Innovation-Led Transitions in Energy Supply*

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I generalize a benchmark model of directed technical change to allow innovations and factors of production (here energy resources) to be substitutes or complements. I show that a dominant sector is forever locked-in under substitutability but researchers' market incentives can drive a transition away from a dominant sector under complementarity. In a calibrated numerical implementation to climate change policy, transitions from coal to gas and then to renewable energy occur in laissez-faire. Optimal policy uses a subsidy for clean R&D to hasten the transition to renewable energy and an increasing emission tax to control residual fossil fuel use. A standalone clean research subsidy is more valuable than a standalone emission tax unless climate change is especially costly or policy cannot be enacted soon. A standalone mandate to use renewable energy can also increase welfare by igniting an energy transition, but only if it is sufficiently large.

JEL: O33, O38, Q41, Q54

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1 Introduction

Economists have long recognized that the first-best climate change policy combines emission taxes with subsidies for research into clean technologies. If political considerations prevent policymakers from deploying both instruments, which should they emphasize? To what degree can one instrument substitute for the other? Models of climate policy that endogenize innovation have reached sharply different conclusions: some have found that emission taxes are far more valuable (Popp, 2006; Fischer and Newell, 2008; Hart, 2019) and others have found that research subsidies are far more valuable (Acemoglu et al., 2016; Greaker et al., 2018).[1] In the recent models, market incentives direct innovation to either fossil or renewable resources. These incentives act to "lock-in" the initially dominant fossil resource. The only drivers of long-run change are resource depletion and policy.[2] Because we are going to run out of atmosphere before we run out of fossil fuels, policy aims to escape fossil lock-in and create clean energy lock-in.

This framing may not be the most apt. Historical experience suggests that technological change, not depletion, has been critical to past transitions between different types of resources (e.g., Flinn, 1959; Marchetti, 1977; Marchetti and Nakicenovic, 1979; Rosenberg, 1983; Grübler, 2004; Fouquet, 2010; Wilson and Grubler, 2011). On this view, the British transition from biomass to coal was driven by technologies such as the steam engine, not by changes in the relative abundance of timber and coal. A model used to study a future transition to renewable energy should allow innovation dynamics thought to drive past transitions in energy supply. If these dynamics might also drive a transition to renewable energy, policy would focus on accelerating and steering that transition rather than on changing which resource is locked-in.

I develop the first model with laissez-faire transitions driven by endogenous innovation decisions. I generate endogenous transitions by generalizing Acemoglu et al. (2012). Each type of energy is produced by combining an energy resource with specialized machines. For instance, coal is combined with steam engines to produce mechanical motion or electricity. A fixed measure of scientists works to improve these machines. Each scientist targets whichever type of machine provides a more valuable patent. Scientists' efforts change the quality of machines from period to period, which in turn changes equilibrium use of each energy resource from period to period. In Acemoglu et al. (2012), the elasticity of substitution

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[1] Historically, economists prioritized emission taxes. Schneider and Goulder (1997) favor emission taxes because they are closer to the primary market failure. Nordhaus (2008, 22) says that proposals to address climate change by providing research support instead of pricing emissions are "not really serious" and fail to recognize "the central economic question about how to slow climate change". Acemoglu et al. (2012) formally show that temporary research subsidies might be sufficient to manage climate change.

[2] Resource economists have long focused on how depletion or exhaustion can induce transitions between resources (e.g., Nordhaus, 1973; Chakravorty and Krulce, 1994; Chakravorty et al., 1997). The emphasis on depletion at the expense of innovation dates back to Jevons (1865), who underestimated the scope for innovation in his famous analysis of the advancing depletion of British coal reserves (Madureira, 2012).

between resources and machines is fixed at unity. Relaxing this restriction, I analytically demonstrate that innovation-led transitions occur if and only if that elasticity of substitution is strictly smaller than unity.

Imagine that there are only two types of energy and that one type of energy initially attracts the majority of scientists and uses more raw resources. I show that three forces determine how each sector's share of research and resource extraction changes in the following period. First, a *market size effect* attracts scientists to whichever sector uses more energy resources. By drawing scientists in, this effect increases that sector's share of resource use in subsequent periods, thereby attracting even more scientists. This positive feedback between extraction and research works to lock in whichever sector is already dominant. Second, a *patent quality effect* drives scientists to the sector where their patent will cover a higher quality machine. This effect draws additional scientists to the sector that dominated research in the previous period, which again works to lock in whichever sector is already dominant. Third, a *supply expansion effect* drives scientists away from the sector with higher quality machines, because these shift out the supply of machine services and thus reduce the price of machine services. This force pushes scientists away from the sector that dominated research effort in the previous period. It is the only force that works against lock-in and in favor of a transition away from the dominant sector.[3]

The elasticity of substitution between resources and machines determines the relative strengths of the patent quality and supply expansion effects. When that elasticity is equal to 1, we have a knife-edge case in which the patent quality and supply expansion effects exactly offset each other. The research allocation is entirely determined by market size effects, so whichever sector initially dominates research and resource supply does so forever. The dominant sector is locked-in, as in Acemoglu et al. (2012) and related literature.[4]

When that elasticity is strictly greater than 1 (machines are "resource-saving"), demand

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[3] The forces generating lock-in are similar to those explored in a related literature on path dependence in technology adoption (e.g., David, 1985; Arthur, 1989; Cowan, 1990). That literature focuses on "dynamic increasing returns" as the source of path dependence, where the likelihood of using a technology increases in the number of times it was used in the past (perhaps through learning-by-doing or network effects). In the present setting, market size and patent quality effects both act like dynamic increasing returns.

[4] The Cobb-Douglas assumption dates to early models of directed technical change (Acemoglu, 2002, 2007). Work on climate and directed technical change since Acemoglu et al. (2012) has used variants of the Cobb-Douglas assumption (Hémous, 2016; van den Bijgaart, 2017; Fried, 2018; Greaker et al., 2018). Acemoglu et al. (2016) develop a setting in which two types of energy technologies compete in each of many product lines. Each product line's production function is Cobb-Douglas. As a result, their setting again generates strong path dependence or lock-in. Subsequent to the present paper, Acemoglu et al. (2019) use a Leontief production function and describe a transition when extraction technologies are fixed. In contrast to the present paper, they predict that renewable resources fully crowd out fossil resources after the transition, which occurs because renewable energy does not require a costly resource input in their setting. Hart (2019) mechanically weakens path dependence by modeling knowledge spillovers between sectors. Innovation interacts multiplicatively with other factors of production, and laissez-faire transitions are still driven by exhaustion.

for machine services is elastic and the price of machine services does not fall by much as technology improves. The patent quality effect dominates the supply expansion effect. Whichever sector dominates research and extraction in some period then does so to an increasing degree in all later periods. The dominant sector is again locked-in.

However, when that elasticity is strictly less than 1 (machines are "resource-using"), demand for machine services is inelastic and the price of machine services falls by a lot as technology improves. The supply expansion effect dominates the patent quality effect. In that case, as the dominant sector becomes more advanced, scientists can begin switching to the other sector. Eventually, their research efforts raise the quality of technology in the dominated sector, which begins increasing that sector's share of extraction via market size effects. The transition in research away from the dominant sector can thereby induce a subsequent transition in energy supply.

To explore the implications for climate change policy, I extend the model to allow resource use to generate carbon dioxide emissions. These emissions eventually raise global temperature and thereby reduce the quantity of final goods produced, all following Nordhaus (2017).[5] I calibrate the model to match market data for coal, natural gas, and emission-free renewables (wind and solar), which compete in electricity generation. Outside estimates suggest that the elasticity of substitution between resources and machines is around 0.4, and I am unable to match market data for elasticities close to or above 1.

Consistent with dominant popular narratives, I find that a transition from coal to gas is underway and that a transition from gas to renewables will eventually follow. The delay until the second transition depends on scientists' productivity. As described analytically, both transitions are innovation-led. Transitions in research proceed swiftly and to completion, but the subsequent transitions in resource use proceed much more slowly and do not eliminate use of other resources. The slowness and incompleteness of transitions in energy supply align well with historical evidence (Smil, 2010, Chapter 2). Fossil fuel use persists in laissez-faire even after the transition to clean energy, eventually warming the planet to dangerous levels.

A policymaker uses a standalone research subsidy to immediately shift all scientists to the clean sector. This shift ignites an energy transition, allowing the policymaker to phase out the subsidy over the ensuing decades without pushing scientists away from the clean sector. However, in contrast to Acemoglu et al. (2012, 2016), the research subsidy cannot drive fossil resource use to trivial levels and thus cannot control long-run warming.

An emission tax can control long-run warming, but, as in Aghion et al. (2016), only very large taxes can shift the near-term research allocation.[6] The policymaker prefers to

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[5] This climate-economy integrated assessment model can be seen as extending Nordhaus (2017) to incorporate the present model of energy use and innovation. Abatement cost is fully endogenous in the present setting, not an exogenously specified function as in Nordhaus (2017) and related models.

[6] In fact, I show that an emission tax has an analytically ambiguous effect on the direction of research, as it both increases the market share of renewables and increases the price of the fossil fuel-using machines that researchers could sell. Hassler et al. (2020) report a similar result. Quantitatively, higher emission taxes do

use a low initial tax that ramps up over time and hastens an energy transition by only a bit.[7] This result contrasts with Acemoglu et al. (2016), where the policymaker has no choice but to use a high emission tax if it wants an energy transition to occur in the absence of a research subsidy. Depending on the specification of scientific progress, the optimal standalone emission tax provides 40–60% of the benefits of the optimal standalone clean research subsidy.[8]

The optimal policy combines a temporarily high research subsidy with an emission tax that starts low and increases over time, eventually increasing quite sharply. The frontloaded subsidy is familiar from previous models with lock-in, but the tax trajectory is quite different.[9] The two policy instruments cannot fully substitute for each other: adding an emission tax to a research subsidy increases the value of policy by 15–20%. The policymaker uses the research subsidy to ignite a transition to clean energy and the emission tax to control the residual fossil fuel use that remains after the transition has occurred. By reducing warming, a portfolio of the two policy instruments raises later centuries' consumption to a level that far exceeds what a research subsidy alone could achieve.

Robustness checks reveal three important caveats to the value of research subsidies. First, they need to be implemented quickly. If no policy can be enacted for another 50 years, then the standalone emission tax is up to 50% more valuable than the standalone research subsidy and can be nearly as valuable as a portfolio of both. Second, research subsidies are more valuable than the emission tax only if the standalone emission tax does not quickly shift research to renewables. If the policymaker has a low utility discount rate, then the policymaker uses a temporarily large standalone emission tax to push scientists to work on renewables. Emission taxes become at least as valuable as research subsidies. Third, research subsidies can be substantially less valuable than emission taxes if the policymaker needs to redirect resource use quickly. When damages are calibrated to an expert survey that implies greater losses from warming (Pindyck, 2019; Lemoine, 2021), the policymaker uses a high and increasing emission tax to quickly redirect resource use.

Finally, I explore the implications of directed technical change for a different type of policy instrument: a mandate to use a minimum share of renewables. Such mandates are common.

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redirect researchers to renewables.

[7] In showing that emission taxes can hasten an innovation-led transition in supply, I extend recent empirical evidence that endogenous innovation can increase the emission reductions from a given emission tax (Aghion et al., 2016; Fried, 2018) or reduce the cost of a given emission cap (Calel, 2020).

[8] Hart (2019) finds that emission taxes are more valuable than research subsidies because research subsidies have difficulty affecting resource use. In that setting, laissez-faire transitions occur only as fossil resources become exhausted (pg 157). In contrast, laissez-faire transitions in energy supply are here triggered by transitions in research, so research subsidies more readily affect resource use. Once research subsidies redirect resource use, they increase consumption beyond what an emission tax achieves.

