t)$ and $j \in (0, h)$, which we have seen implies that $Cov(\epsilon(t), C(t)^{1-\eta})$ is positive. And if α is sufficiently small and $\eta < 1$, then (A-17) is negative for all (h, j) such that $h \in (0, t)$ and $j \in (0, h)$, which we have seen implies that $Cov(\epsilon(t), C(t)^{1-\eta})$ is negative. Therefore if α is sufficiently small, then $Cov(\epsilon(t), C(t)^{1-\eta})$ is positive if and only if $\eta > 1$.

Now consider the main text's decomposition of the social cost of carbon into the deterministic social cost of carbon, the precautionary channel, the damage scaling channel, and the growth insurance channel. Using the expression for $\epsilon(t)$ in equation (A-6) and a second-order expansion of $E[\epsilon(t)]$ around $E[C(w)]$ and $E[z_T(\cdot)]$, we see that $E[\epsilon(t)]$ increases in σ_T. The sum of the precautionary and the damage scaling channels increases in σ_T if and only if they sum to a positive value, and they sum to a positive value if and only if $\eta > 1$. Finally, under the assumptions of the proposition, expression (A-18) increases in σ_T if and only if

$\eta > 1$.

D.3 Proof of Proposition 3

Solving the stochastic differential equation for environmental quality yields:

$$Q(t) = Q(0)\, e^{-\alpha_Q \int_0^t T(j)\,\mathrm{d}j + \left(\mu_Q - \frac{1}{2}\sigma_Q^2\right) t + \sigma_Q\, z_Q(t)}. \tag{A-19}$$

The derivative of time t environmental quality with respect to m is

$$\frac{\mathrm{d}Q(t)}{\mathrm{d}m} = \int_0^t \left.\frac{\mathrm{d}Q(t)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}} \frac{\mathrm{d}T(i)}{\mathrm{d}m}\,\mathrm{d}i. \tag{A-20}$$

Differentiate $Q(t)$ with respect to $T(i)$, for $i \le t$:

$$\begin{aligned}\left.\frac{\mathrm{d}Q(t)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}} &= -\, Q(t)\, \alpha_Q \int_i^t \left.\frac{\mathrm{d}T(k)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}} \mathrm{d}k \\ &= -\, \frac{Q(t)}{C(t)}\, \frac{\alpha_Q}{\alpha}\, \left.\frac{\mathrm{d}C(t)}{\mathrm{d}T(i)}\right|_{M(\cdot)\text{ given}}.\end{aligned}$$

Substitute into equation (A-20), and substitute the definition of $\epsilon(t)$:

$$\frac{\mathrm{d}Q(t)}{\mathrm{d}m} = -\, Q(t)\, \frac{\alpha_Q}{\alpha}\, \epsilon(t).$$

Define $\epsilon_Q(t)$ as the semi-elasticity of environmental quality with respect to a reduction in time 0 emissions:

$$\begin{aligned}\epsilon_Q(t) &\triangleq -\, \frac{1}{Q(t)} \frac{\mathrm{d}Q(t)}{\mathrm{d}m} \\ &= \frac{\alpha_Q}{\alpha}\, \epsilon(t).\end{aligned}$$

Note that $\epsilon_Q(t) > 0$.

Consider the covariance between $\epsilon_Q(t)$ and $Q(t)^{1-\eta_Q}$:

