Estimating the Consequences of Climate Change from Variation in Weather*

Derek Lemoine

University of Arizona and NBER
McClelland Hall 401, 1130 E Helen St, Tucson, AZ, 85721-0108, USA
dlemoine@email.arizona.edu

University of Arizona Working Paper 18-09

November 2018

First version: August 2018

I formally relate the consequences of climate change to time series variation in weather. First, I show that the effects of climate change on adaptation investments can be bounded from below by estimating responses to weather outcomes. The bound becomes tighter when also estimating responses to forecasts. Second, I show that the marginal effect of climate change on long-run payoffs is identical to the average effect of transient weather events. Empirical work should begin estimating the average effect of weather within each climate, which differs from previous approaches.

JEL: D84, H43, Q54

Keywords: climate, weather, information, forecasts, expectations, adjustment, adaptation

*I thank Laura Bakkensen, Olivier Deschênes, Larry Karp, Ashley Langer, Emanuele Massetti, Antony Millner, Robert Pindyck, and Jeffrey Shrader for helpful comments. I also thank seminar participants at the University of Arizona, the University of California Berkeley, and the University of Massachusetts. A previous version circulated as "Sufficient Statistics for the Cost of Climate Change."

1 Introduction

A pressing empirical agenda seeks to estimate the economic costs of climate change. Ignorance of these costs has severely hampered economists' ability to give concrete policy recommendations (Pindyck, 2013). The challenge is that although variation in climate has been primarily cross-sectional, cross-sectional regressions cannot clearly identify the effects of climate.[1] Seeking credible identification, an explosively growing empirical literature has recently explored panel variation in weather.[2] The hope is that variation in transient weather identifies—or at worst bounds—the effects of a change in climate, which manifests itself through weather but differs from a transient weather shock in being repeated period after period and in affecting expectations of weather far out into the future.

I here undertake the first formal analysis that precisely delineates what and how we can learn about the climate from the weather. Linking weather to climate requires analyzing a dynamic model that can capture the distinction between transient and permanent changes in weather. I study an agent (equivalently, firm) who is exposed to stochastic weather outcomes. The agent chooses actions (equivalently, investments) that suit the weather, but adjusting actions from period to period is costly. When choosing actions, the agent knows the current weather, has access to specialized forecasts of the weather some arbitrary number of periods into the future, and relies on knowledge of the climate to generate forecasts at longer horizons. A change in the climate shifts the distribution of potential weather outcomes and alters the agent's expectations about future weather.

I show several novel results. First, I show that estimating the effects of weather on actions understates the long-run effect of climate on actions. Much empirical research has sought to estimate the consequences of climate change for decision variables or functions of decision variables, including productivity (Heal and Park, 2013; Zhang et al., 2018), health (Deschenes, 2014), crime (Ranson, 2014), and energy use (Auffhammer and Aroonruengsawat, 2011; Deschênes and Greenstone, 2011). Many economists have intuited that short-run adaptation responses to weather are likely to be smaller than long-run adaptation responses to climate (e.g., Deschênes and Greenstone, 2007). I show that the critical ingredient for this result is adjustment costs, not expectations of future weather. The actions an agent takes in response to a transient weather shock are constrained by the agent's desire to not change actions too much from period to period, but when the same weather shock is repeated period after period, even a myopic agent eventually achieves a larger change in activity through a sequence of incremental adjustments. I demonstrate that combining short-run adaptation responses to weather realizations with short-run adaptation responses to weather forecasts can better

[1] For many years, empirical analyses did rely on cross-sectional variation in climate to identify the economic consequences of climate change (e.g., Mendelsohn et al., 1994; Schlenker et al., 2005; Nordhaus, 2006). However, cross-sectional analyses fell out of favor due to concerns about omitted variables bias. See Dell et al. (2014) and Auffhammer (2018b) for expositions and Massetti and Mendelsohn (2018) for a review.

[2] For recent reviews, see Dell et al. (2014), Carleton and Hsiang (2016), and Heal and Park (2016). Blanc and Schlenker (2017) discuss the strengths and weaknesses of relying on panel variation in weather.

approximate long-run adaptation to climate. Further, I show that agents respond to forecasts only because they face adjustment costs. Estimating responses to forecasts therefore allows for a nice test: if actions are much less sensitive to forecasts than to weather and agents are patient over the forecasts' timescales, then adjustment costs may be small and responses to weather may approximate responses to climate.

Second, I show that the marginal effect of climate on steady-state expected payoffs is equal to the average treatment effect of weather in the current climate. Much empirical research has sought to estimate the consequences of climate change for flow payoffs such as profits (e.g., Deschênes and Greenstone, 2007) and for variables such as gross output or income that are potentially related to aggregate payoffs (e.g., Dell et al., 2012; Burke et al., 2015; Deryugina and Hsiang, 2017). I show that an easily estimated function of weather is a sufficient statistic for the impact of limited climate change on such variables.[3] This is a surprising and powerful result. Changing the climate is equivalent to changing expected weather in all future periods, yet transient weather shocks identify the marginal consequences of climate. The analysis implies that empirical work should bin locations by climate (e.g., by long-run average temperature) and estimate a single coefficient on weather (e.g., realized temperature) within each bin. The estimated coefficients describe the effect of marginally changing a location's climate on steady-state payoffs, and summing coefficients across bins describes the effect of nonmarginal climate change on steady-state payoffs. Time series variation therefore identifies the consequences of marginal changes in climate and cross-sectional variation identifies the consequences of nonmarginal changes in climate.[4] Care should be taken, however, in extrapolating to very large changes in climate. Estimating the consequences of such large changes will require pushing the available cross-sectional variation beyond the limits of credible identification and may simply be beyond the reach of reduced-form methods.

Figure 1 depicts the intuition underlying the average treatment effect result. Consider estimating the effect of temperature on agricultural profits, as in Deschênes and Greenstone (2007). Each solid curve in the left panel plots profits as a function of current inputs (such as labor and irrigation), conditional on growing season temperature being either typical or hot. Agents maximize profits by choosing inputs at the points labeled a and b. The dotted

[3] I describe the average treatment effect of weather as a sufficient statistic because multiple combinations of structural parameters can yield the same welfare consequences. Estimating the average treatment effect of weather does not recover all deep primitives but does provide a credibly identified estimate of marginal climate impacts (compare Chetty, 2009).

[4] The combination of panel and cross-sectional variation is similar in spirit to, for example, Auffhammer (2018a), except that the suggested approach estimates a coefficient on weather that can vary with the climate rather than estimating a coefficient on weather that varies with both the weather and the climate. (Deryugina and Hsiang (2017) estimate nonmarginal impacts in a different fashion, by allowing the effect of a weather realization to be nonlinear in its frequency.) The use of cross-sectional variation raises the usual concerns about identification. Results in the appendix suggest a sanity test: moving between climates should not have a stronger effect than do extreme weather events within the current climate.

line connecting points a and b then gives the effect on time t profits of time t temperature. Because profits are flat in inputs around point a, small changes in temperature do not have first-order effects on profits through input choices. This is the content of the envelope theorem, as applied by Deschênes and Greenstone (2007) and subsequent literature. If climate differs from weather only through beliefs that affect input choices, then the effects of climate are identified by the effects of transient weather shocks (Hsiang, 2016; Deryugina and Hsiang, 2017).

However, envelope theorem arguments miss the dynamics that distinguish climate from weather. Now imagine that changing inputs imposes adjustment costs, so that time t profits also depend on time $t-1$ inputs. A change in climate means that previous years were hot and subsequent years are also expected to be hot. If last year was hot, then last year's input choices reflect that outcome and it becomes less costly to choose high inputs this year. The dashed curve in the left panel of Figure 1 plots profits in a current hot year conditional on having already adjusted last year's input choices in response to last year's being hot. The inputs that maximize this year's profits increase to point c because they are less constrained by last year's choices. Now consider the implications of agents expecting the subsequent year $t+1$ to once again be hot. Applying more inputs at time t carries the dynamic benefit of reducing time $t+1$ adjustment costs. As a result, the dynamically optimal input choice is point d, where the marginal effect on this year's profit is negative but the marginal effect on expected intertemporal profits is zero (equation (2) below). The dotted line connecting points a and d then gives the change in profit corresponding to permanently increasing temperature. In line with intuition in Deschênes and Greenstone (2007), long-run adjustments potentially make the effects of a permanent change in weather less severe than the effects of a transient change in weather.

But how can we estimate the dotted line connecting points a and d? The right panel of Figure 1 again plots profits as a function of current inputs, but it holds current weather fixed between curves and instead varies only the previous year's input choices. The curve labeled "ss" depicts profits when the typical temperature has occurred many years in a row, so that previous inputs reached a steady state. The other two curves depict this year's profits under the typical temperature outcome but with higher ("H") and lower ("L") choices of inputs in the previous year. The adjustment costs imposed by these past choices constrain this year's choice of inputs and thereby reduce profits.

The dotted curve gives the effect on myopically optimized profits of changing last year's input choices. This curve has a peak at the myopically optimal labor input implied by curve "ss". Around this point (labeled 1), a permanent change in weather does not have first-order effects through past input choices. So the left panel's point c converges to point b. Now imagine that the agent expects the typical temperature to also occur next year. Because this year's input choices do not have first-order effects on next year's profits around point 1, the myopically optimal input choice is also dynamically optimal. So the left panel's point d converges to point c. Combining these results, line a-b converges to line a-d around point

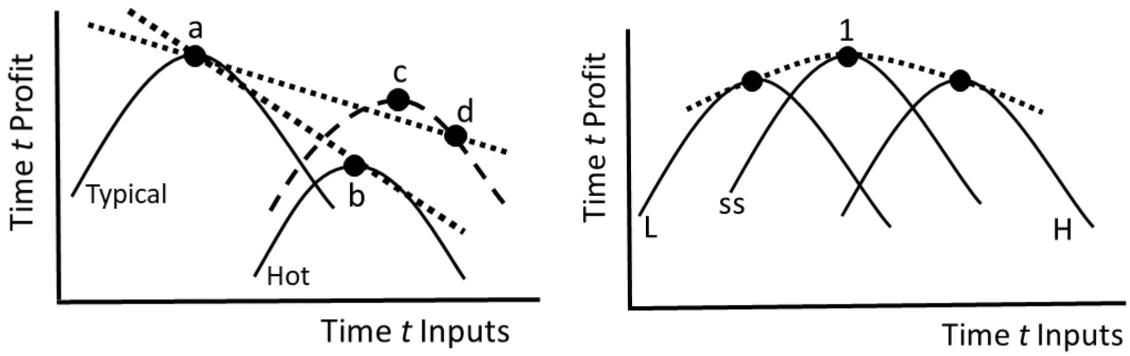

Figure 1: Left: Profits against inputs, conditional on temperature. Line a-b gives the effect on profits of increasing temperature from "typical" to "hot" in the absence of long-run adaptation. Line a-d accounts for adaptation to previous hot years and for expecting next year to again be hot. Right: Profits against inputs, conditional on past input choices. The curve labeled "ss" sets previous inputs to the steady state that would result if the current temperature were maintained forever.

1, so that the treatment effect of a transient weather shock indeed recovers the effect of permanently changing the weather. However, an econometrician may not know which observations in a data set are near a steady state. I show that averaging over potential previous input decisions and potential temperatures can center the estimated marginal effect of temperature around the steady-state inputs corresponding to a location's average temperature. Estimating the average treatment effect of temperature then recovers the effect of a marginal change in that location's climate.

Despite the importance of empirically estimating the costs of climate change and the sharpness of informal debates around the relevance of the recent empirical literature to climate change, there has been remarkably little formal analysis of the link between weather and climate. Previous formal analysis has consisted in appeals to the envelope theorem in static environments (Deschênes and Greenstone, 2007; Hsiang, 2016; Deryugina and Hsiang, 2017), but as described above, a static environment misses the distinction between transient and permanent weather shocks.[5] Envelope theorem intuition has led the literature (i) to of-

[5] A few other papers are also related. First, in an initial expositional analysis, I showed how envelope theorem arguments can fail in a three-period model (Lemoine, 2017). The present work precisely analyzes the consequences of climate change in an infinite-horizon model and constructively shows which types of empirical estimates can be informative about the climate. Second, Kelly et al. (2005) study the cost of having to learn about a change in the climate from an altered sequence of weather as opposed to knowing outright how the climate has changed. I here abstract from learning in order to focus on mechanisms more relevant to the growing empirical literature. Third, calibrated simulations have shown that dynamic responses are critical to the effects of climate on timber markets (Sohngen and Mendelsohn, 1998; Guo and Costello, 2013) and to the cost of increased cyclone risk (Bakkensen and Barrage, 2018). Finally, a few empirical papers have

ten ignore how the effects of transient weather shocks depend on a location's climate and (ii) to often treat the marginal effects of common and uncommon weather events as equally informative about climate change. Because of (i), most empirical literature pools the marginal effects of weather across units that reside in different climate zones, which conflates units for which a weather shock is rare with units for which a weather shock is common.[6] Because of (ii), some empirical literature (e.g., Deryugina and Hsiang, 2017) estimates how payoffs respond to additional days with each type of weather and then combines these estimates with scientific models' projections of how climate change will alter the frequency of each type of weather.[7] In the appendix, I show that this estimator overstates the cost of marginal climate change by capturing the nonlinear consequences of transient weather shocks, which I also show have little bearing on the effects of climate change.

The next section describes the setting. Section 3 solves the dynamic programming problem. Sections 4 and 5 analyze the effects of climate on agents' chosen actions and payoffs, respectively. The final section discusses limitations of the present analysis. The appendix contains additional results, generalizes the analysis, and provides proofs.

2 Setting

An agent is repeatedly exposed to stochastic weather outcomes. The realized weather in period t is w_t. This weather realization imposes two types of costs. A first type of cost arises independently of any actions the agent might take. These unavoidable costs are $\frac{1}{2}\psi(w_t - \bar{w})^2$, where the parameter $\bar{w}$ defines the weather outcome that minimizes unavoidable costs and the parameter $\psi \geq 0$ determines the costliness of any other weather outcome. A second type of cost depends on the agent's actions A_t. These avoidable costs are $\frac{1}{2}\gamma(A_t - w_t)^2$, where $\gamma \geq 0$. They vanish when the agent's actions are well-matched to the weather and potentially become large when the agent's actions are poorly matched to the weather.

In each period, the agent chooses her action A_t. This action may be interpreted as a level of activity (e.g., time spent outdoors, energy used for heating or cooling, irrigation applied to a field) or as a stock of capital (e.g., outdoor gear, size or efficiency of furnace, number or efficiency of irrigation lines). The agent's actions impose two types of costs. First, maintaining A_t imposes costs of $\frac{1}{2}\phi(A_t - \bar{A})^2$, where $\phi \geq 0$. When A_t represents a capital stock, these maintenance costs reflect depreciation. The parameter $\bar{A}$ defines the level of

demonstrated that actions respond to forecasts of future weather (e.g., Neidell, 2009; Rosenzweig and Udry, 2013, 2014; Wood et al., 2014; Shrader, 2017).

[6] Some empirical literature has begun estimating how the effects of weather shocks vary with a location's climate, as summarized in Auffhammer (2018b). The appendix discusses which estimates in Deschênes and Greenstone (2007) come closest to the theoretically recommended approach.

[7] This two-step strategy has also become the dominant approach to estimating the effects of climate on actions (see Carleton and Hsiang, 2016). In the appendix, I show that these estimates do indirectly use some of the information available from responses to forecasts.

activity or capital that is cheapest to sustain. Second, the agent faces a cost of adjusting actions from one period to the next. This cost is $\frac{1}{2}\alpha(A_t - A_{t-1})^2$, where $\alpha \geq 0$. When A_t represents a capital stock, these adjustment costs are investment costs. Relating to the literature on climate adaptation (e.g., Fankhauser et al., 1999; Mendelsohn, 2000), small adjustment costs allow adaptation investments to occur after weather is realized ("reactive" or "ex-post" adaptation), but large adjustment costs require adaptation to occur before weather is realized ("anticipatory" or "ex-ante" adaptation). Maintenance costs make the agent want to choose actions close to $\bar{A}$, and adjustment costs make the agent want to keep actions constant over time.[8]

The agent observes time t weather before selecting her time t action. The agent has access to specialized forecasts of future weather and knows her region's climate, indexed by C and which I will often interpret as temperature. Specialized forecasts extend up to $N \geq 0$ periods ahead. Each period's forecast is an unbiased predictor of later weather. Beyond horizon N, the agent formulates generic forecasts that rely only on knowledge of the climate, not on information germane to that particular time period. For instance, the agent may rely on the local news to predict weather one week out and on forecasts of El Niño conditions to predict weather six months out but relies on knowledge of typical weather to predict weather one year out. Horizon N is therefore the shortest forecast horizon at which the agent receives information beyond knowledge of the climate.

Formally, let f_{it} be the i-period-ahead forecast available in period t. The time t weather realization is a random deviation from the one-period-ahead forecast: $w_t = f_{1(t-1)} + \epsilon_t$, where ϵ_t has mean zero and variance σ^2. Because forecasts are unbiased predictors, any changes in forecasts must be unanticipated: for $i \in \{1, ..., N\}$, $f_{it} = f_{(i+1)(t-1)} + \nu_{it}$, where ν_{it} has mean zero and variance τ_i^2. Forecasts at horizons $i > N$ are $f_{it} = C$.[9] The ν_{it} and ϵ_t are serially uncorrelated, the covariance between ν_{it} and ν_{jt} is δ_{ij}, and the covariance between ϵ_t and ν_{it} is ρ_i.[10] Note that $E_t[w_{t+j}] = f_{jt}$. For notational convenience, collect all specialized forecasts available at time t in a vector F_t of length N.[11]

[8] The general analysis in the appendix does not require allocating either costs or weather impacts in this fashion and allows, among much else, $\bar{A}$ to vary with w_t and A_{t-1} to affect time t payoffs directly.

[9] One might be concerned about a sharp discontinuity in information at horizon N. However, I have left the variances τ_i^2 general. Defining them to decrease in i and to approach zero as i approaches N would allow for the informativeness of the signal about time t weather to increase smoothly from long horizons to short horizons.

[10] Assuming that each shock is serially uncorrelated does not imply that weather and forecasts are serially uncorrelated. For instance, for $t > N$, $Cov_0(w_t, w_{t+1}) = \rho_1 + \sum_{i=1}^{N-1} \delta_{i(i+1)}$.

[11] Climate here controls average weather. One might wonder about the dependence of higher moments of the weather distribution on climate. In fact, the effects of climate change on the variance of the weather are poorly understood and spatially heterogeneous (e.g., Huntingford et al., 2013; Lemoine and Kapnick, 2016). Further, we need to know not just how climate change affects the variance of realized weather but how it affects the forecastability of weather at each horizon: the variance of the weather more than N periods ahead is $\sigma^2 + \sum_{i=1}^{N} \tau_i^2$, so we need to apportion any change in variance between σ^2 and each τ_i^2. The appendix analyzes the consequences of a change in variance and connects these consequences to empirical strategies.

The agent maximizes the present value of payoffs over an infinite horizon. Time t payoffs are:

$$\pi(A_t, A_{t-1}, w_t) = -\frac{1}{2}\gamma(A_t - w_t)^2 - \frac{1}{2}\alpha(A_t - A_{t-1})^2 - \frac{1}{2}\phi(A_t - \bar{A})^2 - \frac{1}{2}\psi(w_t - \bar{w})^2.$$

She chooses time t actions as a function of past actions, current weather, and current forecasts. In order to study an interesting problem, assume that $\gamma + \phi > 0$. The agent solves:

$$\max_{\{A_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E_0 \left[\pi(A_t, A_{t-1}, w_t)\right],$$

where $\beta \in [0, 1)$ is the per-period discount factor, A_{-1} is given, and E_0 denotes expectations at the time 0 information set. The solution satisfies the following Bellman equation:

$$\begin{aligned} V(Z_t, w_t, F_t) =& \max_{A_t} \left\{ \pi(A_t, Z_t, w_t) + \beta E_t \left[V(Z_{t+1}, w_{t+1}, F_{t+1})\right] \right\} \quad (1)\\ \text{s.t.} \quad Z_{t+1} =& A_t \\ w_{t+1} =& f_{1t} + \epsilon_{t+1} \\ f_{i(t+1)} =& f_{(i+1)t} + \nu_{i(t+1)} \quad \text{for } i \in \{1, ..., N\} \\ f_{N(t+1)} =& C + \nu_{N(t+1)} \quad \text{if } N > 0. \end{aligned}$$

The state variable Z_t captures the previous period's actions. Optimal actions satisfy the first-order condition:

$$\frac{\partial \pi(A_t, Z_t, w_t)}{\partial A_t} = -\beta E_t \left[\frac{\partial V(Z_{t+1}, w_{t+1}, F_{t+1})}{\partial Z_{t+1}} \right]. \quad (2)$$

When the right-hand side is nonzero, the myopically optimal point c differs from the dynamically optimal point d in Figure 1.

The setting is sufficiently general to describe many applications of interest. For instance, much empirical literature has studied the effects of weather on energy use. The agent could then be choosing indoor temperature in each period, where maintenance costs reflect energy use and avoidable weather costs reflect thermal comfort. Empirical literature has also studied the effect of weather on agricultural profits. The decision variable could then be irrigation, labor, fertilizer, or crop varieties, maintenance costs reflect the cost of purchasing these in each year, adjustment costs reflect the cost of changing equipment and plans from year to year, and weather costs reflect the deviation in crop yields from their maximum possible value.

The primary specialization in the setting is the assumption of quadratic payoffs. Linear-quadratic models have long been workhorses in economic research because they allow for explicit analytic solutions to the Bellman equation (1). In the appendix, I instead use perturbation methods (Judd, 1996) to generalize the analysis to an arbitrary functional form for $\pi(A_t, A_{t-1}, w_t)$, to vector-valued actions, and to multi-dimensional weather indices.

