Sufficient Statistics for the Cost of Climate Change[*]

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I formally relate the consequences of climate change to time series variation in weather. First, I show that the effects of climate change on adaptation investments can be bounded from below by estimating responses to weather outcomes. The bound becomes tighter when also estimating responses to forecasts. Second, I show that the marginal effect of climate change on long-run payoffs is identical to the average effect of transient weather events. Instead of estimating the marginal effect of weather within distinct weather bins, empirical work should estimate the average effect of weather within each climate.

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1   Introduction

A pressing empirical agenda seeks to estimate the economic costs of climate change. Ignorance of these costs has severely hampered economists' ability to give concrete policy recommendations (Pindyck, 2013). The challenge is that although variation in climate has been primarily cross-sectional, cross-sectional regressions cannot clearly identify the effects of climate.[1] Seeking credible identification, an explosively growing empirical literature has recently explored panel variation in weather.[2] The hope is that variation in transient weather identifies—or at worst bounds—the effects of a change in climate, which manifests itself through weather but differs from a transient weather shock in being repeated period after period and in affecting expectations of weather far out into the future.

I here undertake the first formal analysis that precisely delineates what and how we can learn about the climate from the weather. Linking weather to climate requires analyzing a dynamic model that can capture the distinction between transient and permanent changes in weather. I study an agent who is exposed to stochastic weather outcomes. These weather outcomes impose some costs that are unavoidable and some costs that depend on the agent's actions (equivalently, investments). The agent wants to choose actions that best match the weather, but actions also impose costs: maintaining a given level of activity is costly, and adjusting actions from period to period is costly. When choosing actions, the agent knows the current weather, has access to specialized forecasts of the weather some arbitrary number of periods into the future, and relies on knowledge of the climate to generate forecasts at longer horizons. A change in the climate affects the distribution of realized weather in every period and also affects the agent's expectations of future weather.

I show several novel results. First, I show that estimating the effects of weather on actions understates the long-run effect of climate on actions. Many economists have intuited that short-run adaptation responses to weather are likely to be smaller than long-run adaptation responses to climate (e.g., Deschênes and Greenstone, 2007). I show that the critical factor for this result is adjustment costs, not expectations of future weather. The actions an agent takes in response to a transient weather shock are constrained by the agent's desire to not change actions too much from period to period, but when the same weather shock is repeated period after period, even a myopic agent eventually achieves a larger change in

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[1]For many years, empirical analyses did rely on cross-sectional variation in climate to identify the economic consequences of climate change (e.g., Mendelsohn et al., 1994; Schlenker et al., 2005; Nordhaus, 2006). However, cross-sectional analyses fell out of favor due to concerns about omitted variables bias. See Dell et al. (2014) for an exposition and Massetti and Mendelsohn (2018) for a review.

[2]This literature has estimated the effects of climate on gross domestic product (Dell et al., 2012; Burke et al., 2015), on profits (Deschênes and Greenstone, 2007), and on behavioral variables including productivity (Heal and Park, 2013; Zhang et al., 2018), health (Deschenes, 2014), crime (Ranson, 2014), and energy use (Auffhammer and Aroonruengsawat, 2011; Deschênes and Greenstone, 2011), among many others. For recent reviews, see Dell et al. (2014), Carleton and Hsiang (2016), and Heal and Park (2016). Blanc and Schlenker (2017) discuss the strengths and weaknesses of relying on panel variation in weather.

activity through a sequence of incremental adjustments. I also show that combining short-run adaptation responses to weather realizations with short-run adaptation responses to weather forecasts can better approximate long-run adaptation to climate.

Second, I show that the effect of climate on steady-state payoffs is equal to the average treatment effect of weather around a steady state in the current climate. An easily estimated function of weather is therefore a sufficient statistic for the impact of climate change on variables such as welfare and profits.[3] This is a surprising and powerful result. Changing the climate is equivalent to changing expected weather in all future periods, yet transient weather shocks identify the consequences of climate. This result arises for three reasons. First, the envelope theorem implies that small changes in current actions do not have first-order effects on maximized value. Second, standard representations of adjustment costs imply that small changes in past actions also do not have first-order effects on maximized value around a steady state. Together, these two observations imply that we do not need to consider how expectations of weather affect actions around a steady state. Finally, the treatment effect of weather is linear when payoffs are quadratic and is otherwise approximately linear when the weather has small variance. The average treatment effect of transient weather shocks is then equivalent to the effect of changing the average weather, which in turn is the definition of the effect of changing the climate. This result suggests that reduced-form empirical work should begin estimating the average treatment effect of weather as a function of long-run average weather.[4]

Despite the importance of empirically estimating the costs of climate change and the sharpness of informal debates around the relevance of the recent empirical literature to climate change, there has been remarkably little formal analysis of the link between weather and climate. The most prominent defense of using panel variation to estimate the effects of climate change rests on an appeal to the envelope theorem: if climate differs from weather only via expectations and if expectations matter only via actions, then the envelope theorem suggests that expectations do not matter for the effects of climate on payoffs. This argument dates to Deschênes and Greenstone (2007) and has been most forcefully elaborated in Hsiang (2016) and Deryugina and Hsiang (2017). However, these envelope theorem arguments apply static analysis to an inherently dynamic problem. In fact, climate change can affect predetermined variables that are not subject to the envelope theorem but are themselves actions that were chosen in previous periods based on expectations of weather in the current period

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[3]I describe the average treatment effect of weather as a sufficient statistic because multiple combinations of structural parameters can yield the same welfare consequences. Estimating the average treatment effect of weather does not recover all deep primitives but does provide a credibly identified estimate of climate impacts. See Chetty (2009) for a general treatment of sufficient statistics for welfare analysis.

[4]In contrast, much empirical literature estimates the marginal effect of weather by weather bin (see Carleton and Hsiang, 2016), sometimes allowing the marginal effects to differ by climate zone (e.g., Barreca et al., 2015; Deryugina and Hsiang, 2017; Auffhammer, 2018). The standard practice can identify nonlinearities in the effects of weather on payoffs. In the appendix, I show that nonlinear weather impacts may not indicate anything about the consequences of changing the climate.

and beyond. I here show precisely when researchers can ignore the effects of expectations and show precisely which panel estimators can recover the effects of climate.[5]

   The next section describes the setting. Section 3 solves the dynamic programming problem. Sections 4 and 5 analyze the effects of climate on agents' chosen actions and payoffs, respectively. The final section discusses implications for empirical work. The appendix analyzes a more general setting, provides results about forecasts and nonlinearities, and contains proofs.

2   Setting

An agent is repeatedly exposed to stochastic weather outcomes. The realized weather in period t is w_t. This weather realization imposes two types of costs. A first type of cost arises independently of any actions the agent might take. These unavoidable costs are $\frac{1}{2}\psi(w_t - \bar{w})^2$, where the parameter $\bar{w}$ defines the weather outcome that minimizes unavoidable costs and the parameter $\psi \geq 0$ determines the costliness of any other weather outcome. A second type of cost depends on the agent's actions A_t. These avoidable costs are $\frac{1}{2}\gamma(A_t - w_t)^2$, where $\gamma \geq 0$. They vanish when the agent's actions are well-matched to the weather and potentially become large when the agent's actions are poorly matched to the weather.

   In each period, the agent chooses her action A_t. This action may be interpreted as a level of activity (e.g., time spent outdoors, energy used for heating or cooling, irrigation applied to a field) or as a stock of capital (e.g., outdoor gear, size or efficiency of furnace, number or efficiency of irrigation lines). The agent's actions impose two types of costs. First, maintaining A_t imposes costs of $\frac{1}{2}\phi(A_t - \bar{A})^2$, where $\phi \geq 0$. The parameter $\bar{A}$ defines the level of activity or capital that is cheapest to sustain. It can also be interpreted as the capital stock that would be chosen if weather imposed only unavoidable costs. Second, the agent faces a cost of adjusting actions from one period to the next. This cost is $\frac{1}{2}\alpha(A_t - A_{t-1})^2$, where $\alpha \geq 0$. When A_t represents a capital stock, these adjustment costs are investment costs. Relating to the literature on climate adaptation (e.g., Fankhauser et al., 1999; Mendelsohn, 2000), low adjustment costs allow adaptation investments to occur after weather is realized ("reactive" or "ex-post" adaptation), but large adjustment costs require adaptation to occur

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[5]A few other papers are also related. First, in an expositional analysis, I showed how envelope theorem arguments can fail in a three-period model (Lemoine, 2017). The present work precisely analyzes the consequences of climate change in an infinite-horizon model and constructively shows exactly which types of empirical estimates can be informative about the climate. Second, Kelly et al. (2005) study the cost of having to learn about a change in the climate from an altered sequence of weather as opposed to knowing outright how the climate has changed. I here abstract from learning in order to focus on mechanisms more relevant to the growing empirical literature. Third, calibrated simulations have shown that dynamic responses are critical to the effects of climate on timber markets (Sohngen and Mendelsohn, 1998; Guo and Costello, 2013) and to the cost of increased cyclone risk (Bakkensen and Barrage, 2018). Finally, Shrader (2017) demonstrates the importance of distinguishing adaptation motivated by expectations of future weather in an application to fisheries.

before weather is realized ("anticipatory" or "ex-ante" adaptation). Maintenance costs make the agent want to choose actions close to $\bar{A}$, and adjustment costs make the agent want to keep actions constant over time.

The agent observes time t weather before selecting her time t action. The agent has access to specialized forecasts of future weather and knows her region's climate, indexed by C. Specialized forecasts extend up to $N \geq 0$ periods ahead. Each period's forecast is an unbiased predictor of later weather. Beyond horizon N, the agent formulates generic forecasts that rely only on knowledge of the climate, not on information germane to that particular time period. For instance, the agent may rely on the local news to predict weather one week out but relies on knowledge of typical weather to predict weather one year out. Horizon N is therefore the shortest forecast horizon at which the agent receives information beyond knowledge of the climate.

Formally, let f_{it} be the i-period-ahead forecast available in period t. The time t weather realization is a random deviation from the one-period-ahead forecast: $w_t = f_{1(t-1)} + \epsilon_t$, where ϵ_t has mean zero and variance σ^2. Because forecasts are unbiased predictors, any changes in forecasts must be unanticipated: for $i \in [1, N]$, $f_{it} = f_{(i+1)(t-1)} + \nu_{it}$, where ν_{it} has mean zero and variance τ_i^2. Forecasts at horizons $i > N$ are $f_{it} = C$.[6] The ν_{it} and ϵ_t are serially uncorrelated, the covariance between ν_{it} and ν_{jt} is δ_{ij}, and the covariance between ϵ_t and ν_{it} is ρ_i.[7] Note that $E_t[w_{t+j}] = f_{jt}$. For notational convenience, collect all specialized forecasts available at time t in a vector F_t of length N.[8]

The agent maximizes the present value of payoffs over an infinite horizon. Time t payoffs are:

$$\pi(A_t, A_{t-1}, w_t) = -\frac{1}{2}\gamma(A_t - w_t)^2 - \frac{1}{2}\alpha(A_t - A_{t-1})^2 - \frac{1}{2}\phi(A_t - \bar{A})^2 - \frac{1}{2}\psi(w_t - \bar{w})^2.$$

She chooses time t actions as a function of past actions, current weather, and current forecasts. In order to study an interesting problem, assume that $\gamma + \phi > 0$. The agent solves:

$$\max_{\{A_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E_0\left[\pi(A_t, A_{t-1}, w_t)\right],$$

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[6]One might be concerned about a sharp discontinuity in information at horizon N. However, I have left the variances τ_i^2 general. Defining them to decrease in i and to approach zero as i approaches N would allow for the informativeness of the signal about time t weather to increase smoothly from long horizons to short horizons.

[7]Assuming that each shock is serially uncorrelated does not imply that weather and forecasts are serially uncorrelated. For instance, for $t > N$, $Cov_0(w_t, w_{t+1}) = \rho_1 + \sum_{i=1}^{N-1} \delta_{i(i+1)}$.

[8]The system of weather and forecasts can be written as a vector autoregression. Climate here controls average weather. One might wonder about the dependence of higher moments of the weather distribution on climate. However, the effects of climate change on the variance of the weather are poorly understood and potentially heterogeneous (e.g., Huntingford et al., 2013). Further, we need to know not just how climate change affects the variance of realized weather but how it affects the forecastability of weather at each horizon: the variance of the weather more than N periods ahead is $\sigma^2 + \sum_{i=1}^{N} \tau_i^2$, so we need to apportion any change in variance between σ^2 and each τ_i^2.

where $\beta \in [0, 1)$ is the per-period discount factor, A_{-1} is given, and E_0 denotes expectations at the time 0 information set. The solution satisfies the following Bellman equation:

$$V(Z_t, w_t, F_t) = \max_{A_t} \left\{ \pi(A_t, Z_t, w_t) + \beta E_t \left[V(Z_{t+1}, w_{t+1}, F_{t+1}) \right] \right\} \tag{1}$$

$$\text{s.t.} \quad Z_{t+1} = A_t$$

$$w_{t+1} = f_{1t} + \epsilon_{t+1}$$

$$f_{i(t+1)} = f_{(i+1)t} + \nu_{i(t+1)} \quad \text{for } i \in \{1, ..., N\}$$

$$f_{N(t+1)} = C + \nu_{N(t+1)} \quad \text{if } N > 0.$$

The state variable Z_t summarizes the previous period's actions.

The setting is sufficiently general to describe many applications of interest. For instance, much empirical literature has studied the effects of weather on energy use. The agent could then be choosing indoor temperature in each period, where maintenance costs reflect energy use and avoidable weather costs reflect thermal comfort. Empirical literature has also studied the effect of weather on agricultural profits. The decision variable could then be irrigation, fertilizer inputs, or crop varieties, maintenance costs reflect the cost of purchasing these in each year, adjustment costs reflect the cost of changing equipment and plans from year to year, and weather costs reflect the deviation in crop yields from their maximum possible value.

The primary specialization in the setting is the assumption of quadratic payoffs. Linear-quadratic models have long been workhorses in economic research because they allow for explicit analytic solutions to the Bellman equation (1). The appendix generalizes the analysis to arbitrary functional forms and vector-valued actions by applying perturbation methods (Judd, 1996).

3 Solution

The following proposition describes the value function that solves equation (1):

Proposition 1. *The value function* $V(Z_t, w_t, F_t)$ *has the form:*

$$a_1 Z_t^2 + a_2 w_t^2 + \sum_{i=1}^{N} a_3^i f_{it}^2 + b_1 Z_t w_t + \sum_{i=1}^{N} b_2^i Z_t f_{it} + \sum_{i=1}^{N} b_3^i w_t f_{it} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij} f_{it} f_{jt} + c_1 Z_t + c_2 w_t + \sum_{i=1}^{N} c_3^i f_{it} + d.$$

Optimal actions are:

$$A_t^* = \frac{\alpha A_{t-1} + \gamma w_t + \beta b_1 f_{1t} + \beta \sum_{i<N} b_2^i f_{(i+1)t} + \beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1}. \tag{2}$$

The coefficients are as follows:

1. $a_1 \leq 0$, with $a_1 < 0$ if and only if $\alpha > 0$.

2. $a_2 \leq 0$, with $a_2 < 0$ if and only if $\psi + \gamma(\phi + \alpha) > 0$.

3. $a_3^i \in [\beta^i a_2, 0]$, with $a_3^i < 0$ if and only if both $a_2 < 0$ and $\alpha\beta > 0$ and with $a_3^i > \beta^i a_2$ if and only if $\beta\alpha\gamma > 0$.

4. Each of the b coefficients is positive, with $b_1 > 0$ if and only if $\alpha\gamma > 0$ and $b_2^i, b_3^i, b_4^{ij} > 0$ if and only if $\beta\alpha\gamma > 0$.

5. $c_1 \geq (\leq) \ 0$ if C is sufficiently large (small), and $c_2, c_3^i \geq (\leq) \ 0$ if, in addition, $\bar{w} \geq (\leq) \ 0$.

6. Each a and b coefficient is independent of C.

7. Each c coefficient weakly increases in C, and each c coefficient strictly increases in C if and only if $\beta\alpha\gamma > 0$.

Proof. See appendix.   □

The value function is concave in previous actions ($a_1 \leq 0$), in weather outcomes ($a_2 \leq 0$), and in forecasts ($a_3^i \leq 0$). If $\beta\alpha\gamma > 0$, then each a and b coefficient is nonzero. Several coefficients depend on C, reflecting how climate controls the agent's beliefs about long-run weather. I henceforth omit the asterisk on A_t^* when clear.

4 Effect of Climate on Actions

Now consider how climate change affects the agent's actions, which is of direct relevance to much empirical work and produces results that we will use to analyze the effect of climate on payoffs. Define $\hat{A}_t \triangleq E_0[A_t]$. From equation (2),

$$\hat{A}_t = \frac{\alpha\hat{A}_{t-1} + \gamma C + \beta b_1 C + \beta \sum_{i<N} b_2^i C + \beta b_2^N C + \beta c_1 + \phi\bar{A}}{\gamma + \alpha + \phi - 2\beta a_1}$$

for $t > N$. The following proposition describes long-run behavior:

Proposition 2. *As $t \to \infty$, $\hat{A}_t \to \frac{\gamma}{\gamma+\phi}C + \frac{\phi}{\gamma+\phi}\bar{A} \triangleq A^{ss}$.*

Proof. See appendix.   □

Expected actions converge to a steady state, denoted A^{ss}. This steady-state expected action is a weighted average of the action that minimizes expected weather impacts and the action that minimizes maintenance costs. Steady-state policy fully offsets the avoidable portion of expected weather impacts (determined by the climate C) when there are no maintenance

costs ($\phi = 0$), but steady-state policy becomes unresponsive to the climate as marginal maintenance costs become large relative to marginal avoidable weather costs (as ϕ becomes large relative to γ). Adjustment costs slow the approach to the steady-state expected action, but they do not affect it.