[9] The most comparable work has the tax decline either forever (Greaker et al., 2018) or once lock-in begins working in renewables' favor to crowd out use of fossil resources (Acemoglu et al., 2016). See footnote 28 below.

For instance, around 30 U.S. states and the European Union each mandate a minimum share of renewable electricity. I show that accounting for endogenous innovation responses is critical to the evaluation of a mandate. Sufficiently large mandates (of 20% or more, as are commonly implemented) ignite an energy transition by redirecting research effort through market size effects.[10] If we ignored effects on innovation, we would expect more stringent mandates to bind for longer, but we in fact see the opposite: more stringent mandates actually bind for *less time* than do weaker mandates because they ignite transitions more quickly. Further, welfare is not concave in the stringency of the mandate. Mandates that are large enough to ignite a transition can provide substantial net benefits even though smaller mandates impose net costs and still-smaller mandates provide small net benefits. Accounting for innovation is of first-order importance when evaluating mandates. For instance, mandates that appear to impose hefty costs equivalent to a permanent 20% drop in consumption when ignoring effects on innovation can actually provide benefits equivalent to a permanent 7% increase in consumption.

Formally, I analyze directed technical change when final good production has a nested constant elasticity of substitution structure that allows innovation to complement other inputs. The use of a Cobb-Douglas aggregator for machines and factors of production dates to the earliest models of directed technical change (Acemoglu, 2002, 2007). This paper joins other recent work in noting that Cobb-Douglas assumptions are knife-edge cases with qualitatively special results (e.g., Alvarez-Cuadrado et al., 2017; Baqaee and Farhi, 2019). Complementarities between innovation and factors of production may be common. For instance, Grossman et al. (2017) summarize evidence that capital and labor are complements. The present framework provides one mechanism through which the economy could have transitioned from an era in which unskilled labor was dominant to the modern era in which skilled labor is dominant.[11] Baqaee and Farhi (2019) summarize evidence of complementarities across intermediates throughout supply chains. The present theory of innovation-led transitions could thus explain dynamics in manufacturing activity.

The next section describes the theoretical setting. Section 3 analyzes the relative incentive to research technologies in each sector. Section 4 theoretically describes the economy's

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[10] Some have informally argued that such mandates might allow the energy sector to escape lock-in (e.g., Lehmann and Gawel, 2013). Formal analyses of this channel typically focus on learning-by-doing as the mechanism for technological change (Gerlagh and van der Zwaan, 2006; Kalkuhl et al., 2012), which means that technology matures jointly with energy production. In contrast, here renewable technology begins maturing *before* renewables begin "escaping" lock-in. Fischer and Newell (2008) find that renewable energy mandates are better than renewable R&D subsidies but worse than emission taxes. Clancy and Moschini (2018) show that mandates can induce innovation but do not analyze dynamics. Johnstone et al. (2010) report econometric evidence that mandates to produce renewable energy increase patenting activity.

[11] Acemoglu (2002) explains how shocks to the relative supply of skilled labor interact with the skill premium through endogenous innovation, but his model does not explain how the economy could have transitioned away from an era in which unskilled labor was dominant. I thank Greg Casey for raising this point.

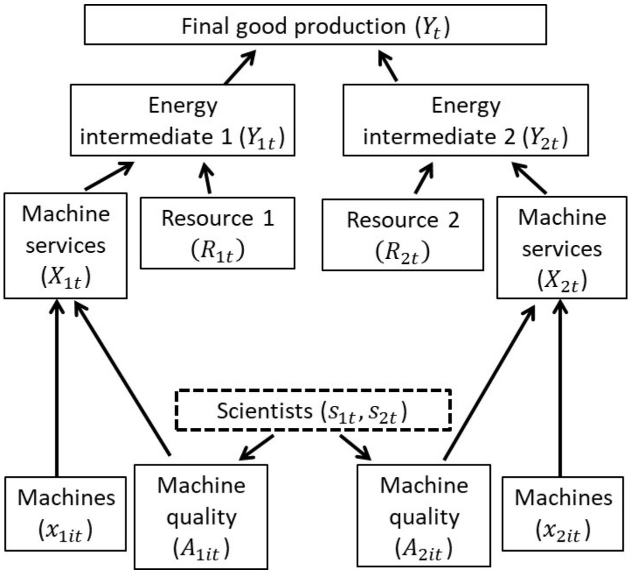

Figure 1: Overview of the theoretical setting, for $N = 2$.

laissez-faire dynamics. Section 5 numerically explores the implications for policies that aim to control future climate change. The final section concludes. The appendix contains details of the calibration, robustness checks, a numerical example, and proofs.

2 Setting

I study a discrete-time economy in which final good production uses multiple types of energy intermediates. Each energy intermediate is generated by combining energy resources with machines. Resources are supplied competitively. A fixed measure of households works as scientists, trying to improve the quality of machines used in producing the energy intermediates. Scientists decide which type of machine to work on. The equilibrium allocation of resources and scientists changes over time as technologies improve. Figure 1 illustrates the model setup, which I now formalize.

The time t final good Y_t is produced competitively from N energy intermediates Y_{jt}, with $j \in \{1, 2, ..., N\}$. The final good is the numeraire in each period. The representative firm's production function takes the familiar constant elasticity of substitution (CES) form:

$$Y_t = A_Y \left(\sum_{j=1}^{N} \nu_j Y_{jt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}.$$

The parameters $\nu_j \in (0, 1)$ are the distribution (or share) parameters, with $\sum_{j=1}^{N} \nu_j = 1$. $A_Y > 0$ is a productivity parameter. I say that resource j is *higher quality* than resource

k if and only if $\nu_j > \nu_k$. The parameter ϵ is the elasticity of substitution. The energy intermediates are gross substitutes ($\epsilon > 1$), consistent with evidence in Papageorgiou et al. (2017).

The energy intermediates Y_{jt} are the energy services produced by combining resource inputs R_{jt} with machine inputs X_{jt}. Production of energy intermediates has the following CES form:

$$Y_{jt} = \left(\kappa R_{jt}^{\frac{\sigma-1}{\sigma}} + (1-\kappa) X_{jt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

The parameter $\kappa \in (0,1)$ is the distribution (or share) parameter. The elasticity of substitution between the resource and machine inputs is σ. I call machines *resource-using* when resources and machines are gross complements ($\sigma < 1$), and I call machines *resource-saving* when resources and machines are gross substitutes ($\sigma > 1$). Resources and machines are less substitutable than are different types of energy intermediates ($\sigma < \epsilon$).

Machine services X_{jt} are produced in a Dixit-Stiglitz environment of monopolistic competition from machines of varying qualities:

$$X_{jt} = \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} \, \mathrm{d}i,$$

where $\alpha \in (0,1)$ and where an implicit fixed factor of production ensures constant returns to scale. The machines x_{jit} that work with resource j at time t are divided into a continuum of types, indexed by i. The quality (or efficiency) of machine x_{jit} is given by A_{jit}. Machines of type i are produced by monopolists who each take the price (p_{jXt}) of machine services as given (each is small) but recognize their ability to influence the price p_{jxit} of machines of type i. The cost of producing a machine is $a > 0$ units of the final good, normalized to $a = \alpha^2$.

Scientists choose which resource they want to study and are then randomly allocated to a machine type i. Each scientist succeeds in innovating with probability $\eta \in (0,1]$. If they fail, scientists earn nothing and the quality of that type of machine is unchanged. Following Acemoglu et al. (2012) and others, successful scientists receive a one-period patent to produce their type of machine. In the numerical implementation, each period will be ten years. Using resource j as an example, successful scientists improve the quality of their machine type to

$$A_{jit} = A_{ji(t-1)} + \gamma A_{ji(t-1)}, \tag{1}$$

where $\gamma > 0$. Scientists are of fixed measure, normalized to 1:[12]

$$1 = \sum_{j=1}^{N} s_{jt}.$$

Firms that enter into production of resource j find a deposit containing one unit of the resource. Firms must pay a fixed cost (in units of the final good) to develop the nth deposit. In equilibrium, all deposits with fixed costs less than p_{jRt} get developed. Order the continuum of deposits by fixed cost. The fixed cost of the nth deposit is then $F_j(n)$, with $F_j(n) = (n/\Psi_j)^{1/\psi}$ for $\psi, \Psi_j > 0$. In equilibrium, $F_j(R_{jt}) = p_{jRt}$. As a result,

$$R_{jt} = \Psi_j p_{jRt}^{\psi}. \tag{2}$$

Resources are therefore supplied isoelastically. I say that resource j is *more accessible* than resource k if and only if $\Psi_j > \Psi_k$. I impose $\psi \geq \alpha/(1-\alpha)$, which ensures that the own-price elasticity of resource supply is greater than the elasticity of machine services with respect to the resource price. To isolate the role of innovation in driving transitions, I assume that the profile of fixed costs does not change over time, as if exploration and innovation in extraction technologies offset depletion over time. This assumption may even understate the effectiveness of innovation at offsetting depletion (Schwerhoff and Stuermer, 2019).

The economy's time t resource constraint is

$$Y_t \geq c_t + a \sum_{j=1}^{N} \int_0^1 x_{jit} \,\mathrm{d}i + \sum_{j=1}^{N} \int_0^{R_{jt}} F_j(n) \,\mathrm{d}n,$$

where $c_t \geq 0$ is the composite consumption good. Households have strictly increasing utility for the consumption good. Scientists therefore each choose their resource type so as to maximize expected earnings.

I study equilibrium outcomes.

Definition 1. *An* equilibrium *is given by sequences of prices for energy intermediates* $(\{p^*_{jt}\}_{j=1}^N)$*, prices for machine services* $(\{p^*_{jXt}\}_{j=1}^N)$*, prices for machines* $(\{p^*_{jxit}\}_{j=1}^N)$*, prices for resources* $(\{p^*_{jRt}\}_{j=1}^N)$*, demands for inputs* $(\{Y^*_{jt}\}_{j=1}^N, \{R^*_{jt}\}_{j=1}^N, \{X^*_{jt}\}_{j=1}^N, \{x^*_{jit}\}_{j=1}^N)$*, and factor allocations* $(\{s^*_{jt}\}_{j=1}^N)$ *such that, in each period t: (i)* $\{Y^*_{jt}\}_{j=1}^N$ *maximizes profits of final good producers, (ii)* $(\{R^*_{jt}\}_{j=1}^N, \{X^*_{jt}\}_{j=1}^N)$ *maximizes profits of energy intermediate producers, (iii)* $(\{p^*_{jxit}\}_{j=1}^N, \{x^*_{jit}\}_{j=1}^N)$ *maximize profits of the producers of each machine i in each sector j, (iv) resource producers enter until they earn zero profits, (v)* $\{s^*_{jt}\}_{j=1}^N$ *maximizes expected earnings of scientists, (vi) prices clear the factor and input markets, and (vii) technologies evolve as in equation* (1)*.*

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[12] Hart (2019) allows for an extensive margin in research. Following Acemoglu et al. (2012, 2016), I keep the total pool of researchers exogenous so that I can focus on implications of the directedness of technical change rather than on well-known externalities in the quantity of research undertaken. See page 55 of Acemoglu et al. (2016) for a discussion.

The equilibrium prices clear all factor markets and all firms maximize profits. If scientists are employed in any two sectors, they receive the same expected reward from both, and if they are not employed in some sector, they receive a greater expected reward in some other sector that has nonzero scientists. Appendix D establishes that the equilibrium is stable in a tâtonnement sense when $N = 2$. Throughout, I drop the asterisks when clear.