3 Solution

The following proposition describes the value function that solves equation (1):

Proposition 1. *The value function $V(Z_t, w_t, F_t)$ has the form:*

$$a_1 Z_t^2 + a_2 w_t^2 + \sum_{i=1}^{N} a_3^i f_{it}^2 + b_1 Z_t w_t + \sum_{i=1}^{N} b_2^i Z_t f_{it} + \sum_{i=1}^{N} b_3^i w_t f_{it} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij} f_{it} f_{jt} + c_1 Z_t + c_2 w_t + \sum_{i=1}^{N} c_3^i f_{it} + d.$$

Optimal actions are:

$$A_t^* = \frac{\alpha A_{t-1} + \gamma w_t + \beta b_1 f_{1t} + \beta \sum_{i=1}^{N-1} b_2^i f_{(i+1)t} + \beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1}. \tag{3}$$

The coefficients are as follows:

1. *$a_1 \leq 0$, with $a_1 < 0$ if and only if $\alpha > 0$.*
2. *$a_2 \leq 0$, with $a_2 < 0$ if and only if $\psi + \gamma(\phi + \alpha) > 0$.*
3. *$a_3^i \in [\beta^i a_2, 0]$, with $a_3^i < 0$ if and only if both $a_2 < 0$ and $\alpha\beta > 0$ and with $a_3^i > \beta^i a_2$ if and only if $\beta\alpha\gamma > 0$.*
4. *Each of the b coefficients is positive, with $b_1 > 0$ if and only if $\alpha\gamma > 0$ and $b_2^i, b_3^i, b_4^{ij} > 0$ if and only if $\beta\alpha\gamma > 0$.*
5. *$c_1 \geq (\leq)\ 0$ if C is sufficiently large (small), and $c_2, c_3^i \geq (\leq)\ 0$ if, in addition, $\bar{w} \geq (\leq)\ 0$.*
6. *Each a and b coefficient is independent of C.*
7. *Each c coefficient weakly increases in C, and each c coefficient strictly increases in C if and only if $\beta\alpha\gamma > 0$.*

Proof. See appendix. □

The value function is concave in previous actions ($a_1 \leq 0$), in weather outcomes ($a_2 \leq 0$), and in forecasts ($a_3^i \leq 0$). If $\beta\alpha\gamma > 0$, then each a and b coefficient is nonzero. Several coefficients depend on C, reflecting how climate controls the agent's beliefs about long-run weather. I henceforth omit the asterisk on A_t^* when clear.

4 Effect of Climate on Actions

Now consider how climate change affects the agent's actions, which is of direct relevance to much empirical work and produces results that we will use to analyze the effect of climate on payoffs. Define $\hat{A}_t \triangleq E_0[A_t]$. From equation (3),

$$\hat{A}_t = \frac{\alpha \hat{A}_{t-1} + \gamma C + \beta b_1 C + \beta \sum_{i<N} b_2^i C + \beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1}$$

for $t > N$. The following proposition describes long-run behavior:

Proposition 2. *As $t \to \infty$, $\hat{A}_t \to \frac{\gamma}{\gamma+\phi} C + \frac{\phi}{\gamma+\phi} \bar{A} \triangleq A^{ss}$.*

Proof. See appendix. □

Expected actions converge to a steady state, denoted A^{ss}. This steady-state expected action is a weighted average of the action that minimizes expected weather impacts and the action that minimizes maintenance costs. Steady-state policy fully offsets the avoidable portion of expected weather impacts (determined by the climate C) when there are no maintenance costs ($\phi = 0$), but steady-state policy becomes unresponsive to the climate as marginal maintenance costs become large relative to marginal avoidable weather costs (as ϕ becomes large relative to γ). Adjustment costs slow the approach to the steady-state expected action, but they do not affect its level.

From Proposition 2, an increase in the climate index affects steady-state expected actions as

$$\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} = \frac{\gamma}{\gamma + \phi} \in [0, 1].$$

As $\gamma \to 0$, there are no avoidable weather impacts, and as $\phi \to \infty$, maintenance costs are too large to justify changing actions on the basis of the climate. In either case, $\mathrm{d}A^{ss}/\mathrm{d}C \to 0$. Steady-state actions otherwise strictly increase with the climate index. But this increase is less than one-for-one when $\phi > 0$: adaptation is less than perfect when maintenance costs deter the agent from fully offsetting the change in climate.

Now consider how we might estimate $\mathrm{d}A^{ss}/\mathrm{d}C$ from data. Reduced-form empirical models can estimate the derivatives $\partial A_t/\partial w_t$ and $\partial A_t/\partial f_{it}$ by regressing observed A_t on weather and forecasts.[12] Imagine that empirical researchers were to then approximate the effect of climate change as

$$\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} \approx \frac{\partial A_t}{\partial w_t} + \sum_{i=1}^{j} \frac{\partial A_t}{\partial f_{it}}, \tag{4}$$

[12]Note that the estimation equation should include either A_{t-1} or time $t-1$ forecasts: time $t-1$ actions can directly affect time t actions (see equation (3)), and the dependence of time $t-1$ actions on time $t-1$ forecasts makes them correlated with time t weather and forecasts. The appendix derives a related omitted variables bias from ignoring time t forecasts.

for $j \in \{0, ..., N\}$. For $\mathrm{d}A^{ss}/\,\mathrm{d}C > 0$ (i.e., for $\gamma > 0$), the bias from this approximation as a fraction of the true effect is

$$Bias(j) = \frac{\frac{\partial A_t}{\partial w_t} + \sum_{i=1}^{j} \frac{\partial A_t}{\partial f_{it}}}{\frac{\mathrm{d}A^{ss}}{\mathrm{d}C}} - 1.$$

$Bias(0)$ is the bias from using only $\partial A_t/\partial w_t$, and $Bias(N)$ is the bias when also using all available forecasts. The approximation underestimates $\mathrm{d}A^{ss}/\,\mathrm{d}C$ if and only if $Bias(j) < 0$ and correctly estimates $\mathrm{d}A^{ss}/\,\mathrm{d}C$ if and only if $Bias(j) = 0$. The following proposition establishes several results about this bias:

Proposition 3. *Assume $\gamma > 0$. Then:*

1. *$Bias(j) \in (-1, 0]$, with $Bias(j) < 0$ if and only if $\alpha > 0$.*
2. *$\frac{\mathrm{d}Bias(j)}{\mathrm{d}j} \geq 0$, $\frac{\mathrm{d}Bias(j)}{\mathrm{d}N} = 0$.*
3. *$\frac{\mathrm{d}Bias(j)}{\mathrm{d}j} \to 0$ as $\beta \to 0$.*
4. *$Bias(j) \to \frac{-\alpha}{\gamma+\alpha+\phi-2\beta a_1}$ as $j, N \to \infty$.*
5. *$\partial A_t/\partial w_t \to 0$, $\partial A_t/\partial f_{it} \to 0$, and $Bias(j) \to -1$ as $\alpha \to \infty$.*
6. *$\mathrm{d}A^{ss}/\,\mathrm{d}C \to 1$ and $Bias(j) \to 0$ as $\gamma \to \infty$.*
7. *$\partial A_t/\partial w_t, \partial A_t/\partial f_{it}$, $\mathrm{d}A^{ss}/\,\mathrm{d}C \to 0$ as either $\gamma \to 0$ or $\phi \to \infty$.*

Proof. See appendix. □

The approximation in (4) never overestimates $\mathrm{d}A^{ss}/\,\mathrm{d}C$ ($Bias(j) \leq 0$, result 1), and it underestimates $\mathrm{d}A^{ss}/\,\mathrm{d}C$ whenever there are nonzero adjustment costs ($\alpha > 0$). The quality of the approximation improves when we include the effects of forecasts in addition to the effects of weather shocks ($\mathrm{d}Bias(j)/\,\mathrm{d}j \geq 0$, result 2), because a weather shock that also affects forecasts is less transient. However, nonzero bias remains even when estimating responses to forecasts at arbitrarily long horizons (i.e., even as $j, N \to \infty$, result 4): the response to current weather and to information about future weather cannot capture how incremental adjustments accumulate over time. The accumulation of incremental adjustments also generates nonzero bias even when agents are myopic.

The bias vanishes in a few special cases. First, as adjustment costs vanish ($\alpha \to 0$, result 1), actions adjust instantaneously to realized weather, so neither expectations nor the slow accrual of incremental adjustments matters for steady-state actions. Second, as avoidable weather impacts become infinitely costly ($\gamma \to \infty$, result 6), the agent tries to exactly match A_t to w_t in every period, regardless of adjustment costs or maintenance costs. Third, when there are no avoidable weather impacts ($\gamma \to 0$, result 7) or maintenance costs are prohibitive ($\phi \to \infty$, result 7), actions become completely insensitive to the climate and also to realized weather and forecasts. In all other cases, the bias is nonzero and becomes large as adjustment costs become large.

Finally, we also see two cases in which $Bias(j) < 0$ but including the effects of forecasts does not improve the quality of the approximation in (4): $\mathrm{d}Bias(j)/\,\mathrm{d}j \to 0$ as either $\beta \to 0$ (result 3) or $\alpha \to \infty$ (result 5).[13] The reason is that actions are not sensitive to forecasts in these cases.[14] First, forecasts enable the agent to take actions that improve future payoffs, but when agents are myopic, they act for the present only. Second, as adjustment costs become very large, agents barely adjust actions on the basis of forecasts. The steady state will change due to the accumulation of tiny changes over a long time horizon, but these effects will not be detectable from responses to forecasts.

5 Effect of Climate on Value

Now consider the expected effect of climate change on intertemporal value and per-period payoffs. From Proposition 1, we have:

$$\begin{aligned} V(Z_t, w_t, F_t) =& V(A^{ss}, C, \boldsymbol{C}) \\ & + [Z_t - A^{ss}]V_Z(A^{ss}, C, \boldsymbol{C}) + [w_t - C]V_w(A^{ss}, C, \boldsymbol{C}) + \sum_{i=1}^{N}[f_{it} - C]V_{f_i}(A^{ss}, C, \boldsymbol{C}) \\ & + [Z_t - A^{ss}]^2 a_1 + [w_t - C]^2 a_2 + \sum_{i=1}^{N}[f_{it} - C]^2 a_3^i + [Z_t - A^{ss}][w_t - C]b_1 \\ & + \sum_{i=1}^{N}[Z_t - A^{ss}][f_{it} - C]b_2^i + \sum_{i=1}^{N}[w_t - C][f_{it} - C]b_3^i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}[w_t - C][f_{it} - C]b_4^{ij}, \end{aligned}$$

where $\boldsymbol{C}$ is an $N \times 1$ vector with all entries equal to C. The envelope theorem and the fact that $\partial\pi(A_t, A_{t-1}, w_t)/\partial A_{t-1} = 0$ around a steady state imply $V_Z(A^{ss}, C, \boldsymbol{C}) = 0$. The

[13] In addition, $\mathrm{d}Bias(j)/\,\mathrm{d}j = 0$ if $\alpha = 0$ because, from result 1 in Proposition 3, $\alpha = 0$ implies that $Bias(j) = 0$ for all j.

[14] From Proposition 1, $\partial A_t/\partial f_{it} \to 0$ as $\beta \to 0$ and, using the solutions for a_1 and b_1 given in the proof, also as $\alpha \to \infty$.

expectation at time 0 of $V(Z_t, w_t, F_t)$ at some future time $t > N$ is:

$$\begin{aligned} E_0[V(Z_t, w_t, F_t)] =& V(A^{ss}, C, \boldsymbol{C}) + E_0[(A_t - A^{ss})^2]a_1 + \sigma^2 a_2 + \sum_{i=1}^{N} \tau_i^2 a_3^i + Cov_0[Z_t, w_t]b_1 \\ &+ \sum_{i=1}^{N} Cov_0[Z_t, f_{it}]b_2^i + \sum_{i=1}^{N} Cov_0[w_t, f_{it}]b_3^i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} Cov_0[w_t, f_{it}]b_4^{ij}. \end{aligned} \tag{5}$$

Recalling from Proposition 1 that each a and b coefficient is independent of C, and recognizing that each covariance is independent of C,[15] we have:

$$\frac{\mathrm{d}E_0[V(Z_t, w_t, F_t)]}{\mathrm{d}C} = \underbrace{\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C}}_{\text{change in ss value}} + \underbrace{2a_1 E_0\left[(Z_t - A^{ss})\left(\frac{\mathrm{d}Z_t}{\mathrm{d}C} - \frac{\mathrm{d}A^{ss}}{\mathrm{d}C}\right)\right]}_{\text{change in transition value}}.$$

We see two components to the expected change in value due to climate change: the change in steady-state value and the change in value along the transition to the steady state.[16]

The next proposition signs the change in transition value:

Proposition 4. *If $\alpha\gamma > 0$, then $\frac{\mathrm{d}E_0[V(Z_t,w_t,F_t)]}{\mathrm{d}C} < \frac{\mathrm{d}V(A^{ss},C,\boldsymbol{C})}{\mathrm{d}C}$ if and only if $A_0 < A^{ss}$. $\frac{\mathrm{d}E_0[V(Z_t,w_t,F_t)]}{\mathrm{d}C} \to \frac{\mathrm{d}V(A^{ss},C,\boldsymbol{C})}{\mathrm{d}C}$ as $\alpha \to 0$, as $\gamma \to 0$, as $t \to \infty$, or as $A_0 \to A^{ss}$.*

Proof. See appendix. □

The transition to a warmer climate imposes costs over and above the change in steady-state value when $A_0 < A^{ss}$ but provides benefits over and above the change in steady-state value when $A_0 > A^{ss}$. When $A_0 < A^{ss}$, the agent is in the process of approaching A^{ss} from below. We already saw that A^{ss} increases in C. Increasing C moves the steady state further away from the current state and therefore requires even more adjustment from the agent. However, when the agent is approaching A^{ss} from above, raising C reduces the total adjustment that the agent will have to undertake before reaching the steady state.

It is reasonable to believe that agents in warmer climates may be approaching their steady-state investment level from below (e.g., by installing air conditioning) and that agents in colder climates may be approaching their steady-state investment level from above (e.g., by installing insulation). We should then expect the cost of adjusting to a warmer climate to be positive in regions with warmer climates and negative in regions with cooler climates. Further, we should expect transition costs (or savings) to be larger in regions that are not

[15] Observe from Proposition 1 that A_t is separable in C, w_t, and f_{it}, and observe that the stochastic terms in w_t and f_{it} are independent of C. Therefore each covariance in equation (5) is independent of C.

[16] Tol et al. (1998) informally draw a similar distinction.

as far along the process of adapting to their baseline climate, whether because these regions have lower incomes, were settled only recently, or have outdated capital stock.

Now consider how climate change affects steady-state value. Using Proposition 1, we have:

$$\begin{aligned}\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C} =& V_w(A^{ss}, C, \boldsymbol{C}) + \sum_{i=1}^{N} V_{f_i}(A^{ss}, C, \boldsymbol{C}) \\ &+ \frac{\mathrm{d}c_1}{\mathrm{d}C} A^{ss} + \frac{\mathrm{d}c_2}{\mathrm{d}C} C + \sum_{i=1}^{N} \frac{\mathrm{d}c_3^i}{\mathrm{d}C} C + \frac{\mathrm{d}d}{\mathrm{d}C}.\end{aligned} \tag{6}$$

The first line recognizes that a change in climate alters average weather and average forecasts. The second line arises because agents anticipate that climate change is permanent: climate change therefore alters the value function itself, beyond altering realized weather and forecasts. For instance, a permanent change in climate can make past adaptation investments more valuable (Proposition 1 showed that $\mathrm{d}c_1/\mathrm{d}C \geq 0$) and can make higher weather outcomes more valuable (or less painful) because they are closer to average weather (Proposition 1 showed that $\mathrm{d}c_2/\mathrm{d}C \geq 0$).

The following proposition describes the net effects of climate change on steady-state value:

Proposition 5.

$$\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C} = \frac{1}{1-\beta} \frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C} = \frac{1}{1-\beta} \left[\frac{\gamma\phi}{\gamma+\phi}(\bar{A} - C) + \psi(\bar{w} - C) \right]. \tag{7}$$

Proof. See appendix. □

Value increases in the climate index if and only if C is sufficiently small. The change in steady-state value is equal to the change in steady-state per-period payoffs, valued as a perpetuity. The first term in brackets reflects the change in the cost of maintaining the adaptation investments chosen for this climate. When the climate is sufficiently cold, a warmer climate may justify investments that require less maintenance, but as the climate becomes sufficiently warm, eventually the chosen investments require more upkeep. This term vanishes as either maintenance costs vanish ($\phi \to 0$) or as the link between actions and weather is broken ($\gamma \to 0$). The second term in brackets reflects the changing cost of unavoidable weather impacts. This term makes a warmer climate valuable when $C < \bar{w}$ but makes a warmer climate costly when $C > \bar{w}$. This term vanishes when weather outcomes impose no unavoidable costs ($\psi \to 0$).

A rapidly growing empirical literature hopes to estimate the cost of climate change from time series variation in weather. From Proposition 1, the marginal effect of weather on value is:

$$\frac{\partial V(Z_t, w_t, F_t)}{\partial w_t} = 2a_2 w_t + b_1 Z_t + \sum_{i=1}^{N} b_3^i f_{it} + c_2.$$

If we average the marginal effect of weather over many observations in a given climate and assume that actions are, on average, close to the steady-state level A^{ss} (as when a location is well-adapted to its current climate), then we obtain the following average treatment effect of weather on value:[17]

$$ATE_w^V(C) \triangleq E_0\left[\frac{\partial V(Z_t, w_t, F_t)}{\partial w_t}\right] = 2a_2 C + b_1 A^{ss} + \sum_{i=1}^{N} b_3^i C + c_2,$$

for $t > N$. Proceeding analogously, we have the average treatment effect of weather on payoffs around a steady state as

$$ATE_w^\pi(C) \triangleq E_0\left[\frac{\mathrm{d}\pi(A_t, A_{t-1}, w_t)}{\mathrm{d}w_t}\right] = E_0\left[\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right],$$

using that $E_0[\partial\pi(A_t, A_{t-1}, w_t)/\partial A_t] = E_0[\partial\pi(A_t, A_{t-1}, w_t)/\partial A_{t-1}] = 0$ around A^{ss}. The next proposition relates these average treatment effects to the marginal effect of climate:

Proposition 6.

$$\frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C} = ATE_w^V(C) = ATE_w^\pi(C)$$

Proof. See appendix. □

This is a surprising result: once all adjustments are complete, the expected change in per-period steady-state payoffs due to a change in climate is identical to the average change in payoffs estimated from weather events around a steady state.[18] The appendix shows that the same result holds for general, non-quadratic payoff functions as long as (i) $\partial\pi(A_t, A_{t-1}, w_t)/\partial A_{t-1} = 0$ at $A_t = A_{t-1}$ and (ii) σ^2 and each τ_i^2 are not too large. When (i) holds (as it does in the main text and in the right panel of Figure 1), the effects of climate on past actions becomes irrelevant for steady-state payoffs and the dynamically optimal action converges to the myopically optimal action, in which case the envelope theorem concludes that the effect of climate on current actions also becomes irrelevant for steady-state payoffs. And when either (ii) holds or payoffs are quadratic, the average treatment effect of weather is approximately linear and thus equivalent to the treatment effect of average weather. The result follows from recognizing that average weather defines the climate.

[17] Relating to the Rubin causal model, the potential outcomes are the realizations of $\partial V/\partial w_t$ if A_{t-1}, w_t, and F_t took on different values.

[18] Further, the appendix shows that the average treatment effect of forecasts can identify the discount factor β and thus yield $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\,\mathrm{d}C$ from Proposition 5.

6 Limitations

I have demonstrated how to estimate the effects of climate change from time series variation in weather. The setting is fairly general, and the appendix generalizes it further. Nonetheless, the results are subject to three main caveats.

First, the present setting omits constraints that could make the short-run effects of weather shocks less severe than the long-run effects of permanently changing the climate. In particular, some have argued that short-run adjustments could be greater than long-run adjustments because some actions may not be sustainable indefinitely (e.g., Fisher et al., 2012; Blanc and Schlenker, 2017; Auffhammer, 2018b), such as water withdrawals from a reservoir. Future work could explore such possibilities by imposing constraints on cumulative deviations in actions from some benchmark value.

Second, the present setting successfully captures the distinction between transient and permanent changes in weather, but global climate change also differs from most weather shocks in its spatial structure. A change in global climate affects weather in every location and thus will have general equilibrium consequences. The present setting has followed most empirical work in abstracting from such effects, but some recent empirical work has begun exploring the implications of changing the weather in many locations simultaneously (e.g., Costinot et al., 2016; Dingel et al., 2018; Gouel and Laborde, 2018).

Finally, the present analysis has held the payoff function constant over time. However, climate change should induce innovations that alter how weather affects payoffs. Some historical studies have begun exploring the interaction between climate and agricultural innovation (e.g., Olmstead and Rhode, 2011; Roberts and Schlenker, 2011). Future work should consider approaches to bounding the scope for innovation.

References

Auffhammer, Maximilian (2018a) "Climate adaptive response estimation: Short and long run impacts of climate change on residential electricity and natural gas consumption using big data," Working Paper 24397, National Bureau of Economic Research.

——— (2018b) "Quantifying economic damages from climate change," *Journal of Economic Perspectives*, Vol. 32, No. 4, pp. 33–52.

Auffhammer, Maximilian and Anin Aroonruengsawat (2011) "Simulating the impacts of climate change, prices and population on California's residential electricity consumption," *Climatic Change*, Vol. 109, No. 1, pp. 191–210.

Bakkensen, Laura and Lint Barrage (2018) "Climate shocks, cyclones, and economic growth: Bridging the micro-macro gap," Working Paper 24893, National Bureau of Economic Research.

Blanc, Elodie and Wolfram Schlenker (2017) "The use of panel models in assessments of climate impacts on agriculture," *Review of Environmental Economics and Policy*, Vol. 11, No. 2, pp. 258–279.

Burke, Marshall, Solomon M. Hsiang, and Edward Miguel (2015) "Global non-linear effect of temperature on economic production," *Nature*, Vol. 527, pp. 235–239.

Carleton, Tamma A. and Solomon M. Hsiang (2016) "Social and economic impacts of climate," *Science*, Vol. 353, No. 6304, p. aad9837.

Chetty, Raj (2009) "Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods," *Annual Review of Economics*, Vol. 1, No. 1, pp. 451–488.

Costinot, Arnaud, Dave Donaldson, and Cory Smith (2016) "Evolving comparative advantage and the impact of climate change in agricultural markets: Evidence from 1.7 million fields around the world," *Journal of Political Economy*, Vol. 124, No. 1, pp. 205–248.

Dell, Melissa, Benjamin F. Jones, and Benjamin A. Olken (2012) "Temperature shocks and economic growth: Evidence from the last half century," *American Economic Journal: Macroeconomics*, Vol. 4, No. 3, pp. 66–95.