From Proposition 2, an increase in the climate index affects steady-state expected actions as

$$\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} = \frac{\gamma}{\gamma + \phi} \in [0, 1].$$

As $\gamma \to 0$, there are no avoidable weather impacts, and as $\phi \to \infty$, maintenance costs are too large to justify changing actions on the basis of the climate. In either case, $\mathrm{d}A^{ss}/\mathrm{d}C \to 0$. Steady-state actions otherwise strictly increase with the climate index. But this increase is less than one-for-one when $\phi > 0$: adaptation is less than perfect when maintenance costs deter the agent from fully offsetting the change in climate.

Now consider how we might estimate $\mathrm{d}A^{ss}/\mathrm{d}C$ from data. Reduced-form empirical models can estimate the derivatives $\partial A_t/\partial w_t$ and $\partial A_t/\partial f_{it}$ by regressing observed A_t on weather and forecasts.[9] Imagine that empirical researchers were to then approximate the effect of climate change as

$$\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} \approx \frac{\partial A_t}{\partial w_t} + \sum_{i=1}^{j} \frac{\partial A_t}{\partial f_{it}}, \tag{3}$$

for $j \in \{0, ..., N\}$. For $\mathrm{d}A^{ss}/\mathrm{d}C > 0$ (i.e., for $\gamma > 0$), the bias from this approximation as a fraction of the true effect is

$$Bias(j) = \frac{\frac{\partial A_t}{\partial w_t} + \sum_{i=1}^{j} \frac{\partial A_t}{\partial f_{it}}}{\frac{\mathrm{d}A^{ss}}{\mathrm{d}C}} - 1.$$

$Bias(0)$ is the bias from using only $\partial A_t/\partial w_t$, and $Bias(N)$ is the bias when also using all available forecasts. The approximation underestimates $\mathrm{d}A^{ss}/\mathrm{d}C$ if and only if $Bias(j) < 0$ and correctly estimates $\mathrm{d}A^{ss}/\mathrm{d}C$ if and only if $Bias(j) = 0$. The following proposition establishes several results about this bias:

Proposition 3. *Assume $\gamma > 0$. Then:*

1. *$Bias(j) \in (-1, 0]$, with $Bias(j) < 0$ if and only if $\alpha > 0$.*

2. *$\frac{\mathrm{d}Bias(j)}{\mathrm{d}j} \geq 0$, $\frac{\mathrm{d}Bias(j)}{\mathrm{d}N} = 0$.*

3. *$\frac{\mathrm{d}Bias(j)}{\mathrm{d}j} \to 0$ as $\beta \to 0$.*

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[9]Note that the estimation equation should include A_{t-1}, because time $t - 1$ actions can directly affect time t actions (see equation (2)) and the dependence of time $t-1$ actions on time $t-1$ forecasts makes them correlated with time t weather and forecasts.

4. $Bias(j) \to \frac{-\alpha}{\gamma+\alpha+\phi-2\beta a_1}$ *as $j, N \to \infty$.*

5. $\partial A_t/\partial w_t \to 0$, $\partial A_t/\partial f_{it} \to 0$, and $Bias(j) \to -1$ as $\alpha \to \infty$.

6. $\mathrm{d}A^{ss}/\mathrm{d}C \to 1$ and $Bias(j) \to 0$ as $\gamma \to \infty$.

7. $\partial A_t/\partial w_t, \partial A_t/\partial f_{it}, \mathrm{d}A^{ss}/\mathrm{d}C \to 0$ as either $\gamma \to 0$ or $\phi \to \infty$.

Proof. See appendix. □

The approximation in (3) never overestimates $\mathrm{d}A^{ss}/\mathrm{d}C$ ($Bias(j) \leq 0$), and it underestimates $\mathrm{d}A^{ss}/\mathrm{d}C$ whenever there are nonzero adjustment costs ($\alpha > 0$). The quality of the approximation improves when we include the effects of forecasts in addition to the effects of weather shocks ($\mathrm{d}Bias(j)/\mathrm{d}j \geq 0$), although the bias with any number of forecasts is independent of the length of the longest forecast horizon ($\mathrm{d}Bias(j)/\mathrm{d}N = 0$).

The approximation in (3) can underestimate $\mathrm{d}A^{ss}/\mathrm{d}C$ for three reasons. First, the approximation misses the effect of changing expectations at horizons longer than N (i.e., it misses the βb_2^N in equation (2)). Second, the approximation misses the change in the policy rule induced by the anticipated permanence of climate change (i.e., it misses the effect of C on c_1 in equation (2)). Third, the approximation misses the accumulated effect of changing the weather period after period: even for a given policy rule, the long-run effect of repeating short-run shocks is greater than the effect of a single short-run shock because incremental adjustments accumulate over time (i.e., the approximation misses the effects on A_{t-1} in equation (2)). The first two reasons make the bias sensitive to the discount factor β and explain why estimating responses to forecasts can be helpful. The third reason is why nonzero bias can arise even when agents are myopic (i.e., even as $\beta \to 0$) and even when estimating responses to forecasts at arbitrarily long horizons (i.e., even as $j, N \to \infty$).

The bias vanishes in a few special cases. First, as adjustment costs vanish ($\alpha \to 0$), actions adjust instantaneously to realized weather, so neither expectations nor the slow accrual of incremental adjustments matters for steady-state actions. Second, as avoidable weather impacts become infinitely costly ($\gamma \to \infty$), the agent tries to exactly match A_t to w_t in every period, regardless of adjustment costs or maintenance costs. Third, when there are no avoidable weather impacts ($\gamma \to 0$) or maintenance costs are prohibitive ($\phi \to \infty$), actions become completely insensitive to the climate and also to realized weather and forecasts. In all other cases, the bias is nonzero and becomes large as adjustment costs become large.

Finally, we also see two cases in which $Bias(j) < 0$ but including the effects of forecasts does not improve the quality of the approximation in (3): $\mathrm{d}Bias(j)/\mathrm{d}j \to 0$ as either $\beta \to 0$ or $\alpha \to \infty$.[10] The reason is that actions are not sensitive to forecasts in these cases.[11] First,

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[10] In addition, $\mathrm{d}Bias(j)/\mathrm{d}j = 0$ if $\alpha = 0$ because, from part 1 of Proposition 3, $\alpha = 0$ implies that $Bias(j) = 0$ for all j.

[11] From Proposition 1, $\partial A_t/\partial f_{it} \to 0$ as $\beta \to 0$ and, using the solutions for a_1 and b_1 given in the proof, also as $\alpha \to \infty$.

forecasts enable the agent to take actions that improve future payoffs, but when agents are myopic, they act for the present only. Second, as adjustment costs become very large, agents barely adjust actions on the basis of forecasts. The steady state will change due to the accumulation of many tiny changes over a very long time horizon, but these effects will not be detectable from responses to forecasts.

5 Effect of Climate on Value

Now consider the expected effect of climate change on intertemporal value and per-period payoffs. From Proposition 1, we have:

$$
\begin{aligned}
V(Z_t, w_t, F_t) =\ & V(A^{ss}, C, \boldsymbol{C}) \\
& + [Z_t - A^{ss}]V_Z(A^{ss}, C, \boldsymbol{C}) + [w_t - C]V_w(A^{ss}, C, \boldsymbol{C}) + \sum_{i=1}^{N}[f_{it} - C]V_{f_i}(A^{ss}, C, \boldsymbol{C}) \\
& + [Z_t - A^{ss}]^2 a_1 + [w_t - C]^2 a_2 + \sum_{i=1}^{N}[f_{it} - C]^2 a_3^i + [Z_t - A^{ss}][w_t - C]b_1 \\
& + \sum_{i=1}^{N}[Z_t - A^{ss}][f_{it} - C]b_2^i + \sum_{i=1}^{N}[w_t - C][f_{it} - C]b_3^i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}[w_t - C][f_{it} - C]b_4^{ij},
\end{aligned}
$$

where $\boldsymbol{C}$ is an $N \times 1$ vector with all entries equal to C. The envelope theorem and the fact that $\partial \pi(A_t, A_{t-1}, w_t)/\partial A_{t-1} = 0$ around a steady state imply $V_Z(A^{ss}, C, \boldsymbol{C}) = 0$. The expectation at time 0 of $V(Z_t, w_t, F_t)$ at some future time $t > N$ is:

$$
\begin{aligned}
E_0[V(Z_t, w_t, F_t)] =\ & V(A^{ss}, C, \boldsymbol{C}) + E_0[(A_t - A^{ss})^2]a_1 + \sigma^2 a_2 + \sum_{i=1}^{N}\tau_i^2 a_3^i + Cov_0[Z_t, w_t]b_1 \\
& + \sum_{i=1}^{N}Cov_0[Z_t, f_{it}]b_2^i + \sum_{i=1}^{N}Cov_0[w_t, f_{it}]b_3^i + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}Cov_0[w_t, f_{it}]b_4^{ij}.
\end{aligned}
$$
(4)

Recalling from Proposition 1 that each a and b coefficient is independent of C, and recognizing that each covariance is independent of C,[12] we have:

$$
\frac{\mathrm{d}E_0[V(Z_t, w_t, F_t)]}{\mathrm{d}C} = \underbrace{\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C}}_{\text{change in ss value}} + \underbrace{2a_1 E_0\left[(Z_t - A^{ss})\left(\frac{\mathrm{d}Z_t}{\mathrm{d}C} - \frac{\mathrm{d}A^{ss}}{\mathrm{d}C}\right)\right]}_{\text{change in transition value}}.
$$

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[12]Observe from Proposition 1 that A_t is separable in C, w_t, and f_{it}, and observe that the stochastic terms in w_t and f_{it} are independent of C. Therefore each covariance in equation (4) is independent of C.

We see two components to the expected change in value due to climate change: the change in steady-state value and the change in value along the transition to the steady state.[13]

The next proposition signs the change in transition value:

Proposition 4. *If $\alpha\gamma > 0$, then $\frac{\mathrm{d}E_0[V(Z_t,w_t,F_t)]}{\mathrm{d}C} < \frac{\mathrm{d}V(A^{ss},C,\boldsymbol{C})}{\mathrm{d}C}$ if and only if $A_0 < A^{ss}$. $\frac{\mathrm{d}E_0[V(Z_t,w_t,F_t)]}{\mathrm{d}C} \to \frac{\mathrm{d}V(A^{ss},C,\boldsymbol{C})}{\mathrm{d}C}$ as $\alpha \to 0$, as $\gamma \to 0$, as $t \to \infty$, or as $A_0 \to A^{ss}$.*

Proof. See appendix. $\square$

The transition to a warmer climate imposes costs over and above the change in steady-state value when $A_0 < A^{ss}$ but provides benefits over and above the change in steady-state value when $A_0 > A^{ss}$. When $A_0 < A^{ss}$, the agent is in the process of approaching A^{ss} from below. We already saw that A^{ss} increases in C. Increasing C moves the steady state further away from the current state and therefore requires even more adjustment from the agent. However, when the agent is approaching A^{ss} from above, raising C reduces the total adjustment that the agent will have to undertake before reaching the steady state.

It is reasonable to believe that agents in warmer climates may be approaching their steady-state investment level from below (e.g., by installing air conditioning) and that agents in colder climates may be approaching their steady-state investment level from above (e.g., by installing insulation). We should then expect the cost of adjusting to a warmer climate to be positive in regions with warmer climates and negative in regions with cooler climates. Further, we should expect transition costs (or savings) to be larger in regions that are not as far along the process of adapting to their baseline climate, whether because these regions have lower incomes, were settled only recently, or have outdated capital stock.

Now consider how climate change affects steady-state value. Using Proposition 1, we have:

$$\frac{\mathrm{d}V(A^{ss},C,\boldsymbol{C})}{\mathrm{d}C} = V_w(A^{ss},C,\boldsymbol{C}) + \sum_{i=1}^{N} V_{f_i}(A^{ss},C,\boldsymbol{C})$$
$$+ \frac{\mathrm{d}c_1}{\mathrm{d}C}A^{ss} + \frac{\mathrm{d}c_2}{\mathrm{d}C}C + \sum_{i=1}^{N} \frac{\mathrm{d}c_3^i}{\mathrm{d}C}C + \frac{\mathrm{d}d}{\mathrm{d}C}. \tag{5}$$

The first line recognizes that a change in climate alters average weather and average forecasts. The second line arises because agents anticipate that climate change is permanent: climate change therefore alters the value function itself, beyond altering realized weather and forecasts. For instance, a permanent change in climate can make past adaptation investments more valuable (Proposition 1 showed that $\mathrm{d}c_1/\mathrm{d}C \geq 0$) and can make higher weather outcomes more valuable (or less painful) because they are closer to average weather (Proposition 1 showed that $\mathrm{d}c_2/\mathrm{d}C \geq 0$).

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[13]Tol et al. (1998) informally draw a similar distinction.

The following proposition describes the net effects of climate change on steady-state value:

Proposition 5.

$$\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C} = \frac{1}{1-\beta}\frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C} = \frac{1}{1-\beta}\left[\frac{\gamma\phi}{\gamma+\phi}(\bar{A}-C) + \psi(\bar{w}-C)\right]. \quad (6)$$

Proof. See appendix.   □

Value increases in the climate index if and only if C is sufficiently small. The change in steady-state value is equal to the change in steady-state per-period payoffs, valued as a perpetuity. The first term in brackets reflects the change in the cost of maintaining the adaptation investments chosen for this climate. When the climate is sufficiently cold, a warmer climate may justify investments that require less maintenance, but as the climate becomes sufficiently warm, eventually the chosen investments require more upkeep. This term vanishes as either maintenance costs vanish ($\phi \to 0$) or as the link between actions and weather is broken ($\gamma \to 0$). The second term in brackets reflects the changing cost of unavoidable weather impacts. This term makes a warmer climate valuable when $C < \bar{w}$ but makes a warmer climate costly when $C > \bar{w}$. This term vanishes when weather outcomes impose no unavoidable costs ($\psi \to 0$).

A rapidly growing empirical literature hopes to estimate the cost of climate change from time series variation in weather. From Proposition 1, the marginal effect of weather on value is:

$$\frac{\partial V(Z_t, w_t, F_t)}{\partial w_t} = 2a_2 w_t + b_1 Z_t + \sum_{i=1}^{N} b_3^i f_{it} + c_2.$$

If we average the marginal effect of weather over many observations in a given climate and assume that expected actions are, on average, close to their steady-state level, then we obtain the following average treatment effect of weather on value:

$$ATE_w^V(C) \triangleq 2a_2 C + b_1 A^{ss} + \sum_{i=1}^{N} b_3^i C + c_2.$$

Proceeding analogously, we have the average treatment effect of weather on payoffs around a steady state as

$$ATE_w^\pi(C) \triangleq E_0\left[\frac{\mathrm{d}\pi(A_t, A_{t-1}, w_t)}{\mathrm{d}w_t}\right] = E_0\left[\frac{\partial\pi(A_t, A_{t-1}, w_t)}{\partial w_t}\right],$$

using that $E_0[\partial\pi(A_t, A_{t-1}, w_t)/\partial A_t] = E_0[\partial\pi(A_t, A_{t-1}, w_t)/\partial A_{t-1}] = 0$ around a steady state. The next proposition relates these average treatment effects to the marginal effect of climate:

Proposition 6.

$$\frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C} = ATE_w^V(C) = ATE_w^\pi(C)$$

Proof. See appendix. □

This is a surprising result: once all adjustments are complete, the expected change in per-period steady-state payoffs due to a change in climate is identical to the average change in payoffs estimated from weather events around a steady state.[14] The appendix shows that the same result holds for general, non-quadratic payoff functions if (i) $\partial\pi(A_t, A_{t-1}, w_t)/\partial A_{t-1} = 0$ when $A_t = A_{t-1}$ and (ii) σ^2 and τ_i^2 are not too large. The envelope theorem holds that the effect of climate on current actions does not matter for the effect of climate on value. When (i) holds (as it does in the main text), the effects of climate on past actions also vanish around a steady state, so that adjustment costs and beliefs about future weather both become irrelevant for value. Finally, when either (ii) holds or payoffs are quadratic, the average treatment effect of weather is approximately linear and thus equivalent to the treatment effect of average weather. The result follows from recognizing that average weather defines the climate.

6   Implications for Empirical Work

A rapidly growing empirical literature seeks to estimate the effects of climate change from panel variation in weather. I now discuss how the present paper's results should influence that research agenda.

First, much empirical research has sought to estimate the consequences of climate change for decision variables or functions of decision variables, including productivity (Heal and Park, 2013; Zhang et al., 2018), health (Deschenes, 2014), crime (Ranson, 2014), and energy use (Auffhammer and Aroonruengsawat, 2011; Deschênes and Greenstone, 2011). Many have recognized that long-run adjustment to a new climate regime may be more complete than the adjustment seen in response to short-run weather shocks.[15] I have formally demonstrated that this intuition relies on adjustment costs, not on forward-looking behavior, and I have shown that empirical work can better approximate the effects of a change in climate by also estimating how actions respond to forecasts of future weather. Further, the appendix shows that modeling forecasts is not optional: ignoring forecasts can act like omitted variables bias when estimating the consequences of weather. Finally, if agents are patient (i.e., if β is

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[14]Further, the appendix shows that the average treatment effect of forecasts can identify the discount factor β and thus yield $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\mathrm{d}C$ from Proposition 5.

[15]Some have argued that short-run adjustments could be greater than long-run adjustments because some actions may not be sustainable indefinitely (e.g., Blanc and Schlenker, 2017), such as water withdrawals from a reservoir. Future work could explore such possibilities by imposing constraints on cumulative deviations in A_t from some benchmark value.

close to 1) over timescales of interest, then responses to weather and to forecasts differ only because of adjustment costs. In this case, estimating the response to forecasts allows for a nice test: if actions are much less sensitive to forecasts than to weather, then adjustment costs may be small and responses to weather may approximate responses to climate.