$$\begin{aligned}&Cov\left[Q(t)^{1-\eta_Q}, \epsilon_Q(t)\right] \\ &\quad= Cov\Bigg[Q(0)^{1-\eta_Q}\, e^{-(1-\eta_Q)\alpha_Q \int_0^t e^{\left(-\phi - \frac{1}{2}\sigma_T^2\right) j + \sigma_T\, z_T(j)} T(0)\,\mathrm{d}j} \\ &\qquad e^{-(1-\eta_Q)\alpha_Q \int_0^t e^{\left(-\phi-\frac{1}{2}\sigma_T^2\right) j + \sigma_T\, z_T(j)} \left[\int_0^j e^{\left(\phi + \frac{1}{2}\sigma_T^2\right) k - \sigma_T\, z_T(k)}\, \phi\, s\, \nu\, \ln\left[\frac{e^{-\delta k} M(0) + \int_0^k \gamma(w)\, C(w)\, e^{-\delta(k-w)}\,\mathrm{d}w + (1-e^{-\delta k}) M_{pre}}{M_{pre}}\right] \mathrm{d}k\right] \mathrm{d}j} \\ &\qquad e^{(1-\eta_Q)\left(\mu_Q - \frac{1}{2}\sigma_Q^2\, t + \sigma_Q\, z_Q(t)\right)}, \\ &\qquad \frac{\alpha_Q}{\alpha}\, \epsilon(t)\Bigg].\end{aligned} \tag{A-21}$$

Assume that $\sigma_T = 0$:

$$Cov\left[Q(t)^{1-\eta_Q}, \epsilon_Q(t)\right]$$
$$= Cov\Bigg[Q(0)^{1-\eta_Q}\, e^{-(1-\eta_Q)\alpha_Q \int_0^t e^{-\phi\, j} T(0)\,\mathrm{d}j}$$
$$e^{-(1-\eta_Q)\alpha_Q \int_0^t e^{-\phi\, j}\left[\int_0^j e^{\phi\, k}\,\phi\, s\, \nu\, \ln\left[\frac{e^{-\delta k} M(0) + \int_0^k \gamma(w)\, C(w)\, e^{-\delta(k-w)}\,\mathrm{d}w + (1-e^{-\delta k}) M_{pre}}{M_{pre}}\right]\mathrm{d}k\right]\mathrm{d}j}$$
$$e^{(1-\eta_Q)\left(\mu_Q - \frac{1}{2}\sigma_Q^2\, t + \sigma_Q\, z_Q(t)\right)},$$
$$\frac{\alpha_Q}{\alpha}\,\epsilon(t)\Bigg].$$

Take a first-order approximation to the first argument in the covariance operator around its exponent evaluated at $E[s]$, $z_C(\cdot) = 0$, and $z_Q(\cdot) = 0$. Drop all nonrandom terms and recognize that $z_Q(t)$ does not appear in the second argument of the covariance operator.

$$Cov\left[Q(t)^{1-\eta_Q}, \epsilon_Q(t)\right]$$
$$\approx \bar{Q}(t)^{1-\eta_Q} Cov\Bigg[-(1-\eta_Q)\alpha_Q \int_0^t e^{-\phi\, j}$$
$$\left[\int_0^j e^{\phi k}\,\phi\, s\, \nu\, \ln\left[\frac{e^{-\delta k} M(0) + \int_0^k \gamma(w)\, C(w)\, e^{-\delta(k-w)}\,\mathrm{d}w + (1-e^{-\delta k}) M_{pre}}{M_{pre}}\right]\mathrm{d}k\right]\mathrm{d}j,$$
$$\frac{\alpha_Q}{\alpha}\,\epsilon(t)\Bigg],$$

where an overbar again represents a value evaluated at the expectation of each random variable. Following the derivation of equations (A-8) and (A-12), the covariance is approximately equal to the sum of the following two expressions, labeled B_1 and B_3 by analogy to A_1 and A_3 from before (note that the old A_2 derived from uncertainty about α, which we here take to be known, so that $B_2 = 0$):

$$\begin{aligned} B_1 \triangleq& -(1-\eta_Q)\, Var(s)\, \alpha_Q^2 \phi\, \nu\, \bar{Q}(t)^{1-\eta_Q} \\ &\int_0^t \int_i^t \int_0^i \frac{e^{-\phi[h-j]}}{e^{\delta j} \bar{M}(j)} \left(1 - E[s] \frac{\mathrm{d}\ln(\bar{M}(k))}{\mathrm{d}s}\right) \mathrm{d}j\, \mathrm{d}h\, \mathrm{d}i \\ &\int_0^t e^{-\phi j} \int_0^j e^{\phi k}\,\phi\,\nu \left[\ln\left(\frac{\bar{M}(k)}{M_{pre}}\right) + E[s] \frac{\mathrm{d}\ln(\bar{M}(k))}{\mathrm{d}s}\right] \mathrm{d}k\, \mathrm{d}j, \end{aligned} \tag{A-22}$$