3 The Equilibrium Direction of Research

The first-order condition for a producer of machine services yields the following demand curve for machines of type i in sector j:

$$x_{jit} = \left(\frac{p_{jXt}}{p_{jxit}}\alpha\right)^{\frac{1}{1-\alpha}} A_{jit}. \tag{3}$$

The monopolist producer of x_{jit} therefore faces an isoelastic demand curve and accordingly marks up its price by a constant fraction over marginal cost: $p_{jxit} = a/\alpha = \alpha$. In equilibrium, the producer of machine type i for use with resource j earns profits of:

$$\pi_{jxit} = (p_{jxit} - a)x_{jit} = \alpha(1-\alpha)p_{jXt}^{\frac{1}{1-\alpha}} A_{jit}.$$

If a scientist succeeds in innovating at time t, she exercises her patent to obtain the monopoly profit π_{jxit}. Her expected reward to choosing to research machines that work with resource type j is therefore

$$\Pi_{jt} = \eta\,\alpha\,(1-\alpha)p_{jXt}^{\frac{1}{1-\alpha}}(1+\gamma)A_{j(t-1)}, \tag{4}$$

where $A_{j(t-1)}$ is the average quality of machines in sector j. This average quality evolves as

$$A_{jt} = \int_0^1 \left[\eta s_{jt}(1+\gamma)A_{ji(t-1)} + (1-\eta s_{jt})A_{ji(t-1)}\right] \mathrm{d}i = (1+\eta\gamma s_{jt})A_{j(t-1)}, \tag{5}$$

where s_{jt} is the measure of scientists working on resource j.

Now consider the relative incentive to research technologies that work with resource j rather than technologies that work with resource k. From equation (4), we have[13]

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{A_{j(t-1)} + \gamma A_{j(t-1)}}{A_{k(t-1)} + \gamma A_{k(t-1)}} \left[\frac{p_{jXt}}{p_{kXt}}\right]^{\frac{1}{1-\alpha}}. \tag{6}$$

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[13] A market size effect would appear here if the quantity of the implicit fixed factor differed by sector. I maintain identical fixed factors in order to focus on market size effects that evolve over time (see equation (10) below). The calibration will absorb differences in fixed factors into the estimated technology parameters.

The intermediate-good producer's first-order conditions for profit-maximization yield

$$p_{jXt} = (1-\kappa) p_{jt} \left[\frac{X_{jt}}{Y_{jt}} \right]^{-1/\sigma} \quad \text{and} \quad p_{jRt} = \kappa \, p_{jt} \left[\frac{R_{jt}}{Y_{jt}} \right]^{-1/\sigma}.$$

The relative incentive to research technologies for use in sector j increases in the relative price of the intermediates and decreases in the machine-intensity of sector j's output. Combining the first-order conditions, we have

$$p_{jXt} = \frac{1-\kappa}{\kappa} \left[\frac{R_{jt}}{X_{jt}} \right]^{1/\sigma} p_{jRt}. \tag{7}$$

From equation (3) and the monopolist's markup, we have

$$x_{jit} = p_{jXt}^{\frac{1}{1-\alpha}} A_{jit}.$$

Substituting into the definition of X_{jt} and using the definition of A_{jt}, we have

$$X_{jt} = p_{jXt}^{\frac{\alpha}{1-\alpha}} A_{jt}. \tag{8}$$

Substitute into equation (7) and solve for equilibrium machine prices:

$$p_{jXt} = \left[p_{jRt} \frac{1-\kappa}{\kappa} \right]^{\frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha}}. \tag{9}$$

Finally, substituting into equation (6) and then using (2), we have:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \underbrace{\frac{(1+\gamma)A_{j(t-1)}}{(1+\gamma)A_{k(t-1)}}}_{\text{patent quality effect}} \underbrace{\left(\frac{(1+\eta\gamma s_{jt})A_{j(t-1)}}{(1+\eta\gamma s_{kt})A_{k(t-1)}} \right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}}}_{\text{supply expansion effect}} \underbrace{\left(\frac{R_{jt}}{R_{kt}} \right)^{\frac{1}{\sigma+\alpha(1-\sigma)} \frac{\psi+\sigma}{\psi}}}_{\text{market size effect}} \left(\frac{\Psi_j}{\Psi_k} \right)^{\frac{-\sigma/\psi}{\sigma+\alpha(1-\sigma)}}. \tag{10}$$

Four terms determine scientists' relative incentive to research machines. The first term is a *patent quality effect* that directs research effort to the sector in which scientists will end up with the patent to better technology.[14] The other channels derive from the relative price of machine services: $(p_{jXt}/p_{kXt})^{1/(1-\alpha)}$ in equation (6). The *supply expansion effect*

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[14] The patent quality effect depends on realized technology $(1+\gamma)A_{j(t-1)}$, not solely on the increment to technology $\gamma A_{j(t-1)}$ produced by a scientist's efforts, which introduces a type of business-stealing distortion. If γ differed by sector and were very small in the more advanced sector, scientists could have a stronger incentive to research machines in the more advanced sector even though their efforts would not improve these machines. This business-stealing distortion vanishes under the assumption of identical γ: by attracting scientists to the more advanced sector, the patent quality effect here also attracts them to the sector where they make the greatest advance.

pushes scientists away from the more advanced sector. From equation (8), the supply of X_{jt} shifts out when its machines' average quality A_{jt} increases, and it shifts out to an especially large degree when α is small. When σ is small (machines are resource-using), the demand curve is steep because the marginal product of additional machines is constrained by the supply of R_{jt}. By shifting out supply, the increase in A_{jt} induces a relatively large decline in the equilibrium price p_{jXt}. However, when σ is large, machines are resource-saving and the demand curve is relatively flat. The increase in A_{jt} then induces a relatively small decline in the equilibrium price p_{jXt}. Improving technology therefore pushes scientists away to a greater degree when the demand curve is steep (σ is small) or the shift in supply is large (α is small) because it reduces p_{jXt} more strongly.

Pause to consider the net effect of a relative improvement in sector j's average technology. We have seen that this relative improvement attracts scientists through the patent quality effect and repels scientists through the supply expansion effect. Combining these channels, the exponent on relative technology is proportional to $(\sigma - 1)(1 - \alpha)$. The supply expansion effect dominates the patent quality effect if and only if $\sigma < 1$. As $\sigma \to 0$, demand for machines becomes perfectly inelastic and the supply expansion effect becomes large. As $\sigma \to \infty$, demand for machines becomes perfectly elastic and the supply expansion effect vanishes. As $\sigma \to 1$, the two effects exactly cancel, so that the incentives to research machines in one sector or the other do not directly depend on the relative quality of technology in each sector.

The remaining machine price channels in equation (10) connect research incentives to resource supply. In particular, we see research directed towards the sector with greater resource use. This is a *market size effect*. It arises for two reasons. First, from equation (7), an increase in R_{jt} shifts out demand for X_{jt}, and does so to an especially large degree when machines and resources are stronger complements (i.e., as σ becomes small). Second, also from equation (7), an increase in p_{jRt} (for given R_{jt}) also shifts out demand for X_{jt} as firms substitute machines for resources. This channel is especially strong when the elasticity of substitution between resources and machines is large, and it vanishes as that elasticity goes to zero. Each of these outward shifts in demand for X_{jt} increases scientists' incentives to work on improving machines in sector j. Therefore, the market size effect draws scientists towards whichever sector is increasing its share of resource supply over time.

Now consider how sector j's share of extraction changes from time t to $t + 1$. Combining the intermediate-good producers' first-order condition for resources with the final-good producers' first-order conditions, we find demand for each resource:

$$p_{jRt} = \kappa\, \nu_j A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_{jt}}{Y_t}\right]^{-1/\epsilon} \left[\frac{R_{jt}}{Y_{jt}}\right]^{-1/\sigma} \quad \text{and} \quad p_{kRt} = \kappa\, \nu_k A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_{kt}}{Y_t}\right]^{-1/\epsilon} \left[\frac{R_{kt}}{Y_{kt}}\right]^{-1/\sigma}. \tag{11}$$

Market-clearing for each resource then implies

$$\left[\frac{R_{jt}}{\Psi_j}\right]^{1/\psi} = \kappa\, \nu_j A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_{jt}}{Y_t}\right]^{-1/\epsilon} \left[\frac{R_{jt}}{Y_{jt}}\right]^{-1/\sigma}, \tag{12}$$

$$\left[\frac{R_{kt}}{\Psi_k}\right]^{1/\psi} = \kappa\, \nu_k A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_{kt}}{Y_t}\right]^{-1/\epsilon} \left[\frac{R_{kt}}{Y_{kt}}\right]^{-1/\sigma}. \tag{13}$$

Demand for sector j's resources (for example) shifts inward as the share of those resources in the production of intermediate good j increases and also shifts inward as the share of intermediate good j in production of the final good increases. Rearranging equations (12) and (13) and then dividing, we have:

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\frac{1}{\sigma}+\frac{1}{\psi}} = \frac{\nu_j}{\nu_k} \left[\frac{\Psi_j}{\Psi_k}\right]^{1/\psi} \left[\frac{Y_{jt}}{Y_{kt}}\right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}}. \tag{14}$$

The change in sector j's share of resource extraction from time t to time $t+1$ therefore has the same sign as the change in sector j's share of intermediate good production. Observe that increasing the average quality of technology A_{jt} increases production of the intermediate good Y_{jt}. Thus, sector j's share of resource extraction tends to increase when the average quality of its technology is advancing relative to sector k. The sector that is advancing more rapidly tends to attract even more scientists in later periods through market size effects, which works to lock in that sector's technological advantage.

4 The Equilibrium Evolution of Resource Use and Technology

I now study the evolution of the economy in a special case with $N = 2$. Label the two sectors as j and k. I show that both the possibility of a transition and the nature of long-run outcomes are sensitive to whether machines are resource-using or resource-saving. I then study three special cases that highlight the relevant dynamics. Appendix C illustrates the main ideas with a numerical example. Section 5 will demonstrate the dynamics in a calibrated application to climate change policy.

The following assumption will be useful for studying transitions. It defines a time t_0 in which sector j dominates research activity with technology that is more advanced than (or not too much less advanced than) sector k's technology:

Assumption 1. *$A_{j(t_0-1)}/A_{k(t_0-1)} > [\Psi_j/\Psi_k]^{\theta}$ and $s^*_{jt_0} > 0.5$ for some time t_0, where $\theta \triangleq 1/[(1-\alpha)(1+\psi)] \in (0,1]$.*

The next lemma establishes one set of structural conditions under which Assumption 1 holds:

Lemma 1. *If $\nu_j = \nu_k$ and $\Psi_j = \Psi_k$, then Assumption 1 holds if (i) $A_{j(t_0-1)} > A_{k(t_0-1)}$ and (ii) either $\sigma > 1$ or σ is not too much smaller than 1.*

Proof. See Appendix E.4. ☐

The *steady state* for this economy has the research allocation fixed forever, so each type of technology improves at a constant rate. Define a *transition in research* as occurring at the first time $t \geq t_0$ at which s_{jt} begins declining, a *transition in extraction* as occurring at the first time $t \geq t_0$ at which R_{jt}/R_{kt} begins declining, and a *transition in technology* as occurring at the first time $t \geq t_0$ at which A_{jt}/A_{kt} begins declining. Finally, define the dominant resource as being *locked-in* from time t_0 when no type of transition occurs after t_0.

Begin by considering the case with $\sigma > 1$:

Proposition 2. *Let $\sigma > 1$.*

1. *If Assumption 1 holds, then resource j is locked-in from time t_0.*
2. *If $s^*_{jt} \in (0.5, 1)$, then $s^*_{j(t+1)} > s^*_{jt}$. If $s^*_{jt} \in (0, 0.5)$, then $s^*_{j(t+1)} < s^*_{jt}$.*
3. *The only stable steady states are at $s_{jt} = 0$ and $s_{jt} = 1$.*

Proof. See Appendix E.5. ☐

If machines are resource-saving, then a transition cannot happen. The economy is locked-in to the dominant sector. The proof shows that sector j increases its share of resource supply whenever it dominates research effort. And when sector j is both increasing its share of resource supply and dominating research effort, the market size and patent quality channels in equation (10) both pull even more scientists towards sector j. Sector j therefore increases its dominance of research effort over time and continually increases its technological advantage over sector k. Sector j's increasing share of resource supply and its increasing share of research activity form a positive feedback loop that prevents sector k from ever catching up: sector j's increasingly improved technology and increasing share of resource extraction both work to attract ever more scientists to sector j, and the improving relative quality of technology in sector j works to increase its share of extraction over time. The economy therefore approaches a corner allocation in research effort.[15] These dynamics are similar to the Cobb-Douglas case analyzed in Acemoglu et al. (2012) and related literature.