Second, much empirical research has sought to estimate the consequences of climate change for flow payoffs such as profits (e.g., Deschênes and Greenstone, 2007) and for variables such as gross output that are potentially related to aggregate payoffs (e.g., Dell et al., 2012; Burke et al., 2015; Deryugina and Hsiang, 2017). I have shown that the average effect of weather in a given climate is a sufficient statistic for the consequences of marginally perturbing the climate. This new result suggests that empirical work should estimate the average effect of weather as a function of long-run average weather, in contrast to the standard approach of estimating the marginal effect of weather within different weather bins and simulating how climate change will alter the frequency of weather in each bin.[16] The suggested approach combines panel and cross-sectional variation: panel variation will identify the average effect of weather within a region's current climate and thus the consequences of marginally changing each location's climate, and cross-sectional variation will identify how that average effect varies across climates and thus the consequences of nonmarginal changes in climate.[17]

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[16]The standard technique emphasizes the curvature of either V or π in w_t, but the appendix shows that this curvature is not necessarily informative about climate change. In particular, the appendix shows that climate impacts can be linear (or even nonexistent) even when weather impacts are arbitrarily nonlinear. Also, note that some empirical work has estimated average effects (e.g., Schlenker and Lobell, 2010; Dell et al., 2012; Wilson, 2017; Colacito et al., 2018), but this is not the dominant practice.

[17]The use of cross-sectional variation raises the usual concerns about identification: if the average effect of weather correlates with unobserved fixed factors, then the nonlinear effects of climate change will not be identified. Similar concerns about combining cross-sectional variation with panel identification apply to the recent empirical studies that estimate how the marginal effects of weather bins vary with the climate (e.g., Barreca et al., 2015; Deryugina and Hsiang, 2017; Auffhammer, 2018). Results in the appendix suggest a sanity test: moving between climates should not have a stronger effect than do extreme weather events within the current climate.

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Appendix

The first section considers what we learn from estimating nonlinear weather impacts, which has been the focus of recent literature that semiparametrically estimates the marginal effect of weather in distinct weather bins. The second section analyzes the average treatment effect of forecasts on payoffs. The third section derives the omitted variables bias from ignoring forecasts in empirical work. The fourth section generalizes the analysis to arbitrary payoff functions. The final section contains proofs.

A   What we do and do not learn from estimating nonlinear weather impacts

Much empirical work has emphasized that weather outcomes have nonlinear effects, and these nonlinear effects often drive the simulated impacts of climate change. We have seen that empirical researchers should instead be estimating the average treatment effect of weather in order to identify the marginal effect of climate. Can the curvature of V and π in w_t tell us something about the curvature of $V(A^{ss}, C, \boldsymbol{C})$ and $\pi(A^{ss}, A^{ss}, C)$ in C? The following proposition provides reason for skepticism:

Proposition 7.

1. $\frac{\partial^2 \pi(A_t, Z_t, w_t)}{\partial w_t^2} \leq \frac{\mathrm{d}^2 \pi(A_t, Z_t, w_t)}{\mathrm{d} w_t^2} \leq \frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w_t^2} \leq \frac{\mathrm{d}^2 \pi(A^{ss}, A^{ss}, C)}{\mathrm{d} C^2} \leq 0$, with:

    (a) $\frac{\partial^2 \pi(A_t, Z_t, w_t)}{\partial w_t^2} < \frac{\mathrm{d}^2 \pi(A_t, Z_t, w_t)}{\mathrm{d} w_t^2}$ if and only if $\beta \alpha \gamma > 0$,

    (b) $\frac{\mathrm{d}^2 \pi(A_t, Z_t, w_t)}{\mathrm{d} w_t^2} < \frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w_t^2}$ if and only if $\gamma > 0$,

    (c) $\frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w_t^2} < \frac{\mathrm{d}^2 \pi(A^{ss}, A^{ss}, C)}{\mathrm{d} C^2}$ if and only if $\alpha \gamma > 0$, and

    (d) $\frac{\mathrm{d}^2 \pi(A^{ss}, A^{ss}, C)}{\mathrm{d} C^2} < 0$ if and only if $\phi \gamma + \psi > 0$.

2. $\frac{\partial^2 \pi(A_t, Z_t, w_t)}{\partial w_t^2} \to \frac{\mathrm{d}^2 \pi(A^{ss}, A^{ss}, C)}{\mathrm{d} C^2}$ as either $\gamma \to 0$ or $\phi \to \infty$.

3. If $\phi + \psi = 0$ and $\alpha > 0$, then $\frac{\partial^2 \pi(A_t, Z_t, w_t)}{\partial w_t^2}, \frac{\mathrm{d}^2 \pi(A_t, Z_t, w_t)}{\mathrm{d} w_t^2}, \frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w_t^2} < 0$ even as $\frac{\mathrm{d}^2 \pi(A^{ss}, A^{ss}, C)}{\mathrm{d} C^2} = 0$.

Proof. See appendix.   □

The proposition relates (i) the curvature of per-period payoffs in w_t holding A_t and Z_t constant ($\partial^2 \pi / \partial w_t^2$), (ii) the curvature of per-period payoffs in w_t when A_t adapts to realizations of w_t ($\mathrm{d}^2 \pi / \mathrm{d} w_t^2$), (iii) the curvature of intertemporal value in w_t, and (iv) the curvature of

per-period steady-state payoffs in C.[1] Empirical work (e.g., Deschênes and Greenstone, 2007) will commonly estimate (ii), because per-period payoffs are observable as profit and actions are often not observable.[2]

The first part of Proposition 7 establishes that the type of curvature estimated as (i), (ii), or (iii) is at least as extreme as the curvature of steady-state payoffs in climate (iv). Empirical estimates should therefore be taken as an upper bound on the nonlinearity of climate impacts. Intuitively, an agent undertakes greater adjustment to a permanent change in climate than to transient weather shocks, and this greater adjustment reduces the impact on payoffs.

The second part of Proposition 7 establishes that the nonlinearity of weather impacts can adequately approximate the nonlinearity of climate impacts when there are no avoidable weather impacts ($\gamma \to 0$) and when maintenance costs become infinitely large ($\phi \to \infty$). In these rather special cases, actions do not adjust to a change in climate. In more general cases, nonlinear weather impacts strictly overestimate the nonlinearity of climate impacts.

But perhaps detecting nonlinear weather impacts tells us something qualitative about climate change? The third part of the proposition establishes that climate impacts can be linear even when weather impacts are nonlinear. In particular, let $\phi = 0$ and $\psi = 0$. Climate then has no effect on steady-state value because there are no unavoidable weather impacts and the agent adjusts her actions to completely offset the avoidable weather impacts from a change in climate ($A^{ss} = C$). However, when $\alpha > 0$, this agent will not choose to completely offset the effects of transient weather events. Transient weather events can then impose arbitrarily nonlinear avoidable costs even though climate change imposes no costs at all in the long run.[3]

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[1]Proposition 5 relates the curvature of per-period steady-state payoffs in C to the curvature of intertemporal value in C: $\frac{\mathrm{d}^2 V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2} = \frac{1}{1-\beta} \frac{\mathrm{d}^2 \pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C^2}$.

[2]Or at least it will estimate (ii) if it conditions on forecasts: the definition of $\mathrm{d}\pi/\mathrm{d}w_t$ used here does not allow Z_t to change with w_t. See Section C for a discussion of the implications of not conditioning on forecasts.

[3]Nonlinear weather impacts do tell us about a less-studied aspect of climate change. From the proof of Proposition 1, we know that $[\beta/(1-\beta)]a_2$ measures the cost of changing the variance of weather outcomes, holding all other variances and covariances constant. Estimating nonlinear weather impacts implies that $a_2 < 0$ instead of $a_2 = 0$. If we expect climate change to change the variance of the weather, then the nonlinearity of weather impacts can measure the cost of this effect. However, care should be taken to apportion any change in variance due to climate change between the variance σ^2 of the weather conditional on forecasts and the variance τ_i^2 of each forecast. This requires detailed modeling of not just how climate change affects the variance of realized weather but of how climate change affects the forecastability of weather at each horizon.

B What forecasts tell us about the effect of climate on value

Now consider how we can use observable variation in forecasts to learn about climate impacts. We saw in the main text that using variation in forecasts can improve an empirical analyst's ability to estimate changes in steady-state actions. We have already seen that $ATE_w^V(C)$ and $ATE_w^\pi(C)$ are sufficient statistics for the effect of climate change on steady-state per-period payoffs. Does using forecasts add anything here? Let $ATE_{f_i}^V(C)$ and $ATE_{f_i}^\pi(C)$ denote the average treatment effect of forecasts at horizon i around a steady state, defined analogously to $ATE_w^V(C)$ and $ATE_w^\pi(C)$. We now have:

Proposition 8.

1. $ATE_{f_i}^V(C) = \beta^i ATE_w^V(C)$.

2. As $N \to \infty$, $ATE_w^V(C) + \sum_{i=1}^N ATE_{f_i}^V(C) \to \frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C}$.

3. $\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C} = \dfrac{ATE_w^V(C)}{1 - \left(\frac{ATE_{f_i}^V(C)}{ATE_w^V(C)}\right)^{\frac{1}{i}}}$.

4. $ATE_{f_i}^\pi(C) = 0$.

Proof. See appendix.   □

The first part of the proposition says that the average treatment effect of forecasts on value is the discounted average treatment effect of weather, which Proposition 6 showed is the average treatment effect of climate on per-period payoffs. We can therefore derive the change in per-period payoffs from either estimate, provided we have an estimate of β in hand. The second and third parts of the proposition show that if we use both types of treatment effects, then we can identify not only the change in steady-state per-period payoffs but also the discount factor β. We can then exactly identify the present value of the change in steady-state value as revealed by the agent's own actions. The final part of the proposition shows that it is critical that the dependent variable be a forward-looking measure of value such as land prices or stock prices. Per-period payoffs (e.g., profits) will not, on average, respond to forecasts around a steady state.

C Omitted variables bias from ignoring forecasts

Most empirical to date work has ignored the existence of weather forecasts. We have seen that forecasts can provide valuable information, but our analysis also implies that ignoring forecasts acts like omitted variables bias when estimating the consequences of weather (see

also Lemoine, 2017; Shrader, 2017).[4] Accounting for forecasts is therefore not optional. The covariance between w_t and f_{it} is, for $t > N$,

$$Cov_0(w_t, f_{it}) = Cov_0\left(\epsilon_t + \sum_{k=1}^{N} \nu_{k(t-k)}, \sum_{j=0}^{N-i} \nu_{(i+j)(t-j)}\right)$$

$$= \rho_i + \sum_{k=1}^{N-i} \delta_{k(i+k)}.$$

In applications, we can reasonably expect each ρ and δ to be positive, with many strictly positive. We can therefore reasonably expect this covariance to be strictly positive. The bias from estimating $\partial V / \partial w_t$ without accounting for F_t is proportional to

$$\sum_{i=1}^{N} \frac{\partial V(Z_t, w_t, F_t)}{\partial f_{it}} Cov_0(w_t, f_{it}).$$

When w_t and f_{it} affect V in the same way, omitting forecasts will generally overestimate the magnitude of $\partial V / \partial w$. A similar analysis applies if the dependent variable were A_t instead of V.[5] Yet since we have previously seen that combining forecasts with weather can generate useful information, one might wonder whether entwining forecasts and weather through omitted variables bias might actually use weather and forecasts in the desired fashion. Unfortunately, this is not generally the case: the ρ and δ terms that are critical to omitted variables bias did not appear in any earlier derivation.[6] Taking advantage of variation in forecasts does require explicitly estimating the effects of weather and of forecasts at each horizon. Empirical work should more carefully consider the informational structure of weather shocks and take care to estimate the treatment effect of interest.

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[4]Previous work has shown that forecasts matter for outcome variables in a variety of contexts, suggesting that we cannot assume that $\partial V / \partial f_{it} = 0$ or that $\partial A_t / \partial f_{it} = 0$. Lave (1963) illustrates the value of rain forecasts to raisin growers, and Wood et al. (2014) find that developing-country farmers with better access to weather information make more changes in their farming practices. Neidell (2009) demonstrates the importance of accounting for forecasts when estimating the health impacts of air pollution. Studying Indian agriculture, Rosenzweig and Udry (2013) show that farmers' investments respond to forecasts (and respond more strongly to more skillful forecasts), and Rosenzweig and Udry (2014) show that forecasts of planting season weather affect migration decisions and thus wages. Shrader (2017) shows that fishers' revenue and effort both respond to seasonal forecasts of El Niño events. Severen et al. (2016) show that land markets capitalize forecasts of climate change.

[5]However, the concern is mitigated if the dependent variable is π and actions are near the steady state. Forecasts affect π only through A_{t-1}, and we saw that the marginal effect of A_{t-1} vanishes around a steady state. Therefore omitted variables bias is not a concern when estimating the consequences of weather on per-period payoffs around a steady state.

[6]In a regression of either value or actions on weather with forecasts acting as the only bias-generating omitted variables, the usual ordinary least squares formula shows that omitted variables bias induces the desired combination of weather and forecasts if and only if $Cov_0(w_t, f_{it}) = Var_0(w_t)$ for all i. There is no reason for this relationship to hold in practice.

D   Generalizing the functional form for payoffs

I now analyze a general functional form for payoffs rather than the quadratic form analyzed in the main text. I also now allow for $K \geq 1$ types of actions, indexed as A_t^k for $k \in \{1, ..., K\}$. Let time t payoffs be $\pi(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, w_t)$, where $\boldsymbol{A}_t \triangleq \{A_t^1, ..., A_t^K\}$ and $\boldsymbol{A}_{t-1} \triangleq \{A_{t-1}^1, ..., A_{t-1}^K\}$. π_{1_k} indicates a partial derivative with respect to A_t^k, π_{2_k} indicates a partial derivative with respect to A_{t-1}^k, and π_3 indicates a partial derivative with respect to w_t. Assume declining marginal benefits of current and past adaptation investments: $\pi_{1_k 1_k} < 0, \pi_{2_k 2_k} \leq 0$. Adjustment costs mean that the marginal benefit of current actions increases in the level of previous actions ($\pi_{1_k 2_k} > 0$). Finally, assume that the effect of weather on payoffs does not depend directly on past adaptation actions: $\pi_{2_k 3} = 0$.

We modify the transition equations for weather and forecasts to multiply each disturbance term by a perturbation parameter $\zeta \geq 0$. The Bellman equation is now:

$$V(\boldsymbol{Z}_t, w_t, F_t; \zeta) = \max_{\boldsymbol{A}_t} \left\{ \pi(\boldsymbol{A}_t, \boldsymbol{Z}_t, w_t) + \beta E_t \left[V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta) \right] \right\}$$

$$\text{s.t.} \quad \boldsymbol{Z}_{t+1} = \boldsymbol{A}_t$$

$$w_{t+1} = f_{1t} + \zeta \epsilon_{t+1}$$

$$f_{i(t+1)} = f_{(i+1)t} + \zeta \nu_{i(t+1)} \quad \text{for } i < N$$

$$f_{N(t+1)} = C + \zeta \nu_{N(t+1)}.$$

We will be especially interested in the following assumption:

Assumption 1. $\pi_{2_k}(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, w_t) = 0$ if $A_{t-1}^k = A_t^k$.

This assumption says that small changes in past actions do not affect payoffs when they match current actions. It will be satisfied by many specifications of adjustment costs, including the specification in the main text.

First consider the deterministic system, with $\zeta = 0$. In this case, all weather and forecasts are simply equal to C. The K first-order conditions are:

$$0 = \pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, C) + \beta V_{1_k}(\boldsymbol{Z}_{t+1}, C, \boldsymbol{C}; 0).$$

The envelope theorem yields:

$$V_{1_k}(\boldsymbol{Z}_t, C, \boldsymbol{C}; 0) = \pi_{2_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, C).$$

Advancing this forward by one timestep and substituting into the first-order conditions, we have the K Euler equations:

$$0 = \pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, C) + \beta \pi_{2_k}(\boldsymbol{A}_{t+1}, \boldsymbol{A}_t, C).$$

A steady state $\bar{\boldsymbol{A}}$ is defined by the following K equations:

$$0 = \pi_{1_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C) + \beta \pi_{2_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C).$$

Assumption 1 would imply that $\pi_{1_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C) = 0$. We also have:

$$\frac{\mathrm{d}\bar{A}^k}{\mathrm{d}C} = \frac{\pi_{1_k 3}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C)}{-\pi_{1_k 1_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C) - (1+\beta)\pi_{1_k 2_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C) - \beta\pi_{2_k 2_k}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C)},$$

where we use $\pi_{2_k 3} = 0$.[7]

We now analyze the policy rule in the full system. The K first-order conditions are:

$$0 = \pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, w_t) + \beta E_t[V_{1_k}(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)].$$

The envelope theorem yields:

$$V_{1_k}(\boldsymbol{Z}_t, w_t, F_{t+1}; \zeta) = \pi_{2_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, w_t).$$

Advancing this forward by one timestep and substituting, we have the K Euler equations:

$$0 = \pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{Z}_t, w_t) + \beta E_t[\pi_{2_k}(\boldsymbol{A}_{t+1}, \boldsymbol{A}_t, w_{t+1})].$$

Approximate the value function via a second-order Taylor series expansion around $\boldsymbol{Z}_t =$

---

[7]In the case with $K = 1$, the steady state is unique if $-\pi_{11} - \beta\pi_{22} > (1+\beta)\pi_{12}$. Now consider stability in the case with $K = 1$. Use the Euler equation to define $A_{t+1}^*(A_t, Z_t)$. Linearizing around $\bar{A}$ gives a first-order difference equation: $A_{t+1} \approx \frac{-\pi_{11}(\bar{A}, \bar{A}, C) - \beta\pi_{22}(\bar{A}, \bar{A}, C)}{\beta\pi_{12}(\bar{A}, \bar{A}, C)}A_t - \frac{1}{\beta}Z_t + D$, for some constant D. Combined with the transition equation for Z, we have a two-dimensional linear system. The product of the eigenvalues is $\frac{1}{\beta} > 1$, and the sum of the eigenvalues is $\frac{-\pi_{11}(\bar{A}, \bar{A}, C) - \beta\pi_{22}(\bar{A}, \bar{A}, C)}{\beta\pi_{12}(\bar{A}, \bar{A}, C)} > 0$. Therefore both eigenvalues are positive and at least one is greater than 1. Forming the characteristic equation and solving for the condition under which the smallest root is less than 1, we find that the system is saddle-path stable if and only if $-\pi_{11}(\bar{A}, \bar{A}, C) - \beta\pi_{22}(\bar{A}, \bar{A}, C) > (1+\beta)\pi_{12}(\bar{A}, \bar{A}, C)$, which is ensured if π satisfies the sufficient condition for uniqueness. Further, if the system is saddle-path stable, then $\mathrm{d}\bar{A}/\mathrm{d}C > 0$ if and only if $\pi_{13}(\bar{A}, \bar{A}, C) > 0$.