——— (2014) "What do we learn from the weather? The new climate-economy literature," *Journal of Economic Literature*, Vol. 52, No. 3, pp. 740–798.

Deryugina, Tatyana and Solomon Hsiang (2017) "The marginal product of climate," Working Paper 24072, National Bureau of Economic Research.

Deschenes, Olivier (2014) "Temperature, human health, and adaptation: A review of the empirical literature," *Energy Economics*, Vol. 46, pp. 606–619.

Deschênes, Olivier and Michael Greenstone (2007) "The economic impacts of climate change: Evidence from agricultural output and random fluctuations in weather," *American Economic Review*, Vol. 97, No. 1, pp. 354–385.

——— (2011) "Climate change, mortality, and adaptation: Evidence from annual fluctuations in weather in the US," *American Economic Journal: Applied Economics*, Vol. 3, No. 4, pp. 152–185.

Dingel, Jonathan I., Kyle C. Meng, and Solomon M. Hsiang (2018) "The spatial structure of productivity, trade, and inequality: Evidence from the global climate," working paper.

Fankhauser, Samuel, Joel B. Smith, and Richard S. J. Tol (1999) "Weathering climate change: some simple rules to guide adaptation decisions," *Ecological Economics*, Vol. 30, No. 1, pp. 67–78.

Fisher, Anthony C., W. Michael Hanemann, Michael J. Roberts, and Wolfram Schlenker (2012) "The economic impacts of climate change: Evidence from agricultural output and random fluctuations in weather: Comment," *American Economic Review*, Vol. 102, No. 7, pp. 3749–3760.

Gouel, Christophe and David Laborde (2018) "The crucial role of international trade in adaptation to climate change," Working Paper 25221, National Bureau of Economic Research.

Guo, Christopher and Christopher Costello (2013) "The value of adaption: Climate change and timberland management," *Journal of Environmental Economics and Management*, Vol. 65, No. 3, pp. 452–468.

Heal, Geoffrey and Jisung Park (2013) "Feeling the heat: Temperature, physiology and the wealth of nations," Working Paper 19725, National Bureau of Economic Research.

——— (2016) "Temperature stress and the direct impact of climate change: A review of an emerging literature," *Review of Environmental Economics and Policy*, Vol. 10, No. 2, pp. 347–362.

Hsiang, Solomon M. (2016) "Climate econometrics," *Annual Review of Resource Economics*, Vol. 8, pp. 43–75.

Huntingford, Chris, Philip D. Jones, Valerie N. Livina, Timothy M. Lenton, and Peter M. Cox (2013) "No increase in global temperature variability despite changing regional patterns," *Nature*, Vol. 500, No. 7462, pp. 327–330.

Judd, Kenneth L. (1996) "Approximation, perturbation, and projection methods in economic analysis," in Hans M. Amman, David A. Kendrick, and John Rust eds. *Handbook of Computational Economics*, Vol. 1: Elsevier, pp. 509–585.

Kelly, David L., Charles D. Kolstad, and Glenn T. Mitchell (2005) "Adjustment costs from environmental change," *Journal of Environmental Economics and Management*, Vol. 50, No. 3, pp. 468–495.

Lemoine, Derek (2017) "Expect above average temperatures: Identifying the economic impacts of climate change," Working Paper 23549, National Bureau of Economic Research.

Lemoine, Derek and Sarah Kapnick (2016) "A top-down approach to projecting market impacts of climate change," *Nature Climate Change*, Vol. 6, No. 1, pp. 51–55.

Massetti, Emanuele and Robert Mendelsohn (2018) "Measuring climate adaptation: Methods and evidence," *Review of Environmental Economics and Policy*, Vol. 12, No. 2, pp. 324–341.

Mendelsohn, Robert (2000) "Efficient adaptation to climate change," *Climatic Change*, Vol. 45, No. 3-4, pp. 583–600.

Mendelsohn, Robert, William D. Nordhaus, and Daigee Shaw (1994) "The impact of global warming on agriculture: A Ricardian analysis," *The American Economic Review*, Vol. 84, No. 4, pp. 753–771.

Neidell, Matthew (2009) "Information, avoidance behavior, and health: The effect of ozone on asthma hospitalizations," *Journal of Human Resources*, Vol. 44, No. 2, pp. 450–478.

Nordhaus, William D. (2006) "Geography and macroeconomics: New data and new findings," *Proceedings of the National Academy of Sciences of the United States of America*, Vol. 103, No. 10, pp. 3510–3517.

Olmstead, Alan L. and Paul W. Rhode (2011) "Responding to climatic challenges: Lessons from U.S. agricultural development," in Gary D. Libecap and Richard H. Steckel eds. *The Economics of Climate Change: Adaptations Past and Present*, pp. 169–194.

Pindyck, Robert S. (2013) "Climate change policy: What do the models tell us?" *Journal of Economic Literature*, Vol. 51, No. 3, pp. 860–872.

Ranson, Matthew (2014) "Crime, weather, and climate change," *Journal of Environmental Economics and Management*, Vol. 67, No. 3, pp. 274–302.

Roberts, Michael J. and Wolfram Schlenker (2011) "The evolution of heat tolerance of corn: Implications for climate change," in GaryD. Libecap and Richard H. Steckel eds. *The Economics of Climate Change: Adaptations Past and Present*, pp. 225–251.

Rosenzweig, Mark R. and Christopher Udry (2014) "Rainfall forecasts, weather, and wages over the agricultural production cycle," *American Economic Review: Papers and Proceedings*, Vol. 104, No. 5, pp. 278–283.

Rosenzweig, Mark and Christopher R. Udry (2013) "Forecasting profitability," Working Paper 19334, National Bureau of Economic Research.

Schlenker, Wolfram, W. Michael Hanemann, and Anthony C. Fisher (2005) "Will U.S. agriculture really benefit from global warming? Accounting for irrigation in the hedonic approach," *American Economic Review*, Vol. 95, No. 1, pp. 395–406.

Shrader, Jeffrey (2017) "Expectations and adaptation to environmental risks," working paper.

Sohngen, Brent and Robert Mendelsohn (1998) "Valuing the impact of large-scale ecological change in a market: The effect of climate change on U.S. timber," *The American Economic Review*, Vol. 88, No. 4, pp. 686–710.

Tol, Richard S.J., Samuel Fankhauser, and Joel B. Smith (1998) "The scope for adaptation to climate change: what can we learn from the impact literature?" *Global Environmental Change*, Vol. 8, No. 2, pp. 109–123.

Wood, Stephen A., Amir S. Jina, Meha Jain, Patti Kristjanson, and Ruth S. DeFries (2014) "Smallholder farmer cropping decisions related to climate variability across multiple regions," *Global Environmental Change*, Vol. 25, pp. 163–172.

Zhang, Peng, Olivier Deschenes, Kyle Meng, and Junjie Zhang (2018) "Temperature effects on productivity and factor reallocation: Evidence from a half million Chinese manufacturing plants," *Journal of Environmental Economics and Management*, Vol. 88, pp. 1–17.

Appendix

The first section analyzes the estimators used in some previous literature that aimed to estimate climate impacts from weather variation. The second section shows what we learn about climate change from estimating nonlinear weather impacts. The third section analyzes the average treatment effect of forecasts on payoffs. The fourth section derives the omitted variables bias from ignoring forecasts in empirical work. The fifth section considers climate change that affects the variance of the weather. The sixth section generalizes the analysis to arbitrary payoff functions, multidimensional action spaces, and multidimensional weather indexes. The final section contains proofs.

A Estimators used in previous literature

I now connect the analysis to three specific ways that researchers have estimated climate impacts from weather impacts. I first analyze the estimators in Deschênes and Greenstone (2007) and Deryugina and Hsiang (2017) for the effects of climate on payoffs, and I conclude with the most common approach to estimating the effects of climate on actions (see Carleton and Hsiang, 2016).

First, consider Deschênes and Greenstone (2007). They estimate agricultural profits $\pi(A_t, A_{t-1}, w_t)$ from variation in weather, where weather is growing season degree-days. In particular, they estimate π at different quantiles of the weather distribution by pooling observations across the United States. They then calculate the change in profit resulting from a change in climate by simulating how average growing season degree-days will change with the climate. In their approach, marginal climate impacts are $\partial\pi/\partial w_t$, evaluated around the average growing season degree-days for a location. This is close to the recommendations that follow from the present analysis. The main distinction is that Deschênes and Greenstone (2007) pool across locations when estimating $\partial\pi/\partial w_t$, so that the marginal effect of increasing growing season degree-days from some level x is identified both from locations in which x is typical and from locations in which x is uncommon. The former locations are well-adapted to x, but the latter locations are not. It is the treatment effect in the former that is truly informative about changing average growing season degree-days; the treatment effect in the latter does not encompass much adaptation.[1] The theoretically recommended approach would estimate $\partial\pi/\partial w_t$ by quantiles of average growing season degree-days, so that each evaluation point recovers the average treatment effect within some climate.[2]

[1] In the main text's setting, note that $\partial\pi/\partial w_t$ depends on adaptation actions A_t, which in turn depend on the climate C. Pooling observations across climate zones estimates a single $\partial\pi/\partial w_t$ from observations in which the true $\partial\pi/\partial w_t$ varies with C.

[2] The state-by-state regressions in Deschênes and Greenstone (2007) come closest to the recommended approach, since the climate may be broadly similar within U.S. states.

Next consider the approach to estimating the effect of climate on payoffs in Deryugina and Hsiang (2017). This paper again pools observations across locations, so the previous discussion applies here as well. But ignore that point and imagine that the pooled regressions successfully recover the marginal effect of weather across the spectrum of weather outcomes for some given location.[3] Deryugina and Hsiang (2017) calculate the marginal effect of climate by estimating the effect on payoffs of having another day in each temperature bin and then simulating how climate change will alter the frequency of days in each temperature bin. Formally, let $p(w_t; C)$ represent the probability density function for weather in climate C. Deryugina and Hsiang (2017) estimate $\pi(A_t, \tilde{A}(w_t), w_t) - \pi(A_t, \tilde{A}(w^0), w^0)$ for each w_t, where w^0 indicates the omitted category and where $\tilde{A}(w_t)$ indicates some incoming action at w_t. They calculate the marginal effect of climate from the following expression:

$$\int_{-\infty}^{\infty} \left[\pi(A_t, \tilde{A}(w_t), w_t) - \pi(A_t, \tilde{A}(w^0), w^0)\right] \frac{\mathrm{d}p(w_t; C)}{\mathrm{d}C} \, \mathrm{d}w_t \triangleq \Gamma.$$

Analyzing, we find

$$\begin{aligned}\Gamma =& \int_{-\infty}^{\infty} \left[\pi(A_t, \tilde{A}(w_t), w_t) - \pi(A_t, \tilde{A}(w^0), w^0)\right] \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t; C)} p(w_t; C) \, \mathrm{d}w_t \\ =& E\left[\left[\pi(A_t, \tilde{A}(w_t), w_t) - \pi(A_t, \tilde{A}(w^0), w^0)\right] \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t; C)}\right] \\ =& Cov\left[\pi(A_t, \tilde{A}(w_t), w_t), \quad \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t; C)}\right],\end{aligned}$$

where the final equality uses $\int \frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C} \, \mathrm{d}w_t = 0$. A second-order Taylor expansion of $\pi(A_t, \tilde{A}(w_t), w_t)$ around the average values of each argument (with average actions assumed to match their

[3] The fact that they do pool observations means that their actual results will differ from the derivation here.

steady-state values, as in the main text) yields:

$$
\begin{aligned}
\Gamma \approx\ & \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial A_t}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[A_t, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
& + \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial A_{t-1}}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[\tilde{A}(w_t), \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
& + \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[w_t, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
& + \frac{1}{2} \left.\frac{\partial^2 \pi(A_t, A_{t-1}, w_t)}{\partial A_t^2}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[(A_t - A^{ss})^2, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
& + \frac{1}{2} \left.\frac{\partial^2 \pi(A_t, A_{t-1}, w_t)}{\partial A_{t-1}^2}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[(\tilde{A}(w_t) - A^{ss})^2, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
& + \frac{1}{2} \left.\frac{\partial^2 \pi(A_t, A_{t-1}, w_t)}{\partial w_t^2}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[(w_t - C)^2, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
& + \left.\frac{\partial^2 \pi(A_t, A_{t-1}, w_t)}{\partial A_t \partial A_{t-1}}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[(A_t - A^{ss})(\tilde{A}(w_t) - A^{ss}), \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
& + \left.\frac{\partial^2 \pi(A_t, A_{t-1}, w_t)}{\partial A_t \partial w_t}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[(A_t - A^{ss})(w_t - C), \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
& + \left.\frac{\partial^2 \pi(A_t, A_{t-1}, w_t)}{\partial A_{t-1} \partial w_t}\right|_{(A^{ss}, A^{ss}, C)} Cov\left[(\tilde{A}(w_t) - A^{ss})(w_t - C), \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right].
\end{aligned}
$$

Now impose the functional form for π from the main text, in which case the expansion of Γ is exact. The second line on the right-hand side is zero. The envelope theorem evaluated around the steady state and the first-order condition together imply that the first line on the right-hand side is also zero. Now assume that w_t is normally distributed with mean C.

A second-order Taylor expansion of the probability term around C then yields:

$$\begin{aligned}\Gamma \approx & \frac{1}{Var[w_t]}\Bigg\{ \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right|_{(A^{ss},A^{ss},C)} Var[w_t] \\ & - \frac{1}{2}[\gamma+\alpha+\phi] Cov\left[(A_t - A^{ss})^2, w_t\right] \\ & - \frac{1}{2}\alpha Cov\left[(\tilde{A}(w_t) - A^{ss})^2, w_t\right] \\ & - \frac{1}{2}[\gamma+\psi] Var[w_t] \\ & + \alpha Cov\left[(A_t - A^{ss})(\tilde{A}(w_t) - A^{ss}), w_t\right] \\ & + \gamma Cov\left[(A_t - A^{ss})(w_t - C), w_t\right] \Bigg\}.\end{aligned}$$

Using the solution for A_t^* from the main text, assuming (in line with much of the empirical literature to date) that $N = 0$, first-order approximating $\tilde{A}(w_t)$ around $w_t = C$, and recognizing that $Cov[w_t^2, w_t] = 2\,Var[w_t]\,C$, we have:

$$\begin{aligned}\Gamma \approx & \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right|_{(A^{ss},A^{ss},C)} \\ & - [\gamma+\alpha+\phi]\frac{(\alpha\tilde{A}'(C)+\gamma)\left(\alpha\tilde{A}(C) - \alpha\tilde{A}'(C)C + \beta b_1 C + \beta c_1 + \phi\bar{A} - (\gamma+\alpha+\phi-2\beta a_1)A^{ss}\right)}{(\gamma+\alpha+\phi-2\beta a_1)^2} \\ & - [\gamma+\alpha+\phi]\frac{(\alpha\tilde{A}'(C)+\gamma)^2 C}{(\gamma+\alpha+\phi-2\beta a_1)^2} \\ & - \alpha\tilde{A}'(C)[\tilde{A}(C) - \tilde{A}'(C)C - A^{ss}] - \alpha(\tilde{A}'(C))^2 C \\ & - \frac{1}{2}[\gamma+\psi] \\ & + \alpha\frac{\alpha\tilde{A}'(C)+\gamma}{\gamma+\alpha+\phi-2\beta a_1}(\tilde{A}(C) - \tilde{A}'(C)C - A^{ss}) \\ & + \alpha\frac{\alpha\tilde{A}(C) - \alpha\tilde{A}'(C)C + \beta b_1 C + \beta c_1 + \phi\bar{A} - (\gamma+\alpha+\phi-2\beta a_1)A^{ss}}{\gamma+\alpha+\phi-2\beta a_1}\tilde{A}'(C) \\ & + 2\alpha\tilde{A}'(C)\frac{\alpha\tilde{A}'(C)+\gamma}{\gamma+\alpha+\phi-2\beta a_1}C \\ & - \gamma\frac{\alpha\tilde{A}'(C)+\gamma}{\gamma+\alpha+\phi-2\beta a_1}C + \gamma\frac{\alpha\tilde{A}(C) - \alpha\tilde{A}'(C)C + \beta b_1 C + \beta c_1 + \phi\bar{A} - (\gamma+\alpha+\phi-2\beta a_1)A^{ss}}{\gamma+\alpha+\phi-2\beta a_1} \\ & + 2\gamma\frac{\alpha\tilde{A}'(C)+\gamma}{\gamma+\alpha+\phi-2\beta a_1}C.\end{aligned}$$

Assuming that $\tilde{A}(C) = A^{ss}$ and simplifying, this becomes:

$$\begin{aligned}
\Gamma \approx & \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right|_{(A^{ss}, A^{ss}, C)} - \frac{1}{2}[\gamma + \psi] \\
& - [\gamma + \alpha + \phi] \frac{(\alpha \tilde{A}'(C) + \gamma)\left(\gamma C + \beta b_1 C + \beta c_1 + \phi \bar{A} - (\gamma + \phi - 2\beta a_1) A^{ss}\right)}{(\gamma + \alpha + \phi - 2\beta a_1)^2} \\
& + (\alpha \tilde{A}'(C) + \gamma) \frac{\gamma C + \beta b_1 C + \beta c_1 + \phi \bar{A} - (\gamma + \phi - 2\beta a_1) A^{ss}}{\gamma + \alpha + \phi - 2\beta a_1} \\
= & \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right|_{(A^{ss}, A^{ss}, C)} - \frac{1}{2}[\gamma + \psi] \\
& + [\alpha \tilde{A}'(C) + \gamma] \frac{\gamma C + \beta b_1 C + \beta c_1 + \phi \bar{A} - (\gamma + \phi - 2\beta a_1) A^{ss}}{\gamma + \alpha + \phi - 2\beta a_1} \frac{-2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1}.
\end{aligned}$$

Using Proposition 2, we have:

$$\begin{aligned}
\Gamma \approx & \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right|_{(A^{ss}, A^{ss}, C)} - \frac{1}{2}[\gamma + \psi] \\
& + [\alpha \tilde{A}'(C) + \gamma] \frac{\beta \overbrace{[b_1 C + c_1 + 2a_1 A^{ss}]}^{V_Z(A^{ss}, C, \boldsymbol{C})}}{\gamma + \alpha + \phi - 2\beta a_1} \frac{-2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1}.
\end{aligned}$$

Note that $\partial V(Z_t, w_t, F_t)/\partial Z_t = b_1 C + c_1 + 2a_1 Z_t$ when $N = 0$, and recall that the envelope theorem and the fact that $\partial \pi(A_t, A_{t-1}, w_t)/\partial A_{t-1} = 0$ around a steady state imply that $V_Z(A^{ss}, C, \boldsymbol{C}) = 0$. Therefore:

$$\Gamma \approx \left.\frac{\partial \pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right|_{(A^{ss}, A^{ss}, C)} - \frac{1}{2}[\gamma + \psi].$$

Proposition 6 then implies that

$$\begin{aligned}
\Gamma \approx & \frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C} - \frac{1}{2}[\gamma + \psi] \\
= & \frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C} + \frac{1}{2}\frac{\partial^2 \pi(A_t, A_{t-1}, w_t)}{\partial w_t^2} \\
\leq & \frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C}.
\end{aligned}$$

The estimator Γ overstates the cost (or understates the benefit) of marginal climate change. Γ adjusts the desired effect $\mathrm{d}\pi(A^{ss}, A^{ss}, C)/\mathrm{d}C$ for the nonlinearity of payoffs in weather. A shift in the distribution of climate makes former extremes more likely, and extremes do tend to be costly. However, when former extremes become more regular, people undertake more preparations for them and they become less costly. The next section will show that nonlinear weather impacts are at best only loosely related to climate impacts and are potentially unrelated to climate impacts.

Finally, much recent literature estimates the effects of climate on actions (or on functions of actions) using the same kind of approach just studied (see Carleton and Hsiang, 2016). This literature estimates $A(\tilde{A}(w_t), w_t, \tilde{F}(w_t)) - A(\tilde{A}(w^0), w^0, \tilde{F}(w^0))$, where $\tilde{F}(w_t)$ indicates some vector of forecasts that correlates with the realized weather w_t (the literature has not explicitly conditioned on forecasts). The literature then calculates the effect of a marginal change in climate as:

$$\int_{-\infty}^{\infty} \left[A(\tilde{A}(w_t), w_t, \tilde{F}(w_t)) - A(\tilde{A}(w^0), w^0, \tilde{F}(w^0))\right] \frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}\,\mathrm{d}w_t \triangleq \Phi.$$

Analyzing, we find

$$\Phi = Cov\left[A(\tilde{A}(w_t), w_t, \tilde{F}(w_t)), \quad \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right].$$

Recall that A_t^* is linear in the main text's setting. We thus have:

$$\begin{aligned}
\Phi =& \frac{\partial A(A_{t-1}, w_t, F_t)}{\partial A_{t-1}} Cov\left[\tilde{A}(w_t), \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] + \frac{\partial A(A_{t-1}, w_t, F_t)}{\partial w_t} Cov\left[w_t, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
&+ \sum_{i=1}^{N} \frac{\partial A(A_{t-1}, w_t, F_t)}{\partial f_{it}} Cov\left[\tilde{f}_{it}, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
=& \frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} Cov\left[\tilde{A}(w_t), \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] + \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} Cov\left[w_t, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] \\
&+ \frac{\beta b_1}{\gamma+\alpha+\phi-2\beta a_1} Cov\left[\tilde{f}_{1t}, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right] + \sum_{i=1}^{N-1} \frac{\beta b_2^i}{\gamma+\alpha+\phi-2\beta a_1} Cov\left[\tilde{f}_{(i+1)t}, \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right].
\end{aligned}$$

Assume that w_t is normally distributed with mean C. A second-order Taylor expansion of the probability term around long-run average weather C yields:

$$\begin{aligned}
\Phi \approx& \frac{1}{Var[w_t]}\left\{\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} Cov\left[\tilde{A}(w_t), w_t\right] + \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} Var[w_t]\right. \\
&\left. + \frac{\beta b_1}{\gamma+\alpha+\phi-2\beta a_1} Cov\left[\tilde{f}_{1t}, w_t\right] + \sum_{i=1}^{N-1} \frac{\beta b_2^i}{\gamma+\alpha+\phi-2\beta a_1} Cov\left[\tilde{f}_{(i+1)t}, w_t\right]\right\}.
\end{aligned}$$

A first-order Taylor expansion of $\tilde{A}(w_t)$ around $w_t = C$ then gives:

$$\Phi \approx \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} + \frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\tilde{A}'(C) + \frac{\beta b_1}{\gamma+\alpha+\phi-2\beta a_1}\frac{Cov\left[\tilde{f}_{1t}, w_t\right]}{Var[w_t]} + \sum_{i=1}^{N-1}\frac{\beta b_2^i}{\gamma+\alpha+\phi-2\beta a_1}\frac{Cov\left[\tilde{f}_{(i+1)t}, w_t\right]}{Var[w_t]}.$$

Note that if t is more than N periods in the future, then $Cov[f_{it}, w_t] = \rho_i + \sum_{j=1}^{N-i}\delta_{j(j+i)}$. We can relate Φ to the main text's estimator by writing it as

$$\Phi \approx \frac{\partial A(A_{t-1}, w_t, F_t)}{\partial w_t} + \frac{\partial A(A_{t-1}, w_t, F_t)}{\partial A_{t-1}}\tilde{A}'(C) + \sum_{i=1}^{N}\frac{\partial A(A_{t-1}, w_t, F_t)}{\partial f_{it}} Corr[\tilde{f}_{it}, w_t]\, Var[f_{it}].$$

Because weather is serially correlated, we should expect that $\tilde{A}'(C) > 0$ and $Corr[\tilde{f}_{it}, w_t] > 0$. Proposition 3 showed that $\partial A_t/\partial w_t < \mathrm{d}A^{ss}/\mathrm{d}w_t$ when $\alpha > 0$. The estimator Φ does not underestimate $\mathrm{d}A^{ss}/\mathrm{d}C$ by as much as would estimating $\partial A_t/\partial w_t$ alone: Φ does account for how previous actions may vary with weather and for how forecasts may vary with weather. However, we should expect that $\tilde{A}'(w_t)$ is small when adjustment costs are large, and we may expect longer-horizon forecasts to be less correlated with weather and, if they do not improve much on knowledge of the background climate, also to have low variance. These arguments suggest that Φ typically underestimates $\mathrm{d}A^{ss}/\mathrm{d}C$ by more than the main text's estimator that uses all available forecasts.