$$\begin{aligned} B_3 \triangleq& (1-\eta_Q)\alpha_Q^2\,\phi\,\nu\,\bar{Q}(t)^{1-\eta_Q} \int_0^t \Bigg[E[s] \int_0^t \int_i^t \int_0^i \frac{e^{-\phi[h-j]} e^{\delta j} \frac{\mathrm{d}\bar{M}(j)}{\mathrm{d}z_C(x)}}{\left(e^{\delta j} \bar{M}(j)\right)^2} \mathrm{d}j\, \mathrm{d}h\, \mathrm{d}i \\ &\int_0^t e^{-\phi j} \int_0^j e^{\phi k}\,\phi\, E[s]\,\nu \frac{\int_0^k \int_0^w \min\{x, v\}\, \gamma(w)\, \frac{\mathrm{d}\bar{C}(w)}{\mathrm{d}z_C(v)}\, e^{-\delta(k-w)}\,\mathrm{d}v\,\mathrm{d}w}{\bar{M}(k)} \mathrm{d}k\, \mathrm{d}j\Bigg] \mathrm{d}x. \end{aligned} \tag{A-23}$$

By Lemma 1, $\mathrm{d}\bar{M}(w)/\mathrm{d}s \leq 0$ if $\alpha\, E[s]$ is sufficiently small. And if $\mathrm{d}\bar{M}(w)/\mathrm{d}s \leq 0$, then equation (A-22) is positive if $\eta_Q > 1$ and $\alpha\, E[s]$ is sufficiently small, because the first two lines are unambiguously positive and the negative term in the third line is small if $\alpha\, E[s]$ is small. By analogous logic, if $\mathrm{d}\bar{M}(w)/\mathrm{d}s \leq 0$, then equation (A-22) is negative if $\eta_Q < 1$ and $\alpha\, E[s]$ is sufficiently small. Therefore, if $\alpha\, E[s]$ is small, then equation (A-22) is positive if and only if $\eta_Q > 1$.

By Lemma 2, $\mathrm{d}\bar{C}(w)/\mathrm{d}z_C(x) \geq 0$ and $\mathrm{d}\bar{M}(w)/\mathrm{d}z_C(x) \geq 0$ if $\alpha\, E[s]$ is sufficiently small. And if $\mathrm{d}\bar{C}(w)/\mathrm{d}z_C(x) \geq 0$ and $\mathrm{d}\bar{M}(w)/\mathrm{d}z_C(x) \geq 0$, then equation (A-23) is negative if and only if $\eta_Q > 1$. Therefore, if $\alpha\, E[s]$ is small, then equation (A-23) is negative if and only if $\eta_Q > 1$.

Therefore, if $\sigma_T = 0$ and $\alpha\, E[s]$ is small, then B_1 and B_3 conflict. The magnitude of B_1 increases with $Var(s)$ and is independent of σ_C, and the magnitude of B_3 increases with σ_C and is independent of $Var(s)$. If $Var(s)/\sigma_C$ is large, then $Cov\left[Q(t)^{1-\eta_Q}, \epsilon_Q(t)\right] > 0$ if and only if $\eta_Q > 1$, and if $Var(s)/\sigma_C$ is small, then $Cov\left[Q(t)^{1-\eta_Q}, \epsilon_Q(t)\right] < 0$ if and only if $\eta_Q > 1$. This establishes the first part of the proposition.

Next, assume that $\sigma_T > 0$, $\sigma_C = 0$, and $Var(s) = 0$. Modifying the proof of Proposition 2 for the case of environmental quality, we find that if α_Q is sufficiently small, then $Cov\left[Q(t)^{1-\eta_Q}, \epsilon_Q(t)\right] > 0$ if and only if $\eta_Q > 1$.

Finally, if the only random variable is $z_Q(t)$, then the covariance in equation (A-21) is zero. This establishes the final part of the proposition.

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