The Process of Self-Discovery:
Learned Helplessness, Self-Efficacy, and Endogenous Overoptimism*

Derek Lemoine

Department of Economics, University of Arizona
McClelland Hall 401, 1130 E Helen St, Tucson, AZ, 85721-0108, USA
dlemoine@email.arizona.edu

I study the origin of people's beliefs about their own abilities, emphasizing the feedback loop between beliefs, effort, and outcomes. Consistent with the modern understanding of depression, I show that agents who are either more pessimistic about their own talent or less sure of their own talent are more prone to low-effort traps. I also show that a principal (such as a parent or manager) does not want an agent (such as a child or employee) to hold beliefs about his own talent that match the principal's beliefs; instead, the principal would like to make the agent overoptimistic and overly uncertain. These results are consistent with empirical findings from the psychology literature about the origin and evolution of children's beliefs, about the importance of self-efficacy beliefs for performance, and about the prevalence of overconfidence.

JEL: D13, D83, I21, J13

Keywords: talent, ability, learning, overconfidence, overoptimism, learned helplessness, depression, pessimism, optimism, effort, parenting, therapy

---

A primary task in life is to learn what we are good at. We learn by engaging in activities and seeing how the world responds. Our efforts in an activity feed back into our beliefs in two ways: applying greater effort tells us more about our ability, and applying greater effort can increase our talent for the activity. Children learn about their talent for academics, sports, and music: trying to perform well tells them how good they may actually be and builds up ability for future efforts. New employees also learn about their aptitude for their jobs. Here, too, applying greater effort early on allows for learning and potentially improves later performance.

This learning process is economically important for at least two reasons. First, depression is now understood to be an outcome of this type of learning. Depressed people have come to doubt their ability and withdraw their efforts across a range of actions. The welfare cost of depression is enormous: depression afflicts around one in six adults in developed countries and contributes more to developed countries' disease burden than does any single class of physical illness (Layard and Clark, 2015). Modern therapies treat depression by changing beliefs and effort choices. It is important to formalize findings about the causes of depressives' beliefs and to thereby identify the structural channels that make modern therapies so effective. Second, agents often try to manipulate each other's beliefs. Of special interest here, a major task of parents and managers is to shape the self-perceptions of children and new employees. Understanding these incentives might be critical for understanding detected regularities in beliefs (such as overconfidence) and for appreciating why early childhood programs have long-lasting benefits.

I develop a formal model of self-discovery that unifies several recent developments in empirical psychology: it generates outcomes consistent with findings about generic overconfidence, with findings central to helplessness theory, with findings about the importance of self-efficacy beliefs for performance, and with findings about parents' influence on their children's beliefs and achievement. Consider an agent who has some uncertain estimate of her own ability to affect the world (her "self-efficacy" or "talent"). Her marginal reward from applying effort increases in her talent: studying pays off more for smarter kids, polishing a paper matters more when the idea is clever, and trying harder at work matters more for a worker who can potentially do the job well. The agent chooses her effort by trading off this marginal reward from effort against the marginal cost of effort, without accounting for how effort feeds back into future beliefs and ability. The agent observes how the world responds to her effort: she receives a test score, learns whether her paper submission was accepted, or obtains feedback from her manager. The agent uses this noisy signal to rationally update her beliefs about her talent, which is not fixed but evolves stochastically and might be increased by effort. The agent's choice of effort determines not only her expected payoff but also her ability to learn about her talent from outcomes such as test scores. An agent who chooses not to exert any effort does not receive any signal of her true ability, whereas an agent who exerts substantial effort receives a test score that is sensitive to her true ability.

I show that this agent demonstrates behavior consistent with modern psychology's find-

ings about learned helplessness and depression. As described below, an important body of work has demonstrated that animals (including people) who learn that they cannot affect their environment display low effort ("motivational deficits") and cease to learn about new ways in which they can affect their environment (display "cognitive deficits"). Closely related work with humans has shown that having a "pessimistic explanatory style" (by which people readily attribute poor results to their own ability) may cause depression. Consider an agent who observes a series of poor results despite exerting effort. In the present setting, she reduces her estimate of her own ability. She rationally chooses lower effort levels in subsequent periods, which allows her talent to decay. If her estimated ability becomes sufficiently low, then the expected reward from effort no longer justifies the cost of applying any nonzero level of effort. The agent stops trying, demonstrating the motivational deficits characteristic of learned helplessness. Because she no longer tries to affect outcomes, the agent can no longer extract information about her true talent or self-efficacy from future outcomes, demonstrating the cognitive deficits emphasized by learned helplessness theory. Further, I show that agents who begin with a more pessimistic or a more uncertain view of their own ability are especially prone to sharp reductions in effort. The reason for the sharp reduction in effort is that such agents tend to attribute poor results to their own pervasive ability rather than to chance, which is consistent with empirical findings about the importance of pessimistic explanatory styles. By locating the origin of pessimistic explanatory styles in the mean and variance of beliefs about one's own ability, I demonstrate the structural mechanisms underpinning the success of cognitive behavioral therapies: when a therapist raises a patient's own self-assessment of her talent or makes the patient more sure of her own ability, the patient becomes more inclined to dismiss bad news as due to chance rather than to her lack of ability and so can better maintain confidence and effort levels in the face of bad shocks.

Thus far we have considered a single agent who learns about her own ability. Now consider the incentives for another agent (the "principal") to manipulate the first agent's beliefs. This principal is a parent who interprets reality for her child or a manager whose praise or condemnation affects her employee.[1] The principal anticipates how the agent will choose effort and learn about his (i.e., the agent's) own ability over time. The principal and the agent's incentives are aligned: the principal wants to maximize the agent's expected payoffs. However, the principal and agent may hold different beliefs about the agent's ability. And, unlike the agent, the principal recognizes how early effort generates information and human capital for later periods. The principal and the agent both see the same outcomes and both update their beliefs rationally, as in the single agent setting.

The psychology literature (described below) emphasizes that parents and managers affect children's and employees' beliefs. I show that these principals often want their agent's initial

---

[1]This relationship can also be interpreted as the relationship between a therapist and her patient or between a "planner" and a "doer" within a single self, as in Thaler and Shefrin (1981) and Fudenberg and Levine (2006).

estimate of his own talent to exceed the principal's estimate of the agent's talent. By making the agent overconfident about his own talent level, the principal corrects the agent's tendency to choose too low an effort level as a result of failing to recognize the implications of effort for future beliefs and talent.[2] However, manipulating the agent's beliefs also imposes a cost: from the principal's perspective, the agent will incorrectly update his beliefs in response to outcomes. I show that the principal's gains from inducing overconfidence typically outweigh these costs.

I also show that the principal has an incentive to manipulate the variance of the agent's beliefs. The variance matters only by affecting how the agent updates beliefs in response to observed test scores. The principal prefers the agent to be more uncertain than herself for two reasons. First, the agent's endogenous choice of effort acts like a call option. The ability to choose zero effort introduces a kink in the agent's payoffs as a function of early news. If the agent receives very good news in an early period, then the agent chooses a high effort level that takes advantage of this news, but if the agent receives very bad news in an early period, then the agent mitigates her losses by choosing zero effort and accepting an outside option. Increasing the variance of the agent's beliefs makes the agent's later effort choices more sensitive to early news, which creates value because the agent is more exposed to good news than to bad news. Second, if the principal successfully induces the agent to overestimate his own ability, then the principal prefers the agent to be more uncertain about his estimate so that the agent's future beliefs will be more sensitive to test outcomes. From the principal's perspective, making her overoptimistic agent more uncertain corrects some of the later informational distortions generated by the initial overoptimism. A principal who can manipulate both estimated ability and uncertainty about that estimate will prefer to make the agent both overoptimistic and oversensitive to new information.

I now describe the relevant findings from several empirical literatures in psychology. These literatures typically focus on the implications of expectations about one's own ability, whereas the present setting also generates predictions for how the variance of beliefs matters. After surveying these psychology literatures, I discuss the relevant economics literature.

Learned helplessness and depression. A suite of findings known collectively as learned helplessness theory underpins the contemporary understanding of depression and the cognitive behavioral therapy techniques used to treat depression. Learned helplessness theory originated in experiments showing that dogs, fish, cats, and rats exposed to uncontrollable shocks lose the motivation to escape later shocks, whereas animals exposed to no shock or to controllable shocks find escaping the same later shocks to be trivially easy (Maier and Seligman, 1976). Parallel phenomena were soon observed in humans. The central thesis is that animals exposed to uncontrollable processes will learn that they have no control over outcomes, and this belief reduces motivation, hampers the ability to learn new causal relationships, and creates emotional disruption. In humans, learned helplessness appears

---

[2]Following Grubb (2015), I will use "overoptimism" to refer to overconfidence about the level of talent, in order to distinguish it from overconfidence in the sense of overprecision.

to be modulated by "explanatory style" (Abramson et al., 1978). People with pessimistic explanatory styles view bad events as reflecting pervasive failures on their own part rather than chance. Empirically, such people are more prone to depression than are people with optimistic explanatory styles, and having a pessimistic explanatory style even seems to cause depression (Seligman, 1991). The modern tools of cognitive behavioral therapy aim to help the depressed by changing the way that they interpret events and thereby raising their perceptions of their self-efficacy (Seligman, 1991; Layard and Clark, 2015).

Children's beliefs. At the same time as learned helplessness theory has demonstrated the importance of beliefs about one's own ability to affect the world, a distinct psychology literature has explored the sources of beliefs about ability. Expectancy-value theories link individuals' performance to their beliefs about how well they will do on a particular task and to the value they attach to succeeding on the task (Eccles (Parsons) et al., 1983; Eccles and Wigfield, 2002).[3] Within this framework, psychologists have especially investigated the sources of children's expectations about their own academic achievement.[4] Children have beliefs about their own ability prior to gaining experience of this ability (Eccles et al., 1993). Where do these beliefs come from? Parents play a key role in determining children's early beliefs: parents interpret reality for their young children, which provides the children information about their own competence and shapes their values (Jacobs and Bleeker, 2004). In particular, young children use their parents as their major source of information about academic ability, even more than grades or teachers (e.g., Nicholls, 1978; Eccles Parsons et al., 1982; Jacobs, 1991; Frome and Eccles, 1998; Spinath and Spinath, 2005). However, most economic models of childhood learning do not represent the child as an active learner or represent parents' roles in shaping that learning (Heckman and Mosso, 2014), despite empirical work in economics having shown that parenting matters for childhood outcomes (Heckman et al., 2006; Heckman and Masterov, 2007).

I show that parents have an incentive to make their children both overoptimistic and overly uncertain.[5] These parenting incentives are consistent with observations in Eccles et al. (1993) and papers cited therein. They report that younger children are more overoptimistic about their own ability than are older children. Children start out very optimistic about themselves and become more pessimistic (and, in many studies, more realistic) as time passes. Young children's initial overoptimism matches the predictions from the current setting. If these children were very confident in their overoptimism, then they would not learn very

---

[3]Expectancy-value theories also emphasize the cost of effort (Eccles and Wigfield, 2002).

[4]This literature should be of special interest to economists given the recent emphasis on the importance of early childhood education for later life outcomes (Currie, 2001; Heckman and Masterov, 2007; Cunha and Heckman, 2010).

[5]We could also interpret the principal as a teacher, in which case we see that teachers have an incentive to manipulate children's beliefs. Indeed, Diamond and Persson (2016) report evidence that teachers manipulate students' test scores in a favorable direction and that these manipulations improve later educational and labor market outcomes. They hypothesize that a higher test score "boosts the student's self-confidence and effort" (p. 3). We will see this hypothesis play out in the present setting.

well. However, while we have seen that parents have an incentive to induce overoptimism, they do not want to induce overprecision. They want their children to eventually learn their true talent, especially as the human capital benefits of additional effort diminish. Young children who are both overoptimistic and oversensitive to shocks should indeed be able to learn from test scores, which diminishes their initial overoptimism over time.

Self-efficacy and performance. A third line of recent research in psychology has emphasized the role of beliefs about one's own self-efficacy in determining performance, with special application to the workplace. Consistent with our setting and results, stronger beliefs about self-efficacy have been shown to increase an agent's chosen level of effort and also the persistence of an agent's effort in the face of bad shocks (Bandura, 1982; Wood and Bandura, 1989; Stajkovic and Luthans, 1998b; Bandura, 2001). This same literature's discussion of learning also matches our setting: beliefs are thought to adjust not to absolute outcomes but to outcomes as filtered through previous beliefs about ability.[6] Further, beliefs about self-efficacy do not improve performance directly, but instead improve performance indirectly by improving effort and persistence (Tenney et al., 2015). This finding is consistent with our setting but not with some related work described below. The importance of beliefs about self-efficacy implies that managers could increase workplace performance if they could increase employees' beliefs about skills they already have (Stajkovic and Luthans, 1998a; Luthans and Youssef, 2007). And managers do have the ability to change beliefs through verbal persuasion and feedback, especially prior to employees having received more direct signals of their own ability (Bandura, 1982; Stajkovic and Luthans, 1998b). I will formally demonstrate that managers have an incentive to persuade employees of their own self-efficacy, even when such beliefs do not themselves directly influence productivity. I will also demonstrate that managers have an incentive to make their employees especially uncertain about their own self-efficacy, so that they do not misallocate effort too severely in the future.

Related economics literature. Numerous studies have found that people tend to be overconfident, with implications for financial markets (Daniel and Hirshleifer, 2015), pricing and contracts (Grubb, 2015), corporate investment (Malmendier and Taylor, 2015a), and entrepreneurship (Astebro et al., 2014), among others.[7] A growing body of economics literature seeks to explain this stylized fact of overconfidence. A number of papers have proposed statistical mechanisms or optimal stopping models that generate systematic overconfidence. The present work is closer to a recent literature that models the incentives to become overconfident.[8] Compte and Postlewaite (2004) assume that confidence directly

---

[6]Of special interest for its fit to our formal setting, Bandura (1982) emphasizes how perceptions of self-efficacy increase after unexpectedly good outcomes and after previous efforts should have improved the agent's skill level. He also discusses how agents are especially subject to negative shocks when they do not yet have much experience, because their beliefs are then held only weakly (i.e., with high variance). Our formal model of learning will capture all of these effects.

[7]For discussions of literature detecting overconfidence, see Rabin and Schrag (1999), Compte and Postlewaite (2004), Benot and Dubra (2011), and Malmendier and Taylor (2015b).