B What we do and do not learn from estimating nonlinear weather impacts

Much empirical work has found that weather outcomes have nonlinear effects. We have seen that empirical researchers should be estimating the average treatment effect of weather in order to identify the marginal effect of climate. Can the curvature of V and π in w_t tell us something about the curvature of $V(A^{ss}, C, \boldsymbol{C})$ and $\pi(A^{ss}, A^{ss}, C)$ in C? The following proposition provides reason for skepticism:

Proposition 7.

1. $\frac{\partial^2 \pi(A_t, Z_t, w_t)}{\partial w_t^2} \leq \frac{\mathrm{d}^2 \pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} \leq \frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w_t^2} \leq \frac{\mathrm{d}^2 \pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C^2} \leq 0$, *with:*

   (a) $\frac{\partial^2 \pi(A_t, Z_t, w_t)}{\partial w_t^2} < \frac{\mathrm{d}^2 \pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2}$ *if and only if* $\beta\alpha\gamma > 0$,

   (b) $\frac{\mathrm{d}^2 \pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} < \frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w_t^2}$ *if and only if* $\gamma > 0$,

(c) $\frac{\partial^2 V(Z_t,w_t,F_t)}{\partial w_t^2} < \frac{\mathrm{d}^2\pi(A^{ss},A^{ss},C)}{\mathrm{d}C^2}$ *if and only if* $\alpha\gamma > 0$, *and*

(d) $\frac{\mathrm{d}^2\pi(A^{ss},A^{ss},C)}{\mathrm{d}C^2} < 0$ *if and only if* $\phi\gamma + \psi > 0$.

2. $\frac{\partial^2\pi(A_t,Z_t,w_t)}{\partial w_t^2} \to \frac{\mathrm{d}^2\pi(A^{ss},A^{ss},C)}{\mathrm{d}C^2}$ *as either* $\gamma \to 0$ *or* $\phi \to \infty$.

3. *If* $\phi + \psi = 0$ *and* $\alpha > 0$, *then* $\frac{\partial^2\pi(A_t,Z_t,w_t)}{\partial w_t^2}, \frac{\mathrm{d}^2\pi(A_t,Z_t,w_t)}{\mathrm{d}w_t^2}, \frac{\partial^2 V(Z_t,w_t,F_t)}{\partial w_t^2} < 0$ *even as* $\frac{\mathrm{d}^2\pi(A^{ss},A^{ss},C)}{\mathrm{d}C^2} = 0$.

Proof. See appendix. □

The proposition relates (i) the curvature of per-period payoffs in w_t holding A_t and Z_t constant ($\partial^2\pi/\partial w_t^2$), (ii) the curvature of per-period payoffs in w_t when A_t adapts to realizations of w_t ($\mathrm{d}^2\pi/\mathrm{d}w_t^2$), (iii) the curvature of intertemporal value in w_t, and (iv) the curvature of per-period steady-state payoffs in C.[4] Empirical work will commonly estimate (ii), because per-period payoffs are observable as profit and actions are often not observable.[5]

The first part of Proposition 7 establishes that the type of curvature estimated as (i), (ii), or (iii) is at least as extreme as the curvature of steady-state payoffs in climate (iv). Empirical estimates should therefore be taken as an upper bound on the nonlinearity of climate impacts. Intuitively, an agent undertakes greater adjustment to a permanent change in climate than to transient weather shocks, and this greater adjustment reduces the impact on payoffs.

The second part of Proposition 7 establishes that the nonlinearity of weather impacts can adequately approximate the nonlinearity of climate impacts when there are no avoidable weather impacts ($\gamma \to 0$) and when maintenance costs become infinitely large ($\phi \to \infty$). In these rather special cases, actions do not adjust to a change in climate. In more general cases, nonlinear weather impacts strictly overestimate the nonlinearity of climate impacts.

But perhaps detecting nonlinear weather impacts tells us something qualitative about climate change? The third part of the proposition establishes that climate impacts can be linear even when weather impacts are nonlinear. In particular, let $\phi = 0$ and $\psi = 0$. Climate then has no effect on steady-state value because there are no unavoidable weather impacts and the agent adjusts her actions to completely offset the avoidable weather impacts from a change in climate ($A^{ss} = C$). However, when $\alpha > 0$, this agent will not choose to completely offset the effects of transient weather events. Transient weather events can then impose

[4] Proposition 5 relates the curvature of per-period steady-state payoffs in C to the curvature of intertemporal value in C: $\frac{\mathrm{d}^2 V(A^{ss},C,\boldsymbol{C})}{\mathrm{d}C^2} = \frac{1}{1-\beta}\frac{\mathrm{d}^2\pi(A^{ss},A^{ss},C)}{\mathrm{d}C^2}$.

[5] Or at least it will estimate (ii) if it conditions on forecasts: the definition of $\mathrm{d}\pi/\mathrm{d}w_t$ used here does not allow Z_t to change with w_t. See Section D for a discussion of the implications of not conditioning on forecasts.

arbitrarily nonlinear costs even though climate change imposes no costs at all in the long run.[6]

C What forecasts tell us about the effect of climate on value

Now consider how we can use observable variation in forecasts to learn about climate impacts. We saw in the main text that using variation in forecasts can improve an empirical analyst's ability to estimate changes in steady-state actions. We have already seen that $ATE_w^V(C)$ and $ATE_w^\pi(C)$ are sufficient statistics for the effect of climate change on steady-state per-period payoffs. Does using forecasts add anything here? Let $ATE_{f_i}^V(C)$ and $ATE_{f_i}^\pi(C)$ denote the average treatment effect of forecasts at horizon i around a steady state, defined analogously to $ATE_w^V(C)$ and $ATE_w^\pi(C)$. We now have:

Proposition 8.

1. $ATE_{f_i}^V(C) = \beta^i ATE_w^V(C)$.
2. *As* $N \to \infty$, $ATE_w^V(C) + \sum_{i=1}^N ATE_{f_i}^V(C) \to \frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C}$.
3. $\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C} = \frac{ATE_w^V(C)}{1 - \left(\frac{ATE_{f_i}^V(C)}{ATE_w^V(C)}\right)^{\frac{1}{i}}}$.
4. $ATE_{f_i}^\pi(C) = 0$.

Proof. See appendix. □

The first part of the proposition says that the average treatment effect of forecasts on value is the discounted average treatment effect of weather, which Proposition 6 showed is the average treatment effect of climate on per-period payoffs. We can therefore derive the change in per-period payoffs from either estimate, provided we have an estimate of β in hand. The second and third parts of the proposition show that if we use both types of treatment effects, then we can identify not only the change in steady-state per-period payoffs but also the discount factor β. We can then exactly identify the present value of the change in steady-state value as revealed by the agent's own actions. The final part of the proposition shows that it is critical that the dependent variable be a forward-looking measure of value such as land prices or stock prices. Per-period payoffs (e.g., profits) will not, on average, respond to forecasts around a steady state.

[6] Below, we will learn what estimating nonlinear weather impacts tells us about the costs of changing the variance of the weather.

D Omitted variables bias from ignoring forecasts

Most empirical to date work has ignored the existence of weather forecasts. We have seen that forecasts can provide valuable information, but our analysis also implies that ignoring forecasts acts like omitted variables bias when estimating the consequences of weather (see also Lemoine, 2017; Shrader, 2017).[7] Accounting for forecasts is therefore not optional. The covariance between w_t and f_{it} is, for $t > N$,

$$\begin{aligned} Cov_0(w_t, f_{it}) =& Cov_0\left(\epsilon_t + \sum_{k=1}^{N} \nu_{k(t-k)}, \sum_{j=0}^{N-i} \nu_{(i+j)(t-j)}\right) \\ =& \rho_i + \sum_{k=1}^{N-i} \delta_{k(i+k)}. \end{aligned}$$

In applications, we can reasonably expect each ρ and δ to be positive, with many strictly positive. We can therefore reasonably expect this covariance to be strictly positive. The bias from estimating $\partial V/\partial w_t$ without accounting for F_t is proportional to

$$\sum_{i=1}^{N} \frac{\partial V(Z_t, w_t, F_t)}{\partial f_{it}} Cov_0(w_t, f_{it}).$$

When w_t and f_{it} affect V in the same way, omitting forecasts will generally overestimate the magnitude of $\partial V/\partial w_t$. A similar analysis applies if the dependent variable were A_t instead of V.[8] Yet since we have previously seen that combining forecasts with weather can generate useful information, one might wonder whether entwining forecasts and weather through omitted variables bias might actually use weather and forecasts in the desired fashion. Unfortunately, this is not generally the case: the ρ and δ terms that are critical to

[7] Previous work has shown that forecasts matter for outcome variables in a variety of contexts, suggesting that we cannot assume that $\partial V/\partial f_{it} = 0$ or that $\partial A_t/\partial f_{it} = 0$. Lave (1963) illustrates the value of rain forecasts to raisin growers, and Wood et al. (2014) find that developing-country farmers with better access to weather information make more changes in their farming practices. Neidell (2009) demonstrates the importance of accounting for forecasts when estimating the health impacts of air pollution. Studying Indian agriculture, Rosenzweig and Udry (2013) show that farmers' investments respond to forecasts (and respond more strongly to more skillful forecasts), and Rosenzweig and Udry (2014) show that forecasts of planting season weather affect migration decisions and thus wages. Shrader (2017) shows that fishers' revenue and effort both respond to seasonal forecasts of El Niño events. Severen et al. (2016) show that land markets capitalize forecasts of climate change.

[8] However, the concern is mitigated if the dependent variable is π and actions are near the steady state. Forecasts affect π only through A_{t-1}, and we saw that the marginal effect of A_{t-1} vanishes around a steady state. Therefore omitted variables bias is not a concern when estimating the consequences of weather on per-period payoffs around a steady state.

omitted variables bias did not appear in any earlier derivation.[9] Taking advantage of variation in forecasts does require explicitly estimating the effects of weather and of forecasts at each horizon. Empirical work should more carefully consider the informational structure of weather shocks and take care to estimate the treatment effect of interest.

E When climate controls the variance of the weather

We now study a case where the climate controls the variance of the weather. The effects of climate change on the variance of the weather are poorly understood and spatially heterogeneous (e.g., Huntingford et al., 2013; Lemoine and Kapnick, 2016), but many are interested in how to think about the costs of greater variance. Extend the previous setting to allow the variance to depend on a distinct climate index D (so that we can study effects on variance without also changing average climate via C). Now let ϵ_t have variance $\sigma^2(D)$ and let each ν_{it} have variance $\tau_i^2(D)$. For simplicity, keep the covariances δ_{ij} and ρ_i independent of D.

First, note from equation (3) that the optimal actions A_t^* are linear in w_t and the f_{it}. It is therefore clear that

$$\frac{\mathrm{d}E_0[A_t]}{\mathrm{d}D} = 0.$$

The variance of weather and forecasts does not affect expected actions because actions are linear in the states when payoffs are quadratic.

Now consider the effect of D on value. From the solution for d in the proof of Proposition 1, the effect on time t value of permanently changing the variance via D is:[10]

$$\frac{\mathrm{d}V(Z_t, w_t, F_t)}{\mathrm{d}D} = \frac{\beta}{1-\beta}\left[a_2 \frac{\mathrm{d}\sigma^2}{\mathrm{d}D} + \sum_{i=1}^{N} a_3^i \frac{\mathrm{d}\tau_i^2}{\mathrm{d}D}\right] \leq 0.$$

Twice-differentiating the value function solution in Proposition 1 with respect to w_t and each f_{it}, we have:

$$\frac{\mathrm{d}V(Z_t, w_t, F_t)}{\mathrm{d}D} = \frac{\beta}{1-\beta}\left[\frac{1}{2}\frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w_t^2}\frac{\mathrm{d}\sigma^2}{\mathrm{d}D} + \frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2 V(Z_t, w_t, F_t)}{\partial f_{it}^2}\frac{\mathrm{d}\tau_i^2}{\mathrm{d}D}\right]. \tag{A-1}$$

[9] In a regression of either value or actions on weather with forecasts acting as the only bias-generating omitted variables, the usual ordinary least squares formula shows that omitted variables bias induces the desired combination of weather and forecasts if and only if $Cov_0(w_t, f_{it}) = Var_0(w_t)$ for all i. There is no reason for this relationship to hold in practice.

[10] Alternately, one could use equation (5) to consider the effect of altered variance on expected value at some future time, as we did with the effect of an altered mean in the main text. However, this analysis becomes more complicated because we have to account for how climate change affects the variance of actions and affects their covariance with weather and forecasts.

If the entire effect of climate change consisted in changing the variance of the one-period-ahead forecast errors ϵ_t, then the curvature of value in weather would identify the cost of that change. However it is more likely that climate change affects the variance of ν_{Nt} than the variance of ϵ_t. In that case, the curvature of value in long-run forecasts identifies the cost of that change. Further, from Proposition 1, those costs become small as β^N decreases. Therefore, if climate change increases the variance of long-run forecasts but does not affect their accuracy,[11] then we can expect the costs of a change in variance to be small.

Finally, Proposition 7 tells us that if $\gamma > 0$, then $\pi(A_t, Z_t, w_t)$ is more concave in w_t than is $V(Z_t, w_t, F_t)$, and we know that $\pi(A_t, Z_t, w_t)$ is independent of f_{it}, once we condition on A_t and Z_t. When applying equation (A-1), it is therefore important to distinguish between estimating the nonlinear effects of weather and forecasts on intertemporal value and estimating the nonlinear effects of weather and forecasts on flow payoffs.

F Generalizing the functional form for payoffs

I now analyze a general functional form for payoffs rather than the quadratic form analyzed in the main text. I also now allow for $K \geq 1$ types of actions, indexed as A_t^k for $k \in \{1, ..., K\}$, and for $J \geq 1$ dimensions of weather, indexed as w_t^j for $j \in \{1, ..., J\}$. Let time t payoffs be $\pi(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, \boldsymbol{w}_t)$, where $\boldsymbol{A}_t \triangleq \{A_t^1, ..., A_t^K\}$, $\boldsymbol{A}_{t-1} \triangleq \{A_{t-1}^1, ..., A_{t-1}^K\}$, and $\boldsymbol{w}_t \triangleq \{w_t^1, ..., w_t^J\}$. π_{1_k} indicates a partial derivative with respect to A_t^k, π_{2_k} indicates a partial derivative with respect to A_{t-1}^k, and π_{3_j} indicates a partial derivative with respect to w_t^j. Assume declining marginal benefits of current and past adaptation investments ($\pi_{1_k 1_k} < 0, \pi_{2_k 2_k} \leq 0$), and assume the presence of smooth adjustment costs, which means that the marginal benefit of current actions increases in the level of previous actions ($\pi_{1_k 2_k} > 0$). Finally, assume that the effect of weather on payoffs does not depend directly on past adaptation actions: $\pi_{2_k 3_j} = 0$.

I modify the transition equations for weather and forecasts to multiply each disturbance term by a perturbation parameter $\zeta \geq 0$. The Bellman equation is now:

$$
\begin{aligned}
V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta) = &\max_{\boldsymbol{A}_t} \left\{ \pi(\boldsymbol{A}_t, \boldsymbol{Z}_t, w_t) + \beta E_t \left[V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta) \right] \right\} \\
\text{s.t.} \quad \boldsymbol{Z}_{t+1} = &\boldsymbol{A}_t \\
w_{t+1}^j = &f_{1t}^j + \zeta \epsilon_{t+1}^j \\
f_{i(t+1)}^j = &f_{(i+1)t}^j + \zeta \nu_{i(t+1)}^j \qquad \text{for } i < N \\
f_{N(t+1)}^j = &C + \zeta \nu_{N(t+1)}^j
\end{aligned}
$$

$\boldsymbol{F}_{t+1}$ now indicates a $J \times N$ matrix of forecasts. In order to avoid excess notation, assume

[11] If climate change does not affect the variance of ϵ_t or of ν_{it} for $i < N$, then it does not affect the accuracy of forecasts made at horizon N.

that the J dimensions of each ϵ_t are uncorrelated with each other and likewise for each ν_t. We will be especially interested in the following assumption:

Assumption 1. $\pi_{2_k}(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, \boldsymbol{w}_t) = 0$ *if* $A^k_{t-1} = A^k_t$.

This assumption says that small changes in past actions do not affect payoffs when they match current actions. It will be satisfied by many specifications of adjustment costs, including the specification in the main text.

First consider the deterministic system, with $\zeta = 0$. In this case, each weather and forecast variable is simply equal to C. The K first-order conditions are:

$$0 = \pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, \boldsymbol{C}) + \beta V_{1_k}(\boldsymbol{Z}_{t+1}, \boldsymbol{C}, \boldsymbol{C}; 0),$$

where I abuse notation in using $\boldsymbol{C}$ for both the $J \times 1$ vector of time t weather and the $J \times N$ matrix of time t forecasts. The envelope theorem yields:

$$V_{1_k}(\boldsymbol{Z}_t, \boldsymbol{C}, \boldsymbol{C}; 0) = \pi_{2_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, \boldsymbol{C}).$$

Advancing this forward by one timestep and substituting into the first-order conditions, we have the K Euler equations:

$$0 = \pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, \boldsymbol{C}) + \beta \pi_{2_k}(\boldsymbol{A}_{t+1}, \boldsymbol{A}_t, \boldsymbol{C}).$$

A steady state $\bar{\boldsymbol{A}}$ is defined by the following K equations:

$$0 = \pi_{1_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C}) + \beta \pi_{2_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C}).$$

Assumption 1 would imply that $\pi_{1_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C}) = 0$. Using $\pi_{2_k 3_j} = 0$,[12] we also have:

$$\frac{\mathrm{d}\bar{A}^k}{\mathrm{d}C^j} = \frac{\pi_{1_k 3_j}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C})}{-\pi_{1_k 1_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C}) - (1+\beta)\pi_{1_k 2_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C}) - \beta \pi_{2_k 2_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C})}.$$

I now analyze the policy rule in the stochastic system. The K first-order conditions are:

$$0 = \pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, \boldsymbol{w}_t) + \beta E_t[V_{1_k}(\boldsymbol{Z}_{t+1}, \boldsymbol{w}_{t+1}, \boldsymbol{F}_{t+1}; \zeta)].$$

[12] In the case with $K = 1$, the steady state is unique if $-\pi_{11} - \beta\pi_{22} > (1+\beta)\pi_{12}$. Now consider stability in the case with $K = 1$ and, for ease of notation, $J = 1$. Use the Euler equation to define $A^*_{t+1}(A_t, Z_t)$. Linearizing around $\bar{A}$ gives a first-order difference equation: $A_{t+1} \approx \frac{-\pi_{11}(\bar{A},\bar{A},C) - \beta\pi_{22}(\bar{A},\bar{A},C)}{\beta\pi_{12}(\bar{A},\bar{A},C)} A_t - \frac{1}{\beta} Z_t + D$, for some constant D. Combined with the transition equation for Z, we have a two-dimensional linear system. The product of the eigenvalues is $\frac{1}{\beta} > 1$, and the sum of the eigenvalues is $\frac{-\pi_{11}(\bar{A},\bar{A},C) - \beta\pi_{22}(\bar{A},\bar{A},C)}{\beta\pi_{12}(\bar{A},\bar{A},C)} > 0$. Therefore both eigenvalues are positive and at least one is greater than 1. Forming the characteristic equation and solving for the condition under which the smallest root is less than 1, we find that the system is saddle-path stable if and only if $-\pi_{11}(\bar{A},\bar{A},C) - \beta\pi_{22}(\bar{A},\bar{A},C) > (1+\beta)\pi_{12}(\bar{A},\bar{A},C)$, which is ensured if π satisfies the sufficient condition for uniqueness. Further, if the system is saddle-path stable, then $\mathrm{d}\bar{A}/\mathrm{d}C > 0$ if and only if $\pi_{13}(\bar{A},\bar{A},C) > 0$.