$\bar{\boldsymbol{A}}$, $w_t = C$, $F_t = \boldsymbol{C}$, and $\zeta = 0$:

$$
\begin{aligned}
V(\boldsymbol{Z}_t, w_t, F_t; \zeta) \approx{}& V(\bar{\boldsymbol{A}}, C, \boldsymbol{C}; 0) + \sum_{k=1}^{K} \left.\frac{\partial V}{\partial Z_t^k}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (Z_t^k - \bar{A}^k) + \left.\frac{\partial V}{\partial w_t}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (w_t - C) \\
&+ \sum_{i=1}^{N} \left.\frac{\partial V}{\partial f_{it}}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (f_{it} - C) + \left.\frac{\partial V}{\partial \zeta}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} \zeta \\
&+ \frac{1}{2} \sum_{k=1}^{K} \left.\frac{\partial^2 V}{\partial Z_t^{k\,2}}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (Z_t^k - \bar{A}^k)^2 + \frac{1}{2} \left.\frac{\partial^2 V}{\partial w_t^2}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (w_t - C)^2 \\
&+ \frac{1}{2} \sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial f_{it}^2}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (f_{it} - C)^2 + \frac{1}{2} \left.\frac{\partial^2 V}{\partial \zeta^2}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} \zeta^2 \\
&+ \sum_{k=1}^{K} \left.\frac{\partial^2 V}{\partial Z_t^k \partial w_t}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (Z_t^k - \bar{A}^k)(w_t - C) \\
&+ \sum_{k=1}^{K} \sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial Z_t^k \partial f_{it}}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (Z_t^k - \bar{A}^k)(f_{it} - C) \\
&+ \sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial w_t \partial f_{it}}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (w_t - C)(f_{it} - C) \\
&+ \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left.\frac{\partial^2 V}{\partial f_{it} \partial f_{jt}}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (f_{it} - C)(f_{jt} - C) \\
&+ \sum_{k=1}^{K} \left.\frac{\partial^2 V}{\partial Z_t^k \partial \zeta}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (Z_t^k - \bar{A}^k)\zeta + \left.\frac{\partial^2 V}{\partial w_t \partial \zeta}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (w_t - C)\zeta \\
&+ \sum_{i=1}^{N} \left.\frac{\partial^2 V}{\partial f_{it} \partial \zeta}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (f_{it} - C)\zeta + \sum_{k=1}^{K-1} \sum_{j=k+1}^{K} \left.\frac{\partial^2 V}{\partial Z_t^k \partial Z_t^j}\right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (Z_t^k - \bar{A}^k)(Z_t^j - \bar{A}^j).
\end{aligned}
$$

Use the envelope theorem to substitute for several of the derivatives and impose $\pi_{2_k 3} = 0$:

$$
\begin{aligned}
V(\boldsymbol{Z}_t, w_t, F_t; \zeta) \approx & V(\bar{\boldsymbol{A}}, C, \boldsymbol{C}; 0) + \sum_{k=1}^{K} \bar{\pi}_{2_k}(Z_t^k - \bar{A}^k) + \bar{\pi}_3(w_t - C) \\
& + \frac{1}{2} \sum_{k=1}^{K} \bar{\pi}_{2_k 2_k}(Z_t^k - \bar{A}^k)^2 + \frac{1}{2} \bar{\pi}_{33}(w_t - C)^2 + \sum_{k=1}^{K-1} \sum_{j=k+1}^{K} \bar{\pi}_{2_k 2_j}(Z_t^k - \bar{A}^k)(Z_t^j - \bar{A}^j) \\
& + \left. \frac{\partial V}{\partial \zeta} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} \zeta + \frac{1}{2} \left. \frac{\partial^2 V}{\partial \zeta^2} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} \zeta^2 \\
& + \sum_{i=1}^{N} \left. \frac{\partial V}{\partial f_{it}} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (f_{it} - C) + \frac{1}{2} \sum_{i=1}^{N} \left. \frac{\partial^2 V}{\partial f_{it}^2} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (f_{it} - C)^2 \\
& + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left. \frac{\partial^2 V}{\partial f_{it} \partial f_{jt}} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (f_{it} - C)(f_{jt} - C) + \sum_{i=1}^{N} \left. \frac{\partial^2 V}{\partial f_{it} \partial \zeta} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} (f_{it} - C)\zeta,
\end{aligned}
$$

where we write $\bar{\pi}$ for $\pi(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C)$. Consider the remaining derivatives. First, we have:

$$
\begin{aligned}
\left. \frac{\partial V(\boldsymbol{Z}_t, w_t, F_t; \zeta)}{\partial f_{it}} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} &= \beta E_t \left[\frac{\partial V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial f_{(i-1)(t+1)}} \right] \text{ for } i > 1, \\
\left. \frac{\partial V(\boldsymbol{Z}_t, w_t, F_t; \zeta)}{\partial f_{1t}} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} &= \beta E_t \left[\frac{\partial V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial w_{t+1}} \right] = \beta E_t[\pi_3(t+1)],
\end{aligned}
$$

where we save notation by using $t + 1$ to stand in for the arguments of π and leaving the conditioning of time t expectations on the evaluation points $\bar{\boldsymbol{A}}$ and C implicit. These imply:

$$
\left. \frac{\partial V(\boldsymbol{Z}_t, w_t, F_t; \zeta)}{\partial f_{it}} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} = \beta^i E_t[\pi_3(t+i)].
$$

Similar derivations yield

$$
\begin{aligned}
\left. \frac{\partial^2 V(\boldsymbol{Z}_t, w_t, F_t; \zeta)}{\partial f_{it}^2} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} &= \beta^i E_t[\pi_{33}(t+i)], \\
\left. \frac{\partial^2 V(\boldsymbol{Z}_t, w_t, F_t; \zeta)}{\partial f_{it} \partial f_{jt}} \right|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} &= 0.
\end{aligned}
$$

Now consider derivatives with respect to ζ:

$$
\begin{aligned}
\frac{\partial V(\boldsymbol{Z}_t, w_t, F_t; \zeta)}{\partial \zeta}\bigg|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} =& \beta E_t\left[\frac{\partial V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial w_{t+1}}\epsilon_{t+1}\right] + \beta \sum_{i=1}^{N} E_t\left[\frac{\partial V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial f_{i(t+1)}}\nu_{i(t+1)}\right] \\
&+ \beta E_t\left[\frac{\partial V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial \zeta}\right] \\
=& \sum_{s=1}^{\infty} \beta^s \left\{Cov_t\left[\pi_3(t+s), \epsilon_{t+s}\right] + \sum_{i=1}^{N} \beta^i Cov_t\left[\pi_3(t+s+i), \nu_{i(t+s)}\right]\right\},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 V(\boldsymbol{Z}_t, w_t, F_t; \zeta)}{\partial \zeta^2}\bigg|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} =& \beta E_t\left[\frac{\partial^2 V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial w_{t+1}^2}\epsilon_{t+1}^2\right] \\
&+ 2\beta \sum_{i=1}^{N} E_t\left[\frac{\partial^2 V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial w_{t+1}\partial f_{i(t+1)}}\epsilon_{t+1}\nu_{i(t+1)}\right] \\
&+ \beta \sum_{i=1}^{N} E_t\left[\frac{\partial^2 V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial f_{i(t+1)}^2}\nu_{i(t+1)}^2\right] \\
&+ 2\beta \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} E_t\left[\frac{\partial^2 V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial f_{i(t+1)}\partial f_{j(t+1)}}\nu_{i(t+1)}\nu_{j(t+1)}\right] \\
&+ \beta E_t\left[\frac{\partial^2 V(\boldsymbol{Z}_{t+1}, w_{t+1}, F_{t+1}; \zeta)}{\partial \zeta^2}\right] \\
=& \sum_{s=1}^{\infty} \beta^s \left\{E_t\left[\pi_{33}(t+s)\right]\sigma^2 + Cov_t\left[\pi_{33}(t+s), \epsilon_{t+s}^2\right]\right. \\
&\left. + \sum_{i=1}^{N}\beta^i E_t\left[\pi_{33}(t+s+i)\right]\tau_i^2 + \sum_{i=1}^{N}\beta^i Cov_t\left[\pi_{33}(t+s+i), \nu_{i(t+s)}^2\right]\right\},
\end{aligned}
$$

$$
\frac{\partial^2 V(\boldsymbol{Z}_t, w_t, F_t; \zeta)}{\partial \zeta \partial f_{it}}\bigg|_{(\bar{\boldsymbol{A}}, C, \boldsymbol{C}, 0)} = \beta^i Cov_t\left[\pi_{33}(t+i), \epsilon_{t+i} + \sum_{s=1}^{i-1}\nu_{(i-s)(t+s)}\right].
$$

Substitute in:

$$
\begin{aligned}
V(\boldsymbol{Z}_t, w_t, F_t; \zeta) \approx & V(\bar{\boldsymbol{A}}, C, \boldsymbol{C}; 0) + \sum_{k=1}^{K} \bar{\pi}_{2_k}(Z_t^k - \bar{A}^k) + \bar{\pi}_3(w_t - C) + \frac{1}{2}\sum_{k=1}^{K} \bar{\pi}_{2_k 2_k}(Z_t^k - \bar{A}^k)^2 \\
& + \frac{1}{2}\bar{\pi}_{33}(w_t - C)^2 + \sum_{k=1}^{K-1}\sum_{j=k+1}^{K} \bar{\pi}_{2_k 2_j}(Z_t^k - \bar{A}^k)(Z_t^j - \bar{A}^j) \\
& + \sum_{i=1}^{N}\beta^i E_t[\pi_3(t+i)](f_{it} - C) + \frac{1}{2}\sum_{i=1}^{N}\beta^i E_t[\pi_{33}(t+i)](f_{it} - C)^2 \\
& + \sum_{i=1}^{N}\beta^i Cov_t\left[\pi_{33}(t+i), \epsilon_{t+i} + \sum_{s=1}^{i-1}\nu_{(i-s)(t+s)}\right](f_{it} - C)\zeta \\
& + \sum_{s=1}^{\infty}\beta^s\left\{Cov_t[\pi_3(t+s), \epsilon_{t+s}] + \sum_{i=1}^{N}\beta^i Cov_t[\pi_3(t+s+i), \nu_{i(t+s)}]\right\}\zeta \\
& + \frac{1}{2}\sum_{s=1}^{\infty}\beta^s\left\{E_t[\pi_{33}(t+s)]\sigma^2 + Cov_t[\pi_{33}(t+s), \epsilon_{t+s}^2]\right. \\
& \qquad\qquad \left. + \sum_{i=1}^{N}\beta^i E_t[\pi_{33}(t+s+i)]\tau_i^2 + \sum_{i=1}^{N}\beta^i Cov_t[\pi_{33}(t+s+i), \nu_{i(t+s)}^2]\right\}\zeta^2.
\end{aligned}
$$

Now consider the time 0 expectation of value at some time $t > N$:

$$
\begin{aligned}
E_0[V(\boldsymbol{Z}_t, w_t, F_t; \zeta)] \approx & V(\bar{\boldsymbol{A}}, C, \boldsymbol{C}; 0) + \sum_{k=1}^{K} \bar{\pi}_{2_k} E_0[Z_t^k - \bar{A}^k] + \frac{1}{2}\sum_{k=1}^{K} \bar{\pi}_{2_k 2_k} E_0[(Z_t^k - \bar{A}^k)^2] \\
& + \sum_{k=1}^{K-1}\sum_{j=k+1}^{K} \bar{\pi}_{2_k 2_j} E_0\left[(Z_t^k - \bar{A}^k)(Z_t^j - \bar{A}^j)\right] \\
& + \frac{1}{2}\bar{\pi}_{33}\left(\sigma^2 + \sum_{i=1}^{N}\tau_i^2\right)\zeta^2 + \frac{1}{2}\sum_{i=1}^{N}\beta^i E_0[\pi_{33}(t+i)]\sum_{j=i}^{N}\tau_j^2\zeta^2 \\
& + \sum_{s=1}^{\infty}\beta^s\left\{Cov_0[\pi_3(t+s), \epsilon_{t+s}] + \sum_{i=1}^{N}\beta^i Cov_0[\pi_3(t+s+i), \nu_{i(t+s)}]\right\}\zeta \\
& + \frac{1}{2}\sum_{s=1}^{\infty}\beta^s\left\{E_0[\pi_{33}(t+s)]\sigma^2 + Cov_0[\pi_{33}(t+s), \epsilon_{t+s}^2]\right. \\
& \qquad\qquad \left. + \sum_{i=1}^{N}\beta^i E_0[\pi_{33}(t+s+i)]\tau_i^2 + \sum_{i=1}^{N}\beta^i Cov_0[\pi_{33}(t+s+i), \nu_{i(t+s)}^2]\right\}\zeta^2.
\end{aligned}
$$

The derivative with respect to climate is:

$$
\frac{\mathrm{d}E_0[V(\boldsymbol{Z}_t, w_t, F_t; \zeta)]}{\mathrm{d}C} \approx \frac{\mathrm{d}V(\bar{\boldsymbol{A}}, C, \boldsymbol{C}; 0)}{\mathrm{d}C} + \sum_{k=1}^{K} \bar{\pi}_{2_k} E_0\left[\frac{\mathrm{d}Z_t^k}{\mathrm{d}C}\right] + \sum_{k=1}^{K} \bar{\pi}_{2_k 2_k} E_0\left[(Z_t^k - \bar{A}^k)\left(\frac{\mathrm{d}Z_t^k}{\mathrm{d}C} - \frac{\mathrm{d}\bar{A}^k}{\mathrm{d}C}\right)\right]
$$

$$
+ \sum_{k=1}^{K-1}\sum_{j=k+1}^{K} \bar{\pi}_{2_k 2_j} E_0\left[\frac{\mathrm{d}Z_t^k}{\mathrm{d}C}(Z_t^j - \bar{A}^j) + (Z_t^k - \bar{A}^k)\frac{\mathrm{d}Z_t^j}{\mathrm{d}C}\right]
$$

$$
+ \frac{1}{2}\bar{\pi}_{333}\left(\sigma^2 + \sum_{i=1}^{N}\tau_i^2\right)\zeta^2 + \frac{1}{2}\sum_{i=1}^{N}\beta^i E_0[\pi_{333}(t+i)]\sum_{j=i}^{N}\tau_j^2\zeta^2
$$

$$
+ \sum_{s=1}^{\infty}\beta^s\left\{Cov_0\left[\pi_{33}(t+s), \epsilon_{t+s}\right] + \sum_{i=1}^{N}\beta^i Cov_0\left[\pi_{33}(t+s+i), \nu_{i(t+s)}\right]\right\}\zeta
$$

$$
+ \frac{1}{2}\sum_{s=1}^{\infty}\beta^s\left\{E_0\left[\pi_{333}(t+s)\right]\sigma^2 + Cov_0\left[\pi_{333}(t+s), \epsilon_{t+s}^2\right]\right.
$$

$$
\left. + \sum_{i=1}^{N}\beta^i E_0\left[\pi_{333}(t+s+i)\right]\tau_i^2 + \sum_{i=1}^{N}\beta^i Cov_0\left[\pi_{333}(t+s+i), \nu_{i(t+s)}^2\right]\right\}\zeta^2.
$$

The terms on the first line are analogous to the terms in the main text, capturing the change in steady-state value and the change in transition value. In the main text, the second term on the first line vanished because Assumption 1 held. The second line arises only for $K > 1$. The remaining lines are new, as they vanish when π is quadratic. They capture how preferences for variance change with climate. $\bar{\pi}_{333} > 0$ means that the agent prefers to attach a weather lottery to a high climate state, analogous to standard interpretations of prudence in consumption. The whole expression is arbitrarily close to the expression in the main text when Assumption 1 holds, $K = 1$, and either π_{333} or ζ is small.