[8]Mobius et al. (2011) show that agents who obtain utility from believing that they have high ability have

improves performance. An agent's confidence in turn depends on the recalled frequency of successes in a task, which the agent can bias so as to increase confidence and thus increase the probability of future success. In the present paper's setting, the agent updates beliefs about talent rationally, and these beliefs matter for performance only by affecting the choice of effort, which is consistent with learned helplessness theory and with empirical findings about self-efficacy. In Bénabou and Tirole (2002), the agent is a quasi-hyperbolic discounter, so each period's self chooses a lower level of effort than earlier selves would have chosen for it. Early selves therefore want to commit later selves to choosing a higher level of effort. One way they can do this is by selectively remembering successes and failures, so as to increase later selves' confidence. Rather than exploring incentives to manipulate the learning process, the present paper emphasizes incentives to manipulate priors, which allows the present paper to address interactions between a principal and an agent who are not part of the same self. Finally, Silva (2015) models the interaction between an advisor and a student. If the student receives a high test score in an early period, then her advisor chooses to begin helping her. However, by affecting later test outcomes, the advisor's help prevents the student from learning her true ability, so her confidence is unduly influenced by the initial high test score. Overconfidence thus arises because an initial high test score leads other actors to muddy the student's contribution to later test outcomes. In contrast, in the present setting, the agent's own choice of effort affects the quality of the signal from a test. Systematic overconfidence emerges from a principal's incentives to change an agent's choice of effort by manipulating the agent's initial beliefs.[9]

Another closely related economics literature has modeled the active role that parents play in child-rearing. Two papers consider how a disconnect between the child and parent's time preference can justify different parenting styles, whether punishing the child for being lazy (Akabayashi, 2006) or restricting the child's choice of occupation (Doepke and Zilibotti, 2014). The present paper also considers parents who are more forward-looking than their children, but it focuses on the parent's incentives to manipulate beliefs. Lizzeri and Siniscalchi (2008) consider a parent who knows the correct way to perform a task and ask when the parent intervenes by correcting the child's actions. These corrections raise short-term payoffs but hamper the child's ability to learn from his actions. In the present setting, both the parent and child are uncertain about the child's true ability and thus are also uncertain about the effort that would be optimal under full information. And the parent cannot manipulate test scores. Instead, both the parent and child learn about the child's talent from test scores and the parent intervenes only by manipulating initial beliefs, which

an incentive to bias their updating process away from Bayesian updating. This bias makes agents overly confident and overly certain. In contrast, we assume Bayesian updating and show that a principal has an incentive to make an agent overly confident and overly uncertain.

[9]There is a growing literature on misspecified beliefs (e.g., Fudenberg et al., 2016), in which the true state is not included in the support of the agent's beliefs. Here, the support always includes the true state, so the agent does learn about his true talent level over time (though that talent level evolves as a function of the agent's choices).

is consistent with the psychology literature's view of children's progress through school. Finally, Dominguez-Martinez and Swank (2009) focus on a parent's incentives to ensure that her child does not shy away from undertaking a task. In their setting, the parent knows the child's true ability but can only send the child a noisy signal. If the child undertakes the task in the first period, then he learns his ability perfectly before the second period. The variance of beliefs plays no role in their setting. In contrast, I abstract from the precise technology through which parents or managers shape beliefs and instead describe their incentives to do so. Further, I emphasize all parties' uncertainty about the agent's talent level and their ability to learn from noisy outcomes.

Outline. The next section describes the single agent setting and demonstrates how it explains the primary results of learned helplessness theory and the effectiveness of modern treatments for depression. Section 2 extends the setting to include a principal and analyzes the principal's incentives to manipulate the agent's beliefs. The subsequent section discusses the emergence of systematic overconfidence. The final section concludes with implications for the emergence of income inequality, for institutional design, and for future work in empirical psychology and experimental economics. The appendix contains additional derivations and proofs.

1   Learning about one's own talent

We begin by developing the single-agent setting. We then consider the development of learned helplessness and the role of pessimistic and optimistic explanatory styles.

1.1   Formal setting

In each period, an agent forms an estimate of her own talent and decides how much effort to apply. Her reward depends on her true talent, on her effort, and on a stochastic shock. Upon observing her reward, the agent updates her beliefs about her true talent before making another effort decision in the following period. The setting can be interpreted in several ways: a student decides how much to study for a test and then observes her grade; an athlete decides how hard to train for or compete in a competition and then observes her finish; or a worker decides how much attention to devote to her job and then observes her productivity.

Formally, at each time t the agent decides upon her effort level $e_t \geq 0$. Her true talent is y_t. Her reward is

$$\pi_t = be_t y_t + \epsilon_t,$$

where $b > 0$ determines the marginal reward to effort and talent and where ϵ_t is a normally distributed, mean-zero shock with variance $\sigma^2 > 0$. The agent's marginal reward to effort

increases in her talent level: additional effort pays off more for talented people.[10] We often refer to π_t as the agent's test score, where the agent values higher scores. The agent's true talent evolves from period to period:

$$y_{t+1} = (1 - \delta)y_t + ae_t + \nu_{t+1}.$$

Her talent decays at rate $\delta \in [0, 1)$ but increases upon the application of effort, with $a \geq 0$ governing the marginal response of talent to effort. For instance, studying for an exam both raises the exam score π_t and can generate knowledge y_{t+1} useful for later exams. The evolution of talent is stochastic, with ν_{t+1} a normally distributed, mean-zero random variable with variance $\omega^2 > 0$.

The agent does not know her true talent y_t.[11] From her perspective, her true talent is an unobserved and imperfectly measured state variable. She updates her beliefs via a Kalman filter, which uses Bayesian inference and the history of observations of π through time t to recursively generate expectations $\hat{E}_t[y_{t+1}]$ of her time $t + 1$ talent y_{t+1}, along with the variance $\widehat{Var}_t[y_{t+1}]$ of her beliefs.[12] Define $\hat{y}_{t+1} \triangleq \hat{E}_t[y_{t+1}]$ and $\hat{\Sigma}_{t+1} \triangleq \widehat{Var}_t[y_{t+1}]$. The agent begins with an estimate $\hat{y}_1 > 0$ and variance $\hat{\Sigma}_1 > 0$.

The Kalman filter application is standard (e.g., Ljungqvist and Sargent, 2004). The prediction step yields the agent's forecast prior to observing the reward π_t:

$$\hat{E}_{t-1}[y_{t+1}] = (1 - \delta)\hat{E}_{t-1}[y_t] + ae_t. \tag{1}$$

The updating step incorporates the time t reward π_t:

$$K_t = (1 - \delta)\hat{\Sigma}_t be_t \left(b^2 e_t^2 \hat{\Sigma}_t + \sigma^2 \right)^{-1}, \tag{2}$$

$$\hat{E}_t[y_{t+1}] = \hat{E}_{t-1}[y_{t+1}] + K_t(\pi_t - be_t \hat{E}_{t-1}[y_t]), \tag{3}$$

where K_t is the Kalman gain, which determines the weight placed on the information in π_t. The agent also updates the variance of her beliefs:

$$\hat{\Sigma}_{t+1} = (1 - \delta)^2 \hat{\Sigma}_t + \omega^2 - (1 - \delta)K_t be_t \hat{\Sigma}_t. \tag{4}$$

Substituting for $\hat{E}_{t-1}[y_{t+1}]$ and K_t in equations (3) and (4) and applying the definition of $\hat{y}_t$,

---

[10]Bénabou and Tirole (2002) motivate the assumption that the marginal reward from effort increases in talent.

[11]Equivalently, the agent does not know the marginal productivity of effort.

[12]Kalman filters recursively generate linear least squares forecasts of unobserved and imperfectly measured state variables. In linear Gaussian settings (like the present one), the Kalman filter is a recursive representation of Bayesian inference.

we have two transition equations in $\hat{\Sigma}_t$, $\hat{y}_t$, and e_t:

$$\hat{y}_{t+1} = (1-\delta)\hat{y}_t + ae_t + (1-\delta)\frac{be_t\hat{\Sigma}_t}{b^2e_t^2\hat{\Sigma}_t + \sigma^2}(be_t(y_t - \hat{y}_t) + \epsilon_t), \tag{5}$$

$$\hat{\Sigma}_{t+1} = \omega^2 + \left(1 - \frac{b^2e_t^2\hat{\Sigma}_t}{b^2e_t^2\hat{\Sigma}_t + \sigma^2}\right)(1-\delta)^2\hat{\Sigma}_t. \tag{6}$$

The agent updates $\hat{y}_t$ to account for the decay of her estimated talent, for the additional talent accumulated through her effort, and for the observed test outcome. Throughout, we only consider cases with $\hat{y}_t > 0$. As long as she exerts effort, the variance of the agent's beliefs declines monotonically towards the per-period variance ω^2 of talent accumulation (assuming $\hat{\Sigma}_1 > \omega^2$).

   In each period, the agent decides whether to exert effort and how much effort to exert. She approaches this decision myopically, failing to account for how effort affects future talent and future information. This assumption is consistent with many settings, including our later interpretation of the agent as a child and with the doer in planner-doer models. If the agent chooses not to exert effort, she receives a payoff $z \geq 0$. If the agent chooses to exert effort, she solves the following maximization problem:

$$\max_{e_t > 0}\left\{\hat{E}_{t-1}[\pi_t] - \frac{1}{2}ce_t^2\right\} = \max_{e_t > 0}\left\{be_t\hat{y}_t - \frac{1}{2}ce_t^2\right\},$$

where $c > 0$ determines the cost of effort. Her optimal effort is then

$$e_t^* = \frac{b}{c}\hat{y}_t. \tag{7}$$

Substituting into her maximization problem and assuming that the agent chooses the outside option z if indifferent, we find that the agent chooses nonzero effort if and only if

$$\hat{y}_t > \sqrt{2\frac{c}{b^2}z}. \tag{8}$$

Substituting her optimal effort choice into equations (5) and (6), we find the evolution of the agent's beliefs when she exerts nonzero effort:

$$\hat{y}_{t+1} = (1-\delta)\hat{y}_t + a\frac{b}{c}\hat{y}_t + (1-\delta)\frac{\frac{b^2}{c}\hat{y}_t\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\left(\frac{b^2}{c}\hat{y}_t(y_t - \hat{y}_t) + \epsilon_t\right), \tag{9}$$

$$\hat{\Sigma}_{t+1} = \omega^2 + \left(1 - \frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\right)(1-\delta)^2\hat{\Sigma}_t. \tag{10}$$

The agent expects her talent to increase over time if and only if $ab/c \geq \delta$. This condition demonstrates the importance of institutions for rewarding talent: if tests reward all equally without regard to talent and effort (i.e., if b is small), then the agent puts forth little effort and does not accumulate skills over time. We henceforth assume that $ab/c \geq \delta$.

1.2   Learned helplessness

Learned helplessness refers to the experimental finding that agents who repeatedly find their actions ineffective eventually stop trying and stop learning about their own effectiveness. In our setting, talent y_t determines the effectiveness of the agent's actions. An agent who chooses nonzero effort in period t will stop trying in period $t+1$ if she receives such a negative reward (i.e., test outcome) in period t that her subsequent talent estimate $\hat{y}_{t+1}$ falls to $\sqrt{2cz}/b$ or below. From equation (9), the agent stops trying in period $t+1$ if and only if

$$\sqrt{2\frac{c}{b^2}z} \geq (1-\delta)\hat{y}_t + a\frac{b}{c}\hat{y}_t + (1-\delta)\frac{\frac{b^2}{c}\hat{y}_t\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\left(\frac{b^2}{c}\hat{y}_t(y_t - \hat{y}_t) + \epsilon_t\right). \qquad (11)$$

Define ψ_t as the deviation in the realized test outcome π_t from the agent's expected outcome:

$$\psi_t \triangleq \frac{b^2}{c}\hat{y}_t(y_t - \hat{y}_t) + \epsilon_t.$$

ψ_t measures the news the agent receives at time t. Rearranging inequality (11), the agent stops trying in period $t+1$ if and only if the period t news ψ_t is worse than a critical value ψ_t^*:

$$e_{t+1} = 0 \iff \psi_t \leq -\frac{1}{K_t}\left\{(1-\delta)\hat{y}_t + a\frac{b}{c}\hat{y}_t - \sqrt{2\frac{c}{b^2}z}\right\} \triangleq \psi_t^*. \qquad (12)$$

The agent gives up trying at time $t+1$ if and only if the news ψ_t is sufficiently bad. The term in braces gives the gap between the deterministic evolution of estimated talent and the threshold at which the agent would give up. The agent gives up when the news is so bad that her updated talent estimate closes this gap.

The following proposition describes when an agent chooses to stop applying effort, assuming that the agent has correct beliefs about the distribution of true talent y_t.[13]

Proposition 1 (Probability of nonzero effort). *If the agent chooses nonzero effort in period t, then $\psi_t^* < 0$. As $\hat{y}_t$ increases, an agent becomes more likely to choose nonzero effort in period $t+1$. As $\hat{\Sigma}_t$ increases, an agent becomes less likely to choose nonzero effort in period $t+1$.*

Proof. See appendix.

□

This proposition has three main results. First, it establishes that the agent stops trying between periods t and $t+1$ only if her news ψ_t^* contains a sufficiently negative shock. As

---

[13]The proof of Proposition 2 will demonstrate analogous results when the distribution of true talent is fixed independently of the agent's beliefs about it.

long as $ab/c \geq \delta$, the agent expects her talent to (weakly) grow over time. Observing a test outcome that matches or exceeds her expectations then gives her no reason to stop trying.

Second, the proposition shows that agents with higher assessments of their own talent are less likely to stop trying between periods t and $t+1$. The proof shows that the effect of a higher talent estimate on the probability of choosing zero effort is proportional to

$$-\frac{1}{K_t}\left\{(1-\delta)+a\frac{b}{c}\right\}-\frac{\psi_t^*}{K_t}\underbrace{\left\{\frac{K_t}{\hat{y}_t}\frac{\sigma^2-\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t+\sigma^2}\right\}}_{\mathrm{d}K_t/\mathrm{d}\hat{y}_t}-\frac{\frac{b^4}{c^2}\hat{y}_t\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t+\sigma^2}\psi_t^*.$$

We see three channels through which an agent's higher estimate matters for the probability of choosing zero effort. The first, negative term captures how an agent who believes that she is more talented (i.e., who has higher $\hat{y}_t$) will require especially negative news if she is to stop trying in the next period. Today's higher talent tends to persist $(1-\delta)$ and today's higher talent can generate greater future talent by inducing additional effort today (ab/c), so increasing today's talent estimate works to increase tomorrow's talent estimate for any given beliefs. The second, ambiguously signed term captures how the agent's estimate of her talent affects how she uses the test result to update her beliefs. An agent with higher estimated talent applies more effort, which increases both the variance of the test outcome and the signal contained in the test outcome. By increasing the variance of the test outcome, her higher talent estimate makes her less sensitive to news, which implies that she requires especially negative news if she is to stop trying in the next period. But increasing the signal contained in the test outcome works in the opposite direction. The former effect dominates when the variance $\hat{\Sigma}_t$ of the agent's beliefs is large relative to the noise σ^2 in the test. The third, positive term captures how, by applying a higher level of effort, an agent with a higher estimate of her own talent increases the variance of test outcomes and thus increases the probability that she will receive a very negative shock. Through this channel, higher talent estimates make agents more likely to choose zero effort in subsequent periods. The proof shows that the net effect of these three competing channels always makes agents with higher talent estimates more likely to choose nonzero effort in the next period.

Finally, the proposition also shows that an agent who is more unsure of her own talent (i.e., who has greater $\hat{\Sigma}_t$) is more likely to stop choosing nonzero effort. First, her greater uncertainty means that her talent estimate responds more strongly to news, so that she stops trying in response to less negative signals of her talent. Second, when the agent has correct beliefs about the distribution of true talent, it is more likely that an agent with greater variance in her beliefs will receive a very negative shock. These two channels work together to make it less likely that agents who are especially uncertain choose nonzero effort in the next period.

The experiments that induce learned helplessness in humans and other animals initially expose the subject to a series of negative shocks. Using subscripts i and j to label agents, the

next proposition shows that agents who are relatively pessimistic and/or relatively unsure of their own talent stop trying sooner.

Proposition 2 (Learned helplessness). *Assume that $\sigma^2 \geq b^4 \hat{y}_t^2 \hat{\Sigma}_t / c^2$. Consider two agents i and j such that $\hat{y}_{ti} \geq \hat{y}_{tj}$ and $\hat{\Sigma}_{ti} \leq \hat{\Sigma}_{tj}$. If these two agents are subject to the same sequence of payoffs $\{\pi_s : \psi_{si}, \psi_{sj} < 0, s \geq t\}$, then agent i chooses nonzero effort for at least as long as agent j does.*

Proof. See appendix.