The dynamics are qualitatively different if $\sigma < 1$. First consider the steady-state research allocation:

Proposition 3. *Let $\sigma < 1$. Then the only steady-state research allocation has $s_{jt} = 0.5$ and the following are true as $t \to \infty$:*

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[15] The only exception is a knife-edge case in which the initial period's equilibrium has scientists equally allocated between the two sectors.

1. $s^*_{jt} \to 0.5$ *(i.e., the steady state is stable).*
2. *If* $\nu_j = \nu_k$ *and* $\Psi_j = \Psi_k$*, then* $R^*_{jt} = R^*_{kt}$ *and* $A_{jt} = A_{kt}$*.*
3. *If* $\nu_j \geq \nu_k$ *and* $\Psi_j \geq \Psi_k$ *with strict inequality for at least one, then* $R^*_{jt} > R^*_{kt}$ *and* $A_{jt} > A_{kt}$*.*
4. R^*_{jt} *and* R^*_{kt} *become constant, and* R^*_{jt}/R^*_{kt} *approaches* $\left[\left(\frac{\nu_j}{\nu_k}\right)^{\psi} \frac{\Psi_j}{\Psi_k}\right]^{\frac{\epsilon}{\epsilon+\psi}}$.

Proof. See Appendix E.6. □

The proposition gives four results. First, the economy approaches a steady-state research allocation in which the average quality of each technology improves at the same rate. The steady state is both unique and stable.[16] Second, if the two resources are of the same quality and accessibility, then the steady state has identical technology and extraction in each. Third, if one sector's resource is of higher quality and more accessible, then that sector dominates resource use and has better technology. Fourth, extraction eventually approaches a constant value in each sector. As discussed previously, resource supply becomes less sensitive to further advances in machine quality as machines becomes more advanced, so resource use cannot grow at a nonzero constant rate for all time. Observe that the long-run share of each resource is not sensitive to the magnitude of σ. These shares are instead completely determined by the characteristics of each resource (specifically, Ψ_j, Ψ_k, ν_j, ν_k, and ψ) and by the elasticity of substitution between the two types of energy (ϵ).

I now analyze the possibility of transitions with resource-using machines:

Proposition 4. *Let* $\sigma < 1$*, and let Assumption 1 hold.*

1. *A transition in extraction occurs only after a transition in research and a transition in technology occurs only after a transition in extraction.*
2. *If resource* j *is relatively accessible* ($\Psi_j \geq \Psi_k$)*, then a transition in technology occurs while sector* j *still provides the larger share of resource supply.*
3. *If* $\nu_j = \nu_k$ *and* $\Psi_j = \Psi_k$*, then a transition in research and a transition in extraction both occur before reaching the steady-state research allocation.*

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[16] A corner allocation cannot persist when $\sigma < 1$. The proof shows that as the average quality of technology in sector j improves, the market size effect becomes negligibly small: resources are not constrained by the availability of machines when machines become very advanced, so further improvements in their average quality do not affect resource use very much. Eventually the supply expansion effect dominates not just the patent quality effect but also the market size effect. Π_{jt}/Π_{kt} then begins to decline. If the allocation of scientists is held fixed at the corner, Π_{jt}/Π_{kt} eventually falls below unity, at which point the corner allocation can no longer be an equilibrium.

Proof. See Appendix E.7. □

If machines are resource-using, then sector j's dominant share of research activity works to push scientists away from sector j through the supply expansion effect in equation (10) even as sector j's improving relative technology works to increase its share of resource supply and thus strengthens the market size effect that pulls scientists towards sector j. The change in the market size effect is especially significant when sector j's technology is still immature, so that sector j can increasingly dominate research effort over time. However, the market size effect becomes less and less sensitive to the quality of sector j's technology as that technology becomes more advanced. The supply expansion effect eventually dominates the market size effect, which pushes scientists back towards sector k. At this point a transition in research occurs. As sector j's share of research continues to fall, a transition in extraction can occur. The transition in extraction is *innovation-led*: it can occur only after the transition in research. Even though research transitions before extraction, sector k does not begin to dominate research effort (triggering a transition in technology) until sometime after the transition in extraction, when both the market size effect and the supply expansion effect work to push scientists towards sector k. Finally, if resource j is relatively accessible, then a transition in technology must happen while sector j still dominates resource supply. Just as the transition in extraction must follow a transition in research, so too a change in the sector that dominates resource supply must follow a change in the sector that dominates research.

The first two parts of the proposition establish that a transition can happen when machines are resource-using. The final part of the proposition describes conditions under which a transition must happen. The sector with more advanced technology can attract the majority of researchers when neither technology is very advanced. However, the relatively backward sector must eventually dominate the research allocation because the steady state has both sectors being equally advanced (see Proposition 3). As the technologies improve, scientists must eventually start switching towards the relatively backward sector, and we already saw that extraction must also start switching towards the relatively backward sector sometime before the relatively backward sector begins to dominate the research allocation. Transitions in research, extraction, and technology must occur sometime before reaching a steady-state research allocation.

I next explore three special cases that highlight the competing effects that drive the evolution of the economy. I structurally ground Assumption 1 in each case.

4.1 Special Case With Only Market Size Effects: $\sigma = 1$

Begin by considering the Cobb-Douglas case studied in previous literature, which arises as $\sigma \to 1$. Let $Y_{jt} = R_{jt}^{\kappa} X_{jt}^{1-\kappa}$ and $Y_{kt} = R_{kt}^{\kappa} X_{kt}^{1-\kappa}$. Equation (10) becomes:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \left(\frac{1+\eta\gamma s_{jt}}{1+\eta\gamma s_{kt}}\right)^{-1} \left(\frac{R_{jt}}{R_{kt}}\right)^{\frac{\psi+1}{\psi}} \left[\frac{\Psi_j}{\Psi_k}\right]^{-1/\psi}. \tag{15}$$

As previously discussed, the patent quality and supply expansion effects exactly cancel, so that the market size effect completely determines the evolution of the research allocation.

How does the market size effect evolve over time? Substituting the Cobb-Douglas forms, equation (14) becomes

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\frac{\psi+1}{\psi}-\kappa\frac{\epsilon-1}{\epsilon}} = \frac{\nu_j}{\nu_k}\left[\frac{\Psi_j}{\Psi_k}\right]^{1/\psi}\left[\frac{X_{jt}}{X_{kt}}\right]^{(1-\kappa)\frac{\epsilon-1}{\epsilon}}.$$

Substituting equation (7) into equation (8) and then using equation (2), we have:

$$X_{jt} = \left[\frac{1-\kappa}{\kappa} R_{jt}^{\frac{\psi+1}{\psi}} \Psi_j^{-1/\psi}\right]^{\alpha} A_{jt}^{1-\alpha}.$$

We then have:

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\Gamma} = \frac{\nu_j}{\nu_k}\left[\frac{\Psi_j}{\Psi_k}\right]^{\frac{1}{\psi}\left[1-\alpha(1-\kappa)\frac{\epsilon-1}{\epsilon}\right]}\left[\frac{A_{jt}}{A_{kt}}\right]^{(1-\alpha)(1-\kappa)\frac{\epsilon-1}{\epsilon}}, \tag{16}$$

where $\Gamma \triangleq \frac{\psi+1}{\psi} - \frac{\epsilon-1}{\epsilon}\left(\kappa + (1-\kappa)\alpha\frac{\psi+1}{\psi}\right) > 0$. Sector j's share of resource extraction increases in the relative quality of sector j's technology. The more that sector j advances relative to sector k, the more that R_{jt}/R_{kt} grows, and the more that R_{jt}/R_{kt} grows, the more that Π_{jt}/Π_{kt} shifts up for any given s_{jt}. The equilibrium s_{jt} must therefore increase as $A_{j(t-1)}/A_{k(t-1)}$ increases.[17]

Using equations (15) and (16) and the result from Appendix D that the total derivative of Π_{jt}/Π_{kt} with respect to s_{jt} is negative, we have $s_{jt} > 0.5$ if and only if

$$\frac{A_{j(t-1)}}{A_{k(t-1)}} > \left[\frac{\nu_j}{\nu_k}\right]^{-\frac{1}{(1-\alpha)(1-\kappa)\frac{\epsilon-1}{\epsilon}}}\left[\frac{\Psi_j}{\Psi_k}\right]^{-\frac{\kappa}{(1-\alpha)(1-\kappa)(\psi+1)}}.$$

If $\Psi_j < \Psi_k$ and $\nu_j < \nu_k$, then Assumption 1 holds when this inequality holds. We now see how lock-in arises: $s_{jt} > 0.5$ implies that $A_{jt}/A_{kt} > A_{j(t-1)}/A_{k(t-1)}$, which ensures that $s_{j(t+1)} > s_{jt}$, which implies that $A_{j(t+1)}/A_{k(t+1)} > A_{jt}/A_{kt}$, and so on. There is a knife-edge case in which $s_{jt} = 0.5$ for all time, but if equilibrium s_{jt} ever takes on any other value, then the economy progresses to a corner allocation in research.

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[17] This result explains why relative technology does not directly affect research incentives in Acemoglu et al. (2012): technology matters in their equation (17) via the same patent quality effect seen here (which they call a "direct productivity effect") and also through their "price effect", but substituting in for relative output prices from their equation (A.3) shows that these two effects exactly cancel. Relative technology ends up playing a role in their setting's equilibrium (see their equation (18)) because relative market size is proportional to the relative quality of technology (see their equation (A.5)). Thus, their Cobb-Douglas assumption generates the same dynamics as does the Cobb-Douglas case analyzed here.

4.2 Special Case Without Market Size Effects: $\sigma = \epsilon$

Now consider a case with $\sigma = \epsilon > 1$. From equation (14), we have

$$\frac{R_{jt}}{R_{kt}} = \left(\frac{\nu_j}{\nu_k} \left[\frac{\Psi_j}{\Psi_k} \right]^{1/\psi} \right)^{\frac{\sigma\psi}{\sigma+\psi}} .$$

The shares of extraction are fixed over time, independently of the quality of technology in either sector. Because R_{jt}/R_{kt} is fixed over time, market size effects cease to steer the evolution of research activity. Substituting for R_{jt}/R_{kt} in equation (10), we have:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{\frac{(1-\alpha)(\epsilon-1)}{(1-\alpha)\epsilon+\alpha}} \left(\frac{1+\eta\gamma s_{jt}}{1+\eta\gamma s_{kt}} \right)^{\frac{-1}{(1-\alpha)\epsilon+\alpha}} \left(\frac{\nu_j}{\nu_k} \right)^{\frac{\epsilon}{(1-\alpha)\epsilon+\alpha}} .$$

As the average quality of technology in sector j improves, the patent quality effect shifts Π_{jt}/Π_{kt} upward and so increases the share of scientists working in sector j. If

$$\frac{A_{j(t-1)}}{A_{k(t-1)}} > \left(\frac{\nu_j}{\nu_k} \right)^{-\frac{\epsilon}{(1-\alpha)(\epsilon-1)}} ,$$

then $s_{jt} > 0.5$. If, in addition, $\nu_j < \nu_k$, then Assumption 1 holds. In that case, sector j is locked-in insofar as its share of research increases towards a corner allocation in which sector j attracts all scientists, but this increasing dominance of research activity does not affect sector j's share of extraction. There is a knife-edge case in which $s_{jt} = 0.5$ for all time, but as with the Cobb-Douglas case analyzed above, if equilibrium s_{jt} ever takes on any other value, then the economy progresses to a corner allocation in research.