We now analyze how steady-state value changes with C. We have:

$$
V(\bar{\boldsymbol{A}}, C, \boldsymbol{C}; 0) = \frac{1}{1-\beta}\pi(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C).
$$

We then have:

$$
\frac{\mathrm{d}V(\bar{\boldsymbol{A}}, C, \boldsymbol{C}; 0)}{\mathrm{d}C} = \frac{1}{1-\beta}\frac{\mathrm{d}\pi(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C)}{\mathrm{d}C}.
$$

That change in steady-state payoffs is:

$$
\frac{\mathrm{d}\pi(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C)}{\mathrm{d}C} = \bar{\pi}_3 + \sum_{k=1}^{K}(\bar{\pi}_{1_k} + \bar{\pi}_{2_k})\frac{\mathrm{d}\bar{A}^k}{\mathrm{d}C}
$$

$$
= \bar{\pi}_3 + \sum_{k=1}^{K}(\bar{\pi}_{1_k} + \bar{\pi}_{2_k})\frac{\bar{\pi}_{1_k 3}}{-\bar{\pi}_{1_k 1_k} - (1+\beta)\bar{\pi}_{1_k 2_k} - \beta\bar{\pi}_{2_k 2_k}}.
$$

Using $\bar{\pi}_{1_k} = -\beta\bar{\pi}_{2_k}$ from the Euler equations, we have:

$$\frac{\mathrm{d}\pi(\bar{\boldsymbol{A}}, \bar{\boldsymbol{A}}, C)}{\mathrm{d}C} = \bar{\pi}_3 + (1-\beta)\sum_{k=1}^{K}\bar{\pi}_{2_k}\frac{\bar{\pi}_{1_k3}}{-\bar{\pi}_{1_k1_k} - (1+\beta)\bar{\pi}_{1_k2_k} - \beta\bar{\pi}_{2_k2_k}}.$$

Now analyze the average treatment effect of weather:

$$ATE_w^\pi(C) \triangleq E_0\left[\frac{\mathrm{d}\pi(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, w_t)}{\mathrm{d}w_t}\right]$$

$$= E_0\left[\pi_3(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, w_t) + \sum_{k=1}^{K}\pi_{1_k}(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, w_t)\frac{\partial A_t^k(\boldsymbol{A}_{t-1}, w_t)}{\partial w_t}\right].$$

If π is quadratic, then π_{1_k}, π_3, and A_t^k are linear and we can pass the expectation operator through to the arguments. The expression then exactly equals $\mathrm{d}\bar{\pi}/\mathrm{d}C$ if we also impose Assumption 1. Now consider the general case. Use a second-order Taylor series approximation to the term in brackets around $\boldsymbol{A}_t = \bar{\boldsymbol{A}}$, $\boldsymbol{A}_{t-1} = \bar{\boldsymbol{A}}$, and $w_t = C$, assume that actions are on average near their steady state, and use $\pi_{2_k3} = 0$ and $\bar{\pi}_{1_k} = -\beta\bar{\pi}_{2_k}$:

$$ATE_w^\pi(C) = \bar{\pi}_3 - \beta\sum_{k=1}^{K}\bar{\pi}_{2_k}\frac{\bar{\pi}_{1_k3}}{-\bar{\pi}_{1_k1_k} - (1+\beta)\bar{\pi}_{1_k2_k} - \beta\bar{\pi}_{2_k2_k}}$$

$$+ \sum_{k=1}^{K}\sum_{j=k}^{K}B_{1_{kj}}Cov_0[A_t^k, A_t^j] + \sum_{k=1}^{K}\sum_{j=k}^{K}B_{2_{kj}}Cov_0[A_{t-1}^k, A_{t-1}^j] + B_3\zeta^2\left[\sigma^2 + \sum_{i=1}^{N}\tau_i^2\right]$$

$$+ \sum_{k=1}^{K}\sum_{j=1}^{K}B_{4_{kj}}Cov_0[A_t^k, A_{t-1}^j] + \sum_{k=1}^{K}B_{5_k}Cov_0[A_t^k, w_t] + \sum_{k=1}^{K}B_{6_k}Cov_0[A_{t-1}^k, w_t],$$

for constants B. The variances and covariances all vanish as $\zeta \to 0$, leaving only the first line. That first line differs from $\mathrm{d}\bar{\pi}/\mathrm{d}C$ by having $-\beta\bar{\pi}_{2_k}$ in place of $(1-\beta)\bar{\pi}_{2_k}$, and it is identical to $\mathrm{d}\bar{\pi}/\mathrm{d}C$ if Assumption 1 holds. Therefore, we have established conditions under which the main result of the paper holds in a general setting: $ATE_w^\pi(C) \approx \mathrm{d}\bar{\pi}/\mathrm{d}C$ when Assumption 1 holds and either π is quadratic or ζ is not too large.[8] The main result of the paper therefore holds under general, non-quadratic payoff functions (and vector-valued actions) as long as i) the variance of weather outcomes is not too large and ii) adjustment costs vanish when current and past actions match each other. These are the same conditions that earlier ensured that the general form of $\mathrm{d}E_0[V(\boldsymbol{Z}_t, w_t, F_t; \zeta)]/\mathrm{d}C$ matched the expression derived in the main text.

[8]If we had instead defined $ATE_w^\pi(C)$ as $E_0[\partial\pi(\boldsymbol{A}_t, \boldsymbol{A}_{t-1}, w_t)/\partial w_t]$, then we would not require Assumption 1. Also, we defined $ATE_w^\pi(C)$ as the average treatment effect conditional on forecasts (so that A_{t-1} did not depend on w_t). The primary result would be unchanged if we had allowed forecasts, and thus A_{t-1}, to reflect the change in weather.

E Proofs and Lemmas

E.1 Proof of Proposition 1

Guess that the value function has the form given in the statement of the proposition. The continuation value becomes:

$$
\begin{aligned}
E_t[V] =& a_1 A_t^2 + a_2 \sigma^2 + a_2 f_{1t}^2 + \sum_i a_3^i \tau_i^2 + \sum_{i<N} a_3^i f_{(i+1)t}^2 + a_3^N C^2 \\
& + b_1 A_t f_{1t} + A_t \sum_{i<N} b_2^i f_{(i+1)t} + A_t b_2^N C \\
& + f_{1t} \sum_{i<N} b_3^i f_{(i+1)t} + f_{1t} b_3^N C + \sum_i \rho_i b_3^i \\
& + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} b_4^{ij} f_{(i+1)t} f_{(j+1)t} + \sum_{i=1}^{N-1} b_4^{iN} f_{(i+1)t} C + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij} \delta_{ij} \\
& + c_1 A_t + c_2 f_{1t} + \sum_{i<N} c_3^i f_{(i+1)t} + c_3^N C + d.
\end{aligned}
$$

The first-order condition is

$$
\begin{aligned}
\gamma(A_t - w_t) + \alpha(A_t - Z_t) + \phi(A_t - \bar{A}) =& \beta E_t[V_Z(Z_{t+1}, w_{t+1}, \boldsymbol{F}_{t+1}] \\
=& \beta \left[2a_1 A_t + b_1 f_{1t} + \sum_{i<N} b_2^i f_{(i+1)t} + b_2^N C + c_1 \right],
\end{aligned}
$$

which implies that optimal actions are

$$
A_t^* = \frac{\alpha Z_t + \gamma w_t + \beta b_1 f_{1t} + \beta \sum_{i<N} b_2^i f_{(i+1)t} + \beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1}.
$$

Substitute A_t^* into the Bellman equation. Matching coefficients to the guessed form of the value function and simplifying, the quadratic coefficients are:

$$
\begin{aligned}
a_1 =& \frac{1}{2} \frac{\alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} - \frac{1}{2}\alpha, \\
a_2 =& \frac{1}{2}\gamma \left[\frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} - 1 \right] - \frac{1}{2}\psi, \\
a_3^1 =& \frac{1}{2} \frac{[\beta b_1]^2}{\gamma + \alpha + \phi - 2\beta a_1} + \beta a_2, \\
a_3^i =& \frac{1}{2} \frac{[\beta b_2^{i-1}]^2}{\gamma + \alpha + \phi - 2\beta a_1} + \beta a_3^{i-1}.
\end{aligned}
$$

Rearrange the solution for a_1:

$$\beta a_1^2 - \frac{1}{2}(\gamma + \alpha + \phi - \beta\alpha)a_1 - \frac{1}{4}\alpha(\gamma + \phi) = 0.$$

Note that a_1 is independent of C. If $\beta = 0$, then the left-hand side is linear in a_1 and the unique solution has $a_1 \leq 0$, with $a_1 < 0$ if and only if $\alpha(\gamma + \phi) > 0$. Recalling that we assumed that $\gamma + \phi > 0$, we then have $a_1 < 0$ if and only if $\alpha > 0$. If $\beta > 0$, then the left-hand side describes a parabola in a_1 that opens up. If $\alpha(\gamma + \phi) = 0$ with $\beta > 0$, then there is a root at zero and a second root that is strictly positive. If $\alpha\beta(\gamma + \phi) > 0$, then the parabola has a strictly negative y-intercept and its roots must be of opposite sign. The two roots are

$$a_1 = \frac{1}{2\beta}\left\{\frac{1}{2}[\gamma + \alpha + \phi - \beta\alpha] \pm \sqrt{\frac{1}{4}(\gamma + \alpha + \phi - \beta\alpha)^2 + \beta\alpha(\gamma + \phi)}\right\}.$$

The second-order condition for A_t^* to be a maximum is

$$0 < \gamma + \alpha + \phi - 2\beta a_1.$$

This is satisfied for $a_1 \leq 0$, using that $\gamma + \phi > 0$. The second-order condition therefore holds at the negative root of a_1. As we solve for the other coefficients, we will find that they are unique conditional on a_1. Therefore the first-order condition is satisfied at only two points, determined by the two roots of a_1. Since we know that the value function is strictly concave in A_t at the point with negative a_1, the point with positive a_1 must either be a saddle point or a minimum. We therefore are only interested in the negative root of a_1 and will henceforth ignore the strictly positive root.

Finding that $a_1 \leq 0$ implies that $a_2 \leq 0$. This inequality is strict if either $\gamma > 0$, $\gamma\alpha > 0$, or $\gamma\phi > 0$. a_2 is independent of C because a_1 is independent of C.

Matching coefficients again, the coefficients on the interaction terms become:

$$b_1 = \alpha \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1},$$

$$b_2^1 = \frac{\alpha \beta b_1}{\gamma + \alpha + \phi - 2\beta a_1},$$

$$b_2^i = \frac{\alpha \beta b_2^{i-1}}{\gamma + \alpha + \phi - 2\beta a_1} \text{ for } i > 1,$$

$$b_3^1 = \gamma \frac{\beta b_1}{\gamma + \alpha + \phi - 2\beta a_1},$$

$$b_3^i = \beta b_2^{i-1} \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \text{ for } i > 1,$$

$$b_4^{1j} = \beta b_2^{j-1} \frac{\beta b_1}{\gamma + \alpha + \phi - 2\beta a_1} + \beta b_3^{j-1},$$

$$b_4^{ij} = \frac{\beta^2 b_2^{i-1} b_2^{j-1}}{\gamma + \alpha + \phi - 2\beta a_1} + \beta b_4^{(i-1)(j-1)} \text{ for } i > 1.$$

Note that we can write

$$b_2^i = \left[\frac{\alpha \beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^i b_1 \quad \text{and} \quad b_3^i = \frac{\gamma}{\alpha} b_2^i,$$

for all $i \in \{1, ..., N\}$. Using $a_1 \leq 0$, we have $b_1 \geq 0$, which implies $b_2^i \geq 0$, which in turn implies $b_3^i \geq 0$, which in turn implies $b_4^{ij} \geq 0$. Clearly $b_1 > 0$ iff $\alpha\gamma > 0$, and each of the other b coefficients is strictly positive iff $b_1 > 0$ and $\beta > 0$. Finally, because a_1 is independent of C, we have that each b coefficient is independent of C.

Now use the solutions for b_2^i and b_1 to analyze a_3^i:

$$
a_3^i = \sum_{k=0}^{i-2} \beta^k \frac{1}{2} \frac{[\beta b_2^{i-1-k}]^2}{\gamma + \alpha + \phi - 2\beta a_1} + \beta^{i-1} a_3^1
$$

$$
= \frac{1}{2} \frac{[\beta b_1]^2}{\gamma + \alpha + \phi - 2\beta a_1} \sum_{k=0}^{i-1} \beta^k \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2(i-1-k)} + \beta^i \left[\frac{1}{2}\gamma \left[\frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} - 1 \right] - \frac{1}{2}\psi \right]
$$

$$
= \frac{1}{2} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2i} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k}
$$

$$
- \frac{1}{2} \beta^i [\gamma + \psi]
$$

$$
= \frac{1}{2} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2i} \frac{1 - \beta^{i+1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2(i+1)}}{1 - \beta \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2}} - \frac{1}{2} \beta^i [\gamma + \psi]
$$

$$
= \beta^i \left\{ \frac{1}{2} \gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \frac{1 - \beta^{i+1} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{2i+2}}{1 - \beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2} - \frac{1}{2} [\gamma + \psi] \right\}. \tag{A-1}
$$

Note that a_3^i is independent of C. The term in braces increases in i, and it strictly increases in i if and only if $\alpha\beta > 0$. It is weakly greater than a_2 and is strictly greater than a_2 if and only if $\alpha\beta\gamma > 0$. As $i \to \infty$, the term in braces goes to:

$$
\frac{1}{2} \gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \frac{1}{1 - \beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2} - \frac{1}{2} [\gamma + \psi]
$$

$$
= \frac{1}{2} \gamma \frac{\gamma(\gamma + \alpha + \phi - 2\beta a_1)}{(\gamma + \alpha + \phi - 2\beta a_1)^2 - (\alpha\beta)^2} - \frac{1}{2} [\gamma + \psi]
$$

$$
= \frac{1}{2} \gamma \frac{\gamma(\gamma + \alpha + \phi - 2\beta a_1)}{\gamma(\gamma + \alpha + \phi - 2\beta a_1) + \alpha^2 + \alpha(\gamma + \phi - 2\beta a_1) + (\phi - 2\beta a_1)(\gamma + \alpha + \phi - 2\beta a_1) - (\alpha\beta)^2} - \frac{1}{2} [\gamma + \psi]
$$

$$
= -\frac{1}{2} \gamma \frac{\alpha(\gamma + \phi - 2\beta a_1) + (\phi - 2\beta a_1)(\gamma + \alpha + \phi - 2\beta a_1) + (1 - \beta^2)\alpha^2}{\gamma(\gamma + \alpha + \phi - 2\beta a_1) + \alpha(\gamma + \phi - 2\beta a_1) + (\phi - 2\beta a_1)(\gamma + \alpha + \phi - 2\beta a_1) + (1 - \beta^2)\alpha^2} - \frac{1}{2}\psi
$$

$$
\leq 0.
$$

This is strictly negative if either $\psi > 0$, $\gamma\phi > 0$, or $\gamma\alpha > 0$, and it is equal to zero otherwise. Therefore, if $\psi + \gamma(\phi + \alpha) + \alpha\beta > 0$, then the term in braces in (A-1) is strictly negative for all finite i, and if $\psi + \gamma(\phi + \alpha) + \alpha\beta = 0$, then the term in braces in (A-1) is zero.

Now match the coefficients on the linear terms:

$$c_1 = \alpha \frac{\beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1},$$

$$c_2 = \gamma \frac{\beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1} + \psi \bar{w},$$

$$c_3^1 = \beta b_1 \frac{\beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1} + \beta b_3^N C + \beta c_2,$$

$$c_3^i = \beta b_2^{i-1} \frac{\beta b_2^N C + \beta c_1 + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1} + \beta \left[b_4^{(i-1)N} C + c_3^{i-1} \right].$$

Solve for c_1:

$$c_1 = \alpha \frac{\beta b_2^N C + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha \beta}.$$

This increases in C, strictly increases in C iff $\alpha \beta \gamma > 0$, and is strictly positive iff $C > -\phi \bar{A}/[\beta b_2^N]$. Substituting into the expression for c_2, we have:

$$c_2 = \gamma \frac{c_1}{\alpha} + \psi \bar{w}.$$

This increases in C, strictly increases in C iff $\alpha \beta \gamma > 0$, and is positive if $c_1 \geq 0$ and $\bar{w} \geq 0$. Substituting for c_1 and for the recursive terms in each c_3^i, we find:

$$c_3^i = \beta^i b_1 \frac{c_1}{\alpha} \sum_{j=0}^{i-1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^j + \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} C + \beta^i \left[b_3^N C + c_2 \right].$$

This too increases in C, strictly increases in C iff $\alpha \beta \gamma > 0$, and is positive if $c_1 \geq 0$ and $\bar{w} \geq 0$.

Finally, matching coefficients yields the constant:

$$d = \frac{1}{2}(\beta b_2^N C + \beta c_1 + \phi \bar{A})\frac{c_1}{\alpha} + \beta a_2 \sigma^2 + \beta \sum_i a_3^i \tau_i^2$$

$$+ \beta c_3^N C + \beta a_3^N C^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij} \delta_{ij} + \sum_i \rho_i b_3^i - \frac{1}{2}\phi \bar{A}^2 - \frac{1}{2}\psi \bar{w}^2 + \beta d.$$

Solving for d yields:

$$d = \frac{1}{1-\beta}\left\{ \frac{1}{2}(\beta b_2^N C + \beta c_1 + \phi \bar{A})\frac{c_1}{\alpha} + \beta a_2 \sigma^2 + \beta \sum_i a_3^i \tau_i^2 \right.$$

$$\left. + \beta c_3^N C + \beta a_3^N C^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij} \delta_{ij} + \sum_i \rho_i b_3^i - \frac{1}{2}\phi \bar{A}^2 - \frac{1}{2}\psi \bar{w}^2 \right\}.$$

E.2   Two Lemmas

The first lemma establishes properties of a_1 that will come in handy in later proofs.

Lemma 9.

   1. $a_1 \to 0$ as $\alpha \to 0$.

   2. $\beta a_1 \to 0$ as $\beta \to 0$.

   3. $a_1 \to -\frac{1}{2}\frac{\gamma+\phi}{1-\beta}$ as $\alpha \to \infty$.

   4. $a_1 \to -\frac{1}{2}\alpha$ as either $\gamma \to \infty$ or $\phi \to \infty$.

Proof. The first claim follows from the analysis in the proof of Proposition 1.