The envelope theorem yields:

$$V_{1_k}(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_{t+1}; \zeta) = \pi_{2_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, \boldsymbol{w}_t).$$

Advancing this forward by one timestep and substituting, we have the K Euler equations:

$$0 = \pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, \boldsymbol{w}_t) + \beta E_t[\pi_{2_k}(\boldsymbol{A}_{t+1}, \boldsymbol{A}_t, \boldsymbol{w}_{t+1})].$$

Approximate the value function via a second-order Taylor series expansion around $\boldsymbol{Z}_t =$

$\bar{\boldsymbol{A}}$, $\boldsymbol{w}_t = \boldsymbol{C}$, $\boldsymbol{F}_t = \boldsymbol{C}$, and $\zeta = 0$:

$$
\begin{aligned}
V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta) \approx & V(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}; 0) + \sum_{k=1}^{K} \left.\frac{\partial V}{\partial Z_t^k}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (Z_t^k - \bar{A}^k) + \sum_{j=1}^{J} \left.\frac{\partial V}{\partial w_t^j}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (w_t^j - C^j) \\
& + \sum_{j=1}^{J}\sum_{i=1}^{N} \left.\frac{\partial V}{\partial f_{it}^j}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (f_{it}^j - C^j) + \left.\frac{\partial V}{\partial \zeta}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} \zeta \\
& + \frac{1}{2}\sum_{k=1}^{K} \left.\frac{\partial^2 V}{\partial Z_t^{k\,2}}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (Z_t^k - \bar{A}^k)^2 + \frac{1}{2}\sum_{j=1}^{J} \left.\frac{\partial^2 V}{\partial w_t^{j\,2}}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (w_t^j - C^j)^2 \\
& + \frac{1}{2}\sum_{j=1}^{J}\sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial f_{it}^{j\,2}}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (f_{it}^j - C^j)^2 + \frac{1}{2} \left.\frac{\partial^2 V}{\partial \zeta^2}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} \zeta^2 \\
& + \sum_{k=1}^{K}\sum_{j=1}^{J} \left.\frac{\partial^2 V}{\partial Z_t^k \partial w_t^j}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (Z_t^k - \bar{A}^k)(w_t^j - C^j) \\
& + \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial Z_t^k \partial f_{it}^j}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (Z_t^k - \bar{A}^k)(f_{it}^j - C^j) \\
& + \sum_{j=1}^{J-1}\sum_{m=j+1}^{J} \left.\frac{\partial^2 V}{\partial w_t^j \partial w_t^m}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (w_t^j - C^j)(w_t^m - C^m) \\
& + \sum_{j=1}^{J}\sum_{m=1}^{J}\sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial w_t^j \partial f_{it}^m}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (w_t^j - C^j)(f_{it}^m - C^m) \\
& + \sum_{j=1}^{J}\sum_{i=1}^{N-1}\sum_{n=i+1}^{N} \left.\frac{\partial^2 V}{\partial f_{it}^j \partial f_{it}^j}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (f_{it}^j - C^j)(f_{it}^j - C^j) \\
& + \sum_{j=1}^{J-1}\sum_{m=j+1}^{J}\sum_{i=1}^{N}\sum_{n=1}^{N} \left.\frac{\partial^2 V}{\partial f_{it}^j \partial f_{nt}^m}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (f_{it}^j - C^j)(f_{nt}^m - C^m) \\
& + \sum_{k=1}^{K} \left.\frac{\partial^2 V}{\partial Z_t^k \partial \zeta}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (Z_t^k - \bar{A}^k)\zeta + \sum_{j=1}^{J} \left.\frac{\partial^2 V}{\partial w_t^j \partial \zeta}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (w_t^j - C^j)\zeta \\
& + \sum_{j=1}^{J}\sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial f_{it}^j \partial \zeta}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (f_{it}^j - C^j)\zeta + \sum_{k=1}^{K-1}\sum_{j=k+1}^{K} \left.\frac{\partial^2 V}{\partial Z_t^k \partial Z_t^j}\right|_{(\bar{\boldsymbol{A}},\boldsymbol{C},\boldsymbol{C},0)} (Z_t^k - \bar{A}^k)(Z_t^j - \bar{A}^j).
\end{aligned}
$$

Use the envelope theorem to substitute for several of the derivatives and impose $\pi_{2_k 3_j} = 0$:

$$
\begin{aligned}
V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta) \approx & V(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}; 0) + \sum_{k=1}^{K} \bar{\pi}_{2_k}(Z_t^k - \bar{A}^k) + \sum_{j=1}^{J} \bar{\pi}_{3_j}(w_t^j - C^j) \\
& + \frac{1}{2}\sum_{k=1}^{K} \bar{\pi}_{2_k 2_k}(Z_t^k - \bar{A}^k)^2 + \frac{1}{2}\sum_{j=1}^{J} \bar{\pi}_{3_j 3_j}(w_t^j - C^j)^2 + \sum_{k=1}^{K-1}\sum_{j=k+1}^{K} \bar{\pi}_{2_k 2_j}(Z_t^k - \bar{A}^k)(Z_t^j - \bar{A}^j) \\
& + \left.\frac{\partial V}{\partial \zeta}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} \zeta + \frac{1}{2}\left.\frac{\partial^2 V}{\partial \zeta^2}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} \zeta^2 \\
& + \sum_{j=1}^{J-1}\sum_{m=j+1}^{J} \bar{\pi}_{3_j 3_m}(w_t^j - C^j)(w_t^m - C^m) \\
& + \sum_{j=1}^{J}\sum_{i=1}^{N} \left.\frac{\partial V}{\partial f_{it}^j}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} (f_{it}^j - C^j) + \frac{1}{2}\sum_{j=1}^{J}\sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial {f_{it}^j}^2}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} (f_{it}^j - C^j)^2 \\
& + \sum_{j=1}^{J}\sum_{i=1}^{N-1}\sum_{n=i+1}^{N} \left.\frac{\partial^2 V}{\partial f_{it}^j \partial f_{nt}^j}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} (f_{it}^j - C^j)(f_{nt}^j - C^j) \\
& + \sum_{j=1}^{J-1}\sum_{m=j+1}^{J}\sum_{i=1}^{N}\sum_{n=1}^{N} \left.\frac{\partial^2 V}{\partial f_{it}^j \partial f_{nt}^m}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} (f_{it}^j - C^j)(f_{nt}^m - C^m) \\
& + \sum_{j=1}^{J}\sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial f_{it}^j \partial \zeta}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} (f_{it}^j - C^j)\zeta,
\end{aligned}
$$

where I write $\bar{\pi}$ for $\pi(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C})$. Consider the remaining derivatives. First, we have:

$$
\begin{aligned}
\left.\frac{\partial V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)}{\partial f_{it}^j}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} =& \beta E_t\left[\frac{\partial V(\boldsymbol{Z}_{t+1}, \boldsymbol{w}_{t+1}, \boldsymbol{F}_{t+1}; \zeta)}{\partial f_{(i-1)(t+1)}^j}\right] \text{ for } i > 1, \\
\left.\frac{\partial V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)}{\partial f_{1t}^j}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} =& \beta E_t\left[\frac{\partial V(\boldsymbol{Z}_{t+1}, \boldsymbol{w}_{t+1}, \boldsymbol{F}_{t+1}; \zeta)}{\partial w_{t+1}^j}\right] = \beta E_t[\pi_{3_j}(t+1)],
\end{aligned}
$$

where I save notation by using $t+1$ to stand in for the arguments of π and leaving the conditioning of time t expectations on the evaluation points $\bar{\boldsymbol{A}}$ and $\boldsymbol{C}$ implicit. These imply:

$$
\left.\frac{\partial V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)}{\partial f_{it}^j}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} = \beta^i E_t[\pi_{3_j}(t+i)].
$$

Similar derivations yield

$$\left.\frac{\partial^2 V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)}{\partial f_{it}^j \partial f_{it}^m}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} = \beta^i E_t[\pi_{3_j 3_m}(t+i)],$$

$$\left.\frac{\partial^2 V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)}{\partial f_{it}^j \partial f_{nt}^m}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} = 0 \text{ for } i \neq n.$$

Now consider derivatives with respect to ζ:

$$\begin{aligned}\left.\frac{\partial V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)}{\partial \zeta}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} &= \sum_{j=1}^{J} \beta E_t \left[\frac{\partial V(\boldsymbol{Z}_{t+1}, \boldsymbol{w}_{t+1}, \boldsymbol{F}_{t+1}; \zeta)}{\partial w_{t+1}^j} \epsilon_{t+1}^j \right] \\ &\quad + \sum_{j=1}^{J} \beta \sum_{i=1}^{N} E_t \left[\frac{\partial V(\boldsymbol{Z}_{t+1}, \boldsymbol{w}_{t+1}, \boldsymbol{F}_{t+1}; \zeta)}{\partial f_{i(t+1)}^j} \nu_{i(t+1)}^j \right] \\ &\quad + \sum_{j=1}^{J} \beta E_t \left[\frac{\partial V(\boldsymbol{Z}_{t+1}, \boldsymbol{w}_{t+1}, \boldsymbol{F}_{t+1}; \zeta)}{\partial \zeta} \right] \\ &= \sum_{j=1}^{J} \sum_{s=1}^{\infty} \beta^s \left\{ Cov_t \left[\pi_{3_j}(t+s), \epsilon_{t+s}^j \right] + \sum_{i=1}^{N} \beta^i Cov_t \left[\pi_{3_j}(t+s+i), \nu_{i(t+s)}^j \right] \right\},\end{aligned}$$

$$\begin{aligned}\left.\frac{\partial^2 V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)}{\partial \zeta^2}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} &= \sum_{j=1}^{J} \sum_{s=1}^{\infty} \beta^s \Bigg\{ E_t \left[\pi_{3_j 3_j}(t+s) \right] (\sigma^j)^2 + Cov_t \left[\pi_{3_j 3_j}(t+s), (\epsilon_{t+s}^j)^2 \right] \\ &\quad + \sum_{i=1}^{N} \beta^i E_t \left[\pi_{3_j 3_j}(t+s+i) \right] (\tau_i^j)^2 \\ &\quad + \sum_{i=1}^{N} \beta^i Cov_t \left[\pi_{3_j 3_j}(t+s+i), (\nu_{i(t+s)}^j)^2 \right] \Bigg\},\end{aligned}$$

$$\left.\frac{\partial^2 V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)}{\partial \zeta \partial f_{it}^j}\right|_{(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}, 0)} = \beta^i Cov_t \left[\pi_{3_j 3_j}(t+i), \epsilon_{t+i}^j + \sum_{s=1}^{i-1} \nu_{(i-s)(t+s)}^j \right].$$

Substitute in:

$$\begin{aligned}
V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta) \approx & V(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}; 0) + \sum_{k=1}^{K} \bar{\pi}_{2_k}(Z_t^k - \bar{A}^k) + \sum_{j=1}^{J} \bar{\pi}_{3_j}(w_t^j - C^j) + \frac{1}{2}\sum_{k=1}^{K} \bar{\pi}_{2_k 2_k}(Z_t^k - \bar{A}^k)^2 \\
& + \frac{1}{2}\sum_{j=1}^{J} \bar{\pi}_{3_j 3_j}(w_t^j - C^j)^2 + \sum_{k=1}^{K-1}\sum_{h=k+1}^{K} \bar{\pi}_{2_k 2_h}(Z_t^k - \bar{A}^k)(Z_t^h - \bar{A}^h) \\
& + \sum_{j=1}^{J}\sum_{i=1}^{N} \beta^i E_t[\pi_{3_j}(t+i)](f_{it}^j - C^j) + \frac{1}{2}\sum_{j=1}^{J}\sum_{i=1}^{N} \beta^i E_t[\pi_{3_j 3_j}(t+i)](f_{it}^j - C^j)^2 \\
& + \sum_{j=1}^{J-1}\sum_{m=j+1}^{J}\sum_{i=1}^{N} \beta^i E_t[\pi_{3_j 3_m}(t+i)](f_{it}^j - C^j)(f_{it}^m - C^m) \\
& + \sum_{j=1}^{J}\sum_{i=1}^{N} \beta^i Cov_t\left[\pi_{3_j 3_j}(t+i), \epsilon_{t+i}^j + \sum_{s=1}^{i-1} \nu_{(i-s)(t+s)}^j\right](f_{it}^j - C^j)\zeta \\
& + \sum_{j=1}^{J}\sum_{s=1}^{\infty} \beta^s \left\{ Cov_t\left[\pi_{3_j}(t+s), \epsilon_{t+s}^j\right] + \sum_{i=1}^{N} \beta^i Cov_t\left[\pi_{3_j}(t+s+i), \nu_{i(t+s)}^j\right] \right\}\zeta \\
& + \frac{1}{2}\sum_{j=1}^{J}\sum_{s=1}^{\infty} \beta^s \bigg\{ E_t\left[\pi_{3_j 3_j}(t+s)\right](\sigma^j)^2 + Cov_t\left[\pi_{3_j 3_j}(t+s), (\epsilon_{t+s}^j)^2\right] \\
& \qquad + \sum_{i=1}^{N} \beta^i E_t\left[\pi_{3_j 3_j}(t+s+i)\right](\tau_i^j)^2 + \sum_{i=1}^{N} \beta^i Cov_t\left[\pi_{3_j 3_j}(t+s+i), (\nu_{i(t+s)}^j)^2\right] \bigg\}\zeta^2.
\end{aligned}$$

Now take the time 0 expectation of value at some time $t > N$:

$$
\begin{aligned}
E_0[V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)] \approx & V(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}; 0) + \sum_{k=1}^{K} \bar{\pi}_{2_k} E_0[Z_t^k - \bar{A}^k] + \frac{1}{2} \sum_{k=1}^{K} \bar{\pi}_{2_k 2_k} E_0[(Z_t^k - \bar{A}^k)^2] \\
& + \sum_{k=1}^{K-1} \sum_{h=k+1}^{K} \bar{\pi}_{2_k 2_h} E_0 \left[(Z_t^k - \bar{A}^k)(Z_t^h - \bar{A}^h) \right] \\
& + \frac{1}{2} \sum_{j=1}^{J} \bar{\pi}_{3_j 3_j} \left((\sigma^j)^2 + \sum_{i=1}^{N} (\tau_i^j)^2 \right) \zeta^2 + \frac{1}{2} \sum_{j=1}^{J} \sum_{i=1}^{N} \beta^i E_0[\pi_{3_j 3_j}(t+i)] \sum_{n=i}^{N} (\tau_n^j)^2 \zeta^2 \\
& + \sum_{j=1}^{J} \sum_{s=1}^{\infty} \beta^s \left\{ Cov_0 \left[\pi_{3_j}(t+s), \epsilon_{t+s}^j \right] + \sum_{i=1}^{N} \beta^i Cov_0 \left[\pi_{3_j}(t+s+i), \nu_{i(t+s)}^j \right] \right\} \zeta \\
& + \frac{1}{2} \sum_{j=1}^{J} \sum_{s=1}^{\infty} \beta^s \Bigg\{ E_0 \left[\pi_{3_j 3_j}(t+s) \right] (\sigma^j)^2 + Cov_0 \left[\pi_{3_j 3_j}(t+s), (\epsilon_{t+s}^j)^2 \right] \\
& \qquad + \sum_{i=1}^{N} \beta^i E_0 \left[\pi_{3_j 3_j}(t+s+i) \right] (\tau_i^j)^2 + \sum_{i=1}^{N} \beta^i Cov_0 \left[\pi_{3_j 3_j}(t+s+i), (\nu_{i(t+s)}^j)^2 \right] \Bigg\}
\end{aligned}
$$

A marginal change in climate can alter several (or all) of the J dimensions of the climate index. The derivative with respect to dimension $m \in \{1, ..., J\}$ of climate is:

$$\frac{\mathrm{d}E_0[V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)]}{\mathrm{d}C^m}$$

$$\approx \frac{\mathrm{d}V(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}; 0)}{\mathrm{d}C^m} + \sum_{k=1}^{K} \bar{\pi}_{2_k} E_0\left[\frac{\mathrm{d}Z_t^k}{\mathrm{d}C^m}\right] + \sum_{k=1}^{K} \bar{\pi}_{2_k 2_k} E_0\left[(Z_t^k - \bar{A}^k)\left(\frac{\mathrm{d}Z_t^k}{\mathrm{d}C^m} - \frac{\mathrm{d}\bar{A}^k}{\mathrm{d}C^m}\right)\right]$$

$$+ \sum_{k=1}^{K-1} \sum_{h=k+1}^{K} \bar{\pi}_{2_k 2_h} E_0\left[\frac{\mathrm{d}Z_t^k}{\mathrm{d}C^m}(Z_t^h - \bar{A}^h) + (Z_t^k - \bar{A}^k)\frac{\mathrm{d}Z_t^h}{\mathrm{d}C^m}\right]$$

$$+ \frac{1}{2}\sum_{j=1}^{J} \bar{\pi}_{3_j 3_j 3_m}\left((\sigma^j)^2 + \sum_{i=1}^{N}(\tau_i^j)^2\right)\zeta^2 + \frac{1}{2}\sum_{j=1}^{J}\sum_{i=1}^{N}\beta^i E_0[\pi_{3_j 3_j 3_m}(t+i)]\sum_{n=i}^{N}(\tau_n^j)^2\zeta^2$$

$$+ \sum_{j=1}^{J}\sum_{s=N+1}^{\infty}\beta^s\left\{Cov_0\left[\pi_{3_j 3_m}(t+s), \epsilon_{t+s}^j\right] + \sum_{i=1}^{N}\beta^i Cov_0\left[\pi_{3_j 3_m}(t+s+i), \nu_{i(t+s)}^j\right]\right\}\zeta$$

$$+ \frac{1}{2}\sum_{j=1}^{J}\sum_{s=N+1}^{\infty}\beta^s\Bigg\{E_0\left[\pi_{3_j 3_j 3_m}(t+s)\right](\sigma^j)^2 + Cov_0\left[\pi_{3_j 3_j 3_m}(t+s), (\epsilon_{t+s}^j)^2\right]$$

$$+ \sum_{i=1}^{N}\beta^i E_0\left[\pi_{3_j 3_j 3_m}(t+s+i)\right](\tau_i^j)^2$$

$$+ \sum_{i=1}^{N}\beta^i Cov_0\left[\pi_{3_j 3_j 3_m}(t+s+i), (\nu_{i(t+s)}^j)^2\right]\Bigg\}\zeta^2.$$

The terms on the first line are analogous to the terms in the main text, capturing the change in steady-state value and the change in transition value. In the main text, the second term on the first line vanished because Assumption 1 held. The second line arises only for $K > 1$. The remaining lines are new, as they vanish when π is quadratic. They capture how preferences for variance change with climate. $\bar{\pi}_{3_j 3_j 3_j} > 0$ means that the agent prefers to attach a weather lottery to a high weather state, analogous to standard interpretations of prudence in consumption. The whole expression is arbitrarily close to the expression in the main text (generalized for $K \geq 1$) when Assumption 1 holds and either each $\pi_{3_j 3_j 3_m}$ is small or ζ is small.

We now analyze how steady-state value changes with C^m. We have:

$$V(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}; 0) = \frac{1}{1-\beta}\pi(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C}).$$

We then have:

$$\frac{\mathrm{d}V(\bar{\boldsymbol{A}}, \boldsymbol{C}, \boldsymbol{C}; 0)}{\mathrm{d}C^m} = \frac{1}{1-\beta}\frac{\mathrm{d}\pi(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, \boldsymbol{C})}{\mathrm{d}C^m}.$$

That change in steady-state payoffs is:

$$\begin{aligned}\frac{\mathrm{d}\pi(\bar{\boldsymbol{A}},\bar{\boldsymbol{A}},\boldsymbol{C})}{\mathrm{d}C^m} =&\bar{\pi}_{3_m}+\sum_{k=1}^{K}(\bar{\pi}_{1_k}+\bar{\pi}_{2_k})\frac{\mathrm{d}\bar{A}^k}{\mathrm{d}C^m}\\ =&\bar{\pi}_{3_m}+\sum_{k=1}^{K}(\bar{\pi}_{1_k}+\bar{\pi}_{2_k})\frac{\bar{\pi}_{1_k3_m}}{-\bar{\pi}_{1_k1_k}-(1+\beta)\bar{\pi}_{1_k2_k}-\beta\bar{\pi}_{2_k2_k}}.\end{aligned}$$

Using $\bar{\pi}_{1_k}=-\beta\bar{\pi}_{2_k}$ from the Euler equations, we have:

$$\frac{\mathrm{d}\pi(\bar{\boldsymbol{A}},\bar{\boldsymbol{A}},\boldsymbol{C})}{\mathrm{d}C^m} =\bar{\pi}_{3_m}+(1-\beta)\sum_{k=1}^{K}\bar{\pi}_{2_k}\frac{\bar{\pi}_{1_k3_m}}{-\bar{\pi}_{1_k1_k}-(1+\beta)\bar{\pi}_{1_k2_k}-\beta\bar{\pi}_{2_k2_k}}.$$

Now analyze the average treatment effect of weather dimension m:

$$\begin{aligned}ATE^{\pi}_{w_m}(\boldsymbol{C}) \triangleq&E_0\left[\frac{\mathrm{d}\pi(\boldsymbol{A}_t,\boldsymbol{A}_{t-1},\boldsymbol{w}_t)}{\mathrm{d}w_t^m}\right]\\ =&E_0\left[\pi_{3_m}(\boldsymbol{A}_t,\boldsymbol{A}_{t-1},\boldsymbol{w}_t)+\sum_{k=1}^{K}\pi_{1_k}(\boldsymbol{A}_t,\boldsymbol{A}_{t-1},\boldsymbol{w}_t)\frac{\partial A_t^k(\boldsymbol{A}_{t-1},\boldsymbol{w}_t)}{\partial w_t^m}\right].\end{aligned}$$

If π is quadratic, then π_{1_k}, π_{3_m}, and A_t^k are linear and we can pass the expectation operator through to the arguments. The expression then exactly equals $\mathrm{d}\bar{\pi}/\,\mathrm{d}C^m$ if we also impose Assumption 1. Now consider the general case. Use a second-order Taylor series approximation to the term in brackets around $\boldsymbol{A}_t=\bar{\boldsymbol{A}}$, $\boldsymbol{A}_{t-1}=\bar{\boldsymbol{A}}$, and $\boldsymbol{w}_t=\boldsymbol{C}$, assume that actions are on average near their steady state, and use $\pi_{2_k3_m}=0$ and $\bar{\pi}_{1_k}=-\beta\bar{\pi}_{2_k}$:

$$\begin{aligned}ATE^{\pi}_{w_m}(\boldsymbol{C}) =&\bar{\pi}_{3_m}-\beta\sum_{k=1}^{K}\bar{\pi}_{2_k}\frac{\bar{\pi}_{1_k3_m}}{-\bar{\pi}_{1_k1_k}-(1+\beta)\bar{\pi}_{1_k2_k}-\beta\bar{\pi}_{2_k2_k}}\\ &+\sum_{k=1}^{K}\sum_{h=k}^{K}B_{1_{kh}}Cov_0[A_t^k,A_t^h]+\sum_{k=1}^{K}\sum_{h=k}^{K}B_{2_{kh}}Cov_0[A_{t-1}^k,A_{t-1}^h]+\zeta^2\sum_{j=1}^{J}B_{3_j}\left[(\sigma^j)^2+\sum_{i=1}^{N}(\tau_i^j)^2\right]\\ &+\sum_{k=1}^{K}\sum_{h=1}^{K}B_{4_{kh}}Cov_0[A_t^k,A_{t-1}^h]+\sum_{j=1}^{J}\sum_{k=1}^{K}B_{5_{kj}}Cov_0[A_t^k,w_t^j]+\sum_{j=1}^{J}\sum_{k=1}^{K}B_{6_{kj}}Cov_0[A_{t-1}^k,w_t^j],\end{aligned}$$

for constants B. The variances and covariances all vanish as $\zeta\to 0$, leaving only the first line. That first line differs from $\mathrm{d}\bar{\pi}/\,\mathrm{d}C^m$ by having $-\beta\bar{\pi}_{2_k}$ in place of $(1-\beta)\bar{\pi}_{2_k}$, and it is identical to $\mathrm{d}\bar{\pi}/\,\mathrm{d}C^m$ if Assumption 1 holds. Therefore, we have established conditions under which the main result of the paper holds in a general setting: $ATE^{\pi}_{w_m}(\boldsymbol{C})\approx\mathrm{d}\bar{\pi}/\,\mathrm{d}C^m$

when Assumption 1 holds and either π is quadratic or ζ is not too large.[13] The main result of the paper therefore holds under general, non-quadratic payoff functions (and vector-valued actions) as long as i) the variance of weather outcomes is not too large and ii) adjustment costs vanish when current and past actions match each other. Further, these are the same conditions that earlier ensured that the general form of $\mathrm{d}E_0[V(\boldsymbol{Z}_t, \boldsymbol{w}_t, \boldsymbol{F}_t; \zeta)]/\,\mathrm{d}C^m$ matched the expression derived in the main text.