4.3 Special Case With Dominant Supply Expansion Effect: $\sigma = 0$

Finally, consider the special case of a Leontief production function for each intermediate good, which arises as $\sigma \to 0$.[18] In order to aid exposition, fix $\psi = \alpha/(1-\alpha)$. Let $Y_{jt} = \min\{R_{jt}, X_{jt}\}$ and $Y_{kt} = \min\{R_{kt}, X_{kt}\}$. In equilibrium, $R_{jt} = X_{jt}$ and $R_{kt} = X_{kt}$. From equation (8), we have:

$$p_{jXt} = \left(\frac{R_{jt}}{A_{jt}} \right)^{\frac{1-\alpha}{\alpha}} .$$

From equation (6), we then have:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{A_{j(t-1)}}{A_{k(t-1)}} \left(\frac{A_{jt}}{A_{kt}} \right)^{-1/\alpha} \left(\frac{R_{jt}}{R_{kt}} \right)^{1/\alpha} . \tag{17}$$

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[18] In subsequent work, Acemoglu et al. (2019) analyze a Leontief production function.

Appendix E.8 shows that

$$\frac{R_{jt}}{R_{kt}} = \left(\frac{\nu_j}{\nu_k} \frac{\left[\Psi_k^{-\frac{1-\alpha}{\alpha}} + A_{kt}^{-\frac{1-\alpha}{\alpha}} \right]}{\left[\Psi_j^{-\frac{1-\alpha}{\alpha}} + A_{jt}^{-\frac{1-\alpha}{\alpha}} \right]} \right)^{\frac{\epsilon\alpha}{\alpha+(1-\alpha)\epsilon}} . \tag{18}$$

If $\nu_j = \nu_k$, then we have $R_{jt} \geq R_{kt}$ if and only if A_{jt} is large enough. Substituting into equation (17), we have:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{A_{j(t-1)}}{A_{k(t-1)}} \left(\frac{A_{jt}}{A_{kt}} \right)^{-1/\alpha} \left(\frac{\nu_j}{\nu_k} \frac{\left[\Psi_k^{-\frac{1-\alpha}{\alpha}} + A_{kt}^{-\frac{1-\alpha}{\alpha}} \right]}{\left[\Psi_j^{-\frac{1-\alpha}{\alpha}} + A_{jt}^{-\frac{1-\alpha}{\alpha}} \right]} \right)^{\frac{\epsilon}{\alpha+(1-\alpha)\epsilon}} . \tag{19}$$

Now consider how Π_{jt}/Π_{kt} evolves when $s_{jt} = 1$. Take logs and differentiate with respect to $A_{j(t-1)}$, holding s_{jt} fixed:

$$\frac{\partial \ln(\Pi_{jt}/\Pi_{kt})}{\partial A_{j(t-1)}} = \frac{1-\alpha}{\alpha} \frac{1}{A_{j(t-1)}} \left\{ -1 + \underbrace{\frac{\epsilon}{\alpha + (1-\alpha)\epsilon} \frac{A_{jt}^{-\frac{1-\alpha}{\alpha}}}{\left[\Psi_j^{-\frac{1-\alpha}{\alpha}} + A_{jt}^{-\frac{1-\alpha}{\alpha}} \right]}}_{\to 0 \text{ as } A_{jt} \to \infty} \right\}.$$

The right-hand term in braces decreases in A_{jt}, going to 0 as $A_{jt} \to \infty$. Therefore the derivative becomes negative as A_{jt} becomes large. A corner allocation can persist for some finite interval when A_{jt} is not too large, but over time the weakening market size effect leads Π_{jt}/Π_{kt} to decrease as A_{jt} continues to grow. As established by Proposition 3, a corner allocation in research cannot persist indefinitely.

Now consider a steady-state research allocation, with $s_{jt} = s$ for all $t \geq t_0$. Because a corner allocation cannot persist, s must be strictly greater than 0 and strictly less than 1. As t increases, $A_{j(t-1)}$ and $A_{k(t-1)}$ become arbitrarily large. From equation (19), we have:

$$\lim_{t\to\infty} \frac{\Pi_{jt}}{\Pi_{kt}} \to \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)} \right)^{-\frac{1}{\alpha}} \left(\frac{\nu_j}{\nu_k} \left[\frac{\Psi_j}{\Psi_k} \right]^{\frac{1-\alpha}{\alpha}} \right)^{\frac{\epsilon}{\alpha+(1-\alpha)\epsilon}} . \tag{20}$$

At an equilibrium with $s \in (0, 1)$, $\Pi_{jt} = \Pi_{kt}$. Then, for t sufficiently large,

$$\left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)} \right)^{\frac{1}{\alpha}} = \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{\nu_j}{\nu_k} \left[\frac{\Psi_j}{\Psi_k} \right]^{\frac{1-\alpha}{\alpha}} \right)^{\frac{\epsilon}{\alpha+(1-\alpha)\epsilon}} .$$

At a steady state, $A_{j(t-1)} = (1+\eta\gamma s)^{\Delta} A_{j(t-1-\Delta)}$ and $A_{k(t-1)} = (1+\eta\gamma(1-s))^{\Delta} A_{k(t-1-\Delta)}$. Therefore the following must hold for all $\Delta \geq 0$:

$$\left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)}\right)^{\frac{1}{\alpha}} = \left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)}\right)^{-\Delta\frac{1-\alpha}{\alpha}} \left(\frac{A_{j(t-1-\Delta)}}{A_{k(t-1-\Delta)}}\right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{\nu_j}{\nu_k}\left[\frac{\Psi_j}{\Psi_k}\right]^{\frac{1-\alpha}{\alpha}}\right)^{\frac{\epsilon}{\alpha+(1-\alpha)\epsilon}}.$$

This implies

$$\left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)}\right)^{-\Delta\frac{1-\alpha}{\alpha}} = 1,$$

which in turn holds if and only if $s = 0.5$. Thus, the steady state research allocation must have $s = 0.5$. From equation (20), $\nu_j \geq \nu_k$ and $\Psi_j \geq \Psi_k$ then imply $A_{j(t-1)} \geq A_{k(t-1)}$ in the steady-state research allocation, with $A_{j(t-1)} > A_{k(t-1)}$ if in addition either $\nu_j > 0.5$ or $\Psi_j > \Psi_k$. Further, from equation (18), R_{jt}/R_{kt} approaches a constant value as t becomes large and $\nu_j \geq \nu_k$ with $\Psi_j \geq \Psi_k$ imply $R_{jt} \geq R_{kt}$, with $R_{jt} > R_{kt}$ if either $\nu_j > \nu_k$ or $\Psi_j > \Psi_k$. These results are familiar from Proposition 3.

Finally, consider an early time t_0 at which $A_{j(t_0-1)}$ and $A_{k(t_0-1)}$ are much smaller than Ψ_j and Ψ_k, respectively, and the economy is not yet at a steady-state research allocation. Equation (19) becomes:

$$\frac{\Pi_{jt_0}}{\Pi_{kt_0}} \approx \left[\left(\frac{A_{j(t_0-1)}}{A_{k(t_0-1)}}\right)^{(1-\alpha)(\epsilon-1)} \left(\frac{1+\eta\gamma s_{jt_0}}{1+\eta\gamma(1-s_{jt_0})}\right)^{-1} \left(\frac{\nu_j}{\nu_k}\right)^{\epsilon}\right]^{\frac{1}{\alpha+(1-\alpha)\epsilon}}. \tag{21}$$

The right-hand side increases in $A_{j(t_0-1)}/A_{k(t_0-1)}$ and decreases in s_{jt_0}. We have that $s_{jt_0} > 0.5$ if and only if[19]

$$\frac{A_{j(t_0-1)}}{A_{k(t_0-1)}} > \left(\frac{\nu_j}{\nu_k}\right)^{\frac{-\epsilon}{(1-\alpha)(\epsilon-1)}}. \tag{22}$$

If $s_{jt_0} > 0.5$, then $A_{j(t_0-1)}/A_{k(t_0-1)}$ increases over time and the right-hand side of equation (21) shifts up over time. As a result, $s_{j(t_0+1)} > s_{jt_0}$. Therefore, sector j can increase its share of research effort over an interval of time with not-too-advanced technology. The reason is that the market size effect increasingly incentivizes scientists to work in sector j (see equation (18)). Eventually sector j's technology becomes sufficiently advanced that the market size effect weakens and the supply expansion effect pushes scientists back towards sector k (see equation (20)). A transition in research thus arises because the sensitivity of R_{jt}/R_{kt} to technological quality diminishes as technology advances, eventually making the supply expansion effect the primary determinant of research activity for some length of time.

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[19] If $\nu_j \leq \nu_k$ and $\Psi_j \leq \Psi_k$, then inequality (22) implies that Assumption 1 holds at t_0.

5 Climate Change Policy

I next use the stylized representation of energy production to generate qualitative insight into how the possibility of laissez-faire transitions affects climate policy. I focus on use of coal, natural gas, and renewables, which compete in electricity and heating. Appendix A details the full model and its calibration.

I extend the theoretical setting in several ways. First, I allow ψ to differ by sector. Second, I model carbon dioxide (CO_2) emissions from coal and natural gas, with other emissions fixed exogenously. Third, I connect these emissions to global temperature, allow greater temperature to reduce economic output, and allow for exogenously increasing total factor productivity, all following the latest (2016R) version of the benchmark DICE model (Nordhaus, 2017). Finally, I allow a policymaker to use policy instruments to affect the market equilibrium. This policymaker seeks to maximize intertemporal welfare, which takes the standard utilitarian, discounted power utility form in per capita consumption. Depending on the scenario, the policymaker can tax greenhouse gas emissions and/or can subsidize R&D into renewable resources. In contrast to standard climate-economy models, the cost of reducing emissions at time t is endogenous: it depends on the supply of each energy resource, on the time t quality of the machines for using each type of resource, and on the substitutability of each type of energy for the other.

As described in Appendix A, I set ϵ to 1.8 based on evidence in Papageorgiou et al. (2017) and Stern (2012) and I set σ to 0.4 based on estimates in Koesler and Schymura (2015) for the elasticity of substitution between energy and value-added in a panel of countries. Marten and Garbaccio (2018) map these estimates into energy supply sectors, and Lemoine (2020) aggregates them into a single energy supply sector. The value of 0.4 is broadly consistent with related evidence in Okagawa and Ban (2008) and Atalay (2017) and is in the ballpark of elasticities of substitution used by computable general equilibrium models of energy use (see Appendix A). Further evidence in support of σ around 0.4 comes from the model's internal dynamics: I am unable to match market data for σ much larger than 0.7 (including $\sigma > 1$), σ around 0.7 generate laissez-faire dynamics inconsistent with expectations of gas displacing coal over time (see Appendix B), and σ much smaller than 0.3 have climate change damages eventually driving consumption to be negative (with any policy that maintains positive consumption becoming infinitely valuable).

The parameter γ determines the timing of laissez-faire transitions. I undertake a bounding analysis on γ in order to gain insight into how that timing affects climate policy. In particular, I explore two values that in practice generate reasonable bounds on laissez-faire dynamics (see below): allocating all scientific research to a single type of energy doubles the quality of its technology over a decade in a *small advances* case ($\gamma = 1$) but septuples that quality in a *large advances* case ($\gamma = 6$). These cases are reasonably consistent with the range of values implied by related literature.[20]

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[20] Ignoring spillovers between sectors, Fried (2018) estimates that marginally increasing the share of scien-

I calibrate the resource supply elasticities to a combination of outside data and outside estimates. I choose the remaining parameters to match market data. In particular, I match recent levelized costs for each type of energy, recent resource consumption, recent shares of R&D spending, and recent economic output. When exploring different values for γ, I recalibrate the other parameters to preserve the match to market data. Appendix A reports that the initial period's emission reductions from small to moderate taxes, a non-targeted moment, are not far from benchmark calibrations.