☐

The proposition tells us that agents who are pessimistic about their talent (small $\hat{y}_t$) and who have less confidence in their talent (large $\hat{\Sigma}_t$) will start choosing zero effort sooner. The proof of Proposition 1 already showed that agents with higher estimates and less uncertainty require worse news in order to stop trying, and the condition that $\sigma^2 \geq b^4 \hat{y}_t^2 \hat{\Sigma}_t / c^2$ (i.e., that the test's noise is at least as large as the variance of the correct test outcome) ensures that agents with greater talent estimates require worse payoffs in order to stop trying.[14] The proof of the proposition shows that talent estimates and uncertainty persist: if two agents see the same rewards π_t and are equally uncertain about their talent, then the agent who began with a higher estimate of her talent maintains a higher estimate, and if two agents see the same rewards π_t and have the same estimate of their talent, then the agent who began as more uncertain about her talent will remain more uncertain. This means that an agent who is more pessimistic and less confident will remain so for a given stream of rewards, so there will always be some possible stream of rewards that would induce her to stop trying but would not induce a more optimistic and confident agent to stop trying.

We also see how learned helplessness can be a trap. As a sequence of worse-than-expected rewards makes an agent increasingly pessimistic about the effect of her effort on outcomes, she reaches a point at which one more similar reward will induce her to stop trying. Once she stops trying, she never resumes trying. Agents who begin with a more pessimistic assessment of their own abilities reach this point faster than agents who begin with a more optimistic assessment of their own abilities.[15] Experiments do not seem to have explored how agents' understanding of testing noise affects their tendency to give up, but our results

---

[14]We would not need this condition if the thought experiment were to subject agents to the same sequence of negative news $\{\psi_s < 0 : s \geq t\}$ rather than to the same sequence of rewards π_s. The condition accounts for the fact that agents with higher talent estimates view a given negative reward as containing more news (because it is more surprising) than do agents with lower talent estimates. The condition is sufficient (but not necessary) to ensure that the effect of greater talent on the news threshold (i.e., making the agent require worse news in order to stop trying) dominates the effect of greater talent on how an agent converts a given reward into news.

[15]Pessimists tend to attribute bad outcomes to their own failures and good outcomes to chance, whereas optimists do the reverse (Seligman, 1991). This pattern suggests that the primary difference between pessimists and optimists lies in their central estimates $\hat{y}_t$, not in the variance $\hat{\Sigma}_t$.

imply that emphasizing the noisy nature of the experiment should increase the number of unfavorable rewards required to make agents stop trying. Learned helplessness may not be a psychological failing but rather a rational response to negative signals in an environment of uncertainty about one's own ability or self-efficacy. Demonstrating learned helplessness may be a sign of rational learning.

1.3   Pessimistic and optimistic explanatory styles

Subsequent to demonstrating learned helplessness and its correlation with pessimism, psychologists began investigating other differences between optimists and pessimists. These investigations revealed that pessimists' self-impressions and effort are more sensitive to bad outcomes (Seligman, 1991).[16] In particular, optimists tend to brush off negative news as bad luck while pessimists tend to respond to negative news by becoming more pessimistic. Emerging from these findings, cognitive behavioral therapy aims to help depressed people by changing the stories that they tell themselves and by encouraging them to try tasks that they have been avoiding because they expect to fail (Layard and Clark, 2015). This therapeutic technique raises pessimists' self-assessments both directly and through experience with the world. It thereby insulates them from bad news.

The following proposition establishes how pessimism and uncertainty affect vulnerability to shocks:

Proposition 3 (Sensitivity to shocks)**.** *Assume that two agents put forth effort at time t. If the two agents have the same variance $\hat{\Sigma}_t$, then the time t growth rate of the more pessimistic agent's talent estimate and effort is more sensitive to time t news. If the two agents have the same talent estimate $\hat{y}_t$, then the time t growth rate of the more uncertain agent's talent estimate and effort is more sensitive to time t news.*

Proof. From equation (7), the growth rate of effort is the same as the growth rate of estimated talent. From equation (9), we have the time t growth rate of the talent estimate as

$$\frac{\hat{y}_{t+1}}{\hat{y}_t} = (1-\delta) + a\frac{b}{c} + \underbrace{(1-\delta)\frac{\frac{b^2}{c}\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}}_{\Gamma}\left(\frac{b^2}{c}\hat{y}_t(y_t - \hat{y}_t) + \epsilon_t\right).$$

The term labeled $\Gamma > 0$ determines sensitivity to time t news ψ_t (and also to time t rewards

---

[16]And optimists' effort choices are more sensitive to good outcomes: "People who believe good events have permanent causes try even harder after they succeed" (Seligman, 1991, Chapter 3). This finding implies that outcomes that increase estimated ability also increase effort, as occurs in our setting.

π_t). Differentiating, we have:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{y}_t} = -2\Gamma \frac{\frac{b^4}{c^2}\hat{y}_t\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2} < 0,$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{\Sigma}_t} = \frac{\Gamma}{\hat{\Sigma}_t}\left\{1 - \frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\right\} > 0.$$

We have that responsiveness to news decreases in $\hat{y}_t$ and increases in $\hat{\Sigma}_t$.

□

Optimism and pessimism affect responsiveness to shocks via the agent's choice of effort. More optimistic agents choose to put forth more effort, which increases both the signal in the test results π_t and the total variance of the test results. The second channel is the important one for the growth rate of the talent estimate. Uncertainty matters in a different way. Agents who are more uncertain about their true talent are more responsive to shocks because they rely less on their prior when updating their beliefs.

One of the great successes of cognitive behavioral therapy is that randomized trials have shown that it not only cures an ongoing instance of depression but also inoculates patients against future depression, in stark contrast to pharmaceutical treatments for depression (Seligman, 1991; Layard and Clark, 2015). We here see how cognitive behavioral therapy achieves this inoculation. First, by inducing pessimists to try harder, it raises the overall variance of outcomes and makes it easier for them to dismiss events as due to chance (and by trying harder, pessimists might also increase their self-assessed talent for later periods).[17] Second, by making pessimists more sure of their own ability, cognitive behavioral therapy reduces the spread of talent levels that pessimists consider possible and thus reduces the degree to which their beliefs about their own efficacy react to shocks.

2   Guiding an agent's beliefs

We now consider the incentives for an altruistic, forward-looking principal to manipulate the beliefs of the previous section's agent. We begin by extending the single-agent setting before analyzing the principal's expected payoffs, her incentives to deter the original agent from choosing nonzero effort, and her incentives to manipulate the original agent's beliefs.

---

[17]And if the patient was avoiding a type of activity or situation entirely, then convincing her to try creates the possibility of receiving favorable news. This favorable news can allow her to escape a zero-effort trap, in which pessimistic beliefs about ability persist indefinitely for lack of new signals.

2.1   Formal setting

Consider a two-period version of the setting from Section 1 and introduce a principal who has an initial opportunity to modify the agent's beliefs. The principal-agent relationship can be interpreted as the relationship between a parent and child, between a manager and a new employee, between a therapist and a patient, or between a planner and a doer within a single self. The principal has her own beliefs about the agent's ability. Whereas the agent begins with an estimate $\hat{y}_1$ of his own ability and variance $\hat{\Sigma}_1$ about this estimate, the principal begins with an estimate $\tilde{y}_1 > 0$ of the agent's ability and variance $\tilde{\Sigma}_1 > 0$ about this estimate. The agent is unaware of the principal's estimate and chooses his effort based on his own estimate. The principal observes the agent's effort. At the end of period 1, both the principal and the agent observe the outcome π_1 and update their beliefs heading into period 2. The agent then chooses his period 2 effort and receives π_2. The principal is altruistic and forward-looking, seeking to maximize the expected sum of the agent's payoffs with discount factor $\beta \in (0, 1]$.

The agent's beliefs evolve as in equations (9) and (10). Applying the Kalman filter to the principal's beliefs with the agent's choice of effort, the principal's estimate of the agent's talent evolves as:

$$\tilde{y}_{t+1} = (1 - \delta)\tilde{y}_t + a\frac{b}{c}\hat{y}_t + (1 - \delta)\frac{\frac{b^2}{c}\hat{y}_t\tilde{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\tilde{\Sigma}_t + \sigma^2}\left(\frac{b^2}{c}\hat{y}_t(y_t - \tilde{y}_t) + \epsilon_t\right).$$

We could similarly derive the evolution of the variance of the principal's beliefs, but we will not need this transition. At the beginning of period 2, the principal's expected payoff is

$$V_2(\hat{y}_2, \tilde{y}_2) = \begin{cases} \tilde{E}_1[\pi_2] - \frac{1}{2}ce_2^2 = be_2\tilde{y}_2 - \frac{1}{2}ce_2^2 = \frac{b^2}{c}\hat{y}_2\left[\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right] & \text{if } \hat{y}_2 > \sqrt{2\frac{c}{b^2}z} \\ z & \text{if } \hat{y}_2 \leq \sqrt{2\frac{c}{b^2}z} \end{cases},$$

where the top line uses the agent's optimal choice of effort, from equation (7) and based on the agent's own estimate $\hat{y}_2$ of his talent.

Now consider the principal's expected payoff at the beginning of period 1. If the agent does not exert effort in this period, then the agent also does not exert effort in the following period and the principal's payoff is $(1 + \beta)z$. If the agent does exert effort in period 1, then the principal's expected total payoff is

$$V_1(\hat{y}_1, \hat{\Sigma}_1, \tilde{y}_1, \tilde{\Sigma}_1) = \tilde{E}_0[\pi_1] - \frac{1}{2}ce_1^2 + \beta\tilde{E}_0\left[V_2(\hat{y}_2, \tilde{y}_2)\right].$$

If the agent exerts effort in period 1, then the principal's expected period 2 payoff (based on initial information) is

$$\tilde{E}_0\left[V_2(\hat{y}_2, \tilde{y}_2)\right] = \tilde{Pr}(\psi_1 \leq \psi_1^*)z + \tilde{Pr}(\psi_1 > \psi_1^*)\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\bigg| \psi_1 > \psi_1^*\right]. \quad (13)$$

The expected period 2 payoff has the structure of a call option: if the news is sufficiently good, the agent takes advantage of his talent, but if the news is sufficiently bad, the agent chooses not to try in period 2 and limits his loss by accepting the outside option z. If the principal and agent had the same beliefs, the option to choose zero effort in period 2 and accept z would increase the principal's expected period 2 value. The agent's period 1 beliefs and effort affect the principal's expected period 2 payoff by affecting the payoffs from effort (via $\hat{y}_2$), by affecting the conditional expectation of payoffs from effort (via the news cutoff ψ_1^*), and by affecting the probability of putting forth effort in period 2 (also via the news cutoff ψ_1^*).

2.2   The principal's expected payoff conditional on period 2 effort

The appendix shows that we can write the principal's expectation of period 2 payoffs conditional on choosing nonzero effort in both periods as

$$
\begin{aligned}
\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\} \middle| \psi_1 > \psi_1^*\right] =& \frac{1}{2}\frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\left((1-\delta)(2\tilde{y}_1 - \hat{y}_1) + a\frac{b}{c}\hat{y}_1\right) \\
&+ \frac{b^2}{c}\left((1-\delta)\hat{y}_1\tilde{K}_1 + a\frac{b}{c}\hat{y}_1\tilde{K}_1 - (1-\delta)(\hat{y}_1 - \tilde{y}_1)\hat{K}_1\right)\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*] \\
&+ \frac{b^2}{c}\hat{K}_1\left(\tilde{K}_1 - \frac{1}{2}\hat{K}_1\right)\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*] \\
&+ \frac{b^2}{c}\tilde{K}_1\left(\frac{b^2}{c}\hat{y}_1(\hat{y}_1 - \tilde{y}_1)\right)\left\{(1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1 + \hat{K}_1\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\right\},
\end{aligned}
$$
$$(14)$$

where K_1 is the Kalman gain from equation (2), with $\hat{K}_1$ using $\hat{\Sigma}_1$ and $\tilde{K}_1$ using $\tilde{\Sigma}_1$. The first line gives the principal's expectation of period 2 payoffs in the absence of news that changes the agent's talent estimate (i.e., conditional on $\psi_1 = 0$). Recall that the agent's period 2 payoffs would be $\frac{1}{2}\frac{b^2}{c}\hat{y}_2^2$. The left-hand parentheses on the first line give the agent's updated talent estimate $\hat{y}_2$, which determines the agent's choice of effort. The right-hand parentheses adjust for how the principal's estimate of period 2 talent may differ from the agent's. If the agent's initial estimate is equal to the principal's ($\hat{y}_1 = \tilde{y}_1$), then the right-hand parentheses also reduce to $\hat{y}_2$ (conditional on $\psi_1 = 0$). However, if the initial estimates differ, then the principal adjusts for decay differently when forming her expectation of period 2 talent. When the principal has the higher opinion of the agent's talents ($\tilde{y}_1 > \hat{y}_1$), the difference in initial estimates increases expected period 2 payoffs by increasing the expected reward corresponding to any choice of effort, but when the principal has the lower opinion of the agent's talents ($\tilde{y}_1 < \hat{y}_1$), the difference in initial estimates reduces expected period 2 payoffs by reducing the expected reward, in which case the principal would expect the agent to exert more effort in period 2 than the principal believes to be optimal.

The second line captures how the expectation of news either creates or destroys value for the principal in period 2. This channel combines two effects. Using properties of a truncated normal distribution, we have that

$$\tilde{E}_0[\psi_1 | \psi_1 > \psi_1^*] = \frac{b^2}{c}\hat{y}_1(\tilde{y}_1 - \hat{y}_1) + \left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2} h(\psi_1^*), \tag{15}$$

where $h(\psi_1^*) \geq 0$ is the hazard rate of ψ_1 around ψ_1^* (i.e., the ratio of the probability density function for ψ_1 to the survival function for ψ_1, evaluated at ψ_1^*). The first term in equation (15) shows how the expectation of news destroys value for the principal when the agent is overly optimistic about his own talent ($\hat{y}_1 > \tilde{y}_1$). In that case, the principal expects the agent to receive a negative shock. The unconditional expectation of news is zero for the agent ($\hat{E}_0[\psi_1] = 0$), but the principal's unconditional expectation of the agent's interpretation of news is negative when the agent is overoptimistic ($\tilde{E}_0[\psi_1] < 0$ if and only if $\hat{y}_1 > \tilde{y}_1$).

The second, positive term in equation (15) conditions the principal's expectation on the agent's decision to choose nonzero effort in period 2. This is a call option effect. A call option confers the right—but not the obligation—to buy an asset at a predefined "strike" price. If the asset's price ends up above that strike price, then the option's holder should choose to exercise the option and buy the asset on the cheap; if the asset's price ends up below that strike price, then the option's holder should not exercise the option. The option's holder can thus take advantage of upside risk without being exposed to downside risk, which makes holding the option more valuable than holding the asset itself. In the same fashion, if the principal and agent have the same beliefs and the agent chooses to work in period 2, then the principal knows that the news must have been either good or at least not too bad ($\psi_1 > \psi_1^*$). This expectation of good news raises the expected payoff from an agent who chooses to work.[18]

The third line is a growth option channel. This line is positive (increasing the principal's value) as long as $\tilde{\Sigma}_1$ is not too much smaller than $\hat{\Sigma}_1$.[19] The classic growth option effect, also called the Oi-Hartman-Abel effect, refers to how uncertainty about, for instance, productivity can increase firms' value because in good times they can expand while in bad times they can contract (Oi, 1961; Hartman, 1972; Abel, 1983). Here the agent's period 2 effort responds to period 1 news. In period 2, the agent's expected payoff is convex in his updated talent estimate $\hat{y}_2$. Additional uncertainty raises the agent's initial expectation of period 2 payoffs

---

[18]The term with $\hat{K}_1$ in the second line of equation (14) captures a wrinkle due to differences in beliefs. If the principal believes that the agent overestimates his own talent ($\hat{y}_1 > \tilde{y}_1$), then this term lowers the principal's expectation of the agent's talent conditional on the agent choosing nonzero effort in period 2.