G Proofs and Lemmas

G.1 Proof of Proposition 1

Guess that the value function has the form given in the statement of the proposition. The continuation value becomes:

$$\begin{aligned}
E_t[V] =& a_1 A_t^2 + a_2\sigma^2 + a_2 f_{1t}^2 + \sum_i a_3^i \tau_i^2 + \sum_{i<N} a_3^i f_{(i+1)t}^2 + a_3^N C^2 \\
&+ b_1 A_t f_{1t} + A_t \sum_{i<N} b_2^i f_{(i+1)t} + A_t b_2^N C \\
&+ f_{1t} \sum_{i<N} b_3^i f_{(i+1)t} + f_{1t} b_3^N C + \sum_i \rho_i b_3^i \\
&+ \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} b_4^{ij} f_{(i+1)t} f_{(j+1)t} + \sum_{i=1}^{N-1} b_4^{iN} f_{(i+1)t} C + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij} \delta_{ij} \\
&+ c_1 A_t + c_2 f_{1t} + \sum_{i<N} c_3^i f_{(i+1)t} + c_3^N C + d.
\end{aligned}$$

The first-order condition is

$$\begin{aligned}
\gamma(A_t - w_t) + \alpha(A_t - Z_t) + \phi(A_t - \bar{A}) =& \beta E_t[V_Z(Z_{t+1}, w_{t+1}, \boldsymbol{F}_{t+1}] \\
=& \beta \left[2a_1 A_t + b_1 f_{1t} + \sum_{i<N} b_2^i f_{(i+1)t} + b_2^N C + c_1 \right],
\end{aligned}$$

which implies that optimal actions are

$$A_t^* = \frac{\alpha Z_t + \gamma w_t + \beta b_1 f_{1t} + \beta \sum_{i<N} b_2^i f_{(i+1)t} + \beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1}.$$

[13] If we had instead defined $ATE_{w_m}^{\pi}(C)$ as $E_0[\partial\pi(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, \boldsymbol{w}_t)/\partial w_t^m]$, then we would not require Assumption 1. Also, we defined $ATE_{w_m}^{\pi}(\boldsymbol{C})$ as the average treatment effect conditional on forecasts (so that each A_{t-1}^k did not depend on $\boldsymbol{w}_t$). The primary result would be unchanged if we had allowed forecasts, and thus each A_{t-1}^k, to reflect the change in weather.

Substitute A_t^* into the Bellman equation. Matching coefficients to the guessed form of the value function and simplifying, the quadratic coefficients are:

$$\begin{aligned}
a_1 =& \frac{1}{2}\frac{\alpha^2}{\gamma+\alpha+\phi-2\beta a_1} - \frac{1}{2}\alpha,\\
a_2 =& \frac{1}{2}\gamma\left[\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} - 1\right] - \frac{1}{2}\psi,\\
a_3^1 =& \frac{1}{2}\frac{[\beta b_1]^2}{\gamma+\alpha+\phi-2\beta a_1} + \beta a_2,\\
a_3^i =& \frac{1}{2}\frac{[\beta b_2^{i-1}]^2}{\gamma+\alpha+\phi-2\beta a_1} + \beta a_3^{i-1}.
\end{aligned}$$

Rearrange the solution for a_1:

$$\beta a_1^2 - \frac{1}{2}(\gamma+\alpha+\phi-\beta\alpha)a_1 - \frac{1}{4}\alpha(\gamma+\phi) = 0.$$

Note that a_1 is independent of C. If $\beta = 0$, then the left-hand side is linear in a_1 and the unique solution has $a_1 \leq 0$, with $a_1 < 0$ if and only if $\alpha(\gamma+\phi) > 0$. Recalling that we assumed that $\gamma+\phi > 0$, we then have $a_1 < 0$ if and only if $\alpha > 0$. If $\beta > 0$, then the left-hand side describes a parabola in a_1 that opens up. If $\alpha(\gamma+\phi) = 0$ with $\beta > 0$, then there is a root at zero and a second root that is strictly positive. If $\alpha\beta(\gamma+\phi) > 0$, then the parabola has a strictly negative y-intercept and its roots must be of opposite sign. The two roots are

$$a_1 = \frac{1}{2\beta}\left\{\frac{1}{2}[\gamma+\alpha+\phi-\beta\alpha] \pm \sqrt{\frac{1}{4}(\gamma+\alpha+\phi-\beta\alpha)^2 + \beta\alpha(\gamma+\phi)}\right\}.$$

The second-order condition for A_t^* to be a maximum is

$$0 < \gamma+\alpha+\phi-2\beta a_1.$$

This is satisfied for $a_1 \leq 0$, using that $\gamma+\phi > 0$. The second-order condition therefore holds at the negative root of a_1. As we solve for the other coefficients, we will find that they are unique conditional on a_1. Therefore the first-order condition is satisfied at only two points, determined by the two roots of a_1. Since we know that the value function is strictly concave in A_t at the point with negative a_1, the point with positive a_1 must either be a saddle point or a minimum. We therefore are only interested in the negative root of a_1 and will henceforth ignore the strictly positive root.

Finding that $a_1 \leq 0$ implies that $a_2 \leq 0$. This inequality is strict if either $\gamma > 0$, $\gamma\alpha > 0$, or $\gamma\phi > 0$. a_2 is independent of C because a_1 is independent of C.

Matching coefficients again, the coefficients on the interaction terms become:

$$
\begin{aligned}
b_1 =& \alpha \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}, \\
b_2^1 =& \frac{\alpha\beta b_1}{\gamma+\alpha+\phi-2\beta a_1}, \\
b_2^i =& \frac{\alpha\beta b_2^{i-1}}{\gamma+\alpha+\phi-2\beta a_1} \text{ for } i>1, \\
b_3^1 =& \gamma \frac{\beta b_1}{\gamma+\alpha+\phi-2\beta a_1}, \\
b_3^i =& \beta b_2^{i-1} \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} \text{ for } i>1, \\
b_4^{1j} =& \beta b_2^{j-1} \frac{\beta b_1}{\gamma+\alpha+\phi-2\beta a_1} + \beta b_3^{j-1}, \\
b_4^{ij} =& \frac{\beta^2 b_2^{i-1} b_2^{j-1}}{\gamma+\alpha+\phi-2\beta a_1} + \beta b_4^{(i-1)(j-1)} \text{ for } i>1.
\end{aligned}
$$

Note that we can write

$$
b_2^i = \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i b_1 \quad \text{and} \quad b_3^i = \frac{\gamma}{\alpha} b_2^i,
$$

for all $i \in \{1, ..., N\}$. Using $a_1 \leq 0$, we have $b_1 \geq 0$, which implies $b_2^i \geq 0$, which in turn implies $b_3^i \geq 0$, which in turn implies $b_4^{ij} \geq 0$. Clearly $b_1 > 0$ iff $\alpha\gamma > 0$, and each of the other b coefficients is strictly positive iff $b_1 > 0$ and $\beta > 0$. Finally, because a_1 is independent of C, we have that each b coefficient is independent of C.

Now use the solutions for b_2^i and b_1 to analyze a_3^i:

$$
\begin{aligned}
a_3^i =& \sum_{k=0}^{i-2} \beta^k \frac{1}{2} \frac{[\beta b_2^{i-1-k}]^2}{\gamma + \alpha + \phi - 2\beta a_1} + \beta^{i-1} a_3^1 \\
=& \frac{1}{2} \frac{[\beta b_1]^2}{\gamma + \alpha + \phi - 2\beta a_1} \sum_{k=0}^{i-1} \beta^k \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2(i-1-k)} + \beta^i \left[\frac{1}{2}\gamma \left[\frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} - 1 \right] - \frac{1}{2}\psi \right] \\
=& \frac{1}{2} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2i} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
& - \frac{1}{2}\beta^i [\gamma + \psi] \\
=& \frac{1}{2} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2i} \frac{1 - \beta^{i+1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2(i+1)}}{1 - \beta \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2}} - \frac{1}{2}\beta^i [\gamma + \psi] \\
=& \beta^i \left\{ \frac{1}{2}\gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \frac{1 - \beta^{i+1} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{2i+2}}{1 - \beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{2}} - \frac{1}{2} [\gamma + \psi] \right\}.
\end{aligned}
\tag{A-2}
$$

Note that a_3^i is independent of C. The term in braces increases in i, and it strictly increases in i if and only if $\alpha\beta > 0$. It is weakly greater than a_2 and is strictly greater than a_2 if and only if $\alpha\beta\gamma > 0$. As $i \to \infty$, the term in braces goes to:

$$
\begin{aligned}
& \frac{1}{2}\gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \frac{1}{1 - \beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2} - \frac{1}{2} [\gamma + \psi] \\
=& \frac{1}{2}\gamma \frac{\gamma(\gamma + \alpha + \phi - 2\beta a_1)}{(\gamma + \alpha + \phi - 2\beta a_1)^2 - (\alpha\beta)^2} - \frac{1}{2} [\gamma + \psi] \\
=& \frac{1}{2}\gamma \frac{\gamma(\gamma + \alpha + \phi - 2\beta a_1)}{\gamma(\gamma + \alpha + \phi - 2\beta a_1) + \alpha^2 + \alpha(\gamma + \phi - 2\beta a_1) + (\phi - 2\beta a_1)(\gamma + \alpha + \phi - 2\beta a_1) - (\alpha\beta)^2} - \frac{1}{2} [\gamma + \psi] \\
=& -\frac{1}{2}\gamma \frac{\alpha(\gamma + \phi - 2\beta a_1) + (\phi - 2\beta a_1)(\gamma + \alpha + \phi - 2\beta a_1) + (1 - \beta^2)\alpha^2}{\gamma(\gamma + \alpha + \phi - 2\beta a_1) + \alpha(\gamma + \phi - 2\beta a_1) + (\phi - 2\beta a_1)(\gamma + \alpha + \phi - 2\beta a_1) + (1 - \beta^2)\alpha^2} - \frac{1}{2}\psi \\
\leq & 0.
\end{aligned}
$$

This is strictly negative if either $\psi > 0$, $\gamma\phi > 0$, or $\gamma\alpha > 0$, and it is equal to zero otherwise. Therefore, if $\psi + \gamma(\phi + \alpha) + \alpha\beta > 0$, then the term in braces in (A-2) is strictly negative for all finite i, and if $\psi + \gamma(\phi + \alpha) + \alpha\beta = 0$, then the term in braces in (A-2) is zero.

Now match the coefficients on the linear terms:

$$\begin{aligned}
c_1 =& \alpha \frac{\beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1}, \\
c_2 =& \gamma \frac{\beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1} + \psi \bar{w}, \\
c_3^1 =& \beta b_1 \frac{\beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1} + \beta b_3^N C + \beta c_2, \\
c_3^i =& \beta b_2^{i-1} \frac{\beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1} + \beta \left[b_4^{(i-1)N} C + c_3^{i-1} \right].
\end{aligned}$$

Solve for c_1:

$$c_1 = \alpha \frac{\beta b_2^N C + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}.$$

This increases in C, strictly increases in C iff $\alpha\beta\gamma > 0$, and is strictly positive iff $C > -\phi\bar{A}/[\beta b_2^N]$. Substituting into the expression for c_2, we have:

$$c_2 = \gamma \frac{c_1}{\alpha} + \psi \bar{w}.$$

This increases in C, strictly increases in C iff $\alpha\beta\gamma > 0$, and is positive if $c_1 \geq 0$ and $\bar{w} \geq 0$. Substituting for c_1 and for the recursive terms in each c_3^i, we find:

$$c_3^i = \beta^i b_1 \frac{c_1}{\alpha} \sum_{j=0}^{i-1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^j + \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} C + \beta^i \left[b_3^N C + c_2 \right].$$

This too increases in C, strictly increases in C iff $\alpha\beta\gamma > 0$, and is positive if $c_1 \geq 0$ and $\bar{w} \geq 0$.

Finally, matching coefficients yields the constant:

$$\begin{aligned}
d =& \frac{1}{2}(\beta b_2^N C + \beta c_1 + \phi \bar{A}) \frac{c_1}{\alpha} + \beta a_2 \sigma^2 + \beta \sum_i a_3^i \tau_i^2 \\
& + \beta c_3^N C + \beta a_3^N C^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij} \delta_{ij} + \sum_i \rho_i b_3^i - \frac{1}{2}\phi \bar{A}^2 - \frac{1}{2}\psi \bar{w}^2 + \beta d.
\end{aligned}$$

Solving for d yields:

$$\begin{aligned}
d =& \frac{1}{1-\beta} \left\{ \frac{1}{2}(\beta b_2^N C + \beta c_1 + \phi \bar{A}) \frac{c_1}{\alpha} + \beta a_2 \sigma^2 + \beta \sum_i a_3^i \tau_i^2 \right. \\
& \left. + \beta c_3^N C + \beta a_3^N C^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij} \delta_{ij} + \sum_i \rho_i b_3^i - \frac{1}{2}\phi \bar{A}^2 - \frac{1}{2}\psi \bar{w}^2 \right\}.
\end{aligned}$$

G.2 Two Lemmas

The first lemma establishes properties of a_1 that will come in handy in later proofs.

Lemma 9.

1. *$a_1 \to 0$ as $\alpha \to 0$.*
2. *$\beta a_1 \to 0$ as $\beta \to 0$.*
3. *$a_1 \to -\frac{1}{2}\frac{\gamma+\phi}{1-\beta}$ as $\alpha \to \infty$.*
4. *$a_1 \to -\frac{1}{2}\alpha$ as either $\gamma \to \infty$ or $\phi \to \infty$.*

Proof. The first claim follows from the analysis in the proof of Proposition 1.

To prove the second claim, first observe that the proof of Proposition 1 showed that $\beta a_1 = 0$ if $\beta = 0$. Then note that as β goes to 0, we have:

$$\lim_{\beta \to 0} \beta a_1 = \frac{1}{2}\left\{\frac{1}{2}[\gamma+\alpha+\phi] \pm \sqrt{\frac{1}{4}(\gamma+\alpha+\phi)^2}\right\} = 0.$$

We now consider the third claim. First assume that $\beta = 0$. We have:

$$\lim_{\alpha\to\infty} a_1 = -\frac{1}{2}(\gamma+\phi).$$

Now assume that $\beta > 0$. Rewrite a_1 as

$$a_1 = \frac{1}{4\beta}\frac{1-\sqrt{1+4\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^2}}}{\frac{1}{\gamma+\alpha+\phi-\beta\alpha}}.$$

We have:

$$\lim_{\alpha\to\infty} a_1 = \frac{0}{0}.$$

Use L'Hôpital's Rule:

$$\begin{aligned}\lim_{\alpha\to\infty} a_1 &= \lim_{\alpha\to\infty}\frac{1}{2\beta(1-\beta)}\frac{\beta(\gamma+\phi) - 2\frac{\beta\alpha(\gamma+\phi)(1-\beta)}{\gamma+\alpha+\phi-\beta\alpha}}{\sqrt{1+4\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^2}}}\\ &= -\frac{1}{2}\frac{\gamma+\phi}{1-\beta}.\end{aligned}$$

Now consider the fourth claim. First assume that $\beta = 0$. We have:

$$\lim_{\gamma\to\infty} a_1 = -\frac{1}{2}\alpha,$$
$$\lim_{\phi\to\infty} a_1 = -\frac{1}{2}\alpha.$$

Now assume that $\beta > 0$. As above, we have:

$$\lim_{\gamma\to\infty} a_1 = \frac{0}{0}.$$

Use L'Hôpital's Rule:

$$\begin{aligned}
\lim_{\gamma\to\infty} a_1 &= \lim_{\gamma\to\infty} \frac{1}{\beta} \frac{-\frac{1}{2}\frac{\frac{\beta\alpha}{(\gamma+\alpha+\phi-\beta\alpha)^2}-2\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^3}}{\sqrt{1+4\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^2}}}}{-\frac{1}{(\gamma+\alpha+\phi-\beta\alpha)^2}} \\
&= \lim_{\gamma\to\infty} \frac{1}{2\beta} \frac{\beta\alpha - 2\frac{\beta\alpha(\gamma+\phi)}{\gamma+\alpha+\phi-\beta\alpha}}{\sqrt{1+4\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^2}}} \\
&= -\frac{1}{2}\alpha.
\end{aligned}$$

The derivation for $\phi \to \infty$ is similar.

□

The second lemma derives a relationship that will be used in several later proofs:

Lemma 10.
$$\frac{\gamma+\alpha+\phi-2\beta a_1}{(\gamma+\phi-2\beta a_1)(\gamma+\alpha+\phi-2\beta a_1-\alpha\beta)} = \frac{1}{\gamma+\phi}$$

Proof. Using the solution for a_1 in the proof of Proposition 1, we have:

$$\gamma+\phi-2\beta a_1 = \frac{1}{2}(\gamma+\phi) + \sqrt{\frac{1}{4}(\gamma+\alpha+\phi-\beta\alpha)^2+\beta\alpha(\gamma+\phi)} - \frac{1}{2}(1-\beta)\alpha,$$

and

$$\gamma+\alpha+\phi-2\beta a_1-\alpha\beta = \frac{1}{2}(\gamma+\phi) + \sqrt{\frac{1}{4}(\gamma+\alpha+\phi-\beta\alpha)^2+\beta\alpha(\gamma+\phi)} + \frac{1}{2}(1-\beta)\alpha.$$

Therefore:

$$
\begin{aligned}
&(\gamma+\phi-2\beta a_1)(\gamma+(1-\beta)\alpha+\phi-2\beta a_1)\\
=&\frac{1}{4}\left(\gamma+\phi\right)^2+\frac{1}{4}(\gamma+\alpha+\phi-\beta\alpha)^2+\beta\alpha(\gamma+\phi)\\
&+(\gamma+\phi)\sqrt{\frac{1}{4}(\gamma+\alpha+\phi-\beta\alpha)^2+\beta\alpha(\gamma+\phi)}-\frac{1}{4}(1-\beta)^2\alpha^2\\
=&\frac{1}{2}\left(\gamma+\phi\right)^2+\frac{1}{2}(1-\beta)\alpha\left(\gamma+\phi\right)+\beta\alpha(\gamma+\phi)+(\gamma+\phi)\sqrt{\frac{1}{4}(\gamma+\alpha+\phi-\beta\alpha)^2+\beta\alpha(\gamma+\phi)}.
\end{aligned}
$$

Substitute for $2\beta a_1$ and factor $\gamma+\phi$:

$$
\begin{aligned}
=&(\gamma+\phi)\left\{\gamma+\phi+(1-\beta)\alpha-2\beta a_1+\beta\alpha\right\}\\
=&(\gamma+\phi)\left\{\gamma+\phi+\alpha-2\beta a_1\right\}.
\end{aligned}
$$

The lemma follows.