   To prove the second claim, first observe that the proof of Proposition 1 showed that $\beta a_1 = 0$ if $\beta = 0$. Then note that as β goes to 0, we have:

$$\lim_{\beta \to 0} \beta a_1 = \frac{1}{2}\left\{\frac{1}{2}[\gamma + \alpha + \phi] \pm \sqrt{\frac{1}{4}(\gamma + \alpha + \phi)^2}\right\} = 0.$$

   We now consider the third claim. First assume that $\beta = 0$. We have:

$$\lim_{\alpha \to \infty} a_1 = -\frac{1}{2}(\gamma + \phi).$$

Now assume that $\beta > 0$. Rewrite a_1 as

$$a_1 = \frac{1}{4\beta}\frac{1 - \sqrt{1 + 4\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^2}}}{\frac{1}{\gamma+\alpha+\phi-\beta\alpha}}.$$

We have:

$$\lim_{\alpha \to \infty} a_1 = \frac{0}{0}.$$

Use L'Hôpital's Rule:

$$\lim_{\alpha \to \infty} a_1 = \lim_{\alpha \to \infty} \frac{1}{2\beta(1-\beta)}\frac{\beta(\gamma+\phi) - 2\frac{\beta\alpha(\gamma+\phi)(1-\beta)}{\gamma+\alpha+\phi-\beta\alpha}}{\sqrt{1 + 4\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^2}}}$$

$$= -\frac{1}{2}\frac{\gamma+\phi}{1-\beta}.$$

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Now consider the fourth claim. First assume that $\beta = 0$. We have:

$$\lim_{\gamma \to \infty} a_1 = -\frac{1}{2}\alpha,$$

$$\lim_{\phi \to \infty} a_1 = -\frac{1}{2}\alpha.$$

Now assume that $\beta > 0$. As above, we have:

$$\lim_{\gamma \to \infty} a_1 = \frac{0}{0}.$$

Use L'Hôpital's Rule:

$$\lim_{\gamma \to \infty} a_1 = \lim_{\gamma \to \infty} \frac{1}{\beta} \frac{-\frac{1}{2} \frac{\frac{\beta\alpha}{(\gamma+\alpha+\phi-\beta\alpha)^2} - 2\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^3}}{\sqrt{1+4\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^2}}}}{-\frac{1}{(\gamma+\alpha+\phi-\beta\alpha)^2}}$$

$$= \lim_{\gamma \to \infty} \frac{1}{2\beta} \frac{\beta\alpha - 2\frac{\beta\alpha(\gamma+\phi)}{\gamma+\alpha+\phi-\beta\alpha}}{\sqrt{1+4\frac{\beta\alpha(\gamma+\phi)}{(\gamma+\alpha+\phi-\beta\alpha)^2}}}$$

$$= -\frac{1}{2}\alpha.$$

The derivation for $\phi \to \infty$ is similar.

$\square$

The second lemma derives a relationship that will be used in several later proofs:

Lemma 10.
$$\frac{\gamma + \alpha + \phi - 2\beta a_1}{(\gamma + \phi - 2\beta a_1)(\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta)} = \frac{1}{\gamma + \phi}$$

Proof. Using the solution for a_1 in the proof of Proposition 1, we have:

$$\gamma + \phi - 2\beta a_1 = \frac{1}{2}(\gamma + \phi) + \sqrt{\frac{1}{4}(\gamma + \alpha + \phi - \beta\alpha)^2 + \beta\alpha(\gamma + \phi)} - \frac{1}{2}(1 - \beta)\alpha,$$

and

$$\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta = \frac{1}{2}(\gamma + \phi) + \sqrt{\frac{1}{4}(\gamma + \alpha + \phi - \beta\alpha)^2 + \beta\alpha(\gamma + \phi)} + \frac{1}{2}(1 - \beta)\alpha.$$

Therefore:

$$(\gamma + \phi - 2\beta a_1)(\gamma + (1 - \beta)\alpha + \phi - 2\beta a_1)$$

$$= \frac{1}{4}(\gamma + \phi)^2 + \frac{1}{4}(\gamma + \alpha + \phi - \beta\alpha)^2 + \beta\alpha(\gamma + \phi)$$

$$+ (\gamma + \phi)\sqrt{\frac{1}{4}(\gamma + \alpha + \phi - \beta\alpha)^2 + \beta\alpha(\gamma + \phi)} - \frac{1}{4}(1 - \beta)^2\alpha^2$$

$$= \frac{1}{2}(\gamma + \phi)^2 + \frac{1}{2}(1 - \beta)\alpha\,(\gamma + \phi) + \beta\alpha(\gamma + \phi) + (\gamma + \phi)\sqrt{\frac{1}{4}(\gamma + \alpha + \phi - \beta\alpha)^2 + \beta\alpha(\gamma + \phi)}.$$

Substitute for $2\beta a_1$ and factor $\gamma + \phi$:

$$= (\gamma + \phi)\left\{\gamma + \phi + (1 - \beta)\alpha - 2\beta a_1 + \beta\alpha\right\}$$

$$= (\gamma + \phi)\left\{\gamma + \phi + \alpha - 2\beta a_1\right\}.$$

The lemma follows.

$$\square$$

E.3   Proof of Proposition 2

The autonomous first-order linear difference equation that determines $\hat{A}_t$ is stable because $\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \in [0, 1)$. The steady state is:

$$A^{ss} = \frac{\left(\gamma + \beta b_1 + \beta \sum_{i < N} b_2^i + \beta b_2^N\right) C + \beta c_1 + \phi\bar{A}}{\gamma + \phi - 2\beta a_1}.$$

Substitute for the coefficients from their solutions in the proof of Proposition 1, solve the geometric series, and simplify:

$$
A^{ss} = \frac{\left(\gamma + \beta b_1 \sum_{i=0}^{N-1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^i + \beta b_1 \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^N \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \right) C}{\gamma + \phi - 2\beta a_1}
$$
$$
+ \frac{\phi\bar{A}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma + \phi - 2\beta a_1}
$$
$$
= \frac{\left(\gamma + \beta b_1 \frac{1-\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^N}{1-\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}} + \beta b_1 \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^N \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \right) C}{\gamma + \phi - 2\beta a_1}
$$
$$
+ \frac{\phi\bar{A}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma + \phi - 2\beta a_1}
$$
$$
= \frac{\left(\gamma + \beta b_1 \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \right) C}{\gamma + \phi - 2\beta a_1} + \frac{\phi\bar{A}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma + \phi - 2\beta a_1}
$$
$$
= \frac{\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}\gamma C + \frac{\phi\bar{A}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}
$$
$$
= \frac{\gamma + \alpha + \phi - 2\beta a_1}{(\gamma + \phi - 2\beta a_1)(\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta)}\left[\gamma C + \phi\bar{A} \right]. \tag{A-2}
$$

Using Lemma 10, we have:

$$
A^{ss} = \frac{\gamma}{\gamma + \phi}C + \frac{\phi}{\gamma + \phi}\bar{A}.
$$

E.4   Proof of Proposition 3

From equation (A-2), we have:

$$
\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} = \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}\frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \phi - 2\beta a_1}.
$$

Using the solution for A_t in Proposition 1 and the solutions for the coefficients given in the proof of that proposition, we have:

$$
\frac{\partial A_t}{\partial w_t} = \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1},
$$
$$
\frac{\partial A_t}{\partial f_{1t}} = \frac{\beta b_1}{\gamma + \alpha + \phi - 2\beta a_1}
$$
$$
= \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1}\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1}.
$$

We also have, for $i > 1$,

$$
\begin{aligned}
\frac{\partial A_t}{\partial f_{it}} &= \frac{\beta b_2^{i-1}}{\gamma + \alpha + \phi - 2\beta a_1} \\
&= \frac{\beta}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{i-1} \frac{\alpha\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \\
&= \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{i}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{\partial A_t}{\partial w_t} + \sum_{i=1}^{j} \frac{\partial A_t}{\partial f_{it}} &= \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \sum_{i=0}^{j} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{i} \\
&= \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left[1 - \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{j+1} \right].
\end{aligned}
$$

Assuming $\gamma > 0$, we now have:

$$
Bias(j) = \left[1 - \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{j+1} \right] \frac{\gamma + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1} - 1.
$$

The term in square brackets is in $(0, 1]$, and the fraction outside of the square brackets is in $(0, 1]$. Therefore $Bias(j) \in (-1, 0]$. As $\alpha \to 0$, $Bias(j) \to 0$. For $\alpha > 0$, the fraction outside of the square brackets is < 1, so $Bias(j) < 0$.

It is clear that $Bias(j)$ is independent of N. The term in parentheses is in $[0, 1)$, so the term in square brackets increases in j. Therefore $\mathrm{d}Bias(j)/\mathrm{d}j \geq 0$. As $\beta \to 0$, $Bias(j)$ becomes constant in j (using Lemma 9).

As $j, N \to \infty$, the term in brackets goes to 1, so $Bias(j) \to -\alpha/[\gamma + \alpha + \phi - 2\beta a_1]$.

Using Lemma 9, note that $\partial A_t/\partial w_t, \partial A_t/\partial f_{it} \to 0$ as $\alpha \to \infty$. Again using Lemma 9, the term in parentheses in $Bias(j)$ goes to 1 as $\alpha \to \infty$ and the fraction outside parentheses goes to 0 as $\alpha \to \infty$. Therefore $Bias(j) \to -1$ as $\alpha \to \infty$.

Using Lemma 9, $\mathrm{d}A^{ss}/\mathrm{d}C \to 1$ as $\gamma \to \infty$. Again using Lemma 9, the fraction outside the square brackets in $Bias(j)$ goes to 1 and the term in square brackets also goes to 1. So $Bias(j) \to 0$ as $\gamma \to \infty$.

Finally, it is easy to see that $\partial A_t/\partial w_t, \partial A_t/\partial f_{it}, \mathrm{d}A^{ss}/\mathrm{d}C \to 0$ as either $\gamma \to 0$ or (using Lemma 9) as $\phi \to \infty$.

E.5   Proof of Proposition 4

Solving the linear difference equation for $\hat{A}_t$ given in the main text, we have:

$$\hat{A}_t = A^{ss} + \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^t \left[\hat{A}_0 - A^{ss} \right].$$

Using the solution for A^{ss} given in Proposition 2, we have:

$$\frac{\mathrm{d}\hat{A}_t}{\mathrm{d}C} = \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^t \right] \frac{\gamma}{\gamma + \phi}.$$

Recalling that $\hat{A}_t \triangleq E_0[A_t]$, the change in transition value is

$$2a_1 E_0 \left[(Z_t - A^{ss}) \left(\frac{\mathrm{d}Z_t}{\mathrm{d}C} - \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} \right) \right]$$

$$= 2a_1 \left\{ (\hat{A}_{t-1} - A^{ss}) \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{t-1} \right] \frac{\gamma}{\gamma + \phi} C + Cov_0 \left[A_{t-1}, \frac{\mathrm{d}A_{t-1}}{\mathrm{d}C} \right] - E_0 \left[(A_{t-1} - A^{ss}) \right] \frac{\gamma}{\gamma +} \right.$$

$$= 2a_1 \left\{ - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{2(t-1)} [A_0 - A^{ss}] \frac{\gamma}{\gamma + \phi} C + Cov_0 \left[A_{t-1}, \frac{\mathrm{d}A_{t-1}}{\mathrm{d}C} \right] \right\}.$$

Using the difference equation for A_t given in Proposition 1 and recognizing that w_t and f_{it} are linearly separable in C and the random variables, we have $Cov_0 \left[A_{t-1}, \frac{\mathrm{d}A_{t-1}}{\mathrm{d}C} \right] = 0$. Therefore the change in transition value is

$$-2a_1 \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{2(t-1)} \frac{\gamma}{\gamma + \phi} C [A_0 - A^{ss}].$$

This is zero if $\alpha\gamma = 0$ and is proportional to $[A_0 - A^{ss}]$ if $\alpha\gamma > 0$, in which case the change in transition value is negative if and only if $A_0 < A^{ss}$. The change in transition value also goes to zero as $A_t \to A^{ss}$ and, because the term in parentheses is < 1, as $t \to \infty$.

E.6   Proof of Proposition 5

I here prove the result through algebraic manipulations. A shorter, cleaner proof would follow the analysis of Section D.

     We begin by analyzing several of the value coefficients derived in the proof of Proposition

1. First, we have:

$$
\begin{aligned}
b_4^{ij} =& \frac{\beta^2 b_2^{i-1} b_2^{j-1}}{\gamma + \alpha + \phi - 2\beta a_1} + \beta b_4^{(i-1)(j-1)} \\
=& \frac{\beta^{i+j}}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{i-1+j-1} [b_1]^2 + \beta b_4^{(i-1)(j-1)} \\
=& \sum_{k=0}^{i-2} \beta^k \frac{\beta^{(i-k)+(j-k)}}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{(i-1-k)+(j-1-k)} [b_1]^2 + \beta^{i-1} b_4^{1(j-(i-1))} \\
=& \frac{\beta^{i+j}}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{(i-1)+(j-1)} [b_1]^2 \sum_{k=0}^{i-2} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
& + \beta^i b_2^{j-i} \frac{\beta b_1}{\gamma + \alpha + \phi - 2\beta a_1} + \beta^i b_3^{j-i}.
\end{aligned}
$$

Note that:

$$
\begin{aligned}
& \beta^i b_2^{j-i} \frac{\beta b_1}{\gamma + \alpha + \phi - 2\beta a_1} \\
=& \frac{\beta^i}{\gamma + \alpha + \phi - 2\beta a_1} [b_1]^2 \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j-i} \beta^{j-i} \\
=& \frac{\beta^{i+j}}{\gamma + \alpha + \phi - 2\beta a_1} [b_1]^2 \beta^{i-1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{(i-1)+(j-1)} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2(i-1)}.
\end{aligned}
$$

We then have:

$$
\begin{aligned}
b_4^{ij} =& \frac{\beta^{i+j}}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{(i-1)+(j-1)} [b_1]^2 \sum_{k=0}^{i-1} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
& + \beta^i b_3^{j-i} \\
=& \frac{\beta^{i+j}}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{i+j} \gamma^2 \sum_{k=0}^{i-1} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
& + \beta^i \frac{\gamma}{\alpha} b_2^{j-i} \\
=& \frac{\beta^{i+j}}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{i+j} \gamma^2 \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k}. \quad \text{(A-3)}
\end{aligned}
$$

Second, analyze the following term, which we will see often:

$$\sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} = \frac{1 - \beta^{-(i+1)} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2(i+1)}}{1 - \beta^{-1} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2}}$$

$$= \frac{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{2} - \beta^{-i} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-2i}}{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{2} - 1}. \tag{A-4}$$

Third, analyze c_3^i further:

$$c_3^i = \beta^i b_1 \frac{c_1}{\alpha} \sum_{j=0}^{i-1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j} + \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} C + \beta^i \left[b_3^N C + c_2 \right]$$

$$= \beta^i b_1 \frac{c_1}{\alpha} \frac{1 - \left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right)^{i}}{1 - \frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}} + \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} C + \beta^i \frac{\gamma}{\alpha} \left[b_2^N C + c_1 \right] + \beta^i \psi \bar{w}. \tag{A-5}$$

Now turn to our expression of interest. Substituting in for the value function derivatives, we have:

$$\frac{\mathrm{d}V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C} = 2a_2 C + b_1 A^{ss} + \sum_{i=1}^{N} b_3^i C + c_2$$

$$+ \sum_{i=1}^{N} \left[2a_3^i C + b_2^i A^{ss} + b_3^i C + \sum_{j=i+1}^{N} b_4^{ij} C + \sum_{j=1}^{i-1} b_4^{ji} C + c_3^i \right]$$

$$+ \frac{\mathrm{d}c_1}{\mathrm{d}C} A^{ss} + \frac{\mathrm{d}c_2}{\mathrm{d}C} C + \sum_{i=1}^{N} \frac{\mathrm{d}c_3^i}{\mathrm{d}C} C + \frac{\mathrm{d}d}{\mathrm{d}C}.$$

This expression is linear in C. We will first analyze the terms without C before analyzing the slope in C. Combining the results gives the statement of the proposition.

Analyzing the terms in $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\mathrm{d}C$ that are independent of C

Using the value function coefficients derived in the proof of Proposition 1 and also using equation (A-5), the terms without C in $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\mathrm{d}C$ are:

$$\left[b_1 \frac{\phi}{\gamma + \phi} \bar{A} + \psi \bar{w} + \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \right]$$

$$+ \sum_{i=1}^{N} b_2^i \frac{\phi}{\gamma + \phi} \bar{A}$$

$$+ \sum_{i=1}^{N} \beta^i b_1 \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i \right] \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \phi - 2\beta a_1}$$

$$+ \sum_{i=1}^{N} \beta^i \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}$$

$$+ \sum_{i=1}^{N} \beta^i \psi \bar{w}$$

$$+ \frac{\alpha\beta b_2^N}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \frac{\phi}{\gamma + \phi} \bar{A}$$

$$+ \frac{\beta}{1 - \beta} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}$$

$$+ \frac{\beta}{1 - \beta} \beta^N b_1 \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^N \right] \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \phi - 2\beta a_1}$$

$$+ \frac{\beta}{1 - \beta} \beta^N \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}$$

$$+ \frac{\beta}{1 - \beta} \beta^N \psi \bar{w}.$$

Apply Lemma 10 to the third, seventh, and eighth lines, cancel the second line with part of the third, and solve the geometric series:

$$
= \left[b_1 \frac{\phi}{\gamma + \phi} \bar{A} + \psi \bar{w} + \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \right]
$$

$$
+ \beta \frac{1 - \beta^N}{1 - \beta} b_1 \frac{\phi \bar{A}}{\gamma + \phi}
$$

$$
+ \beta \frac{1 - \beta^N}{1 - \beta} \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ \beta \frac{1 - \beta^N}{1 - \beta} \psi \bar{w}
$$

$$
+ \frac{\alpha\beta b_2^N}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \frac{\phi}{\gamma + \phi} \bar{A}
$$

$$
+ \frac{\beta}{1 - \beta} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N \frac{\phi \bar{A}}{\gamma + \phi}
$$

$$
+ \frac{\beta}{1 - \beta} \beta^N b_1 \frac{\phi \bar{A}}{\gamma + \phi} \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^N \right]
$$

$$
+ \frac{\beta}{1 - \beta} \beta^N \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ \frac{\beta}{1 - \beta} \beta^N \psi \bar{w}.
$$

Cancel the final two lines and part of the third-to-last line with earlier lines:

$$
= b_1 \frac{\phi}{\gamma + \phi} \bar{A} + \psi \bar{w} + \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ \frac{\beta}{1 - \beta} b_1 \frac{\phi \bar{A}}{\gamma + \phi}
$$

$$
+ \frac{\beta}{1 - \beta} \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ \frac{\beta}{1 - \beta} \psi \bar{w}
$$

$$
+ \frac{\alpha\beta b_2^N}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \frac{\phi}{\gamma + \phi} \bar{A}
$$

$$
+ \frac{\beta}{1 - \beta} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N \frac{\phi \bar{A}}{\gamma + \phi}
$$

$$
- \frac{\beta}{1 - \beta} \beta^N b_1 \frac{\phi \bar{A}}{\gamma + \phi} \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^N .
$$