[19]We can obtain a more precise condition that will be useful later. This line is positive if and only if $\tilde{K}_1 \geq \frac{1}{2}\hat{K}_1$. Define $K_1|_{\frac{1}{2}\hat{\Sigma}_1}$ as the Kalman gain corresponding to variance $\frac{1}{2}\hat{\Sigma}_1$. From equation (2), it is easy to see that $\frac{1}{2}\hat{K}_1 > K_1|_{\frac{1}{2}\hat{\Sigma}_1}$. And from the proof of Proposition 1, the Kalman gain increases in its variance Σ. Therefore, $\tilde{K}_1 = \frac{1}{2}\hat{K}_1$ implies that $\tilde{\Sigma}_1 > \frac{1}{2}\hat{\Sigma}_1$, and we have that $\tilde{K}_1 \geq \frac{1}{2}\hat{K}_1$ implies $\tilde{\Sigma}_1 > \frac{1}{2}\hat{\Sigma}_1$.

because if he learns that he is talented, then he will try harder, and if he learns that he is not talented, then he will try less or even not try at all. If the agent and the principal have the same initial variance in their beliefs about the agent's talent (so that $\hat{K}_1 = \tilde{K}_1$), then this growth option effect also raises the principal's expected value. However, if the agent is much more uncertain about his own talent than is the principal ($\hat{K}_1 >> \tilde{K}_1$), then, from the principal's perspective, the agent's effort decisions are too sensitive to period 1 news. In that case, the principal's expected cost of effort rises much more than does the principal's expected benefit of effort, so that the growth option effect can reduce the principal's expected value.

The final line adjusts the agent's news to reflect that the agent does not read the news correctly (from the principal's perspective). The term in braces gives the principal's expectation of how the agent's talent estimate (and thus effort) will evolve. This term is positive as long as the agent is not too overoptimistic. The terms outside braces give the difference between the principal's read on news and the agent's read on news and how, via $\tilde{K}_1$, the principal's estimate of the agent's talent reacts to this difference. A principal believes that her overly optimistic agent (with $\hat{y}_1 > \tilde{y}_1$) does not properly appreciate the significance of a good test outcome and reads too much into a bad test outcome. Reading the same news with the principal's beliefs then increases value when the term in braces is positive. Analogously, a principal believes that her overly pessimistic agent (with $\hat{y}_1 < \tilde{y}_1$) reads too much into a good test outcome while not reading enough into a bad test outcome. Reading the same news with the principal's beliefs then decreases value.[20]

2.3   Will a principal deter an agent from trying?

Before considering the principal's incentive to manipulate the agent's beliefs, we first ask whether a principal would deter her agent from trying in the first period.

Proposition 4 (Principals often allow an agent to try). *There exist $\alpha_1 > 1, \alpha_2 \in (1, 2)$ such that if $\hat{y}_1 \in [\tilde{y}_1, \alpha_1 \tilde{y}_1]$ and $\hat{\Sigma}_1 < \alpha_2 \tilde{\Sigma}_1$, then a principal does not want to deter an agent who wants to exert nonzero effort in period 1. α_1 increases in the discount factor β.*

Proof. See appendix.

$\square$

If the principal and the agent have the same beliefs, then the principal never wants to deter an agent who finds it optimal to exert nonzero effort in period 1: the agent chooses the period 1 effort that maximizes period 1 payoffs, and considering period 2 payoffs increases the value

---

[20]The principal also believes that good news is less likely than an overly optimistic agent believes and more likely than an overly pessimistic agent believes, which reduces the principal's expected value relative to an optimistic agent and raises her expected value relative to a pessimistic agent. These effects are captured by evaluating news with $\tilde{E}_0[\psi_1]$ rather than $\hat{E}_0[\psi_1]$. See the discussion of equation (15).

of effort by introducing human capital benefits and generating information. However, if the principal and the agent have different estimates of the agent's talent or different degrees of uncertainty about the agent's talent, then the agent's effort decisions can be unattractive to the principal: the agent may base his period 1 effort on an incorrect talent estimate, and the agent updates to what the principal considers to be an incorrect talent estimate and thus chooses an incorrect effort level in period 2. If the agent has an opinion of her own talent than is higher than the principal deems warranted, then the principal will still generally allow the agent to try. But the principal may deter an agent with a notably wrong self-assessment or an agent who will react too strongly to the test results.

2.4   Endogenous overoptimism

Now consider the principal's incentives to alter the beliefs of an agent who chooses nonzero effort in period 1:[21]

Proposition 5 (Incentives to manipulate the agent's beliefs). *Consider an agent who has the same beliefs as the principal ($\hat{y}_1 = \tilde{y}_1, \hat{\Sigma}_1 = \tilde{\Sigma}_1$), and let the hazard rate of ψ_1 be small around ψ_1^*. The principal would gain from increasing the variance $\hat{\Sigma}_1$ of the agent's beliefs, with the gains going to zero as the hazard rate goes to zero. If $b^4\hat{y}_1^2\hat{\Sigma}_1/c^2$ is either less than or not too much larger than σ^2, then the principal would gain from increasing the agent's talent estimate $\hat{y}_1$. This gain increases in a and can be strictly positive even for $a = 0$.*

Proof. See appendix.

$\square$

When the agent has the same beliefs as the principal, the principal's incentive to manipulate the agent's beliefs arises only from how those beliefs affect period 2 outcomes: the agent chooses his period 1 effort to maximize his expected period 1 payoff, and the principal cannot improve upon this choice. But the agent's myopia makes him tend to undersupply period 1 effort once we consider the effect of effort on period 2 information and ability. The agent's underprovision of effort grows more severe as a increases because the human capital benefits of early effort become more important, but this underprovision exists even when effort does not feed back into ability (i.e., even when $a = 0$).

The principal can correct for the agent's underprovision of effort by raising the agent's estimate of his own talent. But two distortions arise. First, the overoptimistic agent will choose nonzero effort in period 2 in some cases when the principal would prefer the agent not to apply effort at all. Second, even when the principal believes that the agent should

---

[21]The hazard rate at $\psi_1 = 0$ is 0.7 and is smaller for $\psi_1 < 0$. Proposition 1 showed that for an agent who chooses nonzero effort, $\psi_1^* < 0$. Therefore the hazard rate of ψ_1 around ψ_1^* is less than 0.7. The assumption that the hazard rate is small therefore just means that the agent needs to receive fairly negative news in order to stop trying in period 2.

choose nonzero effort, the overoptimistic agent's incorrect period 2 beliefs will lead him to choose period 2 effort suboptimally (from the principal's perspective). The assumption that the hazard rate of ψ_1 is small around ψ_1^* means that the principal is unlikely to want the agent to refuse to try in period 2, so the first distortion is small. And the (sufficient, not necessary) condition that the agent's perception of the variance $b^4 \hat{y}_1^2 \hat{\Sigma}_1 / c^2$ of a perfectly measured reward is either less than or not too much larger than the measurement noise σ^2 means that the overoptimistic agent is relatively responsive to period 1 news, which mitigates the second distortion.[22] In this case, the principal would like to make the agent overoptimistic.

The principal would also like to make the agent overly uncertain (i.e., underprecise), when the probability that the agent will stop trying in period 2 is small. The agent's period 2 beliefs about his own talent are more sensitive to period 1 news (i.e., to period 1 deviations from expected outcomes) when the agent is more uncertain. The more uncertain agent is more likely to stop trying in period 2 (see Proposition 2). This effect reduces the principal's expected value and is strong when the hazard rate of ψ_1 around ψ_1^* is large. However, the more uncertain agent is also more likely to end up wanting to choose an unusually high level of effort in period 2. The additional spread in the agent's effort levels creates value because the agent's option to stop trying allows the agent to take advantage of especially good news while mitigating the consequences of especially bad news. This effect increases the principal's expected value and dominates the first effect when the hazard rate of ψ_1 around ψ_1^* is small. As the hazard rate goes to zero, the option to stop trying becomes irrelevant, and the principal then does not want to distort the variance of beliefs for an agent who has the same talent estimate as the principal.[23]

The next proposition shows that there is another channel that can drive the principal to make the agent especially uncertain:

Proposition 6 (Manipulating variance to correct mistaken beliefs). *Let the hazard rate of ψ_1 around ψ_1^* be approximately zero. Then the principal would gain from increasing the variance $\hat{\Sigma}_1$ of the agent's beliefs. These gains are proportional to $(\hat{y}_1 - \tilde{y}_1)^2$.*

Proof. See appendix. □

---

[22]Formally, this assumption ensures that $\mathrm{d}\hat{K}_1 / \mathrm{d}\hat{y}_1$ is either positive or small in magnitude, which in turn also ensures that raising the agent's talent estimate indeed generates informational benefits.

[23]In our analysis of equation (14), we saw that the principal's value is also increased through a growth option effect. One might think that this effect would make the principal want to distort the agent's beliefs in a direction of greater uncertainty: for instance, Easley and Kiefer (1988) show that if an agent's continuation value is convex in an uncertain variable, then that agent may want to choose actions that increase the variance of later beliefs. However, it is easy to see in equation (14) that, ignoring the option to choose zero effort, the growth option effect is maximized when $\hat{\Sigma}_1 = \tilde{\Sigma}_1$. For other values of $\hat{\Sigma}_1$, the agent's incorrect learning leads to incorrect effort choices, which imposes a cost on the principal. The proof of Proposition 5 shows that the principal's desire to distort the agent's beliefs actually relates to how the price of a call option increases in the variance of the underlying asset's price process.

The assumption that the hazard rate is approximately zero means that we can ignore the agent's option to choose zero effort in period 2. In this case, we already saw that the principal's incentive to manipulate the variance of the agent's beliefs disappears when the principal and the agent share the same talent estimate. However, we now see that the principal wants to make the agent more uncertain when their talent estimates differ. This channel derives from the terms with $\hat{K}_1$ on the second and fourth lines of equation (14). These terms capture how the agent's ability to learn from period 1 outcomes reduces the distortion in period 2 beliefs. The more uncertain the agent is, the more he relies on period 1 outcomes when forming period 2 beliefs. In the principal's view, making the agent rely more heavily on period 1 outcomes is good because these are unbiased reports of the agent's ability, whereas the agent's prior is biased (in the principal's eyes) when $\hat{y}_1 \neq \tilde{y}_1$.[24]

Figure 1 plots examples of the talent estimates that the principal would wish to impart to the agent.[25] The diagonal (dashed) depicts the agent's talent estimate $\hat{y}_1$ that would be equal to the principal's estimate $\tilde{y}_1$. We see that the principal's optimal estimates are always above the diagonal: the principal wishes to make the agent overoptimistic. Further, as we increase the variance of the agent's beliefs (moving from circles to solid to crosses in the plot), we see that the principal wishes to make the agent even more overoptimistic. For any of the initial estimates plotted along the horizontal axis, the principal's maximized value (not shown) is greatest along the line corresponding to the case in which the agent is most uncertain. The principal wants to make the agent overoptimistic in order to increase early effort, but these mistaken beliefs also distort second period effort. As we have seen, the principal can mitigate this distortion by increasing the variance of the agent's beliefs, so that the agent responds more strongly to period 1 outcomes. These results show how the principal wants to make the agent's beliefs both overoptimistic and underprecise.

The shaded region depicts the set of talent estimates for which the agent would choose zero effort in period 1. We see that the principal often wants to prevent an agent from choosing zero effort even when an agent who had the same beliefs as the principal would choose zero effort. The principal might allow the agent to choose zero effort in period 1 only if the principal herself believes that the agent is especially low-talented. But if this pessimistic principal could make her agent especially uncertain about his own talent, then the agent will learn quickly from the period 1 outcome. In this case, even very pessimistic principals prevent their agents from choosing zero effort (crosses), despite the likelihood of bad news.

Our results are consistent with the psychology literature's findings about early childhood beliefs. As described in the introduction, the psychology literature reports that parents strongly influence young children's self-assessments and that young children begin school

---

[24]Proposition 6 does not imply that the principal wants to raise $\hat{\Sigma}_1$ to infinity: as $\hat{\Sigma}_1$ increases, the hazard rate of ψ_1 around ψ_1^* increases, which eventually violates the condition of the proposition.

[25]I plot a case with $\delta = 0.05$, $b = 1$, $a = 0.05$, $c = 0.5$, $\sigma = 0.5$, $\omega = 0.2$, $z = 0.5$, $\beta = 0.95$, and $\tilde{\Sigma}_1 = 1$. I show three values for the variance $\hat{\Sigma}_1$ of the agent's beliefs: 1/4, 1, and 4.
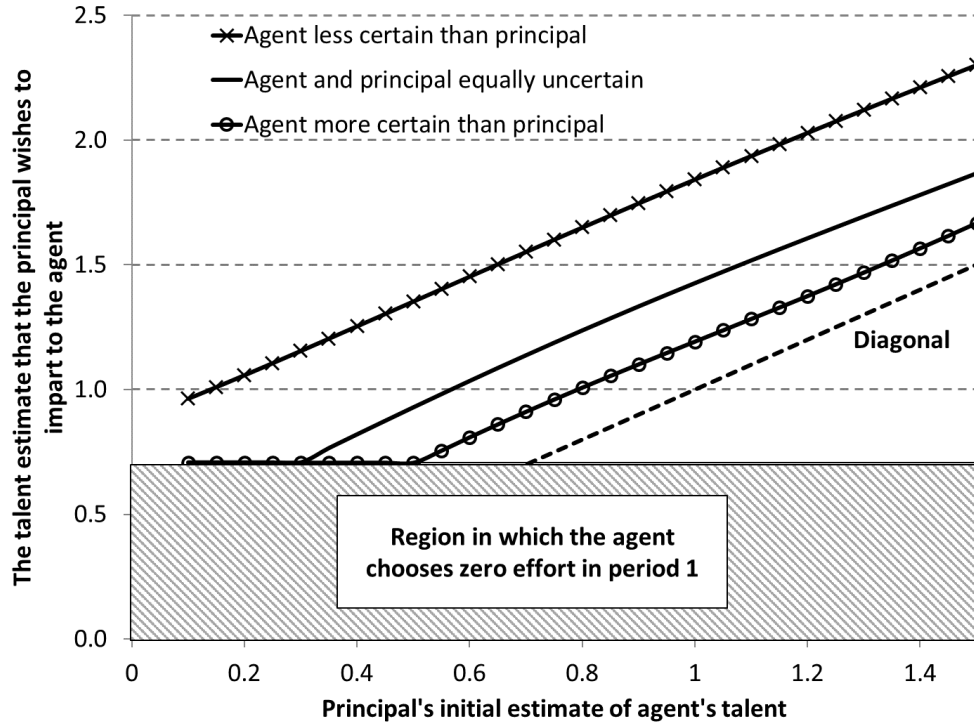
Figure 1: The initial talent estimate $\hat{y}_1$ that the principal wishes to impart to her agent, conditional on the variance $\hat{\Sigma}_1$ of the agent's beliefs. The horizontal axis gives the principal's initial estimate $\tilde{y}_1$ of the agent's talent. The lines describe cases in which the agent's beliefs have greater variance than the principal's beliefs ($\hat{\Sigma}_1 > \tilde{\Sigma}_1$, crosses), the same variance as the principal's beliefs ($\hat{\Sigma}_1 = \tilde{\Sigma}_1$, solid), and less variance than the principal's beliefs ($\hat{\Sigma}_1 < \tilde{\Sigma}_1$, circles). The dashed line is the diagonal, showing the agent's talent estimate that would match the principal's estimate.

rather overoptimistic about their own abilities. They become more pessimistic (and realistic) over time, as they receive grades and feedback from teachers. This pattern matches the incentives outlined in Proposition 5. Parents want their children to begin overoptimistic so that they apply themselves more early on, when human capital accumulation is rapid and information is at a premium. However, parents do not want their children to remain so overoptimistic indefinitely, misapplying their efforts at later stages of school and life. Parents thus want their children to be sufficiently uncertain that they learn about themselves from experiences such as grades. Most children should become more pessimistic with age, as test scores work to correct their initial overoptimism. From the parents' perspective, this pattern optimizes the gains from additional early childhood effort against the cost of later mistaken beliefs.