□

G.3 Proof of Proposition 2

The autonomous first-order linear difference equation that determines $\hat{A}_t$ is stable because $\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\in[0,1)$. The steady state is:

$$
A^{ss}=\frac{\left(\gamma+\beta b_1+\beta\sum_{i<N}b_2^i+\beta b_2^N\right)C+\beta c_1+\phi\bar{A}}{\gamma+\phi-2\beta a_1}.
$$

Substitute for the coefficients from their solutions in the proof of Proposition 1, solve the geometric series, and simplify:

$$\begin{aligned}
A^{ss} =& \frac{\left(\gamma + \beta b_1 \sum_{i=0}^{N-1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i + \beta b_1 \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\right) C}{\gamma+\phi-2\beta a_1} \\
&+ \frac{\phi\bar{A}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\phi-2\beta a_1} \\
=& \frac{\left(\gamma + \beta b_1 \frac{1-\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N}{1-\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}} + \beta b_1 \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\right) C}{\gamma+\phi-2\beta a_1} \\
&+ \frac{\phi\bar{A}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\phi-2\beta a_1} \\
=& \frac{\left(\gamma + \beta b_1 \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\right) C}{\gamma+\phi-2\beta a_1} + \frac{\phi\bar{A}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\phi-2\beta a_1} \\
=& \frac{\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\alpha+\phi-2\beta a_1-\alpha}\gamma C + \frac{\phi\bar{A}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\alpha+\phi-2\beta a_1-\alpha} \\
=& \frac{\gamma+\alpha+\phi-2\beta a_1}{(\gamma+\phi-2\beta a_1)(\gamma+\alpha+\phi-2\beta a_1-\alpha\beta)} \left[\gamma C + \phi\bar{A}\right]. \qquad \text{(A-3)}
\end{aligned}$$

Using Lemma 10, we have:

$$A^{ss} = \frac{\gamma}{\gamma+\phi} C + \frac{\phi}{\gamma+\phi}\bar{A}.$$

G.4 Proof of Proposition 3

From equation (A-3), we have:

$$\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} = \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\phi-2\beta a_1}.$$

Using the solution for A_t in Proposition 1 and the solutions for the coefficients given in the proof of that proposition, we have:

$$\begin{aligned}
\frac{\partial A_t}{\partial w_t} =& \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}, \\
\frac{\partial A_t}{\partial f_{1t}} =& \frac{\beta b_1}{\gamma+\alpha+\phi-2\beta a_1} \\
=& \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} \frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}.
\end{aligned}$$

We also have, for $i > 1$,

$$\begin{aligned}
\frac{\partial A_t}{\partial f_{it}} =& \frac{\beta b_2^{i-1}}{\gamma+\alpha+\phi-2\beta a_1}\\
=& \frac{\beta}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i-1}\frac{\alpha\gamma}{\gamma+\alpha+\phi-2\beta a_1}\\
=& \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial A_t}{\partial w_t} + \sum_{i=1}^{j}\frac{\partial A_t}{\partial f_{it}} =& \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\sum_{i=0}^{j}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i}\\
=& \frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left[1-\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{j+1}\right].
\end{aligned}$$

Assuming $\gamma > 0$, we now have:

$$Bias(j) = \left[1-\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{j+1}\right]\frac{\gamma+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1} - 1.$$

The term in square brackets is in $(0, 1]$, and the fraction outside of the square brackets is in $(0, 1]$. Therefore $Bias(j) \in (-1, 0]$. As $\alpha \to 0$, $Bias(j) \to 0$. For $\alpha > 0$, the fraction outside of the square brackets is < 1, so $Bias(j) < 0$.

It is clear that $Bias(j)$ is independent of N. The term in parentheses is in $[0, 1)$, so the term in square brackets increases in j. Therefore $\mathrm{d}Bias(j)/\mathrm{d}j \geq 0$. As $\beta \to 0$, $Bias(j)$ becomes constant in j (using Lemma 9).

As $j, N \to \infty$, the term in brackets goes to 1, so $Bias(j) \to -\alpha/[\gamma + \alpha + \phi - 2\beta a_1]$.

Using Lemma 9, note that $\partial A_t/\partial w_t, \partial A_t/\partial f_{it} \to 0$ as $\alpha \to \infty$. Again using Lemma 9, the term in parentheses in $Bias(j)$ goes to β as $\alpha \to \infty$ and the fraction outside parentheses goes to 0 as $\alpha \to \infty$. Therefore $Bias(j) \to -1$ as $\alpha \to \infty$.

Using Lemma 9, $\mathrm{d}A^{ss}/\mathrm{d}C \to 1$ as $\gamma \to \infty$. Again using Lemma 9, the fraction outside the square brackets in $Bias(j)$ goes to 1 and the term in square brackets also goes to 1. So $Bias(j) \to 0$ as $\gamma \to \infty$.

Finally, it is easy to see that $\partial A_t/\partial w_t, \partial A_t/\partial f_{it}$, $\mathrm{d}A^{ss}/\mathrm{d}C \to 0$ as either $\gamma \to 0$ or (using Lemma 9) as $\phi \to \infty$.

G.5 Proof of Proposition 4

Solving the linear difference equation for $\hat{A}_t$ given in the main text, we have:

$$\hat{A}_t = A^{ss} + \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^t \left[\hat{A}_0 - A^{ss} \right].$$

Using the solution for A^{ss} given in Proposition 2, we have:

$$\frac{\mathrm{d}\hat{A}_t}{\mathrm{d}C} = \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^t \right] \frac{\gamma}{\gamma + \phi}.$$

Recalling that $\hat{A}_t \triangleq E_0[A_t]$, the change in transition value is

$$\begin{aligned}
&2a_1 E_0 \left[(Z_t - A^{ss}) \left(\frac{\mathrm{d}Z_t}{\mathrm{d}C} - \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} \right) \right] \\
=&2a_1 \Bigg\{ (\hat{A}_{t-1} - A^{ss}) \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{t-1} \right] \frac{\gamma}{\gamma + \phi} C + Cov_0 \left[A_{t-1}, \frac{\mathrm{d}A_{t-1}}{\mathrm{d}C} \right] - E_0 \left[(A_{t-1} - A^{ss}) \right] \frac{\gamma}{\gamma +} \\
=&2a_1 \Bigg\{ - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{2(t-1)} [A_0 - A^{ss}] \frac{\gamma}{\gamma + \phi} C + Cov_0 \left[A_{t-1}, \frac{\mathrm{d}A_{t-1}}{\mathrm{d}C} \right] \Bigg\}.
\end{aligned}$$

Using the difference equation for A_t given in Proposition 1 and recognizing that w_t and f_{it} are linearly separable in C and the random variables, we have $Cov_0 \left[A_{t-1}, \frac{\mathrm{d}A_{t-1}}{\mathrm{d}C} \right] = 0$. Therefore the change in transition value is

$$-2a_1 \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{2(t-1)} \frac{\gamma}{\gamma + \phi} C \, [A_0 - A^{ss}].$$

This is zero if $\alpha\gamma = 0$ and is proportional to $[A_0 - A^{ss}]$ if $\alpha\gamma > 0$, in which case the change in transition value is negative if and only if $A_0 < A^{ss}$. The change in transition value also goes to zero as $A_t \to A^{ss}$ and, because the term in parentheses is < 1, as $t \to \infty$.

G.6 Proof of Proposition 5

I here prove the result through algebraic manipulations. A shorter, cleaner proof would follow the analysis of Section F.

We begin by analyzing several of the value coefficients derived in the proof of Proposition

1. First, we have:

$$
\begin{aligned}
b_4^{ij} =& \frac{\beta^2 b_2^{i-1} b_2^{j-1}}{\gamma+\alpha+\phi-2\beta a_1} + \beta b_4^{(i-1)(j-1)} \\
=& \frac{\beta^{i+j}}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i-1+j-1} [b_1]^2 + \beta b_4^{(i-1)(j-1)} \\
=& \sum_{k=0}^{i-2} \beta^k \frac{\beta^{(i-k)+(j-k)}}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{(i-1-k)+(j-1-k)} [b_1]^2 + \beta^{i-1} b_4^{1(j-(i-1))} \\
=& \frac{\beta^{i+j}}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{(i-1)+(j-1)} [b_1]^2 \sum_{k=0}^{i-2} \beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
& + \beta^i b_2^{j-i} \frac{\beta b_1}{\gamma+\alpha+\phi-2\beta a_1} + \beta^i b_3^{j-i}.
\end{aligned}
$$

Note that:

$$
\begin{aligned}
& \beta^i b_2^{j-i} \frac{\beta b_1}{\gamma+\alpha+\phi-2\beta a_1} \\
=& \frac{\beta^i}{\gamma+\alpha+\phi-2\beta a_1} [b_1]^2 \beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j-i} \beta^{j-i} \\
=& \frac{\beta^{i+j}}{\gamma+\alpha+\phi-2\beta a_1} [b_1]^2 \beta^{i-1} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{(i-1)+(j-1)} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2(i-1)}.
\end{aligned}
$$

We then have:

$$
\begin{aligned}
b_4^{ij} =& \frac{\beta^{i+j}}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{(i-1)+(j-1)} [b_1]^2 \sum_{k=0}^{i-1} \beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
& + \beta^i b_3^{j-i} \\
=& \frac{\beta^{i+j}}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i+j} \gamma^2 \sum_{k=0}^{i-1} \beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
& + \beta^i \frac{\gamma}{\alpha} b_2^{j-i} \\
=& \frac{\beta^{i+j}}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i+j} \gamma^2 \sum_{k=0}^{i} \beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}. \qquad \text{(A-4)}
\end{aligned}
$$

Second, analyze the following term, which we will see often:

$$\sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} = \frac{1-\beta^{-(i+1)}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2(i+1)}}{1-\beta^{-1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2}}$$
$$= \frac{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2}-\beta^{-i}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2i}}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2}-1}. \tag{A-5}$$

Third, analyze c_3^i further:

$$c_3^i = \beta^i b_1 \frac{c_1}{\alpha} \sum_{j=0}^{i-1} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j} + \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} C + \beta^i \left[b_3^N C + c_2\right]$$
$$= \beta^i b_1 \frac{c_1}{\alpha} \frac{1-\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^{i}}{1-\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}} + \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} C + \beta^i \frac{\gamma}{\alpha}\left[b_2^N C + c_1\right] + \beta^i \psi \bar{w}. \tag{A-6}$$

Now turn to our expression of interest. Substituting in for the value function derivatives, we have:

$$\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C} = 2a_2 C + b_1 A^{ss} + \sum_{i=1}^{N} b_3^i C + c_2$$
$$+ \sum_{i=1}^{N}\left[2a_3^i C + b_2^i A^{ss} + b_3^i C + \sum_{j=i+1}^{N} b_4^{ij} C + \sum_{j=1}^{i-1} b_4^{ji} C + c_3^i\right]$$
$$+ \frac{\mathrm{d}c_1}{\mathrm{d}C} A^{ss} + \frac{\mathrm{d}c_2}{\mathrm{d}C} C + \sum_{i=1}^{N} \frac{\mathrm{d}c_3^i}{\mathrm{d}C} C + \frac{\mathrm{d}d}{\mathrm{d}C}.$$

This expression is linear in C. We will first analyze the terms without C before analyzing the slope in C. Combining the results gives the statement of the proposition.

Analyzing the terms in $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\,\mathrm{d}C$ that are independent of C

Using the value function coefficients derived in the proof of Proposition 1 and also using equation (A-6), the terms without C in $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\,\mathrm{d}C$ are:

$$
\begin{aligned}
&\left[b_1 \frac{\phi}{\gamma+\phi}\bar{A} + \psi\bar{w} + \gamma \frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\right] \\
&+ \sum_{i=1}^{N} b_2^i \frac{\phi}{\gamma+\phi}\bar{A} \\
&+ \sum_{i=1}^{N} \beta^i b_1 \frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left[1-\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\right]\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\phi-2\beta a_1} \\
&+ \sum_{i=1}^{N} \beta^i \gamma \frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+ \sum_{i=1}^{N} \beta^i \psi\bar{w} \\
&+ \frac{\alpha\beta b_2^N}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi}{\gamma+\phi}\bar{A} \\
&+ \frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} b_2^N \frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+ \frac{\beta}{1-\beta}\beta^N b_1 \frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left[1-\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^N\right]\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\phi-2\beta a_1} \\
&+ \frac{\beta}{1-\beta}\beta^N \gamma \frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+ \frac{\beta}{1-\beta}\beta^N \psi\bar{w}.
\end{aligned}
$$

Apply Lemma 10 to the third, seventh, and eighth lines, cancel the second line with part of the third, and solve the geometric series:

$$
\begin{aligned}
=&\left[b_1\frac{\phi}{\gamma+\phi}\bar{A}+\psi\bar{w}+\gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\right]\\
&+\beta\frac{1-\beta^N}{1-\beta}b_1\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\beta\frac{1-\beta^N}{1-\beta}\gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+\beta\frac{1-\beta^N}{1-\beta}\psi\bar{w}\\
&+\frac{\alpha\beta b_2^N}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi}{\gamma+\phi}\bar{A}\\
&+\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\frac{\beta}{1-\beta}\beta^N b_1\frac{\phi\bar{A}}{\gamma+\phi}\left[1-\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^N\right]\\
&+\frac{\beta}{1-\beta}\beta^N\gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+\frac{\beta}{1-\beta}\beta^N\psi\bar{w}.
\end{aligned}
$$

Cancel the final two lines and part of the third-to-last line with earlier lines:

$$
\begin{aligned}
=&b_1\frac{\phi}{\gamma+\phi}\bar{A}+\psi\bar{w}+\gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+\frac{\beta}{1-\beta}b_1\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\frac{\beta}{1-\beta}\gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+\frac{\beta}{1-\beta}\psi\bar{w}\\
&+\frac{\alpha\beta b_2^N}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi}{\gamma+\phi}\bar{A}\\
&+\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\frac{\phi\bar{A}}{\gamma+\phi}\\
&-\frac{\beta}{1-\beta}\beta^N b_1\frac{\phi\bar{A}}{\gamma+\phi}\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^N.
\end{aligned}
$$

Combine the first four lines and substitute b_2^N into the final line:

$$
\begin{aligned}
=&\frac{1}{1-\beta}b_1\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\frac{1}{1-\beta}\gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+\frac{1}{1-\beta}\psi\bar{w}\\
&+\frac{\alpha\beta b_2^N}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi}{\gamma+\phi}\bar{A}\\
&+\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\frac{\phi\bar{A}}{\gamma+\phi}\\
&-\frac{\beta}{1-\beta}\frac{\phi\bar{A}}{\gamma+\phi}b_2^N.
\end{aligned}
$$

Combine the final two lines, substitute for b_1 in the first line, and apply Lemma 10 to the second line:

$$
\begin{aligned}
=&\frac{1}{1-\beta}\frac{\gamma\alpha}{\gamma+\alpha+\phi-2\beta a_1}\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\frac{1}{1-\beta}\gamma\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1}\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\frac{1}{1-\beta}\psi\bar{w}\\
&+\frac{\alpha\beta b_2^N}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi}{\gamma+\phi}\bar{A}\\
&-\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi\bar{A}}{\gamma+\phi}b_2^N.
\end{aligned}
$$

Combine the first two lines and cancel the final two lines:

$$
=\frac{1}{1-\beta}\frac{\gamma\phi}{\gamma+\phi}\bar{A}+\frac{1}{1-\beta}\psi\bar{w}. \tag{A-7}
$$

Analyzing the slope of $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\,\mathrm{d}C$ in C

The slope of $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\,\mathrm{d}C$ in C is:

$$\begin{aligned}\frac{\mathrm{d}^2V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2} =& 2a_2 + b_1\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \sum_{i=1}^{N} b_3^i + \frac{\mathrm{d}c_2}{\mathrm{d}C} \\ &+ \sum_{i=1}^{N}\left[2a_3^i + b_2^i\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + b_3^i + \sum_{j=i+1}^{N} b_4^{ij} + \sum_{j=1}^{i-1} b_4^{ji} + \frac{\mathrm{d}c_3^i}{\mathrm{d}C}\right] \\ &+ \frac{\mathrm{d}c_1}{\mathrm{d}C}\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \frac{\mathrm{d}c_2}{\mathrm{d}C} + \sum_{i=1}^{N}\frac{\mathrm{d}c_3^i}{\mathrm{d}C} \\ &+ \frac{\beta}{1-\beta}\left\{\frac{\gamma+\alpha+\phi-2\beta a_1}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}\beta[b_2^N]^2 + 2\frac{\mathrm{d}c_3^N}{\mathrm{d}C} + 2a_3^N\right\}.\end{aligned}$$

Differentiate equation (A-6), and use Lemma 10 and equation (A-4):

$$\begin{aligned}\sum_{i=1}^{N}\frac{\mathrm{d}c_3^i}{\mathrm{d}C} =& \sum_{i=1}^{N}\beta^i\left[1-\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\right]\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N \\ &+ \sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}b_4^{jN} \\ &+ \sum_{i=1}^{N}\beta^i\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N \\ =& \beta\frac{1-\beta^N}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N \\ &- \frac{\gamma}{\gamma+\phi}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^2 b_2^N\sum_{i=0}^{N-1}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i \\ &+ \sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\ &+ \beta\frac{1-\beta^N}{1-\beta}\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N.\end{aligned}$$

Using this expression and $\mathrm{d}c_3^N/\mathrm{d}C$, we then have:

$$\frac{\mathrm{d}c_1}{\mathrm{d}C}\frac{\mathrm{d}A^{ss}}{\mathrm{d}C}+\frac{\mathrm{d}c_2}{\mathrm{d}C}+\sum_{i=1}^{N}\frac{\mathrm{d}c_3^i}{\mathrm{d}C}+\frac{\beta}{1-\beta}\left\{\frac{\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\beta[b_2^N]^2+2\frac{\mathrm{d}c_3^N}{\mathrm{d}C}+2a_3^N\right\}$$

$$\begin{aligned}
=&\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\left[\frac{\gamma}{\gamma+\phi}+\frac{\gamma}{\alpha}\right]\\
&+\beta\frac{1+\beta^N}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N\\
&-\frac{\gamma}{\gamma+\phi}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^2 b_2^N\sum_{i=0}^{N-1}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\\
&+\sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+\beta\frac{1+\beta^N}{1-\beta}\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\\
&+\beta\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}[b_2^N]^2\\
&-2\frac{\beta}{1-\beta}\beta^N\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^N\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N\\
&+2\frac{\beta}{1-\beta}\beta^N\sum_{j=1}^{N-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}b_2^N\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N\sum_{k=0}^{N}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}-\frac{\beta}{1-\beta}\beta^N\left[\gamma+\psi\right].
\end{aligned}$$

Do the summation in the third line and simplify:

$$
\begin{aligned}
&\frac{\mathrm{d}c_1}{\mathrm{d}C}\frac{\mathrm{d}A^{ss}}{\mathrm{d}C}+\frac{\mathrm{d}c_2}{\mathrm{d}C}+\sum_{i=1}^{N}\frac{\mathrm{d}c_3^i}{\mathrm{d}C}+\frac{\beta}{1-\beta}\left\{\frac{\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\beta[b_2^N]^2+2\frac{\mathrm{d}c_3^N}{\mathrm{d}C}+2a_3^N\right\}\\
=&\frac{1+\beta^{N+1}}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N\\
&+\sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}\frac{(1+\beta^N)(\gamma+\alpha+\phi-2\beta a_1)+(1-\beta)\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\\
&+\beta\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}[b_2^N]^2\\
&-\frac{\beta}{1-\beta}\frac{2(\gamma+\alpha+\phi-2\beta a_1-\alpha\beta)-(1-\beta)\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\gamma}{\gamma+\phi}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{N+1}b_2^N\\
&+2\frac{\beta}{1-\beta}\beta^N\sum_{j=1}^{N-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}b_2^N\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{N}\sum_{k=0}^{N}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}-\frac{\beta}{1-\beta}\beta^N\left[\gamma+\psi\right].
\end{aligned}
\tag{A-8}
$$

Now analyze the other terms in $\mathrm{d}V(A^{ss},C,\boldsymbol{C})/\,\mathrm{d}C$, substituting in for the coefficients derived in the proof of Proposition 1:

$$
\begin{aligned}
&\left[2a_2 + b_1\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \sum_{i=1}^{N} b_3^i + \frac{\mathrm{d}c_2}{\mathrm{d}C}\right] + \sum_{i=1}^{N}\left[2a_3^i + b_2^i\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + b_3^i + \sum_{j=i+1}^{N} b_4^{ij} + \sum_{j=1}^{i-1} b_4^{ji} + \frac{\mathrm{d}c_3^i}{\mathrm{d}C}\right]\\
=&\gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} - [\gamma+\psi] + \frac{\gamma}{\gamma+\phi}b_1\sum_{i=0}^{N}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i\\
&+ 2\frac{\gamma}{\alpha}b_1\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\sum_{i=0}^{N-1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i + \frac{\gamma}{\alpha}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\\
&+ \sum_{i=1}^{N}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2i}\sum_{k=0}^{i}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&- \sum_{i=1}^{N}\beta^i[\gamma+\psi] + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} b_4^{ij}\\
&+ \sum_{i=1}^{N}\beta^i\left[1-\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\right]\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N\\
&+ \sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}b_4^{jN} + \sum_{i=1}^{N}\beta^i\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N.
\end{aligned}
$$

Solving some of the geometric series and simplifying, this becomes:

$$
\begin{aligned}
&\left[2a_2 + b_1 \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \sum_{i=1}^{N} b_3^i + \frac{\mathrm{d}c_2}{\mathrm{d}C}\right] + \sum_{i=1}^{N}\left[2a_3^i + b_2^i \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + b_3^i + \sum_{j=i+1}^{N} b_4^{ij} + \sum_{j=1}^{i-1} b_4^{ji} + \frac{\mathrm{d}c_3^i}{\mathrm{d}C}\right] \\
=&\frac{\gamma}{\gamma+\phi} b_1 \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+ \frac{1}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} b_2^N \frac{-(\gamma+\alpha+\phi-2\beta a_1+\alpha\beta)(1-\beta)+\beta(1-\beta^N)(\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+ \frac{\gamma}{\alpha} b_1 \frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left[2-\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^N\right] \\
&+ \sum_{i=0}^{N} \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2i} \sum_{k=0}^{i} \beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+ 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} b_4^{ij} \\
&+ \frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{N+1} b_2^N \\
&+ \beta\frac{1-\beta^N}{1-\beta}\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} b_2^N \\
&+ \sum_{i=1}^{N} \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} - \frac{1-\beta^{N+1}}{1-\beta}\left[\gamma+\psi\right].
\end{aligned}
$$

Substituting for b_1 and b_2^N and simplifying, we have:

$$
\begin{aligned}
&\left[2a_2 + b_1 \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \sum_{i=1}^{N} b_3^i + \frac{\mathrm{d}c_2}{\mathrm{d}C}\right] + \sum_{i=1}^{N}\left[2a_3^i + b_2^i \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + b_3^i + \sum_{j=i+1}^{N} b_4^{ij} + \sum_{j=1}^{i-1} b_4^{ji} + \frac{\mathrm{d}c_3^i}{\mathrm{d}C}\right] \\
=&\frac{\gamma}{\gamma+\phi}\frac{\gamma\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+ \frac{1}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} b_2^N \\
&\quad \frac{-(\gamma+\alpha+\phi-2\beta a_1+\alpha\beta)(1-\beta)+\beta(1-\beta^N)(\gamma+\alpha+\phi-2\beta a_1-\alpha\beta-\alpha\beta)}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+ 2\gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+ \sum_{i=0}^{N}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2i}\sum_{k=0}^{i}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+ 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} b_4^{ij} \\
&+ \frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{N+1} b_2^N \\
&+ \frac{\beta}{1-\beta}\frac{\gamma}{\alpha}\frac{(1-\beta^N)(\gamma+\alpha+\phi-2\beta a_1)-(1-\beta)\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} b_2^N \\
&+ \sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j} b_4^{jN} - \frac{1-\beta^{N+1}}{1-\beta}\left[\gamma+\psi\right]. \qquad \text{(A-9)}
\end{aligned}
$$