5.1 Laissez-Faire Outcomes

Figure 2 plots the laissez-faire trajectories of research (top) and extraction (middle and bottom) in the cases with small (left) and large (right) advances. A transition towards gas occurs over the next decades, with gas proceeding to dominate energy supply for the next century or more.[21] Eventually, a second transition occurs, driving resource use away from gas and towards renewables. This double transition is consistent with many energy analysts' discussions of future market dynamics. The transitions are innovation-led, with gas and then renewables coming to dominate the research allocation decades before they dominate resource supply. The calibration requires coal to have the most advanced technology and renewables to have the least advanced technology. The transitions work to narrow these gaps in turn. The timing of the transition to renewables is sensitive to the rapidity with which science progresses. The transition occurs just over 400 years in the future (after the end of the plot) in the case of small advances but occurs only 100 years from now in the case of large advances.

The optimistic message is that an innovation-led transition to renewables does occur. The pessimistic message is that it is insufficient to avoid climate catastrophe. The bottom panels of Figure 2 show that improvements in each resource's technologies and in total factor productivity drive a large expansion in total resource use, so that coal and gas consumption can be substantial even when these resources provide only a small share of total supply.[22] And it is the level of fossil resource use that determines emissions and global climate change.

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tists improves technology by 426% over 5 years at the initial level of renewable scientists used here, implying a γ of around 8 for our 10-year timestep. This estimate is close to the large advances cases. In the calibration of Acemoglu et al. (2019), each scientist expects to advance technology by 11% over 5 years at the initial level of renewable scientists used here, implying γ of around 0.2 for our 10-year timestep. This value is close to the small advances cases. Acemoglu et al. (2016) also estimate an innovation production function, but the mapping to the present paper is less clear.

[21] The calibration ignores current policies, which include various subsidies for each kind of research and resource production and include air pollution regulations (see Acemoglu et al., 2019). The initial decline in renewables' share of research might reflect that current policies inflate the initial, calibrated level of research into renewables.

[22] The drawn-out nature and incompleteness of the transitions from coal to gas and from gas to renewables accord with evidence from history (Smil, 2010, Chapter 2).

In fact, coal and gas use are greater over the next 200 years in the large advances case, despite the relatively quick transition to renewables.

The left panel of Figure 3 plots warming under each trajectory. Warming from 2015 to 2095 is 3.3°C (4.8°C) in the case of small (large) advances. These values bracket the mean projection in the IPCC's RCP8.5 scenario.[23] By 2400, warming reaches enormous levels, despite the transition to clean energy seen in the case of large advances. The right panel of Figure 3 shows that consumption per capita increases over the next centuries. In the case with small advances, consumption per capita subsequently plateaus as growth in total factor productivity slows and as warming becomes more severe. Further warming then gradually reduces consumption per capita. In the case with large advances, improvements in renewable technologies generate a boom that lasts until the losses from warming become especially severe.[24]

Calculating the channels from equation (10), the patent quality effect strongly pushes scientists away from renewables over the next two centuries, but the supply expansion effect dominates it because σ is less than 1. The market size effect also pushes scientists away from renewables over the next two centuries. Renewables' market share does not begin increasing until after the transition in research is underway. That transition in research is ignited by the supply expansion effect becoming large enough (because gas technology becomes advanced enough) to overwhelm both the patent quality effect and the market size effect. As renewables gain market share, they tilt the market size effect just enough to attract more scientists to renewables. This shift in the research allocation further increases renewables' share of supply, compounding the change in the market size effect and generating a self-reinforcing process that culminates in renewable resources dominating both research and supply.[25]

5.2 Optimal Policy

We have seen that a transition to renewable resources does eventually occur in laissez-faire. However, that transition is too late to avoid substantial warming, and residual fossil fuel use after the transition drives further warming. Now consider how a policymaker would control warming by taxing emissions and/or subsidizing renewable R&D.[26]

To gain intuition, consider two resources, labeled j and k, with use of fossil resource j taxed at τ_t per unit and researchers in clean sector k subsidized at rate ω_t. Letting p_{jRt}

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[23] See Table 2.1 at `https://ar5-syr.ipcc.ch/topic_futurechanges.php`.

[24] In the final period, warming reduces output by 99% in the case with large advances but reduces output by only 86% in the case with small advances.

[25] In the case of large advances, the post-2250 reduction in renewables' share of research occurs once the supply expansion effect begins pushing scientists out of renewables.

[26] This is a difficult program to solve, involving a bilevel programming problem with both equilibrium and complementarity constraints. Appendix A describes the solution method.

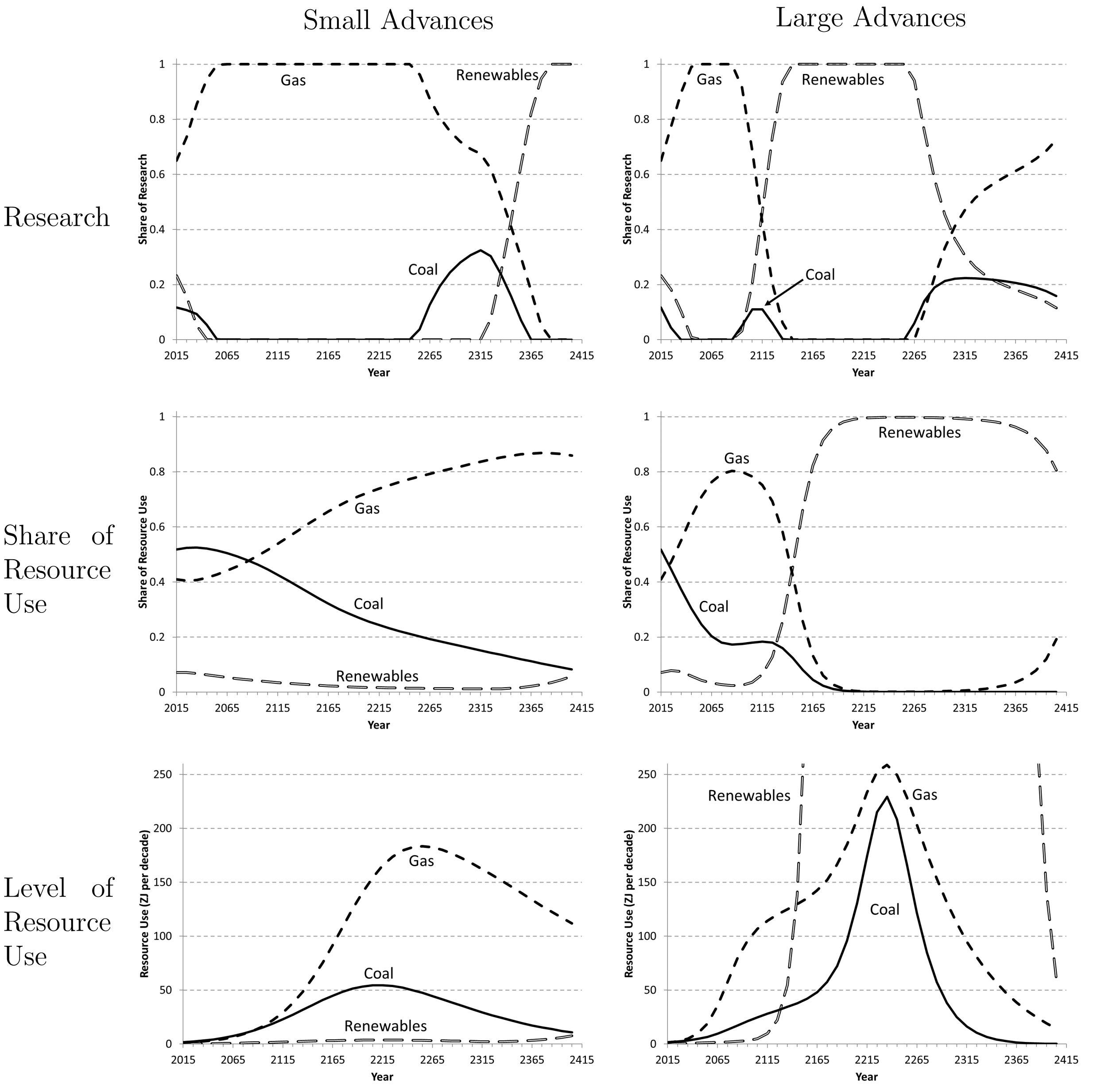
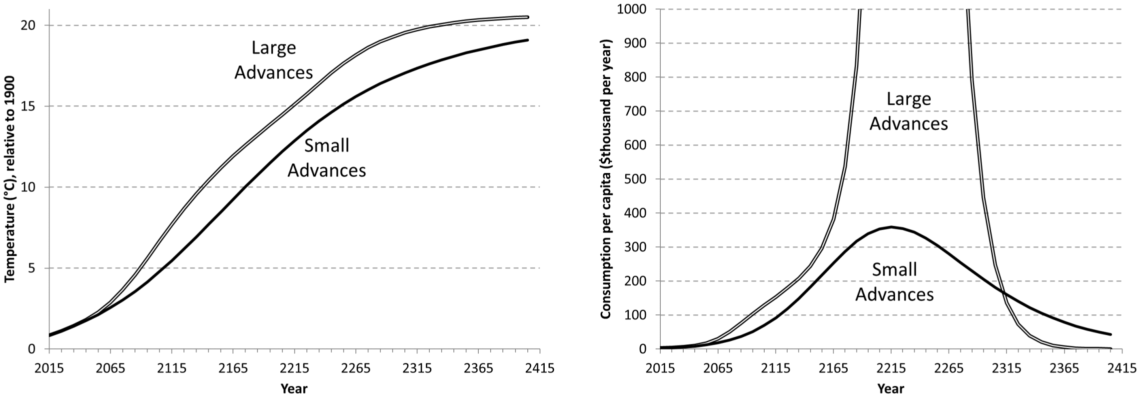


Figure 2: Laissez-faire resource use and research allocation for small advances ($\gamma = 1$, left) and large advances ($\gamma = 6$, right).



(a) Temperature (b) Consumption per capita

Figure 3: Temperature and consumption per capita in laissez-faire.

indicate the consumer price of resource j, equation (2) becomes

$$R_{jt} = \Psi_j [p_{jRt} - \tau_t]^{\psi}.$$

Equation (14) then becomes

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\frac{1}{\sigma}} \frac{(R_{jt}/\Psi_j)^{\frac{1}{\psi}} + \tau_t}{(R_{kt}/\Psi_k)^{\frac{1}{\psi}}} = \frac{\nu_j}{\nu_k} \left[\frac{Y_{jt}}{Y_{kt}}\right]^{\frac{1}{\sigma} - \frac{1}{\epsilon}}. \tag{23}$$

For given technologies, increasing τ_t reduces equilibrium R_{jt} and increases equilibrium R_{kt}. Equation (10) becomes:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{A_{j(t-1)}}{(1+\omega_t) A_{k(t-1)}} \left(\frac{(1+\eta\gamma s_{jt}) A_{j(t-1)}}{(1+\eta\gamma s_{kt}) A_{k(t-1)}}\right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}} \left(\frac{R_{jt}}{R_{kt}}\right)^{\frac{1}{\sigma+\alpha(1-\sigma)}} \left(\frac{(R_{jt}/\Psi_j)^{1/\psi} + \tau_t}{(R_{kt}/\Psi_k)^{1/\psi}}\right)^{\frac{\sigma}{\sigma+\alpha(1-\sigma)}}.$$

Increasing τ_t has ambiguous effects on the allocation of research. On the one hand, by increasing the consumer price of fossil resources, a higher tax works to increase the price of fossil machine services and thus to increase the incentive to improve fossil technologies. On the other hand, by reducing fossil resource use and increasing clean resource use, a higher tax works to push researchers towards the clean sector through the market size effect.[27] In contrast, increasing ω_t works directly to drive scientists towards the clean sector. Through the right-hand side of equation (23), this effect works to reduce R_{jt} and to increase R_{kt}, which further drives scientists to the clean sector through the market size effect.