3   Discussion: Generic overconfidence

Economists typically emphasize the role of overly optimistic self-assessments in economic life. The present paper has focused on the emergence of overoptimism in settings with an experienced principal and an unexperienced agent. Such settings are consistent with relationships between parents and children and between managers and new employees. How does the present setting explain the stylized fact of generic overoptimism among adults?

If agents believe that test scores are especially noisy, then initial overoptimism among children can mechanically persist into adulthood. However, our results are also consistent with a more intriguing explanation. A prominent line of research in behavioral economics develops "planner-doer" models of the self (e.g., Thaler and Shefrin, 1981; Fudenberg and Levine, 2006). In each period, a person is of two minds: the "doer" part of herself wants to select an action to maximize instantaneous payoffs, but the "planner" portion of herself is forward-looking and wants to manipulate the doer into selecting her own preferred actions. This type of setting has been used to explain the effect of pensions on saving and the preference for commitment devices. In our case, the planner corresponds to the principal and the doer corresponds to the agent. Parents act as the forward-looking planner for children, but adults eventually develop their own planner, who retains the altruistic parents' incentive to manipulate the beliefs of the myopic (or selfish) doer. Just as parents want to make their myopic children overoptimistic, so too later planners want to manipulate the beliefs of the doer so as to make them systematically overoptimistic. Adults' overoptimistic beliefs may thus reflect how their parents' distortions are refreshed by their own internal planner.[26]

---

[26]The major difference between our setting and planner-doer models is that we have modeled the agent as updating his beliefs after a one-time intervention by the principal. In a planner-doer analogue to our setting, the planner might be able to intervene on a more regular basis. There are multiple ways to model the potential for repeated interventions, and the exact way in which our results translate would likely depend on this choice.

4   Conclusion

We have seen that rational learning about one's own abilities can explain psychologists' findings about depression, about the evolution of children's beliefs, and about the value of inculcating self-efficacy beliefs in the workplace. Cognitive behavioral therapy inoculates patients against depression by raising their perceptions of their own ability and making them more sure of their own ability. Principals (such as a parent or manager) have an incentive to manipulate the beliefs of an agent (such as a child or new employee): making the agent overoptimistic increases his effort, which increases human capital for later periods and provides information about his own talent, and making the agent more uncertain than the principal corrects the agent's tendency to retain distorted beliefs into the future.

These results have implications for the emergence and persistence of inequality. First, children who receive positive test scores early on (whether because of luck or because of greater talent) will tend to choose higher levels of effort, which can make them even more talented in the future. This mechanism for generating inequality is similar to one discussed in Piketty (1995). Second, if parents' discount factors (i.e., incentives to manipulate beliefs) or technologies (i.e., ability to manipulate beliefs) correlate positively with income, then higher-income parents will move their children towards a combination of beliefs and effort that tends to leave them better off once they become adults. Inequality will then tend to persist across generations.[27,28]

Our results also highlight the importance of institutions for rewarding talent and effort. Many debate whether we should reward children who achieve better outcomes or reward all children for trying. Recent discussions of "helicopter parenting" reflect concerns about not exposing children to the effects of their own efforts. The present setting highlights the importance of connecting rewards to effort and talent: if test scores are only weakly related to effort and ability, then the agent does not choose a high level of effort and her human capital does not accrue as quickly (or may even decay over time). But the present setting also highlights the risks of making test scores too revealing: because early effort has long-run benefits, it can be socially beneficial to allow children with mistakenly optimistic beliefs about their own abilities to persist in those mistakes for some time. This tension between incentivizing optimal effort from both the more talented and the less talented appears in policy debates about, for instance, tracking elementary school students according to their

---

[27]Development economists have recently hypothesized that "aspirations traps" can generate persistent poverty (Ray, 2006; Duflo, 2012; Genicot and Ray, 2014). These traps arise from the interaction between effort choices and aspirations in much the way that we have seen persistent low-effort, low-reward outcomes arise from the interaction between effort choices and beliefs about ability. The social determination of aspirations acts much like the postulated correlation between discount factors and income.

[28]Bandura et al. (2001) find that parents with high beliefs about their own self-efficacy are especially likely to raise children who have strong beliefs in their own self-efficacy, which eventually affects their children's career choices. This intergenerational transmission of self-efficacy acts like the postulated correlation between belief-shaping technologies and income.

abilities.

Finally, the present setting has emphasized the importance of the variance of beliefs and of principals' incentives to manipulate that variance. The psychology literature has studied the evolution of children's central estimates of their own talent, but the variance of their beliefs should play a critical role in this learning process. Future work in psychology should study the evolution of this variance and explore how it covaries with initial overoptimism. Further, learned helplessness theory has emphasized the importance of pessimistic explanatory styles for depression, where pessimistic explanatory styles refer to the types of conclusions people draw about themselves from events in the world. We have seen that these explanatory styles are consistent with both a low central estimate of one's own ability and a high degree of uncertainty about one's own ability. Future work should study the relative importance of overly pessimistic self-assessments and overly uncertain self-assessments for the emergence of depression and for the cognitive behavioral therapy techniques used to treat depression.

References

Abel, Andrew B. (1983) "Optimal investment under uncertainty," *The American Economic Review*, Vol. 73, No. 1, pp. 228–233.

Abramson, Lyn Y., Martin E. Seligman, and John D. Teasdale (1978) "Learned helplessness in humans: Critique and reformulation," *Journal of Abnormal Psychology*, Vol. 87, No. 1, pp. 49–74.

Akabayashi, Hideo (2006) "An equilibrium model of child maltreatment," *Journal of Economic Dynamics and Control*, Vol. 30, No. 6, pp. 993–1025.

Astebro, Thomas, Holger Herz, Ramana Nanda, and Roberto A. Weber (2014) "Seeking the roots of entrepreneurship: Insights from behavioral economics," *Journal of Economic Perspectives*, Vol. 28, No. 3, pp. 49–70.

Bandura, Albert (1982) "Self-efficacy mechanism in human agency," *American Psychologist*, Vol. 37, No. 2, pp. 122–147.

———— (2001) "Social cognitive theory: An agentic perspective," *Annual Review of Psychology*, Vol. 52, No. 1, pp. 1–26.

Bandura, Albert, Claudio Barbaranelli, Gian Vittorio Caprara, and Concetta Pastorelli (2001) "Self-efficacy beliefs as shapers of children's aspirations and career trajectories," *Child Development*, Vol. 72, No. 1, pp. 187–206.

Bénabou, Roland and Jean Tirole (2002) "Self-confidence and personal motivation," *The Quarterly Journal of Economics*, Vol. 117, No. 3, pp. 871–915.

Benot, Jean-Pierre and Juan Dubra (2011) "Apparent overconfidence," *Econometrica*, Vol. 79, No. 5, pp. 1591–1625.

Compte, Olivier and Andrew Postlewaite (2004) "Confidence-enhanced performance," *American Economic Review*, Vol. 94, No. 5, pp. 1536–1557.

Cunha, Flavio and James J. Heckman (2010) "Investing in our young people," Working Paper 16201, National Bureau of Economic Research.

Currie, Janet (2001) "Early childhood education programs," *The Journal of Economic Perspectives*, Vol. 15, No. 2, pp. 213–238.

Daniel, Kent and David Hirshleifer (2015) "Overconfident investors, predictable returns, and excessive trading," *Journal of Economic Perspectives*, Vol. 29, No. 4, pp. 61–88.

Diamond, Rebecca and Petra Persson (2016) "The long-term consequences of teacher discretion in grading of high-stakes tests," Working Paper 22207, National Bureau of Economic Research.

Doepke, Matthias and Fabrizio Zilibotti (2014) "Parenting with style: Altruism and paternalism in intergenerational preference transmission," Working Paper 20214, National Bureau of Economic Research.

Dominguez-Martinez, Silvia and Otto H. Swank (2009) "A simple model of self-assessment," *The Economic Journal*, Vol. 119, No. 539, pp. 1225–1241.

Duflo, Esther (2012) "Hope as capability," Harvard, May.

Easley, David and Nicholas M. Kiefer (1988) "Controlling a stochastic process with unknown parameters," *Econometrica*, Vol. 56, No. 5, pp. 1045–1064.

Eccles, Jacquelynne S. and and Allan Wigfield (2002) "Motivational beliefs, values, and goals," *Annual Review of Psychology*, Vol. 53, No. 1, pp. 109–132.

Eccles, Jacquelynne, Allan Wigfield, Rena D. Harold, and Phyllis Blumenfeld (1993) "Age and gender differences in children's self- and task perceptions during elementary school," *Child Development*, Vol. 64, No. 3, pp. 830–847.

Eccles (Parsons), Jacquelynne, Terry F. Adler, Robert Futterman, Susan B. Goff, Caroline B. Kaczala, Judith L. Meece, and Carol Midgley (1983) "Expectancies, values, and academic behaviors," in Janet T. Spence ed. *Achievement and Achievement Motives: Psychological and Sociological Approaches*, San Francisco: W. H. Freeman and Company, pp. 75–146.

Eccles Parsons, Jacquelynne, Terry F. Adler, and Caroline M. Kaczala (1982) "Socialization of achievement attitudes and beliefs: Parental influences," *Child Development*, Vol. 53, No. 2, pp. 310–321.

Frome, Pamela M. and Jacquelynne S. Eccles (1998) "Parents' influence on children's achievement-related perceptions," *Journal of Personality and Social Psychology*, Vol. 74, No. 2, pp. 435–452.

Fudenberg, Drew and David K. Levine (2006) "A dual-self model of impulse control," *The American Economic Review*, Vol. 96, No. 5, pp. 1449–1476.

Fudenberg, Drew, Gleb Romanyuk, and Philipp Strack (2016) "Active learning with misspecified beliefs," working paper.

Genicot, Garance and Debraj Ray (2014) "Aspirations and inequality," Working Paper 19976, National Bureau of Economic Research.

Grubb, Michael D. (2015) "Overconfident consumers in the marketplace," *Journal of Economic Perspectives*, Vol. 29, No. 4, pp. 9–36.

Hartman, Richard (1972) "The effects of price and cost uncertainty on investment," *Journal of Economic Theory*, Vol. 5, No. 2, pp. 258–266.

Heckman, James J. and Dimitriy V. Masterov (2007) "The productivity argument for investing in young children," *Applied Economic Perspectives and Policy*, Vol. 29, No. 3, pp. 446–493.

Heckman, James J. and Stefano Mosso (2014) "The economics of human development and social mobility," *Annual Review of Economics*, Vol. 6, pp. 689–733.

Heckman, James J., Jora Stixrud, and Sergio Urzua (2006) "The effects of cognitive and noncognitive abilities on labor market outcomes and social behavior," *Journal of Labor Economics*, Vol. 24, No. 3, pp. 411–482.

Jacobs, Janis E. (1991) "Influence of gender stereotypes on parent and child mathematics attitudes," *Journal of Educational Psychology*, Vol. 83, No. 4, pp. 518–527.

Jacobs, Janis E. and Martha M. Bleeker (2004) "Girls' and boys' developing interests in math and science: Do parents matter?" *New Directions for Child and Adolescent Development*, No. 106, pp. 5–21.

Layard, Richard and David M. Clark (2015) *Thrive: How Better Mental Health Care Transforms Lives and Saves Money*, Princeton, New Jersey: Princeton University Press.

Lizzeri, Alessandro and Marciano Siniscalchi (2008) "Parental guidance and supervised learning," *The Quarterly Journal of Economics*, Vol. 123, No. 3, pp. 1161–1195.

Ljungqvist, Lars and Thomas J. Sargent (2004) *Recursive Macroeconomic Theory*, Cambridge, Mass: The MIT Press, 2nd edition.

Luthans, Fred and Carolyn M. Youssef (2007) "Emerging positive organizational behavior," *Journal of Management*, Vol. 33, No. 3, pp. 321–349.

Maier, Steven F. and Martin E. Seligman (1976) "Learned helplessness: Theory and evidence," *Journal of Experimental Psychology: General*, Vol. 105, No. 1, pp. 3–46.

Malmendier, Ulrike and Timothy Taylor (2015a) "Behavioral CEOs: The role of managerial overconfidence," *Journal of Economic Perspectives*, Vol. 29, No. 4, pp. 37–60.

——— (2015b) "On the verges of overconfidence," *Journal of Economic Perspectives*, Vol. 29, No. 4, pp. 3–8.

Mobius, Markus M., Muriel Niederle, Paul Niehaus, and Tanya S. Rosenblat (2011) "Managing self-confidence: Theory and experimental evidence," Working Paper 17014, National Bureau of Economic Research.

Nicholls, John G. (1978) "The development of the concepts of effort and ability, perception of academic attainment, and the understanding that difficult tasks require more ability," *Child Development*, Vol. 49, No. 3, pp. 800–814.

Oi, Walter Y. (1961) "The desirability of price instability under perfect competition," *Econometrica*, Vol. 29, No. 1, pp. 58–64.

Piketty, Thomas (1995) "Social mobility and redistributive politics," *The Quarterly Journal of Economics*, Vol. 110, No. 3, pp. 551–584.

Rabin, Matthew and Joel L. Schrag (1999) "First impressions matter: A model of confirmatory bias," *The Quarterly Journal of Economics*, Vol. 114, No. 1, pp. 37–82.

Ray, Debraj (2006) "Aspirations, poverty and economic change," in R. Bénabou, A. Banerjee, and D. Mookherjee eds. *What Have We Learnt About Poverty*: Oxford University Press.

Seligman, Martin E. P. (1991) *Learned Optimism*, New York: Alfred A. Knopf.

Silva, Francisco (2015) "Inducing overconfidence," Technical report.

Spinath, Birgit and Frank M. Spinath (2005) "Development of self-perceived ability in elementary school: the role of parents' perceptions, teacher evaluations, and intelligence," *Cognitive Development*, Vol. 20, No. 2, pp. 190–204.

Stajkovic, Alexander D. and Fred Luthans (1998a) "Self-efficacy and work-related performance: A meta-analysis," *Psychological Bulletin*, Vol. 124, No. 2, pp. 240–261.

———— (1998b) "Social cognitive theory and self-efficacy: Going beyond traditional motivational and behavioral approaches," *Organizational Dynamics*, Vol. 26, No. 4, pp. 62–74.

Tenney, Elizabeth R., Jennifer M. Logg, and Don A. Moore (2015) "(Too) optimistic about optimism: The belief that optimism improves performance," *Journal of Personality and Social Psychology*, Vol. 108, No. 3, pp. 377–399.

Thaler, Richard H. and H. M. Shefrin (1981) "An economic theory of self-control," *Journal of Political Economy*, Vol. 89, No. 2, pp. 392–406.

Wood, Robert and Albert Bandura (1989) "Social cognitive theory of organizational management," *Academy of Management Review*, Vol. 14, No. 3, pp. 361–384.