Combine the first four lines and substitute b_2^N into the final line:

$$
\begin{aligned}
=&\frac{1}{1-\beta}b_1\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\frac{1}{1-\beta}\gamma\frac{\phi\bar{A}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\\
&+\frac{1}{1-\beta}\psi\bar{w}\\
&+\frac{\alpha\beta b_2^N}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi}{\gamma+\phi}\bar{A}\\
&+\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\frac{\phi\bar{A}}{\gamma+\phi}\\
&-\frac{\beta}{1-\beta}\frac{\phi\bar{A}}{\gamma+\phi}b_2^N.
\end{aligned}
$$

Combine the final two lines, substitute for b_1 in the first line, and apply Lemma 10 to the second line:

$$
\begin{aligned}
=&\frac{1}{1-\beta}\frac{\gamma\alpha}{\gamma+\alpha+\phi-2\beta a_1}\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\frac{1}{1-\beta}\gamma\frac{\gamma+\alpha+\phi-2\beta a_1-\alpha}{\gamma+\alpha+\phi-2\beta a_1}\frac{\phi\bar{A}}{\gamma+\phi}\\
&+\frac{1}{1-\beta}\psi\bar{w}\\
&+\frac{\alpha\beta b_2^N}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi}{\gamma+\phi}\bar{A}\\
&-\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\phi\bar{A}}{\gamma+\phi}b_2^N.
\end{aligned}
$$

Combine the first two lines and cancel the final two lines:

$$
=\frac{1}{1-\beta}\frac{\gamma\phi}{\gamma+\phi}\bar{A}+\frac{1}{1-\beta}\psi\bar{w}. \tag{A-6}
$$

Analyzing the slope of $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\mathrm{d}C$ **in** C

The slope of $\mathrm{d}V(A^{ss}, C, \boldsymbol{C})/\mathrm{d}C$ in C is:

$$
\begin{aligned}
\frac{\mathrm{d}^2 V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2} =& 2a_2 + b_1 \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \sum_{i=1}^{N} b_3^i + \frac{\mathrm{d}c_2}{\mathrm{d}C} \\
&+ \sum_{i=1}^{N} \left[2a_3^i + b_2^i \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + b_3^i + \sum_{j=i+1}^{N} b_4^{ij} + \sum_{j=1}^{i-1} b_4^{ji} + \frac{\mathrm{d}c_3^i}{\mathrm{d}C} \right] \\
&+ \frac{\mathrm{d}c_1}{\mathrm{d}C} \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \frac{\mathrm{d}c_2}{\mathrm{d}C} + \sum_{i=1}^{N} \frac{\mathrm{d}c_3^i}{\mathrm{d}C} \\
&+ \frac{\beta}{1-\beta} \left\{ \frac{\gamma + \alpha + \phi - 2\beta a_1}{[\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta]^2} \beta [b_2^N]^2 + 2 \frac{\mathrm{d}c_3^N}{\mathrm{d}C} + 2a_3^N \right\}.
\end{aligned}
$$

Differentiate equation (A-5), and use Lemma 10 and equation (A-3):

$$
\begin{aligned}
\sum_{i=1}^{N} \frac{\mathrm{d}c_3^i}{\mathrm{d}C} =& \sum_{i=1}^{N} \beta^i \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i \right] \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N \\
&+ \sum_{i=1}^{N} \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} \\
&+ \sum_{i=1}^{N} \beta^i \frac{\gamma}{\alpha} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N \\
=& \beta \frac{1 - \beta^N}{1 - \beta} \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N \\
&- \frac{\gamma}{\gamma + \phi} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^2 b_2^N \sum_{i=0}^{N-1} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i \\
&+ \sum_{i=1}^{N} \beta^i \sum_{j=1}^{i-1} \beta^{-j} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j+N} \sum_{k=0}^{j} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
&+ \beta \frac{1 - \beta^N}{1 - \beta} \frac{\gamma}{\alpha} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N.
\end{aligned}
$$

Using this expression and $\mathrm{d}c_3^N / \mathrm{d}C$, we then have:

$$
\begin{aligned}
&\frac{\mathrm{d}c_1}{\mathrm{d}C}\frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \frac{\mathrm{d}c_2}{\mathrm{d}C} + \sum_{i=1}^{N}\frac{\mathrm{d}c_3^i}{\mathrm{d}C} + \frac{\beta}{1-\beta}\left\{\frac{\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\beta[b_2^N]^2 + 2\frac{\mathrm{d}c_3^N}{\mathrm{d}C} + 2a_3^N\right\} \\
=&\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N\left[\frac{\gamma}{\gamma+\phi}+\frac{\gamma}{\alpha}\right] \\
&+\beta\frac{1+\beta^N}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N \\
&-\frac{\gamma}{\gamma+\phi}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^2 b_2^N\sum_{i=0}^{N-1}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i \\
&+\sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+\beta\frac{1+\beta^N}{1-\beta}\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N \\
&+\beta\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}[b_2^N]^2 \\
&-2\frac{\beta}{1-\beta}\beta^N\left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^N\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N \\
&+2\frac{\beta}{1-\beta}\beta^N\sum_{j=1}^{N-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}b_2^N\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N\sum_{k=0}^{N}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}-\frac{\beta}{1-\beta}\beta^N[\gamma+\psi].
\end{aligned}
$$

Do the summation in the third line and simplify:

$$
\frac{dc_1}{dC}\frac{dA^{ss}}{dC} + \frac{dc_2}{dC} + \sum_{i=1}^{N}\frac{dc_3^i}{dC} + \frac{\beta}{1-\beta}\left\{\frac{\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\beta[b_2^N]^2 + 2\frac{dc_3^N}{dC} + 2a_3^N\right\}
$$

$$
=\frac{1+\beta^{N+1}}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N
$$

$$
+\sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}
$$

$$
+\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}\frac{(1+\beta^N)(\gamma+\alpha+\phi-2\beta a_1)+(1-\beta)\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N
$$

$$
+\beta\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}[b_2^N]^2
$$

$$
-\frac{\beta}{1-\beta}\frac{2(\gamma+\alpha+\phi-2\beta a_1-\alpha\beta)-(1-\beta)\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\frac{\gamma}{\gamma+\phi}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{N+1}b_2^N
$$

$$
+2\frac{\beta}{1-\beta}\beta^N\sum_{j=1}^{N-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k}
$$

$$
+\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}b_2^N\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^N\sum_{k=0}^{N}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} - \frac{\beta}{1-\beta}\beta^N[\gamma+\psi].
$$

$$(\text{A-7})$$

Now analyze the other terms in $dV(A^{ss},C,\boldsymbol{C})/dC$, substituting in for the coefficients

derived in the proof of Proposition 1:

$$
\left[2a_2 + b_1 \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \sum_{i=1}^{N} b_3^i + \frac{\mathrm{d}c_2}{\mathrm{d}C} \right] + \sum_{i=1}^{N} \left[2a_3^i + b_2^i \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + b_3^i + \sum_{j=i+1}^{N} b_4^{ij} + \sum_{j=1}^{i-1} b_4^{ji} + \frac{\mathrm{d}c_3^i}{\mathrm{d}C} \right]
$$

$$
= \gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} - [\gamma + \psi] + \frac{\gamma}{\gamma + \phi} b_1 \sum_{i=0}^{N} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^i
$$

$$
+ 2\frac{\gamma}{\alpha} b_1 \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \sum_{i=0}^{N-1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^i + \frac{\gamma}{\alpha} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N
$$

$$
+ \sum_{i=1}^{N} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2i} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k}
$$

$$
- \sum_{i=1}^{N} \beta^i [\gamma + \psi] + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij}
$$

$$
+ \sum_{i=1}^{N} \beta^i \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i \right] \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N
$$

$$
+ \sum_{i=1}^{N} \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} + \sum_{i=1}^{N} \beta^i \frac{\gamma}{\alpha} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N.
$$

Solving some of the geometric series and simplifying, this becomes:

$$
\left[2a_2 + b_1 \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \sum_{i=1}^{N} b_3^i + \frac{\mathrm{d}c_2}{\mathrm{d}C} \right] + \sum_{i=1}^{N} \left[2a_3^i + b_2^i \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + b_3^i + \sum_{j=i+1}^{N} b_4^{ij} + \sum_{j=1}^{i-1} b_4^{ji} + \frac{\mathrm{d}c_3^i}{\mathrm{d}C} \right]
$$

$$
= \frac{\gamma}{\gamma + \phi} b_1 \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ \frac{1}{1 - \beta} \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N \frac{-(\gamma + \alpha + \phi - 2\beta a_1 + \alpha\beta)(1 - \beta) + \beta(1 - \beta^N)(\gamma + \alpha + \phi - 2\beta a_1 - \alpha}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ \frac{\gamma}{\alpha} b_1 \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left[2 - \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^N \right]
$$

$$
+ \sum_{i=0}^{N} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2i} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k}
$$

$$
+ 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij}
$$

$$
+ \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{N+1} b_2^N
$$

$$
+ \beta \frac{1 - \beta^N}{1 - \beta} \frac{\gamma}{\alpha} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N
$$

$$
+ \sum_{i=1}^{N} \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} - \frac{1 - \beta^{N+1}}{1 - \beta} [\gamma + \psi].
$$

Substituting for b_1 and b_2^N and simplifying, we have:

$$
\left[2a_2 + b_1 \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + \sum_{i=1}^{N} b_3^i + \frac{\mathrm{d}c_2}{\mathrm{d}C} \right] + \sum_{i=1}^{N} \left[2a_3^i + b_2^i \frac{\mathrm{d}A^{ss}}{\mathrm{d}C} + b_3^i + \sum_{j=i+1}^{N} b_4^{ij} + \sum_{j=1}^{i-1} b_4^{ji} + \frac{\mathrm{d}c_3^i}{\mathrm{d}C} \right]
$$

$$
= \frac{\gamma}{\gamma + \phi} \frac{\gamma \alpha}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ \frac{1}{1 - \beta} \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N
$$

$$
\frac{-(\gamma + \alpha + \phi - 2\beta a_1 + \alpha\beta)(1 - \beta) + \beta(1 - \beta^N)(\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta - \alpha\beta)}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ 2\gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ \sum_{i=0}^{N} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2i} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k}
$$

$$
+ 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} b_4^{ij}
$$

$$
+ \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{N+1} b_2^N
$$

$$
+ \frac{\beta}{1 - \beta} \frac{\gamma}{\alpha} \frac{(1 - \beta^N)(\gamma + \alpha + \phi - 2\beta a_1) - (1 - \beta)\alpha}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N
$$

$$
+ \sum_{i=1}^{N} \beta^i \sum_{j=1}^{i-1} \beta^{-j} b_4^{jN} - \frac{1 - \beta^{N+1}}{1 - \beta} \left[\gamma + \psi \right]. \tag{A-8}
$$

Now combine (A-7) and (A-8) and simplify:

$$
\begin{aligned}
&\frac{\mathrm{d}^2 V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2} \\
=&\frac{1}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}b_2^N\frac{2\beta(\gamma+\alpha+\phi-2\beta a_1-\alpha)}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+2\sum_{i=1}^{N}\beta^i\sum_{j=1}^{i-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+2\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}\frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}b_2^N \\
&+\beta\frac{\beta}{1-\beta}\frac{\gamma+\alpha+\phi-2\beta a_1}{[\gamma+\alpha+\phi-2\beta a_1-\alpha\beta]^2}[b_2^N]^2 \\
&-2\frac{\beta}{1-\beta}\frac{\gamma}{\gamma+\phi}\frac{\gamma+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta}\left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^{N+1}b_2^N \\
&+2\frac{\beta}{1-\beta}\beta^N\sum_{j=1}^{N-1}\beta^{-j}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{j+N}\sum_{k=0}^{j}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+\frac{\beta}{1-\beta}\frac{\gamma}{\alpha}b_2^N\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{N}\sum_{k=0}^{N}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+\frac{\gamma}{\gamma+\phi}\frac{\gamma\alpha}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+2\gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1-\alpha\beta} \\
&+\sum_{i=0}^{N}\frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1}\left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2i}\sum_{k=0}^{i}\beta^{-k}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
&+2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}b_4^{ij} \\
&-\frac{1}{1-\beta}\left[\gamma+\psi\right].
\end{aligned}
$$

Substitute for b_4^{ij} from equation (A-3), simplify, and rearrange:

$$
\begin{aligned}
&\frac{\mathrm{d}^2 V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2} \\
={}& \gamma \frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left\{ \frac{\gamma}{\gamma + \phi} + 2\beta \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \right\} \\
&+ \frac{1}{1-\beta} \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N \frac{2\beta(\gamma + \alpha + \phi - 2\beta a_1 - \alpha)}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \\
&+ 2\frac{\beta}{1-\beta} \frac{\gamma}{\alpha} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N \\
&+ \beta \frac{\beta}{1-\beta} \frac{\gamma\alpha}{[\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta]^2} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^N b_2^N \\
&- 2\frac{\beta}{1-\beta} \frac{\gamma}{\gamma + \phi} \frac{\gamma + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{N+1} b_2^N \\
&+ 2\sum_{i=1}^{N} \beta^i \sum_{j=1}^{i-1} \beta^{-j} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j+N} \sum_{k=0}^{j} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
&+ 2\frac{\beta}{1-\beta} \beta^N \sum_{j=1}^{N-1} \beta^{-j} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j+N} \sum_{k=0}^{j} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
&+ \frac{1}{1-\beta} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2N} \sum_{k=0}^{N} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
&+ \sum_{i=0}^{N-1} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2i} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
&+ 2\sum_{j=1}^{N-1} \sum_{i=j+1}^{N} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j+i} \sum_{k=0}^{j} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
&- \frac{1}{1-\beta} [\gamma + \psi].
\end{aligned}
$$

Substitute from equation (A-4) and simplify:

$$
\frac{\mathrm{d}^2 V(A^{Ss}, C, \boldsymbol{C})}{\mathrm{d}C^2}
$$

$$
= \gamma \frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left\{ \frac{\gamma}{\gamma + \phi} + 2\beta \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \right\}
$$

$$
+ \frac{1}{1 - \beta} \frac{\gamma}{\gamma + \phi} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N \frac{2\beta(\gamma + \alpha + \phi - 2\beta a_1 - \alpha)}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}
$$

$$
+ 2 \frac{\beta}{1 - \beta} \frac{\gamma}{\alpha} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N
$$

$$
+ \beta \frac{\beta}{1 - \beta} \frac{\gamma\alpha}{[\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta]^2} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^N b_2^N
$$

$$
- 2 \frac{\beta}{1 - \beta} \frac{\gamma}{\gamma + \phi} \frac{\gamma + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^{N+1} b_2^N
$$

$$
+ 2 \sum_{i=1}^{N} \beta^i \sum_{j=1}^{i-1} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^N \frac{\left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-j} - \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j+2}}{1 - \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2}
$$

$$
+ 2 \frac{\beta}{1 - \beta} \beta^N \sum_{j=1}^{N-1} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^N \frac{\left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-j} - \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j+2}}{1 - \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2}
$$

$$
+ \frac{\beta^N}{1 - \beta} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \frac{1 - \beta^{1+N} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2(1+N)}}{1 - \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2}
$$

$$
+ \sum_{i=0}^{N-1} \beta^i \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \frac{1 - \beta^{1+i} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{2(1+i)}}{1 - \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2}
$$

$$
+ 2 \sum_{j=1}^{N-1} \sum_{i=j+1}^{N} \beta^j \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^i \frac{\left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-j} - \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{j+2}}{1 - \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2}
$$

$$
- \frac{1}{1 - \beta} [\gamma + \psi].
$$

Solving the geometric series, repeatedly using Lemma 10, and working through tedious

algebra (available upon request) then yields:

$$\frac{\mathrm{d}^2 V(A^{ss}, C, \boldsymbol{C})}{\mathrm{d}C^2} = -\frac{1}{1-\beta}\left[\frac{\gamma\phi}{\gamma+\phi} + \psi\right].$$

It is straightforward to show that

$$\frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C} = \frac{\gamma\phi}{\gamma+\phi}(\bar{A} - C) + \psi(\bar{w} - C).$$

The proposition follows from these last two expressions and (A-6).