Now combine (A-8) and (A-9) and simplify:

$$
\begin{aligned}
&\frac{\mathrm{d}^2 V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2} \\
=&\frac{1}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} b_2^N \frac{2\beta(\gamma+\alpha+\phi-2\beta a_1-\alpha)}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+2\sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+2\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} b_2^N \\
&+\beta\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}[b_2^N]^2 \\
&-2\frac{\beta}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\gamma+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{N+1} b_2^N \\
&+2\frac{\beta}{1-\beta}\beta^N\sum_{j=1}^{N-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+\frac{\beta}{1-\beta}\frac{\gamma}{\alpha} b_2^N\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{N}\sum_{k=0}^{N}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+\frac{\gamma}{\gamma+\phi}\frac{\gamma\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+2\gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+\sum_{i=0}^{N}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2i}\sum_{k=0}^{i}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} b_4^{ij} \\
&-\frac{1}{1-\beta}[\gamma+\psi]\,.
\end{aligned}
$$

Substitute for b_4^{ij} from equation (A-4), simplify, and rearrange:

$$
\begin{aligned}
&\frac{\mathrm{d}^2 V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2}\\
=&\gamma\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left\{\frac{\gamma}{\gamma+\phi}+2\beta\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\right\}\\
&+\frac{1}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N\frac{2\beta(\gamma+\alpha+\phi-2\beta a_1-\alpha)}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+2\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\\
&+\beta\frac{\beta}{1-\beta}\frac{\gamma\alpha}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^N b_2^N\\
&-2\frac{\beta}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\gamma+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{N+1} b_2^N\\
&+2\sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+2\frac{\beta}{1-\beta}\beta^N\sum_{j=1}^{N-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+\frac{1}{1-\beta}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2N}\sum_{k=0}^{N}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+\sum_{i=0}^{N-1}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2i}\sum_{k=0}^{i}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+2\sum_{j=1}^{N-1}\sum_{i=j+1}^{N}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+i}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&-\frac{1}{1-\beta}\left[\gamma+\psi\right].
\end{aligned}
$$

Substitute from equation (A-5) and simplify:

$$
\begin{aligned}
&\frac{\mathrm{d}^2 V(A^{Ss}, C, \boldsymbol{C})}{\mathrm{d}C^2}\\
=&\gamma\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left\{\frac{\gamma}{\gamma+\phi}+2\beta\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\right\}\\
&+\frac{1}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N\frac{2\beta(\gamma+\alpha+\phi-2\beta a_1-\alpha)}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+2\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\\
&+\beta\frac{\beta}{1-\beta}\frac{\gamma\alpha}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^N b_2^N\\
&-2\frac{\beta}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\gamma+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{N+1} b_2^N\\
&+2\sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N\frac{\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-j}-\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+2}}{1-\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}\\
&+2\frac{\beta}{1-\beta}\beta^N\sum_{j=1}^{N-1}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N\frac{\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-j}-\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+2}}{1-\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}\\
&+\frac{\beta^N}{1-\beta}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\frac{1-\beta^{1+N}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2(1+N)}}{1-\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}\\
&+\sum_{i=0}^{N-1}\beta^i\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\frac{1-\beta^{1+i}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2(1+i)}}{1-\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}\\
&+2\sum_{j=1}^{N-1}\sum_{i=j+1}^{N}\beta^j\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i\frac{\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-j}-\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+2}}{1-\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}\\
&-\frac{1}{1-\beta}[\gamma+\psi]\,.
\end{aligned}
$$

Solving the geometric series, repeatedly using Lemma 10, and working through tedious algebra (available upon request) then yields:

$$\frac{\mathrm{d}^2 V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2} = -\frac{1}{1-\beta}\left[\frac{\gamma\phi}{\gamma+\phi} + \psi\right].$$

It is straightforward to show that

$$\frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C} = \frac{\gamma\phi}{\gamma+\phi}(\bar{A} - C) + \psi(\bar{w} - C).$$

The proposition follows from these last two expressions and (A-7).

G.7 Proof of Proposition 6

Using the value function coefficients derived in the proof of Proposition 1 and applying Lemma 10, we have:

$$
\begin{aligned}
ATE_w^V(C) =& 2a_2 C + b_1 A^{ss} + \sum_{i=1}^{N} b_3^i C + c_2 \\
=& \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1} C - (\gamma+\psi)C + b_1\left[\frac{\gamma}{\gamma+\phi}C + \frac{\phi}{\gamma+\phi}\bar{A}\right] + \sum_{i=1}^{N}\frac{\gamma}{\alpha} b_2^i C \\
&+ \gamma\frac{\beta b_2^N C + \phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} + \psi\bar{w} \\
=& \frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \psi(\bar{w}-C) \\
&- \gamma\frac{\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1}C + \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}C \\
&+ \gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
=& \frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \psi(\bar{w}-C) \\
&- \gamma\frac{\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}C + \gamma\frac{\alpha\beta[\gamma+\phi-2\beta a_1]}{[\gamma+\alpha+\phi-2\beta a_1][\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]}C \\
&+ \frac{\gamma\phi\bar{A}}{\gamma+\phi}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1} \\
=& \frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \psi(\bar{w}-C) \\
&+ \gamma\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1}C\left[\frac{2\beta a_1}{\gamma+\phi} + \frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\right] \\
=& \frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \psi(\bar{w}-C) \\
&+ \frac{\gamma}{\gamma+\phi}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1}C\left[2\beta a_1 + \beta\alpha - \frac{\beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1}\right].
\end{aligned}
$$

Use $a_1 = \frac{1}{2}\frac{\alpha^2}{\gamma+\alpha+\phi-2\beta a_1} - \frac{1}{2}\alpha$ from the proof of Proposition 1:

$$\begin{aligned}
ATE_w^V(C) =& \frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \psi(\bar{w}-C)\\
&+ \frac{\gamma}{\gamma+\phi}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1}C\left[\frac{\beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1} - \beta\alpha + \beta\alpha - \frac{\beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1}\right]\\
=& \frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \psi(\bar{w}-C)\\
=& \frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C}.
\end{aligned}$$

To obtain $ATE_w^\pi(C)$, partially differentiate $\pi(A_t, A_{t-1}, w_t)$ with respect to w_t and then take expectations (and impose the assumption that expected actions are around a steady state):

$$ATE_w^\pi(C) = \gamma(A^{ss}-C) - \psi(C-\bar{w}).$$

Substituting for A^{ss} yields

$$ATE_w^\pi(C) = \frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C}.$$

G.8 Proof of Proposition 7

Differentiating $\pi(A_t, Z_t, w_t)$, we have:

$$\frac{\partial^2\pi(A_t, Z_t, w_t)}{\partial w_t^2} = -\gamma - \psi$$

and

$$\frac{\mathrm{d}^2\pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} = \frac{\partial^2\pi(A_t, Z_t, w_t)}{\partial w_t^2} - \frac{\partial A_t}{\partial w_t}\left[-1 + \frac{\partial A_t}{\partial w_t}(\gamma+\alpha+\phi)\right].$$

From Proposition 1, we have:

$$\frac{\mathrm{d}^2\pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} = \frac{\partial^2\pi(A_t, Z_t, w_t)}{\partial w_t^2} + \gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\frac{-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1}.$$

Note that

$$\frac{\partial^2\pi(A_t, Z_t, w_t)}{\partial w_t^2} - \frac{\mathrm{d}^2\pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} = -\gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\frac{-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1} \leq 0.$$

Using Lemma 9, we see that the inequality is strict if and only if $\gamma > 0$, $\beta > 0$, and $\alpha > 0$. From Proposition 1, we have:

$$\frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w^2} = 2a_2 = \gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} - \gamma - \psi.$$

Note that

$$\frac{\mathrm{d}^2\pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} - \frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w^2} = -\gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\frac{\gamma+\alpha+\phi}{\gamma+\alpha+\phi-2\beta a_1} \leq 0.$$

Using our assumption that that $\gamma + \phi > 0$, the inequality is strict if and only if $\gamma > 0$. Using Proposition 5, Lemma 10, and the value function coefficients derived in the proof of Proposition 1, we have:

$$\begin{aligned}\frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w^2} - \frac{\mathrm{d}^2\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C^2} =& \gamma\left[\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} - 1\right] + \frac{\gamma\phi}{\gamma+\phi}\\ =& -\gamma\frac{\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1} + \gamma\frac{\phi}{\gamma+\phi}\\ =& -\gamma\frac{\alpha-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1} + \gamma\frac{\phi}{\gamma+\phi}\frac{-(\gamma+\phi)+\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1}\\ =& -\gamma\frac{\gamma}{\gamma+\phi}\frac{\alpha-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1}\\ \leq& 0.\end{aligned}$$

The inequality is strict if and only if $\alpha\gamma > 0$. Proposition 5 implies that $\mathrm{d}\pi(A^{ss}, A^{ss}, w_t)/\,\mathrm{d}C^2 \leq 0$, with the inequality strict if and only if either $\psi > 0$ or $\gamma\phi > 0$. We have established the first part of the proposition. To prove the second part of the proposition, note that none of the inequalities above are strict if $\gamma = 0$ and note that $\gamma\phi/[\gamma+\phi] \to \gamma$ as $\phi \to \infty$. To prove the third part of the proposition, note that $\phi = 0$ implies $\gamma > 0$ (by our assumption that $\gamma + \phi > 0$) and, from Proposition 1, $\gamma\alpha > 0$ implies $a_2 < 0$, but $\phi, \psi = 0$ implies $\mathrm{d}^2\pi(A^{ss}, A^{ss}, C)/\,\mathrm{d}C^2 = 0$ from Proposition 5.

G.9 Proof of Proposition 8

Defining $ATE^V_{f_i}(C)$ in the analogous fashion as $ATE^V_w(C)$, we have:

$$ATE^V_{f_i}(C) \triangleq 2a_3^i C + b_2^i A^{ss} + b_3^i C + \sum_{j=i+1}^{N} b_4^{ij} C + \sum_{j=1}^{i-1} b_4^{ji} C + c_3^i.$$

Using the value function coefficients derived in the proof of Proposition 1, we have:

$$
\begin{aligned}
ATE^V_{f_i}(C) =& \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2i}\sum_{k=0}^{i}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} C\\
&-\beta^i\left[\gamma+\psi\right]C\\
&+b_1\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\left[\frac{\gamma}{\gamma+\phi}C+\frac{\phi}{\gamma+\phi}\bar{A}\right]\\
&+\frac{\gamma}{\alpha}b_1\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i C\\
&+C\sum_{j=i+1}^{N}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i+j}\sum_{k=0}^{i}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+C\sum_{j=1}^{i-1}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i+j}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+\beta^i b_1\frac{c_1}{\alpha}\left[1-\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\right]\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\phi-2\beta a_1}+\beta^i\frac{\gamma}{\alpha}\left[b_2^N C+c_1\right]+\beta^i\psi\bar{w}\\
&+\beta^i\frac{\gamma}{\alpha}b_2^N C\frac{1}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}\\
&\quad\left\{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2\sum_{j=1}^{i-1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^j-\sum_{j=1}^{i-1}\beta^{-j}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-j}\right\}.
\end{aligned}
$$

Combine the first and fifth lines, combine the ψ terms, and substitute for c_1 in the third-to-last line:

$$\begin{aligned}
ATE^V_{f_i}(C) =& -\beta^i\gamma C + \beta^i\psi[\bar{w} - C]\\
&+ b_1\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\left[\frac{\gamma}{\gamma+\phi}C + \frac{\phi}{\gamma+\phi}\bar{A}\right]\\
&+ \frac{\gamma}{\alpha}b_1\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i C\\
&+ C\sum_{j=i}^{N}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i+j}\sum_{k=0}^{i}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+ C\sum_{j=1}^{i-1}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i+j}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}\\
&+ \beta^i b_1\frac{\beta b_2^N C + \phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left[1-\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\right]\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\phi-2\beta a_1}\\
&+ \beta^i\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N C + \beta^i\gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+ \beta^i\frac{\gamma}{\alpha}b_2^N C\frac{1}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}\\
&\quad\left\{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2\sum_{j=1}^{i-1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^j - \sum_{j=1}^{i-1}\beta^{-j}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-j}\right\}.
\end{aligned}$$

Apply Lemma 10 to the sixth and seventh lines, combine the sixth line with the second and seventh lines, and solve the geometric series in k:

$$\begin{aligned}
ATE^V_{f_i}(C) =& - \beta^i \gamma C + \beta^i \psi[\bar{w} - C] \\
&+ b_1 \left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right)^i \frac{\gamma}{\gamma+\phi} C \\
&+ \frac{\gamma}{\alpha} b_1 \left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right)^i C \\
&+ C \sum_{j=i}^{N} \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^{i+j} \frac{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2 - \beta^{-i} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2i}}{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2 - 1} \\
&+ C \sum_{j=1}^{i-1} \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^{i+j} \frac{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2 - \beta^{-j} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2j}}{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2 - 1} \\
&- b_1 \frac{\beta b_2^N C}{\gamma+\phi} \left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right)^i \\
&+ \beta^i \frac{\gamma}{\gamma+\phi} \frac{1}{\alpha} \frac{[\gamma+\alpha+\phi-2\beta a_1 - \alpha][\gamma+\alpha+\phi-2\beta a_1] + \beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1} b_2^N C \\
&+ \beta^i \gamma \frac{\phi \bar{A}}{\gamma+\phi} \\
&+ \beta^i \frac{\gamma}{\alpha} b_2^N C \frac{1}{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2 - 1} \\
&\quad \left\{ \beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2 \sum_{j=1}^{i-1} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^j - \sum_{j=1}^{i-1} \beta^{-j} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-j} \right\}.
\end{aligned}$$

Simplify, and solve the geometric series in j:

$$
\begin{aligned}
ATE^V_{f_i}(C) =& -\beta^i\gamma C + \beta^i\psi[\bar{w}-C] \\
&+ b_1\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\frac{\gamma}{\gamma+\phi}C \\
&+ \frac{\gamma}{\alpha}b_1\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i C \\
&+ C\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i\frac{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&\quad\left(1-\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N\right) \\
&- C\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\beta^i}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}\left(1-\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{N-i+1}\right) \\
&+ C\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\frac{\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha} \\
&\quad\left(1-\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-(i-1)}\right) \\
&- b_1\frac{\beta b_2^N C}{\gamma+\phi}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i \\
&+ \beta^i\frac{\gamma}{\gamma+\phi}\frac{1}{\alpha}\frac{[\gamma+\alpha+\phi-2\beta a_1-\alpha][\gamma+\alpha+\phi-2\beta a_1]+\beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1}b_2^N C \\
&+ \beta^i\gamma\frac{\phi\bar{A}}{\gamma+\phi} \\
&+ \beta^i\frac{\gamma}{\alpha}b_2^N C\frac{1}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1} \\
&\quad\left\{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2\sum_{j=1}^{i-1}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^j-\sum_{j=1}^{i-1}\beta^{-j}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-j}\right\}.
\end{aligned}
$$

Substitute b_2^N using its solution from the proof of Proposition 1, substitute the solution for b_1 from the proof of Proposition 1, solve the geometric series in the final line, and simplify:

$$
\begin{aligned}
ATE^V_{f_i}(C) =& -\beta^i\gamma C + \beta^i\psi[\bar{w} - C] + \beta^i\gamma\frac{\phi\bar{A}}{\gamma+\phi} \\
&+ \frac{\gamma\alpha}{\gamma+\alpha+\phi-2\beta a_1}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\frac{\gamma}{\gamma+\phi}C \\
&+ \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i C \\
&+ C\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i\frac{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \\
&- \frac{\gamma}{\alpha}\frac{(\gamma+\alpha+\phi-2\beta a_1)^2-\beta\alpha^2}{(\gamma+\alpha+\phi-2\beta a_1-\alpha)(\gamma+\alpha+\phi-2\beta a_1-\alpha\beta)}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i\frac{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-} \\
&- C\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\beta^i}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1} \\
&+ C\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1-\alpha}\frac{\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1} \\
&- C\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\frac{\beta^i}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha} \\
&- C\frac{\gamma}{\gamma+\phi}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{i+1}b_2^N \\
&+ \beta^i\frac{\gamma}{\gamma+\phi}\frac{1}{\alpha}\frac{[\gamma+\alpha+\phi-2\beta a_1-\alpha][\gamma+\alpha+\phi-2\beta a_1]+\beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1}b_2^N C \\
&+ \beta^i\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1-\alpha}\frac{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}b_2^N C \\
&+ \beta^i\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{1}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2-1}b_2^N C.
\end{aligned}
$$

Apply Lemma 10 and simplify:

$$
\begin{aligned}
ATE^V_{f_i}(C) =& -\beta^i\gamma C + \beta^i\psi[\bar{w}-C] + \beta^i\gamma\frac{\phi\bar{A}}{\gamma+\phi} + \beta^i\frac{\gamma^2}{\gamma+\phi}C \\
&+ \frac{\gamma^2}{\gamma+\phi}\frac{\beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}{(\gamma+\alpha+\phi-2\beta a_1)^2-\beta\alpha^2}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i C \\
&+ \frac{\gamma^2}{\gamma+\phi}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i C \\
&- \frac{\gamma^2}{\gamma+\phi}\frac{(\gamma+\alpha+\phi-2\beta a_1)^2-\alpha\beta(\gamma+\alpha+\phi-2\beta a_1)}{(\gamma+\alpha+\phi-2\beta a_1)^2-\beta\alpha^2}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i C \\
&+ \beta^i\frac{\gamma}{\gamma+\phi}\frac{1}{\alpha}\frac{[\gamma+\alpha+\phi-2\beta a_1-\alpha][\gamma+\alpha+\phi-2\beta a_1]+\beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1}b_2^N C \\
&- \beta^i\frac{\gamma}{\gamma+\phi}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\frac{\beta\alpha^2}{(\gamma+\alpha+\phi-2\beta a_1)^2-\beta\alpha^2}b_2^N C \\
&- \beta^i\frac{\gamma}{\gamma+\phi}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\alpha}\frac{(\gamma+\alpha+\phi-2\beta a_1)^2}{(\gamma+\alpha+\phi-2\beta a_1)^2-\beta\alpha^2}b_2^N C.
\end{aligned}
$$

Combine the final three lines, combine the third and fourth lines, and simplify the first line:

$$
\begin{aligned}
ATE^V_{f_i}(C) =& \beta^i\frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \beta^i\psi[\bar{w}-C] \\
&+ \frac{\gamma^2}{\gamma+\phi}\frac{\beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}{(\gamma+\alpha+\phi-2\beta a_1)^2-\beta\alpha^2}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i C \\
&- \frac{\gamma^2}{\gamma+\phi}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\frac{\beta\alpha^2}{(\gamma+\alpha+\phi-2\beta a_1)^2-\beta\alpha^2}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^i C.
\end{aligned}
$$

Cancel the final two lines:

$$
\begin{aligned}
ATE^V_{f_i}(C) =& \beta^i\frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \beta^i\psi[\bar{w}-C] \\
=& \beta^i ATE^V_w(C).
\end{aligned}
$$

The second equation follows from the proof of Proposition 6. We have proved the first part of Proposition 8.

To prove the second part of Proposition 8, use the above result and Proposition 6 to see that

$$
ATE^V_w(C) + \sum_{i=1}^{\infty} ATE^V_{f_i}(C) = \sum_{i=0}^{\infty}\beta^i ATE^V_w(C) = \frac{1}{1-\beta}ATE^V_w(C) = \frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C}.
$$

To prove the third part of Proposition 8, note that $ATE^V_{f_i}(C)/ATE^V_w(C) = \beta^i$ and use Proposition 6.

The final part of Proposition 8 follows straightforwardly from the fact that $\pi(A_t, At-1, w_t)$ is independent of F_t other than through A_t (and as noted before it is easy to show that $E_0[\partial\pi(A_t, A_{t-1}, w_t)/\partial A_t] = 0$ around a steady state).

References from the Appendix

Carleton, Tamma A. and Solomon M. Hsiang (2016) "Social and economic impacts of climate," *Science*, Vol. 353, No. 6304, p. aad9837.

Deryugina, Tatyana and Solomon Hsiang (2017) "The marginal product of climate," Working Paper 24072, National Bureau of Economic Research.

Deschênes, Olivier and Michael Greenstone (2007) "The economic impacts of climate change: Evidence from agricultural output and random fluctuations in weather," *American Economic Review*, Vol. 97, No. 1, pp. 354–385.

Huntingford, Chris, Philip D. Jones, Valerie N. Livina, Timothy M. Lenton, and Peter M. Cox (2013) "No increase in global temperature variability despite changing regional patterns," *Nature*, Vol. 500, No. 7462, pp. 327–330.

Lave, Lester B. (1963) "The value of better weather information to the raisin industry," *Econometrica*, Vol. 31, No. 1/2, pp. 151–164.

Lemoine, Derek (2017) "Expect above average temperatures: Identifying the economic impacts of climate change," Working Paper 23549, National Bureau of Economic Research.

Lemoine, Derek and Sarah Kapnick (2016) "A top-down approach to projecting market impacts of climate change," *Nature Climate Change*, Vol. 6, No. 1, pp. 51–55.

Neidell, Matthew (2009) "Information, avoidance behavior, and health: The effect of ozone on asthma hospitalizations," *Journal of Human Resources*, Vol. 44, No. 2, pp. 450–478.

Rosenzweig, Mark R. and Christopher Udry (2014) "Rainfall forecasts, weather, and wages over the agricultural production cycle," *American Economic Review: Papers and Proceedings*, Vol. 104, No. 5, pp. 278–283.

Rosenzweig, Mark and Christopher R. Udry (2013) "Forecasting profitability," Working Paper 19334, National Bureau of Economic Research.

Severen, Christopher, Christopher Costello, and Olivier Deschenes (2016) "A forward looking Ricardian approach: Do land markets capitalize climate change forecasts?" Working Paper 22413, National Bureau of Economic Research.

Shrader, Jeffrey (2017) "Expectations and adaptation to environmental risks," working paper.

Wood, Stephen A., Amir S. Jina, Meha Jain, Patti Kristjanson, and Ruth S. DeFries (2014) "Smallholder farmer cropping decisions related to climate variability across multiple regions," *Global Environmental Change*, Vol. 25, pp. 163–172.