Appendix: Additional proofs and derivations

A   Derivation of equation (14)

We here derive the principal's expectation of period 2 payoffs conditional on the agent choosing nonzero effort in both periods.

$$\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\bigg|\psi_1 > \psi_1^*\right]$$

$$=\frac{b^2}{c}\tilde{E}_0\left[\hat{y}_2\tilde{y}_2\bigg|\psi_1 > \psi_1^*\right] - \frac{1}{2}\frac{b^2}{c}\tilde{E}_0\left[\hat{y}_2^2\bigg|\psi_1 > \psi_1^*\right]$$

$$=\frac{b^2}{c}\tilde{E}_0\bigg[\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\left((1-\delta)\tilde{y}_1 + a\frac{b}{c}\hat{y}_1\right)$$

$$+ \hat{K}_1\underbrace{\left(\frac{b^2}{c}\hat{y}_1(y_1 - \hat{y}_1) + \epsilon_1\right)}_{\psi_1}\left((1-\delta)\tilde{y}_1 + a\frac{b}{c}\hat{y}_1\right)$$

$$+ \tilde{K}_1\left(\frac{b^2}{c}\hat{y}_1\underbrace{(y_1 - \tilde{y}_1)}_{y_1 - \hat{y}_1 + \hat{y}_1 - \tilde{y}_1} + \epsilon_1\right)\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)$$

$$+ \tilde{K}_1\left(\frac{b^2}{c}\hat{y}_1\underbrace{(y_1 - \tilde{y}_1)}_{y_1 - \hat{y}_1 + \hat{y}_1 - \tilde{y}_1} + \epsilon_1\right)\hat{K}_1\underbrace{\left(\frac{b^2}{c}\hat{y}_1(y_1 - \hat{y}_1) + \epsilon_1\right)}_{\psi_1}\bigg|\psi_1 > \psi_1^*\bigg]$$

$$- \frac{1}{2}\frac{b^2}{c}\tilde{E}_0\bigg[\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)^2 + \left(\hat{K}_1\underbrace{\left(\frac{b^2}{c}\hat{y}_1(y_1 - \hat{y}_1) + \epsilon_1\right)}_{\psi_1}\right)^2$$

$$+ 2\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\hat{K}_1\underbrace{\left(\frac{b^2}{c}\hat{y}_1(y_1 - \hat{y}_1) + \epsilon_1\right)}_{\psi_1}\bigg|\psi_1 > \psi_1^*\bigg]$$

$$
\begin{aligned}
=&\frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\left((1-\delta)\tilde{y}_1 + a\frac{b}{c}\hat{y}_1\right)\\
&+ \frac{b^2}{c}\hat{K}_1\left((1-\delta)\tilde{y}_1 + a\frac{b}{c}\hat{y}_1\right)\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\\
&+ \frac{b^2}{c}\tilde{K}_1\frac{b^2}{c}\hat{y}_1(\hat{y}_1 - \tilde{y}_1)\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\\
&+ \frac{b^2}{c}\tilde{K}_1\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\\
&+ \frac{b^2}{c}\tilde{K}_1\frac{b^2}{c}\hat{y}_1(\hat{y}_1 - \tilde{y}_1)\hat{K}_1\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\\
&+ \frac{b^2}{c}\tilde{K}_1\hat{K}_1\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*]\\
&- \frac{1}{2}\frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)^2\\
&- \frac{1}{2}\frac{b^2}{c}\hat{K}_1^2\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*]\\
&- \frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\hat{K}_1\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\\
=&\frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\left((1-\delta)\left(\tilde{y}_1 - \frac{1}{2}\hat{y}_1\right) + \frac{1}{2}a\frac{b}{c}\hat{y}_1\right)\\
&- \frac{b^2}{c}(1-\delta)(\hat{y}_1 - \tilde{y}_1)\hat{K}_1\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\\
&+ \frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\left(\frac{b^2}{c}\hat{y}_1(\hat{y}_1 - \tilde{y}_1)\right)\tilde{K}_1\\
&+ \frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\tilde{K}_1\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\\
&+ \frac{b^2}{c}\left(\frac{b^2}{c}\hat{y}_1(\hat{y}_1 - \tilde{y}_1)\right)\tilde{K}_1\hat{K}_1\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\\
&+ \frac{b^2}{c}\hat{K}_1\left(\tilde{K}_1 - \frac{1}{2}\hat{K}_1\right)\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*].
\end{aligned}
$$

B   Proof of Proposition 1

First, assume that an agent chooses zero effort in period t, which implies that she chooses zero effort in period $t+1$. Raising $\hat{y}_t$ makes her more likely to choose nonzero effort in period t and thus also in period $t+1$. Raising $\hat{\Sigma}_t$ has no effect.

For the rest of the proof, assume that the agent chooses nonzero effort in period 1. By

inequality (8) and our assumption that $ab/c \geq \delta$, the expression in braces in equation (12) is negative. Therefore $\psi_t^* < 0$.

From inequality (12), the agent stops applying effort in period $t + 1$ if and only if

$$\pi_t \leq \psi_t^* + \frac{b^2}{c}\hat{y}_t^2 \triangleq \pi_t^*. \tag{16}$$

If the distribution of true talent were fixed independently of the agent's beliefs, then changing a parameter would make the agent more likely to choose nonzero effort in period $t + 1$ if and only if changing that parameter would reduce π_t^*. However, we consider the case in which the agent has correct beliefs about the distribution of true talent. Define ξ_t as a normally distributed random variable with mean 0 and variance $b^4\hat{y}_t^2\hat{\Sigma}_t/c^2 + \sigma^2$. Then $\pi_t = b^2\hat{y}_t^2/c + \xi_t$. We find that the agent stops applying effort in period $t+1$ if and only if $\xi_t \leq \psi_t^*$. By properties of the normal distribution, we have that the probability that the agent stops applying effort in period $t + 1$ is

$$Pr(\xi_t \leq \psi_t^*) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\psi_t^*}{\sqrt{2(b^4\hat{y}_t^2\hat{\Sigma}_t/c^2 + \sigma^2)}}\right)\right].$$

Now consider how the probability that the agent stops applying effort in period $t + 1$ changes in $\hat{y}_t$:

$$\frac{\mathrm{d}[Pr(\xi_t \leq \psi_t^*)]}{\mathrm{d}\hat{y}_t} \propto \frac{\mathrm{d}\psi_t^*}{\mathrm{d}\hat{y}_t} - \frac{\frac{b^4}{c^2}\hat{y}_t\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\psi_t^*. \tag{17}$$

Differentiate ψ_t^* with respect to $\hat{y}_t$:

$$\frac{\mathrm{d}\psi_t^*}{\mathrm{d}\hat{y}_t} = -\frac{\psi_t^*}{K_t}\frac{\mathrm{d}K_t}{\mathrm{d}\hat{y}_t} - \frac{1}{K_t}\left\{(1 - \delta) + a\frac{b}{c}\right\}. \tag{18}$$

Recall that we assume $ab/c \geq \delta$. Substituting optimal effort from equation (7) into equation (2) and then differentiating with respect to $\hat{y}_t$, we have:

$$\frac{\mathrm{d}K_t}{\mathrm{d}\hat{y}_t} = (1 - \delta)\frac{\frac{b^2}{c}\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\left\{1 - 2\frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\right\} = \frac{K_t}{\hat{y}_t}\left\{1 - 2\frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\right\} \in \left(-\frac{K_t}{\hat{y}_t}, \frac{K_t}{\hat{y}_t}\right).$$

If $\mathrm{d}K_t/\mathrm{d}\hat{y}_t \leq 0$, then $\mathrm{d}\psi_t^*/\mathrm{d}\hat{y}_t < 0$. Consider the case where $\mathrm{d}K_t/\mathrm{d}\hat{y}_t > 0$. Then the first term on the right-hand side of equation (18) is positive while the second term is negative. The first term is at its largest possible value when $\mathrm{d}K_t/\mathrm{d}\hat{y}_t$ is at its maximum. Thus,

$$\frac{\mathrm{d}\psi_t^*}{\mathrm{d}\hat{y}_t} < -\frac{\psi_t^*}{\hat{y}_t} - \frac{1}{K_t}\left\{(1 - \delta) + a\frac{b}{c}\right\} = -\frac{1}{K_t}\sqrt{2\frac{c}{b^2}}z \leq 0.$$

We have shown that $d\psi_t^*/d\hat{y}_t < 0$, which will be useful for later results. Substitute equation (18) into (17) and multiply the final term by $\hat{y}_t/\hat{y}_t$:

$$
\begin{aligned}
\frac{d[Pr(\xi_t \leq \psi_t^*)]}{d\hat{y}_t} \propto & -\frac{\psi_t^*}{\hat{y}_t}\left\{1 - 2\frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\right\} - \frac{1}{K_t}\left\{1 - \delta + a\frac{b}{c}\right\} - \frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\frac{\psi_t^*}{\hat{y}_t} \\
= & -\frac{\psi_t^*}{\hat{y}_t}\left(1 - \frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\right) - \frac{1}{K_t}\left\{1 - \delta + a\frac{b}{c}\right\} \\
= & -\frac{1}{K_t}\frac{\sqrt{2\frac{c}{b^2}z}}{\hat{y}_t}\left(1 - \frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\right) - \frac{1}{K_t}\left\{1 - \delta + a\frac{b}{c}\right\}\frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2} \\
< & \; 0,
\end{aligned}
$$

where we substitute for ψ_t^* from equation (12) and use $ab/c \geq \delta$. This establishes the part of the proposition considering an increase in $\hat{y}_t$.

Now consider how the probability that the agent stops applying effort in period $t+1$ changes in $\hat{\Sigma}_t$:

$$
\frac{d[Pr(\xi_t \leq \psi_t^*)]}{d\hat{\Sigma}_t} \propto \frac{d\psi_t^*}{d\hat{\Sigma}_t} - \frac{1}{2}\frac{\frac{b^4}{c^2}\hat{y}_t^2}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\psi_t^*. \tag{19}
$$

Differentiate ψ_t^* with respect to $\hat{\Sigma}_t$:

$$
\frac{d\psi_t^*}{d\hat{\Sigma}_t} = -\frac{\psi_t^*}{K_t}\frac{dK_t}{d\hat{\Sigma}_t} > 0.
$$

Substituting optimal effort from equation (7) into equation (2) and then differentiating with respect to $\hat{\Sigma}_t$, we have:

$$
\frac{dK_t}{d\hat{\Sigma}_t} = (1 - \delta)\frac{\frac{b^2}{c}\hat{y}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\left\{1 - \frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2}\right\} > 0.
$$

Thus $d\psi_t^*/d\hat{\Sigma}_t > 0$ and, by equation (19) and $\psi_t^* < 0$, $d[Pr(\xi_t \leq \psi_t^*)]/d\hat{\Sigma}_t > 0$. This establishes the part of the proposition considering an increase in $\hat{\Sigma}_t$.

C   Proof of Proposition 2

Recall that an agent stops trying between periods t and $t+1$ if and only if $\pi_t \leq \pi_t^*$, where π_t^* was defined in equation (16). Consider how π_t^* changes in the agent's talent estimate $\hat{y}_t$:

$$
\begin{aligned}
\frac{\mathrm{d}\pi_t^*}{\mathrm{d}\hat{y}_t} =& \frac{\mathrm{d}\psi_t^*}{\mathrm{d}\hat{y}_t} + 2\frac{b^2}{c}\hat{y}_t \\
=& -\frac{1}{K_t}\frac{\sqrt{2\frac{c}{b^2}z}}{\hat{y}_t}\frac{\sigma^2 - \frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2} - 2\frac{1}{K_t}\left\{1 - \delta + a\frac{b}{c}\right\}\frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t + \sigma^2} + 2\frac{b^2}{c}\hat{y}_t \\
=& -\frac{\sqrt{2\frac{c}{b^2}z}}{\hat{y}_t}\frac{\sigma^2 - \frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{(1-\delta)\frac{b^2}{c}\hat{y}_t\hat{\Sigma}_t} - 2\left\{1 - \delta + a\frac{b}{c}\right\}\frac{\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{(1-\delta)\frac{b^2}{c}\hat{y}_t\hat{\Sigma}_t} + 2\frac{b^2}{c}\hat{y}_t \\
\propto& -\frac{\sqrt{2\frac{c}{b^2}z}}{\hat{y}_t}\frac{\sigma^2 - \frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^2}{c}\hat{y}_t\hat{\Sigma}_t} - 2\left\{1 - \delta + a\frac{b}{c}\right\}\frac{b^2}{c}\hat{y}_t + 2(1-\delta)\frac{b^2}{c}\hat{y}_t \\
=& -\frac{\sqrt{2\frac{c}{b^2}z}}{\hat{y}_t}\frac{\sigma^2 - \frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t}{\frac{b^2}{c}\hat{y}_t\hat{\Sigma}_t} - 2a\frac{b}{c}\frac{b^2}{c}\hat{y}_t \\
\propto& -\frac{\sqrt{2\frac{c}{b^2}z}}{\hat{y}_t}\left[\sigma^2 - \frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t\right] - 2a\frac{b}{c}\frac{b^4}{c^2}\hat{y}_t^2\hat{\Sigma}_t,
\end{aligned}
$$

where we use results from the proof of Proposition 1. The derivative is negative if $\sigma^2 \geq b^4\hat{y}_t^2\hat{\Sigma}_t/c^2$, as assumed in the proposition. And note that $\psi_s < 0$ for $s \geq t$ implies that $\hat{y}_s \leq \hat{y}_t$ and $\hat{\Sigma}_s \leq \hat{\Sigma}_t$. Therefore, $\sigma^2 \geq b^4\hat{y}_t^2\hat{\Sigma}_t/c^2$ implies that $\sigma^2 \geq b^4\hat{y}_s^2\hat{\Sigma}_s/c^2$ and $\mathrm{d}\pi_s^*/\hat{y}_s < 0$, for $s \geq t$. Differentiating π_t^* with respect to $\hat{\Sigma}_t$, we have

$$
\frac{\mathrm{d}\pi_t^*}{\mathrm{d}\hat{\Sigma}_t} = \frac{\mathrm{d}\psi_t^*}{\mathrm{d}\hat{\Sigma}_t} > 0,
$$

where the sign follows from the proof of Proposition 1.

   We have shown that π_t^* decreases in $\hat{y}_t$ under the condition of the proposition and always increases in $\hat{\Sigma}_t$. In that case, all time t rewards that make agent i choose zero effort at time $t+1$ also make agent j choose zero effort at time $t+1$, and some time t rewards can make agent j choose zero effort at time $t+1$ without leading agent i to do the same. We have established that if $\hat{y}_{ti} \geq \hat{y}_{tj}$ and $\hat{\Sigma}_{ti} \leq \hat{\Sigma}_{tj}$, then agent j chooses nonzero effort at time $t+1$ only if agent i also does.