E.7   Proof of Proposition 6

Using the value function coefficients derived in the proof of Proposition 1 and applying Lemma 10, we have:

$$
\begin{aligned}
ATE_w^V(C) =\,& 2a_2 C + b_1 A^{ss} + \sum_{i=1}^{N} b_3^i C + c_2 \\[2mm]
=\,& \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} C - (\gamma + \psi)C + b_1 \left[\frac{\gamma}{\gamma + \phi} C + \frac{\phi}{\gamma + \phi} \bar{A} \right] + \sum_{i=1}^{N} \frac{\gamma}{\alpha} b_2^i C \\[2mm]
& + \gamma \frac{\beta b_2^N C + \phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} + \psi \bar{w} \\[2mm]
=\,& \frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\gamma\phi}{\gamma + \phi} (\bar{A} - C) + \psi(\bar{w} - C) \\[2mm]
& - \gamma \frac{\phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1} C + \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} C \\[2mm]
& + \gamma \frac{\phi \bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \\[2mm]
=\,& \frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\gamma\phi}{\gamma + \phi} (\bar{A} - C) + \psi(\bar{w} - C) \\[2mm]
& - \gamma \frac{\phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} C + \gamma \frac{\alpha\beta[\gamma + \phi - 2\beta a_1]}{[\gamma + \alpha + \phi - 2\beta a_1][\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta]} C \\[2mm]
& + \frac{\gamma\phi \bar{A}}{\gamma + \phi} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}{\gamma + \alpha + \phi - 2\beta a_1} \\[2mm]
=\,& \frac{\gamma\phi}{\gamma + \phi} (\bar{A} - C) + \psi(\bar{w} - C) \\[2mm]
& + \gamma \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}{\gamma + \alpha + \phi - 2\beta a_1} C \left[\frac{2\beta a_1}{\gamma + \phi} + \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \right] \\[2mm]
=\,& \frac{\gamma\phi}{\gamma + \phi} (\bar{A} - C) + \psi(\bar{w} - C) \\[2mm]
& + \frac{\gamma}{\gamma + \phi} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}{\gamma + \alpha + \phi - 2\beta a_1} C \left[2\beta a_1 + \beta\alpha - \frac{\beta\alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} \right].
\end{aligned}
$$

Use $a_1 = \frac{1}{2} \frac{\alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} - \frac{1}{2}\alpha$ from the proof of Proposition 1:

$$
\begin{aligned}
ATE^V_w(C) =& \frac{\gamma\phi}{\gamma + \phi}(\bar{A} - C) + \psi(\bar{w} - C) \\
& + \frac{\gamma}{\gamma + \phi} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}{\gamma + \alpha + \phi - 2\beta a_1} C \left[\frac{\beta\alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} - \beta\alpha + \beta\alpha - \frac{\beta\alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} \right] \\
=& \frac{\gamma\phi}{\gamma + \phi}(\bar{A} - C) + \psi(\bar{w} - C) \\
=& \frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C}.
\end{aligned}
$$

To obtain $ATE^\pi_w(C)$, partially differentiate $\pi(A_t, A_{t-1}, w_t)$ with respect to w_t and then take expectations (and impose the assumption that expected actions are around a steady state):

$$
ATE^\pi_w(C) = \gamma(A^{ss} - C) - \psi(C - \bar{w}).
$$

Substituting for A^{ss} yields

$$
ATE^\pi_w(C) = \frac{\mathrm{d}\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C}.
$$

E.8   Proof of Proposition 7

Differentiating $\pi(A_t, Z_t, w_t)$, we have:

$$
\frac{\partial^2\pi(A_t, Z_t, w_t)}{\partial w_t^2} = -\gamma - \psi
$$

and

$$
\frac{\mathrm{d}^2\pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} = \frac{\partial^2\pi(A_t, Z_t, w_t)}{\partial w_t^2} - \frac{\partial A_t}{\partial w_t}\left[-1 + \frac{\partial A_t}{\partial w_t}(\gamma + \alpha + \phi) \right].
$$

From Proposition 1, we have:

$$
\frac{\mathrm{d}^2\pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} = \frac{\partial^2\pi(A_t, Z_t, w_t)}{\partial w_t^2} + \gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \frac{-2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1}.
$$

Note that

$$
\frac{\partial^2\pi(A_t, Z_t, w_t)}{\partial w_t^2} - \frac{\mathrm{d}^2\pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} = -\gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} \frac{-2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1} \leq 0.
$$

Using Lemma 9, we see that the inequality is strict if and only if $\gamma > 0$, $\beta > 0$, and $\alpha > 0$. From Proposition 1, we have:

$$
\frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w^2} = 2a_2 = \gamma \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1} - \gamma - \psi.
$$

Note that

$$\frac{\mathrm{d}^2\pi(A_t, Z_t, w_t)}{\mathrm{d}w_t^2} - \frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w^2} = -\gamma\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1}\frac{\gamma+\alpha+\phi}{\gamma+\alpha+\phi-2\beta a_1} \leq 0.$$

Using our assumption that that $\gamma + \phi > 0$, the inequality is strict if and only if $\gamma > 0$. Using Proposition 5, Lemma 10, and the value function coefficients derived in the proof of Proposition 1, we have:

$$\begin{aligned}
\frac{\partial^2 V(Z_t, w_t, F_t)}{\partial w^2} - \frac{\mathrm{d}^2\pi(A^{ss}, A^{ss}, C)}{\mathrm{d}C^2} =& \gamma\left[\frac{\gamma}{\gamma+\alpha+\phi-2\beta a_1} - 1\right] + \frac{\gamma\phi}{\gamma+\phi}\\
=& -\gamma\frac{\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1} + \gamma\frac{\phi}{\gamma+\phi}\\
=& -\gamma\frac{\alpha-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1} + \gamma\frac{\phi}{\gamma+\phi}\frac{-(\gamma+\phi)+\gamma+\alpha+\phi-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1}\\
=& -\gamma\frac{\gamma}{\gamma+\phi}\frac{\alpha-2\beta a_1}{\gamma+\alpha+\phi-2\beta a_1}\\
\leq& 0.
\end{aligned}$$

The inequality is strict if and only if $\alpha\gamma > 0$. Proposition 5 implies that $\mathrm{d}\pi(A^{ss}, A^{ss}, w_t)/\,\mathrm{d}C^2 \leq 0$, with the inequality strict if and only if either $\psi > 0$ or $\gamma\phi > 0$. We have established the first part of the proposition. To prove the second part of the proposition, note that none of the inequalities above are strict if $\gamma = 0$ and note that $\gamma\phi/[\gamma+\phi] \to \gamma$ as $\phi \to \infty$. To prove the third part of the proposition, note that $\phi = 0$ implies $\gamma > 0$ (by our assumption that $\gamma + \phi > 0$) and, from Proposition 1, $\gamma\alpha > 0$ implies $a_2 < 0$, but $\phi, \psi = 0$ implies $\mathrm{d}^2\pi(A^{ss}, A^{ss}, C)/\,\mathrm{d}C^2 = 0$ from Proposition 5.

E.9   Proof of Proposition 8

Defining $ATE_{f_i}^V(C)$ in the analogous fashion as $ATE_w^V(C)$, we have:

$$ATE_{f_i}^V(C) \triangleq 2a_3^i C + b_2^i A^{ss} + b_3^i C + \sum_{j=i+1}^{N} b_4^{ij} C + \sum_{j=1}^{i-1} b_4^{ji} C + c_3^i.$$

Using the value function coefficients derived in the proof of Proposition 1, we have:

$$
\begin{aligned}
ATE_{f_i}^V(C) =& \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{2i} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} C \\
& - \beta^i \left[\gamma+\psi\right] C \\
& + b_1 \left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i \left[\frac{\gamma}{\gamma+\phi}C + \frac{\phi}{\gamma+\phi}\bar{A}\right] \\
& + \frac{\gamma}{\alpha}b_1 \left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right)^i C \\
& + C \sum_{j=i+1}^{N} \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i+j} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
& + C \sum_{j=1}^{i-1} \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1}\right]^{i+j} \sum_{k=0}^{j} \beta^{-k} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2k} \\
& + \beta^i b_1 \frac{c_1}{\alpha} \left[1 - \left(\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right)^i\right] \frac{\gamma+\alpha+\phi-2\beta a_1}{\gamma+\phi-2\beta a_1} + \beta^i \frac{\gamma}{\alpha} \left[b_2^N C + c_1\right] + \beta^i \psi\bar{w} \\
& + \beta^i \frac{\gamma}{\alpha} b_2^N C \frac{1}{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2 - 1} \\
& \quad \left\{\beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2 \sum_{j=1}^{i-1} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^j - \sum_{j=1}^{i-1} \beta^{-j} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-j}\right\}.
\end{aligned}
$$

Combine the first and fifth lines, combine the ψ terms, and substitute for c_1 in the third-to-last line:

$$
\begin{aligned}
ATE_{f_i}^V(C) = & -\beta^i \gamma C + \beta^i \psi[\bar{w} - C] \\
& + b_1 \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i \left[\frac{\gamma}{\gamma + \phi} C + \frac{\phi}{\gamma + \phi} \bar{A} \right] \\
& + \frac{\gamma}{\alpha} b_1 \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i C \\
& + C \sum_{j=i}^{N} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{i+j} \sum_{k=0}^{i} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
& + C \sum_{j=1}^{i-1} \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{i+j} \sum_{k=0}^{j} \beta^{-k} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-2k} \\
& + \beta^i b_1 \frac{\beta b_2^N C + \phi\bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \left[1 - \left(\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i \right] \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \phi - 2\beta a_1} \\
& + \beta^i \frac{\gamma}{\alpha} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} b_2^N C + \beta^i \gamma \frac{\phi\bar{A}}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \\
& + \beta^i \frac{\gamma}{\alpha} b_2^N C \frac{1}{\beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2 - 1} \\
& \quad \left\{ \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2 \sum_{j=1}^{i-1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^j - \sum_{j=1}^{i-1} \beta^{-j} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-j} \right\}.
\end{aligned}
$$

Apply Lemma 10 to the sixth and seventh lines, combine the sixth line with the second and seventh lines, and solve the geometric series in k:

$$
\begin{aligned}
ATE_{f_i}^V(C) = & -\beta^i \gamma C + \beta^i \psi[\bar{w} - C] \\
& + b_1 \left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right)^i \frac{\gamma}{\gamma+\phi} C \\
& + \frac{\gamma}{\alpha} b_1 \left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right)^i C \\
& + C \sum_{j=i}^{N} \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^{i+j} \frac{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2 - \beta^{-i}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2i}}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2 - 1} \\
& + C \sum_{j=1}^{i-1} \frac{\gamma^2}{\gamma+\alpha+\phi-2\beta a_1} \left[\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right]^{i+j} \frac{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2 - \beta^{-j}\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^{-2j}}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2 - 1} \\
& - b_1 \frac{\beta b_2^N C}{\gamma+\phi} \left(\frac{\alpha\beta}{\gamma+\alpha+\phi-2\beta a_1} \right)^i \\
& + \beta^i \frac{\gamma}{\gamma+\phi} \frac{1}{\alpha} \frac{[\gamma+\alpha+\phi-2\beta a_1 - \alpha][\gamma+\alpha+\phi-2\beta a_1] + \beta\alpha^2}{\gamma+\alpha+\phi-2\beta a_1} b_2^N C \\
& + \beta^i \gamma \frac{\phi\bar{A}}{\gamma+\phi} \\
& + \beta^i \frac{\gamma}{\alpha} b_2^N C \frac{1}{\beta\left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1}\right]^2 - 1} \\
& \quad \left\{ \beta \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^2 \sum_{j=1}^{i-1} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^j - \sum_{j=1}^{i-1} \beta^{-j} \left[\frac{\alpha}{\gamma+\alpha+\phi-2\beta a_1} \right]^{-j} \right\}.
\end{aligned}
$$

Simplify, and solve the geometric series in j:

$$
\begin{aligned}
ATE_{f_i}^V(C) = & -\beta^i \gamma C + \beta^i \psi[\bar{w} - C] \\
& + b_1 \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i \frac{\gamma}{\gamma + \phi} C \\
& + \frac{\gamma}{\alpha} b_1 \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i C \\
& + C \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^i \frac{\beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2}{\beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2 - 1} \frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \\
& \left(1 - \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^N \right) \\
& - C \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta} \frac{\beta^i}{\beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2 - 1} \left(1 - \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{N-i+1} \right) \\
& + C \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^i}{\beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2 - 1} \frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha} \\
& \left(1 - \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-(i-1)} \right) \\
& - b_1 \frac{\beta b_2^N C}{\gamma + \phi} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i \\
& + \beta^i \frac{\gamma}{\gamma + \phi} \frac{1}{\alpha} \frac{[\gamma + \alpha + \phi - 2\beta a_1 - \alpha][\gamma + \alpha + \phi - 2\beta a_1] + \beta\alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N C \\
& + \beta^i \gamma \frac{\phi \bar{A}}{\gamma + \phi} \\
& + \beta^i \frac{\gamma}{\alpha} b_2^N C \frac{1}{\beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2 - 1} \\
& \left\{ \beta \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^2 \sum_{j=1}^{i-1} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^j - \sum_{j=1}^{i-1} \beta^{-j} \left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1} \right]^{-j} \right\}.
\end{aligned}
$$

Substitute b_2^N using its solution from the proof of Proposition 1, substitute the solution for b_1 from the proof of Proposition 1, solve the geometric series in the final line, and simplify:

$$
ATE_{f_i}^V(C) = -\beta^i \gamma C + \beta^i \psi[\bar{w} - C] + \beta^i \gamma \frac{\phi \bar{A}}{\gamma + \phi}
$$

$$
+ \frac{\gamma \alpha}{\gamma + \alpha + \phi - 2\beta a_1}\left(\frac{\alpha \beta}{\gamma + \alpha + \phi - 2\beta a_1}\right)^i \frac{\gamma}{\gamma + \phi} C
$$

$$
+ \frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1}\left(\frac{\alpha \beta}{\gamma + \alpha + \phi - 2\beta a_1}\right)^i C
$$

$$
+ C\frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha \beta}\left[\frac{\alpha \beta}{\gamma + \alpha + \phi - 2\beta a_1}\right]^i \frac{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2}{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2 - 1}\frac{\alpha \beta}{\gamma + \alpha + \phi - 2\beta a_1}
$$

$$
- \frac{\gamma}{\alpha}\frac{(\gamma + \alpha + \phi - 2\beta a_1)^2 - \beta \alpha^2}{(\gamma + \alpha + \phi - 2\beta a_1 - \alpha)(\gamma + \alpha + \phi - 2\beta a_1 - \alpha \beta)}\left[\frac{\alpha \beta}{\gamma + \alpha + \phi - 2\beta a_1}\right]^i \frac{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2}{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2 -}
$$

$$
- C\frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha \beta}\frac{\beta^i}{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2 - 1}
$$

$$
+ C\frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}\frac{\left[\frac{\alpha \beta}{\gamma + \alpha + \phi - 2\beta a_1}\right]^i}{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2 - 1}
$$

$$
- C\frac{\gamma^2}{\gamma + \alpha + \phi - 2\beta a_1}\frac{\beta^i}{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2 - 1}\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}
$$

$$
- C\frac{\gamma}{\gamma + \phi}\left(\frac{\alpha \beta}{\gamma + \alpha + \phi - 2\beta a_1}\right)^{i+1} b_2^N
$$

$$
+ \beta^i \frac{\gamma}{\gamma + \phi}\frac{1}{\alpha}\frac{[\gamma + \alpha + \phi - 2\beta a_1 - \alpha][\gamma + \alpha + \phi - 2\beta a_1] + \beta \alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N C
$$

$$
+ \beta^i \frac{\gamma}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}\frac{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2}{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2 - 1} b_2^N C
$$

$$
+ \beta^i \frac{\gamma}{\alpha}\frac{\gamma + \alpha + \phi - 2\beta a_1}{\gamma + \alpha + \phi - 2\beta a_1 - \alpha \beta}\frac{1}{\beta\left[\frac{\alpha}{\gamma + \alpha + \phi - 2\beta a_1}\right]^2 - 1} b_2^N C.
$$

Apply Lemma 10 and simplify:

$$
\begin{aligned}
ATE^V_{f_i}(C) = & - \beta^i \gamma C + \beta^i \psi[\bar{w} - C] + \beta^i \gamma \frac{\phi \bar{A}}{\gamma + \phi} + \beta^i \frac{\gamma^2}{\gamma + \phi} C \\
& + \frac{\gamma^2}{\gamma + \phi} \frac{\beta \alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}{(\gamma + \alpha + \phi - 2\beta a_1)^2 - \beta \alpha^2} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i C \\
& + \frac{\gamma^2}{\gamma + \phi} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i C \\
& - \frac{\gamma^2}{\gamma + \phi} \frac{(\gamma + \alpha + \phi - 2\beta a_1)^2 - \alpha\beta(\gamma + \alpha + \phi - 2\beta a_1)}{(\gamma + \alpha + \phi - 2\beta a_1)^2 - \beta \alpha^2} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^i C \\
& + \beta^i \frac{\gamma}{\gamma + \phi} \frac{1}{\alpha} \frac{[\gamma + \alpha + \phi - 2\beta a_1 - \alpha][\gamma + \alpha + \phi - 2\beta a_1] + \beta \alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} b_2^N C \\
& - \beta^i \frac{\gamma}{\gamma + \phi} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\beta \alpha^2}{(\gamma + \alpha + \phi - 2\beta a_1)^2 - \beta \alpha^2} b_2^N C \\
& - \beta^i \frac{\gamma}{\gamma + \phi} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha}{\alpha} \frac{(\gamma + \alpha + \phi - 2\beta a_1)^2}{(\gamma + \alpha + \phi - 2\beta a_1)^2 - \beta \alpha^2} b_2^N C.
\end{aligned}
$$

Combine the final three lines, combine the third and fourth lines, and simplify the first line:

$$
\begin{aligned}
ATE^V_{f_i}(C) = & \beta^i \frac{\gamma \phi}{\gamma + \phi} (\bar{A} - C) + \beta^i \psi[\bar{w} - C] \\
& + \frac{\gamma^2}{\gamma + \phi} \frac{\beta \alpha^2}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}{(\gamma + \alpha + \phi - 2\beta a_1)^2 - \beta \alpha^2} \left(\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right)^i C \\
& - \frac{\gamma^2}{\gamma + \phi} \frac{\gamma + \alpha + \phi - 2\beta a_1 - \alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \frac{\beta \alpha^2}{(\gamma + \alpha + \phi - 2\beta a_1)^2 - \beta \alpha^2} \left[\frac{\alpha\beta}{\gamma + \alpha + \phi - 2\beta a_1} \right]^i C.
\end{aligned}
$$

Cancel the final two lines:

$$
\begin{aligned}
ATE^V_{f_i}(C) = & \beta^i \frac{\gamma \phi}{\gamma + \phi} (\bar{A} - C) + \beta^i \psi[\bar{w} - C] \\
= & \beta^i ATE^V_w(C).
\end{aligned}
$$

The second equation follows from the proof of Proposition 6. We have proved the first part of Proposition 8.

To prove the second part of Proposition 8, use the above result and Proposition 6 to see that

$$
ATE^V_w(C) + \sum_{i=1}^{\infty} ATE^V_{f_i}(C) = \sum_{i=0}^{\infty} \beta^i ATE^V_w(C) = \frac{1}{1 - \beta} ATE^V_w(C) = \frac{dV(A^{ss}, C, \boldsymbol{C})}{dC}.
$$

To prove the third part of Proposition 8, note that $ATE_{f_i}^V(C)/ATE_w^V(C) = \beta^i$ and use Proposition 6.

The final part of Proposition 8 follows straightforwardly from the fact that $\pi(A_t, At-1, w_t)$ is independent of F_t other than through A_t (and as noted before it is easy to show that $E_0[\partial\pi(A_t, A_{t-1}, w_t)/\partial A_t] = 0$ around a steady state).

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