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[27] If larger τ_t does increase the clean sector's share of research, then the right-hand side of equation (23) falls, further increasing the clean sector's share of resource use.

The top panels of Figure 4 plot welfare-maximizing policy choices, with emission taxes on the left and research subsidies on the right. Solid red (hollow blue) lines indicate the case with small (large) advances. The tax ramps up over time, and does so much faster when the policymaker cannot also use a research subsidy.[28] In contrast, the research subsidy starts out large but phases out over the next decades. The optimal research subsidy is not visibly affected by whether or not a tax is available.

The middle panels show how these policies affect the clean sector's share of research (left) and of resource use (right). A policymaker uses an emission tax to bring the transitions in research and resource use slightly forward in time, with the trajectories still qualitatively similar to the cases without policy. In contrast, a policymaker uses a clean research subsidy to immediately shift all research activity to the clean sector, which continues to dominate the research allocation even after the subsidy is withdrawn. The rapid improvement in technology substantially increases near-term renewable resource use. Renewables' share of supply soon approaches 100%, but the level of coal and gas use nonetheless also increases (as seen in laissez-faire for the case of large advances). Renewables achieve their greatest share of near-term resource use when a policymaker combines the research subsidy with an emission tax. In this case, the policymaker can also reduce long-run use of fossil resources.

The bottom panels plot the consequences for global temperature, for the case of small advances (left) and for the case of large advances (right). All policies limit warming over the coming two centuries. However, the welfare-maximizing research policy can actually increase long-term warming unless it is paired with an emission tax: the research policy can stimulate a transition to renewables that reduces near-term emissions, but it cannot control the residual fossil fuel use that drives long-term warming. These results indicate that policy must make fossil resources more expensive, not just make clean energy cheaper, if it is to limit warming (see also Hassler et al., 2020). The standalone research subsidy enables much greater consumption than does the standalone emission tax once renewables dominate supply, after 100 (50) years in the case of small (large) advances. By limiting warming more effectively, a portfolio of both instruments allows far greater consumption in later centuries.

We have seen that outcomes are sensitive to the type of instrument that a policymaker

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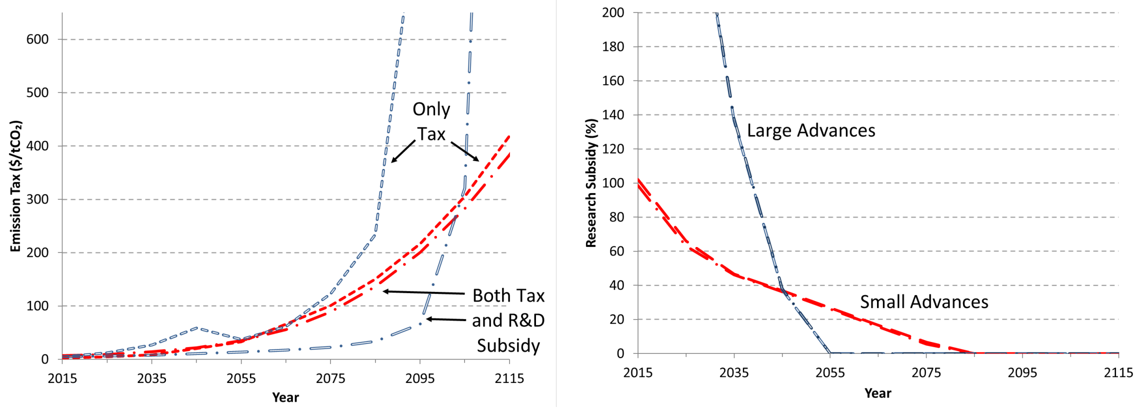
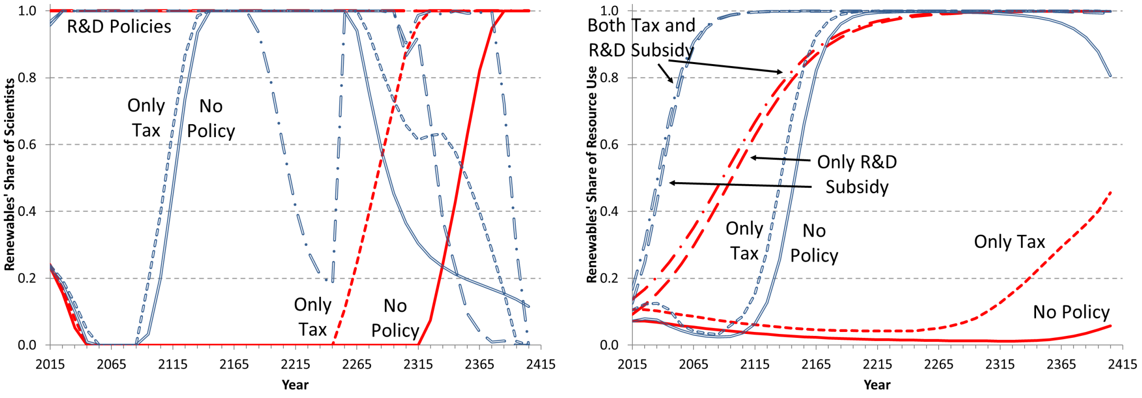
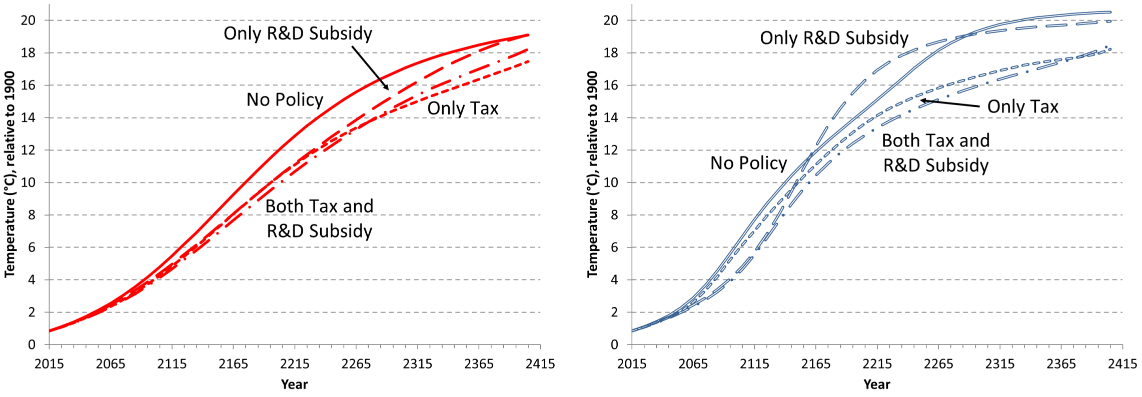
[28] This tax is quite different from the most closely related work (although direct comparisons are complicated by previous work not expressing taxes per unit of emissions). Greaker et al. (2018) show that the model of Acemoglu et al. (2012) implies emission taxes that are low and declining. They also show that an especially small elasticity of substitution between intermediates can generate an increasing tax, albeit one that increases at a declining rate (in contrast to the present paper's sharply increasing tax). The ϵ used here is between the values explored in Greaker et al. (2018). In Acemoglu et al. (2016), the optimal tax is hump-shaped. They interpret the declining portion as reflecting the eventual diminished role of fossil resources (pg 88). In the present setting, lock-in does not work in renewables' favor to such a degree, so that substantial fossil use remains even after a transition to renewables. The optimal tax is indeed hump-shaped over the full 400-year horizon, but the late decline in the tax is here merely an artifact of the policymaker anticipating the end of the world in 2415.



(a) Tax Policy ($\$/\text{tCO}_2$)) (b) Clean Research Subsidy (%)



(c) Research Shares (d) Resource Shares



(e) Temperature, for Small Advances (f) Temperature, for Large Advances

Figure 4: Welfare-maximizing policies. Solid, red lines are for the case of small advances ($\gamma = 1$), and hollow, blue lines are for the case of large advances ($\gamma = 6$). The first row plots only the coming century.

Table 1: Welfare-maximizing outcomes for different policy instruments.

	Policy Scenario			
Model Version	**No policy**	**Emission tax**	**Research subsidy**	**Both instruments**
Emission Tax in 2015 (\$ per tCO_2)				
Small Advances	-	2.5	-	7.3
Large Advances	-	4.8	-	5.1
Renewables' Share of Resources in 2015 (%)				
Small Advances	7.2	10.7	9.2	13.7
Large Advances	7.2	10.5	12.5	16.7
Renewables' Share of Scientists in 2015 (%)				
Small Advances	23.3	24.1	100	100
Large Advances	23.3	23.8	96.7	95.7
Temperature in 2115 (°C, relative to 1900)				
Small Advances	5.5	4.8	5.0	4.7
Large Advances	7.7	7.0	5.7	5.5
*Balanced Growth Equivalent Consumption Gain (%)**				
Small Advances	-	1.6	2.6	3.1
Large Advances	-	4.8	12.3	14.4

*Relative to the no-policy scenario. The balanced growth equivalent translates changes in welfare into the constant relative difference in consumption between two counterfactual consumption trajectories that grow at the same constant rate (Mirrlees and Stern, 1972).

can use. Table 1 undertakes further comparisons. Figure 4 showed that the end-of-century tax is larger when the policymaker cannot also use a research subsidy, but the top panel of Table 1 shows that today's welfare-maximizing tax is larger when the policymaker combines the tax with a research subsidy. The tax and research subsidy are complements in the near term. Figure 4 also showed that renewables' share of the next decades' resource use is greater under a standalone research policy than under a standalone tax policy, but the second panel of Table 1 shows that the tax policy increases renewables' share of today's resource use by a bit more when scientists take only small advances. In this case, a research policy takes time to affect resource use.

The most important near-term difference between the policies is their effect on the allocation of research. The third panel of Table 1 shows that an emission tax does direct scientists towards the clean sector, but the welfare-maximizing tax increases the clean sector's share of scientists by less than 1 percentage point.[29] In contrast, a policymaker who can use a research subsidy immediately redirects all scientists to the clean sector.

All policies reduce warming over the century relative to laissez-faire, with the combined policy reducing warming the most (fourth panel).[30] Which of the two standalone policies most strongly limits warming depends on the nature of scientific advances. If scientists take only small steps, the standalone tax policy limits this century's warming by more, but if scientists take large steps, the standalone research policy limits this century's warming by more. Nonetheless, the bottom panel shows that the standalone tax is only 62% as valuable as the standalone research subsidy when scientists take small advances. And the standalone tax policy provides a mere 39% of the benefit of a standalone research policy when scientists take large advances, reflecting that large advances enable research policy to more readily affect near-term resource use. A policymaker always benefits from adding an emission tax to a research policy, as the tax increases value by 20% (17%) in the case of small (large) advances.[31]

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[29] Scientists are only slightly responsive to the emission tax. With small (large) advances and a tax of \$150 per tCO_2, the clean sector's share of research is only 0.35 (0.28), not too far above the laissez-faire share of 0.23. See also footnote 34.

[30] Laissez-faire temperature in 2115 is similar to DICE-2016R, but optimal temperature is greater than the value of 3.7°C from DICE-2016R. The optimal temperature in the high-damage case below is fairly close to that in DICE-2016R (see Tables A-3 and A-4). The difference in Table 1 stems from oil emissions being exogenous here and from the implied marginal abatement cost curve. The exogenous emissions increase year 2115 temperature by 1.4°C. With the first period's technology, the policymaker can here obtain low levels of emission reductions at less cost than in DICE-2016R but finds high levels of emission reductions to be more expensive than in DICE-2016R.