   Now proceed by induction. Consider some time $s \geq t$. Assume that $\hat{y}_{si} \geq \hat{y}_{sj}$ and $\hat{\Sigma}_{si} \leq \hat{\Sigma}_{sj}$, so that agent j chooses nonzero effort at time $s+1$ only if agent i also does. Also assume that $\hat{y}_{sj} > \sqrt{2cz}/b$ so that both agents choose nonzero effort at time s. And assume that $\pi_s > \pi_{si}^*, \pi_{sj}^*$ so that both agents choose nonzero effort at time $s+1$. This assumption

implies that $\psi_s > \psi_{si}^*, \psi_{sj}^*$. Write equation (9) with fixed rewards π_s and differentiate:

$$
\begin{aligned}
\left.\frac{\mathrm{d}\hat{y}_{s+1}}{\mathrm{d}\hat{y}_s}\right|_{\pi_s \text{ fixed}} &= (1-\delta) + a\frac{b}{c} + \frac{\mathrm{d}K_s}{\mathrm{d}\hat{y}_s}\psi_s - 2K_s\hat{y}_s\frac{b^2}{c} \\
&> (1-\delta) + a\frac{b}{c} + \frac{\mathrm{d}K_s}{\mathrm{d}\hat{y}_s}\psi_s^* - 2K_s\hat{y}_s\frac{b^2}{c} \\
&\geq (1-\delta) + a\frac{b}{c} + \frac{K_s}{\hat{y}_s}\psi_s^* - 2K_s\hat{y}_s\frac{b^2}{c} \\
&= \frac{\sqrt{2cz/b^2}}{\hat{y}_s} - 2K_s\hat{y}_s\frac{b^2}{c} \\
&\geq 1 - 2(1-\delta)\frac{\frac{b^4}{c^2}\hat{y}_s^2\hat{\Sigma}_s}{\sigma^2 + \frac{b^4}{c^2}\hat{y}_s^2\hat{\Sigma}_s} \\
&\geq \delta,
\end{aligned}
$$

where we know $\delta \geq 0$. The first inequality follows from recognizing that $\psi_s > \psi_s^*$ under the assumption that the agent chooses nonzero effort at time s, the second inequality follows from the proof of Proposition 1, and the third inequality follows from inequality (8) and the condition that the agent chooses nonzero effort in period s. The final inequality follows from the proposition's condition that $\sigma^2 \geq b^4\hat{y}_t^2\hat{\Sigma}_t/c^2$, which we have seen implies that $\sigma^2 \geq b^4\hat{y}_s^2\hat{\Sigma}_s/c^2$. We also have

$$
\left.\frac{\mathrm{d}\hat{y}_{s+1}}{\mathrm{d}\hat{\Sigma}_s}\right|_{\pi_s \text{ fixed}} = \frac{\mathrm{d}K_s}{\mathrm{d}\hat{\Sigma}_s}\psi_s < 0.
$$

Putting these pieces together, we see that agent i's higher value of $\hat{y}_s$ and smaller value of $\hat{\Sigma}_s$ ensure that $\hat{y}_{(s+1)i} \geq \hat{y}_{(s+1)j}$. Now differentiate equation (10):

$$
\frac{\mathrm{d}\hat{\Sigma}_{s+1}}{\mathrm{d}\hat{y}_s} = -\frac{2\frac{b^4}{c^2}\hat{y}_s\hat{\Sigma}_s\sigma^2}{[\frac{b^4}{c^2}\hat{y}_s^2\hat{\Sigma}_s + \sigma^2]^2}(1-\delta)^2\hat{\Sigma}_s < 0,
$$

$$
\begin{aligned}
\frac{\mathrm{d}\hat{\Sigma}_{s+1}}{\mathrm{d}\hat{\Sigma}_s} &= \frac{\sigma^2}{\frac{b^4}{c^2}\hat{y}_s^2\hat{\Sigma}_s + \sigma^2}(1-\delta)^2 - \frac{\frac{b^4}{c^2}\hat{y}_s^2\sigma^2}{[\frac{b^4}{c^2}\hat{y}_s^2\hat{\Sigma}_s + \sigma^2]^2}(1-\delta)^2\hat{\Sigma}_s \\
&\propto 1 - \frac{\frac{b^4}{c^2}\hat{y}_s^2\hat{\Sigma}_s}{\frac{b^4}{c^2}\hat{y}_s^2\hat{\Sigma}_s + \sigma^2} > 0.
\end{aligned}
$$

Agent i's higher value of $\hat{y}_s$ and smaller value of $\hat{\Sigma}_s$ ensure that $\hat{\Sigma}_{(s+1)i} \leq \hat{\Sigma}_{(s+1)j}$. We have established that $\hat{y}_{si} \geq \hat{y}_{sj}$ and $\hat{\Sigma}_{si} \leq \hat{\Sigma}_{sj}$ imply that $\hat{y}_{(s+1)i} \geq \hat{y}_{(s+1)j}$ and $\hat{\Sigma}_{(s+1)i} \leq \hat{\Sigma}_{(s+1)j}$. Combining with previous results, agent j chooses nonzero effort at time $s + 2$ only if agent i also does. By induction, we have that agent i chooses nonzero effort for at least as long as agent j does.

D   Proof of Proposition 4

If a principal deters an agent from trying in period 1, then the period 1 portion of the principal's payoffs changes by

$$z - \frac{b^2}{c}\hat{y}_1\left(\tilde{y}_1 - \frac{1}{2}\hat{y}_1\right).$$

If the agent chooses to try in period 1, then $\frac{1}{2}\frac{b^2}{c}\hat{y}_1^2 > z$, from inequality (8). Therefore, the above condition is negative if it is negative at $z = \frac{1}{2}\frac{b^2}{c}\hat{y}_1^2$ (so that the agent is indifferent between trying or not in period 1). The change in period 1 payoffs makes deterring the agent unattractive if

$$0 \geq \frac{1}{2}\frac{b^2}{c}\hat{y}_1^2 - \frac{b^2}{c}\hat{y}_1\left(\tilde{y}_1 - \frac{1}{2}\hat{y}_1\right) = \frac{b^2}{c}\hat{y}_1\left(\hat{y}_1 - \tilde{y}_1\right).$$

If $\tilde{y}_1 \geq \hat{y}_1$, then the principal's change in period 1 payoffs never favors deterring an agent who chooses nonzero effort. If $\tilde{y}_1 < \hat{y}_1$, then the principal's change in period 1 payoffs might favor deterring an agent who chooses nonzero effort, but the principal may nevertheless choose not to deter this agent if doing so would reduce the principal's expected period 2 payoffs by a sufficiently large amount.

Now consider how deterring an agent changes the principal's expected period 2 payoffs. Using equation (13), if a principal deters an agent from trying in period 1, then the change in the principal's expected period 2 payoff (using initial information) is

$$\tilde{Pr}(\psi_1 > \psi_1^*)\left\{z - \tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\Big|\psi_1 > \psi_1^*\right]\right\}.$$

If the agent chooses to try in period 1, then $\frac{1}{2}\frac{b^2}{c}\hat{y}_1^2 > z$, from inequality (8). Therefore, the above condition is negative if it is negative at $z = \frac{1}{2}\frac{b^2}{c}\hat{y}_1^2$ (so that the agent is indifferent between trying or not in period 1). The change in expected period 2 payoffs dissuades a principal from deterring the agent in period 1 if

$$0 \geq \tilde{Pr}(\psi_1 > \psi_1^*)\left\{\frac{1}{2}\frac{b^2}{c}\hat{y}_1^2 - \tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\Big|\psi_1 > \psi_1^*\right]\right\}.$$

Using equation (14), this condition becomes

$$\begin{aligned}
0 \geq& \frac{1}{2}\frac{b^2}{c}\hat{y}_1^2 - \frac{1}{2}\frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\left((1-\delta)(2\tilde{y}_1 - \hat{y}_1) + a\frac{b}{c}\hat{y}_1\right) \\
&- \frac{b^2}{c}\left((1-\delta)\hat{y}_1\tilde{K}_1 + a\frac{b}{c}\hat{y}_1\tilde{K}_1 - (1-\delta)(\hat{y}_1 - \tilde{y}_1)\hat{K}_1\right)\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*] \\
&- \frac{b^2}{c}\hat{K}_1\left(\tilde{K}_1 - \frac{1}{2}\hat{K}_1\right)\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*] \\
&- \frac{b^2}{c}\tilde{K}_1\left(\frac{b^2}{c}\hat{y}_1(\hat{y}_1 - \tilde{y}_1)\right)\left\{(1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1 + \hat{K}_1\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]\right\}. \qquad (20)
\end{aligned}$$

Using properties of a truncated normal distribution, we have that

$$\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*] = \frac{b^2}{c}\hat{y}_1(\tilde{y}_1 - \hat{y}_1) + \left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\frac{\phi(\lambda)}{1 - \Phi(\lambda)},$$

$$\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*] = \tilde{E}_0\left[\psi_1^2 - \tilde{E}_0[\psi_1]^2 + \tilde{E}_0[\psi_1]^2|\psi_1 > \psi_1^*\right] = \tilde{Var}_0[\psi_1|\psi_1 > \psi_1^*] + \tilde{E}_0[\psi_1]^2$$

$$= \frac{b^4}{c^2}\hat{y}_1^2(\tilde{y}_1 - \hat{y}_1)^2 + \left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]\left\{1 - \frac{\phi(\lambda)}{1 - \Phi(\lambda)}\left[\frac{\phi(\lambda)}{1 - \Phi(\lambda)} - \lambda\right]\right\},$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the density function and cumulative distribution function for the standard normal distribution and where

$$\lambda = \frac{\psi_1^* - \frac{b^2}{c}\hat{y}_1(\tilde{y}_1 - \hat{y}_1)}{\left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}}.$$

Substituting into inequality (20) and combining terms, we have:

$$0 \geq \frac{1}{2}\frac{b^2}{c}\hat{y}_1^2 - \frac{1}{2}\frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\left((1-\delta)(2\tilde{y}_1 - \hat{y}_1) + a\frac{b}{c}\hat{y}_1\right)$$

$$- \frac{b^2}{c}\left((1-\delta)\hat{y}_1\tilde{K}_1 + a\frac{b}{c}\hat{y}_1\tilde{K}_1 - (1-\delta)(\hat{y}_1 - \tilde{y}_1)\hat{K}_1\right)\left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\frac{\phi(\lambda)}{1 - \Phi(\lambda)}$$

$$- \frac{b^4}{c^2}(1-\delta)(\hat{y}_1 - \tilde{y}_1)^2\hat{K}_1\hat{y}_1$$

$$- \frac{b^2}{c}\hat{K}_1\left(\tilde{K}_1 - \frac{1}{2}\hat{K}_1\right)\left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]\left\{1 - \frac{\phi(\lambda)}{1 - \Phi(\lambda)}\left[\frac{\phi(\lambda)}{1 - \Phi(\lambda)} - \lambda\right]\right\}$$

$$+ \frac{1}{2}\frac{b^2}{c}\hat{K}_1^2\frac{b^4}{c^2}\hat{y}_1^2(\tilde{y}_1 - \hat{y}_1)^2$$

$$- \frac{b^2}{c}\left(\frac{b^2}{c}\hat{y}_1(\hat{y}_1 - \tilde{y}_1)\right)\tilde{K}_1\hat{K}_1\left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\frac{\phi(\lambda)}{1 - \Phi(\lambda)}. \tag{21}$$

Using the assumption that $ab/c \geq \delta$, the top line on the right-hand side is negative if $(1-\delta)(2\tilde{y}_1 - \hat{y}_1) + a\frac{b}{c}\hat{y}_1 \geq \hat{y}_1$. This holds if and only if

$$\tilde{y}_1 \geq \hat{y}_1\left(1 - \frac{\frac{ab}{c} - \delta}{2(1-\delta)}\right).$$

Using the assumption that $ab/c \geq \delta$, the right-hand side is less than $\hat{y}_1$. Therefore, there exists $\alpha_3 > 1$ such that if $\hat{y}_1 \leq \alpha_3\tilde{y}_1$, then the first line of inequality (21) is negative. There exists $\alpha_4 > 1$ such that if $\hat{y}_1 \leq \alpha_4\tilde{y}_1$, then the second line of inequality (21) is negative. The third line is always negative. There exists $\alpha_2 \in (1,2)$ such that the fourth line is negative if

$\hat{\Sigma}_1 < \alpha_2 \tilde{\Sigma}_1$ (see footnote 19). The fifth line is positive but is small for $|\hat{y}_1 - \tilde{y}_1|$ small. The final line is negative if and only if $\hat{y}_1 \geq \tilde{y}_1$.

If $\hat{y}_1 = \tilde{y}_1$, then deterring the agent can only reduce the principal's period 1 payoffs and reduces the principal's period 2 payoffs if $\hat{\Sigma}_1 < \alpha_2 \tilde{\Sigma}_1$. If $\hat{y}_1 > \tilde{y}_1$, then deterring the agent might increase the principal's period 1 payoffs but reduces the principal's period 2 payoffs if $\hat{y}_1 \leq \min\{\alpha_3, \alpha_4\}\tilde{y}_1$ and $\hat{\Sigma}_1 < \alpha_2 \tilde{\Sigma}_1$. Because the maximum period 1 gain from deterring an agent is proportional to $\hat{y}_1 - \tilde{y}_1$, there is some $\alpha_1 \in (1, \min\{\alpha_3, \alpha_4\}]$ such that if $\hat{y}_1 \in [\tilde{y}_1, \alpha_1 \tilde{y}_1]$, then the potential period 1 gains from deterring an agent are smaller than the discounted losses from period 2. When the discount factor β is larger, the principal places more weight on the change in period 2 payoffs and so α_1 is larger.

<div style="text-align: right">□</div>

E   Proof of Proposition 5

Begin by analyzing how the principal's period 1 payoffs change with beliefs. The principal's period 1 payoffs are:

$$\frac{b^2}{c}\hat{y}_1 \left[\tilde{y}_1 - \frac{1}{2}\hat{y}_1\right].$$

This is independent of $\hat{\Sigma}_1$. Differentiating with respect to $\hat{y}_1$, we have:

$$\frac{b^2}{c}\left[\tilde{y}_1 - \hat{y}_1\right].$$

Around $\hat{y}_1 = \tilde{y}_1$, this is zero.

Thus, we need only consider how the principal's expected value of period 2 payoffs changes in $\hat{y}_1$ and $\hat{\Sigma}_1$. Define $\epsilon_1^*(y_1)$ as the value of ϵ_1 such that $\psi_1 = \psi_1^*$ given true period 1 talent y_1, and write $\tilde{f}_x(\cdot)$ as the density of random variable x under the principal's beliefs. We have:

$$\tilde{E}_0\left[V_2(\hat{y}_2, \tilde{y}_2)\right] = \int_{-\infty}^{\infty} \int_{-\infty}^{\epsilon_1^*(y_1)} z\, \tilde{f}_\epsilon(\epsilon_1)\, \mathrm{d}\epsilon_1\, \tilde{f}_y(y_1)\, \mathrm{d}y_1$$
$$+ \int_{-\infty}^{\infty} \int_{\epsilon_1^*(y_1)}^{\infty} \frac{b^2}{c}\hat{y}_2 \left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\} \tilde{f}_\epsilon(\epsilon_1)\, \mathrm{d}\epsilon_1\, \tilde{f}_y(y_1)\, \mathrm{d}y_1.$$

Note that $\hat{y}_1$ and $\tilde{y}_2$ are functions of y_1 and ϵ_1. Differentiate with respect to a parameter x, which could be $\hat{y}_1$ or $\hat{\Sigma}_1$:

$$\frac{\mathrm{d}\tilde{E}_0\left[V_2(\hat{y}_2, \tilde{y}_2)\right]}{\mathrm{d}x} = \int_{-\infty}^{\infty} \left[z - \frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\Big|_{\epsilon_1^*(y_1)}\right] \tilde{f}_\epsilon(\epsilon_1^*(y_1))\frac{\mathrm{d}\epsilon_1^*(y_1)}{\mathrm{d}x}\, \tilde{f}_y(y_1)\, \mathrm{d}y_1$$
$$+ \int_{-\infty}^{\infty} \int_{\epsilon_1^*(y_1)}^{\infty} \frac{\mathrm{d}\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\right]}{\mathrm{d}x} \tilde{f}_\epsilon(\epsilon_1)\, \mathrm{d}\epsilon_1\, \tilde{f}_y(y_1)\, \mathrm{d}y_1.$$

$\epsilon^*(y_1)$ is the value of the random shock at which agents are just indifferent between working or not. Thus, around $\hat{y}_1 = \tilde{y}_1$ and $\hat{\Sigma}_1 = \tilde{\Sigma}_1$, the integrand on the top line is zero: agents optimize their period 2 effort "optimally" conditional on their beliefs, and those beliefs are the same as their principals' beliefs. Small changes in the probability of exerting zero effort do not have a first-order effect. Rewriting the bottom line using a conditional expectation, we have:

$$\frac{\mathrm{d}\tilde{E}_0\left[V_2(\hat{y}_2, \tilde{y}_2)\right]}{\mathrm{d}x} = \tilde{Pr}(\psi_1 > \psi_1^*) \frac{\mathrm{d}\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\Big|\psi_1 > \psi_1^*\right]}{\mathrm{d}x}.$$

We can therefore analyze a principal's incentives to manipulate her agent's initial beliefs by analyzing the derivative of the conditional expectation.

Begin by considering the principal's gain from increasing the agent's talent estimate $\hat{y}_1$. Differentiating equation (14) with respect to $\hat{y}_1$ and then setting $\hat{y}_1 = \tilde{y}_1$ and $\hat{\Sigma}_1 = \tilde{\Sigma}_1$ yields:

$$\frac{\mathrm{d}\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\Big|\psi_1 > \psi_1^*\right]}{\mathrm{d}\hat{y}_1}\Bigg|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1} = \frac{b^2}{c}a\frac{b}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)$$

$$+ \frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\left(\frac{b^2}{c}\hat{y}_1\right)K_1$$

$$+ \frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)\frac{\mathrm{d}K_1}{\mathrm{d}\hat{y}_1}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]$$

$$+ \frac{b^2}{c}a\frac{b}{c}K_1\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]$$

$$+ \frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)K_1\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{y}_1}\Bigg|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1}$$

$$+ \frac{b^2}{c}\left(\frac{b^2}{c}\hat{y}_1\right)K_1^2\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]$$

$$+ \frac{b^2}{c}K_1\frac{\mathrm{d}K_1}{\mathrm{d}\hat{y}_1}\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*]$$

$$+ \frac{b^2}{c}\frac{1}{2}K_1^2\frac{\mathrm{d}\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{y}_1}\Bigg|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1}, \quad (22)$$

where we write K_1 for $\tilde{K}_1 = \hat{K}_1$.

Differentiating the conditional expectation given in the proof of Proposition 4, we have

$$
\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{y}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1} = -\frac{b^2}{c}\hat{y}_1 + \frac{b^4}{c^2}\hat{y}_1\tilde{\Sigma}_1\left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{-1/2}\frac{\phi(\lambda)}{1-\Phi(\lambda)}
$$
$$
+ \left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\left[\frac{-\lambda\phi(\lambda)}{1-\Phi(\lambda)} + \frac{\phi(\lambda)^2}{[1-\Phi(\lambda)]^2}\right]\frac{\mathrm{d}\lambda}{\mathrm{d}\hat{y}_1},
$$

where we use $\phi'(\lambda) = -\lambda\phi(\lambda)$. Note that the hazard rate of ψ_1 around ψ_1^* is $\phi(\lambda)/[1 - \Phi(\lambda)]$. This goes to zero as the density $\phi(\lambda)$ goes to zero. And by properties of the normal distribution, $\phi'(\lambda)$ also goes to zero as $\phi(\lambda)$ goes to zero. Therefore $-\lambda\phi(\lambda)/[1 - \Phi(\lambda)]$ is small when the hazard rate is small around ψ_1^*. Under the assumption that the hazard rate is small around ψ_1^*, we have:

$$
\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{y}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1} \approx -\frac{b^2}{c}\hat{y}_1. \tag{23}
$$

Differentiating the other conditional expectation given in the proof of Proposition 4, we have

$$
\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{y}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1} = 2\frac{b^4}{c^2}\hat{y}_1\tilde{\Sigma}_1\left\{1 - \frac{\phi(\lambda)}{1-\Phi(\lambda)}\left[\frac{\phi(\lambda)}{1-\Phi(\lambda)} - \lambda\right]\right\}
$$
$$
+ \left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]\left\{-\left(\frac{-\lambda\phi(\lambda)}{1-\Phi(\lambda)} + \frac{\phi(\lambda)^2}{[1-\Phi(\lambda)]^2}\right)\left[\frac{\phi(\lambda)}{1-\Phi(\lambda)} - \lambda\right]\right.
$$
$$
\left.- \frac{\phi(\lambda)}{1-\Phi(\lambda)}\left[\frac{-\lambda\phi(\lambda)}{1-\Phi(\lambda)} + \frac{\phi(\lambda)^2}{[1-\Phi(\lambda)]^2} - 1\right]\right\}\frac{\mathrm{d}\lambda}{\hat{y}_1},
$$

where we use $\phi'(\lambda) = -\lambda\phi(\lambda)$. Under the assumption that the hazard rate is small around ψ_1^*, we have:

$$
\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{y}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1} \approx 2\frac{b^4}{c^2}\hat{y}_1\tilde{\Sigma}_1. \tag{24}
$$

Substituting (23) and (24) into (22), we have:

$$
\left.\frac{\mathrm{d}\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2-\frac{1}{2}\hat{y}_2\right\}\Big|\,\psi_1>\psi_1^*\right]}{\mathrm{d}\hat{y}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1}
\approx \frac{b^2}{c}a\frac{b}{c}\left((1-\delta)\hat{y}_1+a\frac{b}{c}\hat{y}_1\right)
$$

$$
+\frac{b^2}{c}\left((1-\delta)\hat{y}_1+a\frac{b}{c}\hat{y}_1\right)\frac{\mathrm{d}K_1}{\mathrm{d}\hat{y}_1}\tilde{E}_0[\psi_1|\psi_1>\psi_1^*]
$$

$$
+\frac{b^2}{c}a\frac{b}{c}K_1\tilde{E}_0[\psi_1|\psi_1>\psi_1^*]
$$

$$
+\frac{b^2}{c}\left(\frac{b^2}{c}\hat{y}_1\right)K_1^2\tilde{E}_0[\psi_1|\psi_1>\psi_1^*]
$$

$$
+\frac{b^2}{c}K_1\frac{\mathrm{d}K_1}{\mathrm{d}\hat{y}_1}\tilde{E}_0[\psi_1^2|\psi_1>\psi_1^*]
$$

$$
+\frac{b^2}{c}K_1^2\frac{b^4}{c^2}\hat{y}_1\tilde{\Sigma}_1.
$$

Recall that we assume $ab/c\geq\delta$. From the proof of Proposition 1, the assumption that $b^4\hat{y}_1^2\hat{\Sigma}_1/c^2$ is either less than or not much larger than σ^2 means that $\mathrm{d}K_1/\mathrm{d}\hat{y}_1$ is either positive or small. And around $\hat{y}_1=\tilde{y}_1$, $\tilde{E}_0[\psi_1|\psi_1>\psi_1^*]$ is positive (because $\psi_1^*<0$ and $\tilde{E}_0[\psi_1]=0$). Therefore, all lines are either positive or small, which means that the derivative is positive. And it is easy to see by inspection that the principal's gain from raising $\hat{y}_1$ above $\tilde{y}_1$ increases in a under the given conditions and that this gain can be strictly positive when $a=0$. We have established the part of the proposition showing the principal's gain from raising the agent's talent estimate $\hat{y}_1$.

Now consider the principal's gain from increasing the variance $\hat{\Sigma}_1$ of the agent's talent estimate. Differentiating equation (14) with respect to $\hat{\Sigma}_1$ yields:

$$
\left.\frac{\mathrm{d}\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2-\frac{1}{2}\hat{y}_2\right\}\Big|\,\psi_1>\psi_1^*\right]}{\mathrm{d}\hat{\Sigma}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1}
=\frac{b^2}{c}\left((1-\delta)\hat{y}_1+a\frac{b}{c}\hat{y}_1\right)K_1\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1>\psi_1^*]}{\mathrm{d}\hat{\Sigma}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1}
$$

$$
+\frac{b^2}{c}\frac{1}{2}K_1^2\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1^2|\psi_1>\psi_1^*]}{\mathrm{d}\hat{\Sigma}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1}, \quad (25)
$$

where we write K_1 for $\tilde{K}_1=\hat{K}_1$.

Differentiating the conditional expectation given in the proof of Proposition 4, we have

$$
\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1>\psi_1^*]}{\mathrm{d}\hat{\Sigma}_1}=\left[\frac{-\lambda\phi(\lambda)}{1-\Phi(\lambda)}+\frac{\phi(\lambda)^2}{[1-\Phi(\lambda)]^2}\right]\frac{\mathrm{d}\psi_1^*}{\mathrm{d}\hat{\Sigma}_1}, \quad (26)
$$

where we use $\phi'(\lambda) = -\lambda\phi(\lambda)$ and $\mathrm{d}\lambda/\mathrm{d}\hat{\Sigma}_1 = [\mathrm{d}\psi_1^*/\mathrm{d}\hat{\Sigma}_1][\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2]^{-1/2}$. We saw in the proof of Proposition 1 that $\mathrm{d}\psi_1^*/\mathrm{d}\hat{\Sigma}_1 > 0$. We also saw in Proposition 1 that $\psi_1^* < 0$, which implies that $\lambda < 0$. Therefore $\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]/\mathrm{d}\hat{\Sigma}_1 > 0$.

Differentiating the other conditional expectation given in the proof of Proposition 4, we have

$$
\begin{aligned}
\frac{\mathrm{d}\tilde{E}_0[\psi_1^2|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{\Sigma}_1} &= \left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\left\{-\left(\frac{-\lambda\phi(\lambda)}{1-\Phi(\lambda)} + \frac{\phi(\lambda)^2}{[1-\Phi(\lambda)]^2}\right)\left[\frac{\phi(\lambda)}{1-\Phi(\lambda)} - \lambda\right]\right. \\
&\quad \left. -\frac{\phi(\lambda)}{1-\Phi(\lambda)}\left[\frac{-\lambda\phi(\lambda)}{1-\Phi(\lambda)} + \frac{\phi(\lambda)^2}{[1-\Phi(\lambda)]^2} - 1\right]\right\}\frac{\mathrm{d}\psi_1^*}{\mathrm{d}\hat{\Sigma}_1} \\
&= \left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\left[-2\frac{\phi(\lambda)}{1-\Phi(\lambda)} + \lambda\right]\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{\Sigma}_1} \\
&\quad + \left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\frac{\phi(\lambda)}{1-\Phi(\lambda)}\frac{\mathrm{d}\psi_1^*}{\mathrm{d}\hat{\Sigma}_1},
\end{aligned}
\tag{27}
$$

where we use $\phi'(\lambda) = -\lambda\phi(\lambda)$ and $\mathrm{d}\lambda/\mathrm{d}\hat{\Sigma}_1 = [\mathrm{d}\psi_1^*/\mathrm{d}\hat{\Sigma}_1][\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2]^{-1/2}$. Note that $\lambda = \psi_1^*/[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2]^{1/2}$ at $\hat{y}_1 = \tilde{y}_1$. Substitute equation (27), the expression for λ, and the definition of ψ_1^* from equation (12) into equation (25):

$$
\begin{aligned}
&\left.\frac{\mathrm{d}\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{\tilde{y}_2 - \frac{1}{2}\hat{y}_2\right\}\middle|\psi_1 > \psi_1^*\right]}{\mathrm{d}\hat{\Sigma}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1} \\
&= \frac{1}{2}\frac{b^2}{c}\left((1-\delta)\hat{y}_1 + a\frac{b}{c}\hat{y}_1\right)K_1\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{\Sigma}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1} \\
&\quad + \frac{1}{2}\frac{b^2}{c}K_1\sqrt{2\frac{c}{b^2}}z\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{\Sigma}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1} \\
&\quad + \frac{1}{2}\frac{b^2}{c}K_1^2\left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\frac{\phi(\lambda)}{1-\Phi(\lambda)}\frac{\mathrm{d}\psi_1^*}{\mathrm{d}\hat{\Sigma}_1} \\
&\quad - \frac{b^2}{c}K_1^2\left[\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1 + \sigma^2\right]^{1/2}\frac{\phi(\lambda)}{1-\Phi(\lambda)}\left.\frac{\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]}{\mathrm{d}\hat{\Sigma}_1}\right|_{\hat{y}_1=\tilde{y}_1,\hat{\Sigma}_1=\tilde{\Sigma}_1}.
\end{aligned}
$$

We saw in the proof of Proposition 1 that $\mathrm{d}\psi_1^*/\mathrm{d}\hat{\Sigma}_1 > 0$, and we saw above that $\mathrm{d}\tilde{E}_0[\psi_1|\psi_1 > \psi_1^*]/\mathrm{d}\hat{\Sigma}_1 > 0$. The first three lines on the right-hand side of the equation are therefore positive, and the final line is negative. Using equation (26), note that each of the first three lines on the right-hand side of the equation goes to zero with the hazard rate of ψ_1 around ψ_1^* and the fourth line goes to zero with the square of the hazard rate of ψ_1 around ψ_1^*.

Therefore we have that equation (25) is strictly positive for a small hazard rate of ψ_1 around ψ_1^*, and it is easy to see that it goes to zero as that hazard rate goes to zero. We have established the part of the proposition showing the principal's gain from raising the variance $\hat{\Sigma}_1$ of the agent's talent estimate.

$\square$

F Proof of Proposition 6

The proof of Proposition 5 showed that we need only consider how $\hat{\Sigma}_1$ affects the principal's expectation of period 2 value. Differentiating equation (14) with respect to $\hat{\Sigma}_1$ and letting the hazard rate of ψ_1 around ψ_1^* go to zero yields:

$$
\left. \frac{\mathrm{d}\tilde{E}_0\left[\frac{b^2}{c}\hat{y}_2\left\{ \tilde{y}_2 - \frac{1}{2}\hat{y}_2 \right\} \middle| \psi_1 > \psi_1^* \right]}{\mathrm{d}\hat{\Sigma}_1} \right|_{h(\psi_1^*)\approx 0}
$$

$$
= \frac{b^2}{c}(\hat{y}_1 - \tilde{y}_1)\left(\frac{b^2}{c}\hat{y}_1\tilde{K}_1 - (1-\delta) \right)\frac{\mathrm{d}\hat{K}_1}{\mathrm{d}\hat{\Sigma}_1}\tilde{E}_0[\psi_1 | \psi_1 > \psi_1^*]
$$

$$
= \frac{b^2}{c}(\hat{y}_1 - \tilde{y}_1)^2(1-\delta)^2 \frac{\frac{b^4}{c^2}\hat{y}_1^2}{\frac{b^4}{c^2}\hat{y}_1^2\hat{\Sigma}_1 + \sigma^2}\left\{ 1 - \frac{\frac{b^4}{c^2}\hat{y}_1^2\hat{\Sigma}_1}{\frac{b^4}{c^2}\hat{y}_1^2\hat{\Sigma}_1 + \sigma^2} \right\}\left\{ 1 - \frac{\frac{b^4}{c^2}\hat{y}_1^2\tilde{\Sigma}_1}{\frac{b^4}{c^2}\hat{y}_1^2\hat{\Sigma}_1 + \sigma^2} \right\}.
$$

This expression is strictly greater than zero if $\hat{y}_1 \neq \tilde{y}_1$. It is proportional to $(\hat{y}_1 - \tilde{y}_1)^2$.

$\square$