[31] The truly optimal policy portfolio also includes a subsidy for machine production and the ability to steer research between coal and gas technologies. Appendix B shows that including the optimal machine subsidy does not change the primary conclusions. The ability to direct fossil research is unlikely to be important because the depicted optimal policy does not leave any scientists working on either fossil resource.

5.3 Alternate Policymaking Environments

Table 2 reports the effects on welfare of varying the policymaking environment. Appendix B reports effects on the variables from Table 1 and reports robustness checks involving model calibration.

The first rows in each panel of Table 2 repeat results familiar from the previous section. The second rows in each panel delay the implementation of policy for 50 years. From there, the tax policy is not that different from the base case. However, the research policy does change. Delay makes it harder for the research policy to shift resource supply because renewable technologies fall further behind gas technologies in the period of delay.[32] The optimal research policy does not immediately move all scientists to the renewable sector once the period of delay is over. Because the research policy takes too long to affect resource use, the standalone tax policy is more valuable than the standalone research policy. In the case of small advances, the standalone emission tax is 50% more valuable than the standalone research policy and is nearly as valuable as the combined policy.

The third rows use a lower utility discount rate (pure rate of time preference). The base specification follows DICE-2016R in discounting utility at 1.5% per year, but this alternate specification follows Stern (2007) in discounting utility at 0.1% per year. The more patient policymaker attaches greater weight to losses from future warming. As a result, policy produces much larger benefits. The initial emission tax is dramatically larger than in the base specification, now \$863 (\$751) per tCO_2 in the case of small (large) advances. Most scientists soon work in the renewable sector, and all do within a century. In the case of small advances, the tax falls over the century as scientists no longer require such a strong incentive. When a research subsidy is also available, the policymaker uses the research subsidy to shift scientists and reduces the initial tax all the way to \$20 (\$11) per tCO_2. Because both policies soon shift research, the standalone emission tax is now as valuable as (70% more valuable than) the standalone research subsidy. Combining the two increases the value of the policy program by around 15% in either case.

The fourth rows calibrate climate change damages to the survey of experts in Pindyck (2019), as implemented in Lemoine (2021).[33] The marginal losses from warming are nearly ten times larger than in the base specification from DICE-2016R (see Appendix A). Because climate change is much more expensive, the gains from policy are much larger. The policymaker designs an emission tax that starts off large (around \$450 per tCO_2 in both cases) and shifts all researchers to the renewable sector within this century. In contrast to

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[32] Finding that delaying policy reduces the effectiveness of a research subsidy as the renewable sector falls further behind echoes Acemoglu et al. (2012, 2016), but here the question is more nuanced. In those papers, the renewable sector is becoming more locked-in over time. Here delay also brings closer the dates at which the renewable sector would endogenously begin attracting more scientists and increasing its share of supply.

[33] Pindyck (2019) asks experts for the probability of consumption losses from climate change exceeding various thresholds in fifty years. Lemoine (2021) combines the resulting distribution of losses with a distribution for warming to obtain a distribution of damages per unit of warming.

Table 2: Balanced growth equivalent gain under alternate policymaking environments.

	Policy Scenario		
Specification	**Emission tax**	**Research subsidy**	**Both instruments**
Small Advances			
Base	1.6	2.6	3.1
50-Year Delay	1.3	0.9	1.4
Less Discounting*	77.0	77.8	89.5
Higher Damages**	58.4	31.7	59.9
Large Advances			
Base	4.8	12.3	14.4
50-Year Delay	4.5	4.0	7.5
Less Discounting*	827.7	490.6	1007.4
Higher Damages**	99.3	105.3	126.9

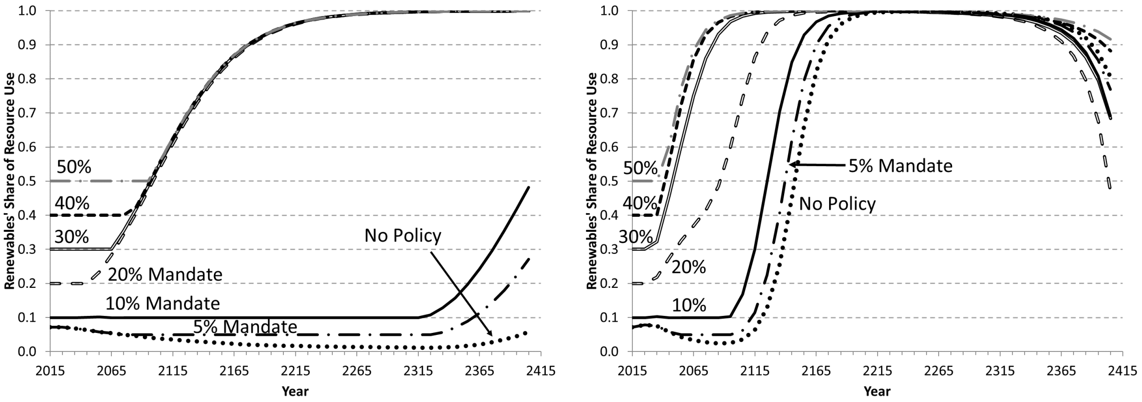
* Pure rate of time preference reduced from 1.5% to 0.01% per year, as in Stern (2007).

** Damages increased to calibration of Lemoine (2021), from survey evidence in Pindyck (2019).

the case of low discounting, the standalone emission tax increases over the century even for small advances: the primary rationale for the large tax is here to quickly increase renewables' share of resource use, not to increase renewables' long-run share of resource use by shifting near-term research. As a result, renewables dominate supply by the end of the century and temperature remains far lower than in any other scenario studied, reaching only 3.6°C (4.1°C) in 100 years. And while the initial tax does drop in the presence of a research subsidy (to around \$80 or \$100 per tCO_2), its subsequent trajectory contrasts with the case of low discounting, very quickly rising to levels comparable to the case of a standalone emission tax in order to shift resource use. In the case of small advances, the standalone emission tax is twice as valuable as the standalone research subsidy and is nearly as valuable as the combined policy. When even moderate warming imposes high costs, the policymaker becomes primarily concerned with shifting resource supply rapidly. The emission tax is better suited to this task when research takes time to pay off.

5.4 Mandates to use Renewable Energy

I have thus far examined emission taxes and research subsidies, but the possibility of endogenous innovation-led transitions has important implications for policies that mandate a minimum share of renewable resource use. Such policies are common, as around 30 U.S. states have Renewable Portfolio Standards and the U.S. Congress has considered Clean Energy Standards.



(a) Small Advances

(b) Large Advances

Figure 5: Renewables' share of resource use under various mandates.

Table 3: Balanced growth equivalent gain under various mandates, with research endogenously responding to the mandate and with research fixed at the laissez-faire trajectory.

	Renewable Energy Mandate					
	5%	10%	20%	30%	40%	50%
Small Advances						
Endogenous research	0.03	-0.13	2.61	2.26	0.69	-3.25
Fixed research	0	0.050	-1.50	-6.34	–	–
Large Advances						
Endogenous research	-0.04	-0.80	-3.98	2.39	7.36	7.77
Fixed research	0.04	0.04	-1.76	-7.22	-19.73	–

Dashes indicate cases where consumption eventually becomes negative under the mandate. The balanced growth equivalent translates changes in welfare into the constant relative difference in consumption between two counterfactual consumption trajectories that grow at the same constant rate (Mirrlees and Stern, 1972).

A binding mandate pushes scientists towards the clean sector through the market size effect. Figure 5 plots the share of renewable resource use under mandates ranging from 5% to 50%. All mandates hasten the transition in resource use. Moreover, the mandates eventually make themselves nonbinding by redirecting scientists to the clean sector. Mandates of 5% or 10% bind for a long time and only speed up the transition by a bit. An analysis that ignored innovation would predict that larger mandates would bind for longer, but we instead see that larger mandates bind for less time. A mandate of 10% retains many scientists working in the fossil sectors, but the larger mandates leave very few (if any) scientists in the fossil sectors. The large mandates rapidly improve clean technology, which incentivizes even more renewable resource use than the mandate requires. Sufficiently large mandates can thus quickly ignite an energy transition that would have otherwise occurred centuries later.

Mandates might avoid substantial warming, but they are crude instruments. Are they worth it? Table 3 compares the benefits from various mandates, for cases in which scientists respond to the mandate (as in Figure 5) and for cases in which scientists are fixed at the laissez-faire allocation (plotted in Figure 2). Very small mandates (here represented by 5%) can provide small benefits, but larger mandates that are not quite large enough to ignite a transition (here represented by 10%) always impose small costs. Intriguingly, still-larger mandates can provide substantial benefits relative to a no-policy world. In fact, comparing to Table 1, large mandates can improve welfare by more than even the welfare-maximizing emission tax policy. The reason for these benefits is twofold. First, the temperature trajectory is more similar to the trajectory seen under the standalone research policy, which limits warming over this century (but is not effective at limiting long-run warming). Second, binding mandates substantially increase total resource use, which can increase near-term consumption. (Very large mandates impose net costs because they distort supply to such a degree that near-term consumption falls.) Even though mandates can be superior to standalone emission taxes and standalone research subsidies, they are not superior to the combination of emission taxes and research subsidies. The best mandates (of 21% and 45%) provide benefits of 2.6% and 8.0% for the cases of small and large advances, which are smaller than the benefits of 3.1% and 14.4% reported for the combined portfolio in Table 1.[34]

Conventional cost-benefit analyses take a static perspective on mandates, analogous to the experiments with research fixed to the laissez-faire trajectory. We see that such analyses can be highly misleading. An analysis that ignored endogenous research responses would

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[34] The mandates demonstrate tipping behavior: the mandates are either large enough to quickly ignite a transition or they are not. Around the threshold, a small increase in the mandate can lead to a qualitatively different economic trajectory. As a result, there are two local maxima in welfare, one with a very small mandate and one with a large mandate. For the same reason, the tax policy can also have two local optima, one with a tax high enough to quickly ignite a transition (by shifting near-term research) and one with a far smaller tax. For instance, the former optimum yields a year 2015 tax of \$314 per tCO_2 in the base specification with small advances, but the latter optimum yields greater welfare (and thus was the one reported in Section 5.2). For small advances, the high-tax optimum does yield greater welfare in the cases with low discounting or high damages.

assess the larger mandates to be rather costly, but we instead see that they can be rather beneficial. For instance, an analysis that ignored endogenous research responses would predict that a 40% mandate would eventually drive consumption to negative levels in the case with small advances (effectively generating infinite losses) or would impose severe balanced growth equivalent (BGE) losses of 20% in the case of large advances. However, the correct analysis would predict minor BGE benefits of 0.69% in the case of small advances and substantial BGE benefits of 7.4% in the case of large advances. The endogenous response of research to mandates dominates the welfare evaluation, changing the sign of the welfare effect even when costs might otherwise appear to be enormous.

6 Conclusions

We have seen that complementarities between innovation and factors of production are critical to the possibility of innovation-led transitions in factor use. These complementarities eventually push scientists away from the more advanced sector, and the redirection of scientific effort eventually redirects factor use away from the dominant sector. In a calibrated numerical implementation, I find that laissez-faire use of energy resources eventually does transition towards clean renewable resources from emission-intensive coal and gas. Nonetheless, warming reaches dangerous levels because use of coal and gas persists beyond the transition. A temporary emission tax, a temporary subsidy to clean energy research, and a temporary mandate to use clean energy can each quickly ignite the transition if they are sufficiently large, but only emission taxes can control the residual fossil fuel use that remains after the transition has occurred. Future work should integrate this model of innovation-led transitions with leading models of resource depletion and of innovation in resource extraction technologies (e.g., Schwerhoff and Stuermer, 2019) to assess the relative importance of each mechanism in recent and historical data. Future work should also assess the importance of complementarities in driving innovation-led transitions in non-energy sectors of the economy.

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