General Equilibrium Rebound from Energy Efficiency Innovation*

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Energy efficiency improvements "rebound" when economic responses undercut their direct energy savings. I show that general equilibrium channels typically amplify rebound by making consumption goods cheaper but typically dampen rebound by increasing demand for non-energy inputs to production and by changing the size of the energy supply sector. Improvements in the efficiency of the energy supply sector generate especially large rebound because they make energy cheaper in all other sectors. Quantitatively, improving the efficiency of U.S. non-energy supply sectors by 1% would reduce U.S. energy use by 0.58%, with rebound of 28%. General equilibrium channels increase those savings by 19%; however, they reduce the savings from improving the efficiency of the energy supply sector by 65%.

JEL: D58, O31, O33, Q41

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1 Introduction

Industrial economies have become much better at converting energy into useful work. For instance, the energy intensity of U.S. output fell by 35% from 1985 to 2011,[1] and the energy intensity of British output has fallen by 80% since 1850 (Fouquet and Pearson, 2003). However, economists since Jevons (1865) have wondered whether improvements in energy efficiency might actually increase aggregate use of energy resources. Further, environmental economists have been especially concerned with the possibility of such large "rebound" effects because of the prominent externalities associated with energy use. Most formal analyses of rebound effects have focused on partial equilibrium settings that hold some prices fixed. Yet Jevons was especially worried about general equilibrium channels,[2] and his concern has been reinforced by computable general equilibrium models that suggest the potential for strong rebound effects through "economy-wide" or "indirect" channels (Allan et al., 2009; Turner, 2013). As a result, many have called for theoretical research to illuminate the channels through which economy-wide rebound arises (e.g., Dimitropoulos, 2007; Turner, 2013; Borenstein, 2015).

I fill the gap in the theoretical literature by developing an analytically tractable general equilibrium framework for studying the implications of improved energy efficiency. I disentangle the channels through which improvements in energy efficiency affect total energy resource use and I sign the effect in a range of cases. The modeled economy contains an arbitrary number of sectors that produce distinct consumption goods. In the baseline model, each consumption good is produced competitively by combining a non-energy input (here interpreted as labor) with energy resources, using a constant elasticity of substitution (CES) technology. Each household values leisure and consumption. Households sell labor in return for a wage. Energy resources are converted to useful work via energy conversion technologies that are specific to each consumption good sector. Energy resources are themselves produced by an additional sector that also employs a CES technology in energy resources and labor. I study how an improvement in the quality of some sector's energy conversion technology affects prices and energy resource consumption throughout the economy.

An engineering estimate of the effects of an efficiency improvement would hold the production of energy services (e.g., useful work or lighting) fixed and calculate the energy resources displaced by the improvement in efficiency. "Rebound" is the percentage of these engineering savings lost through economic responses. A partial equilibrium analysis of rebound holds the prices of consumption goods, energy resources, and labor fixed. In this case with fixed prices, I show the result familiar from previous literature (e.g., Saunders, 1992; Sorrell and Dim-

[1] https://energy.gov/eere/analysis/energy-intensity-indicators-highlights

[2] Jevons (1865, VII p. 141) wrote, "Now, if the quantity of coal used in a blast-furnace, for instance, be diminished in comparison with the yield, the profits of the trade will increase, new capital will be attracted, the price of pig-iron will fall, but the demand for it increase; and eventually the greater number of furnaces will more than make up for the diminished consumption of each." This story hinges on changes in prices. I will formally map it into an output price channel.

itropoulos, 2008): rebound is proportional to the elasticity of substitution between energy resources and non-energy inputs to production. This elasticity captures how firms substitute towards energy resources when improved technology reduces its effective cost. When energy resources and non-energy inputs are gross substitutes in production (i.e., when this elasticity is greater than 1), rebound is greater than 100%. In this case, an efficiency improvement is said to "backfire," actually increasing consumption of energy resources.

In general equilibrium, all prices adjust to the improved energy conversion technology. Improving the technology in some sector k reduces the cost of producing that sector's consumption good and thus reduces the equilibrium price of its consumption good. This lower price reduces sector k's demand for inputs, but households' substitution towards sector k's newly cheap consumption good increases its scale and thus its demand for inputs. Sector k's changing demand for inputs affects aggregate demand for energy resources and labor. Any change in aggregate demand for labor in turn affects hours of labor supplied and the wage. And these changes in turn affect demand for all consumption goods (through changes in income) and input costs for all consumption goods. Finally, the energy supply sector must expand or contract to match any changes in resource demand from the consumption good sectors. These expansions and contractions change the energy supply sector's own demand for energy resources and labor, with implications for energy resource use in all other sectors.

I show that sectoral elasticities of substitution between energy resources and non-energy inputs are critical to general equilibrium rebound, which is consistent with computable general equilibrium models' well-known sensitivity to these parameters (see Broberg et al., 2015). However, I also highlight other critical parameters. For instance, computable general equilibrium modelers typically do not emphasize (and even do not consistently report) the elasticity of substitution between consumption good sectors. I show that this parameter's relation to unity and to sectoral elasticities of substitution is critical to the sign and magnitude of general equilibrium channels. I also show that the general equilibrium consequences of the energy supply sector are especially important to evaluations of rebound. By amplifying other sectors' changes in energy resource use, the energy supply sector's responses can increase energy savings even beyond what an engineering analysis would predict, a case of negative rebound or "super-conservation" (Saunders, 2008).[3]

I use the new theoretical results to quantitatively investigate general equilibrium consequences of increased energy efficiency in the U.S. economy. Instead of calibrating a fully specified computable general equilibrium model or restricting attention to partial equilibrium consequences estimable through program evaluation methods, I quantify and decom-

[3]I also show that rebound can be especially severe when improved technology arises in the energy supply sector. Because an improvement in the efficiency of energy production leads all other producers to substitute towards energy resources, backfire occurs for a much broader set of conditions than when new technology arises in some consumption good sector. This result formalizes and confirms the results of numerical models that have emphasized rebound from the energy supply sector (see Allan et al., 2007; Sorrell, 2007; Hanley et al., 2009; Turner, 2009).

pose general equilibrium consequences using knowledge only of factor shares and elasticities of substitution. I find that simultaneously improving the efficiency of energy resource use in all non-energy supply sectors by 1% reduces total energy resource use by 0.58%, with rebound reducing engineering savings by nearly 30%. If not for the energy supply sector's response, general equilibrium channels would undercut energy resource savings by 1.5%; however, once we account for this response, general equilibrium considerations in fact increase energy resource savings by an additional 19% beyond what a partial equilibrium analysis would predict. Improved efficiency generates the greatest reductions in overall energy resource use when it arises in commercial real estate or construction, with the latter even demonstrating negative rebound or "super-conservation". Improved efficiency can backfire when it arises in electronics, metals, or some transportation sectors. Consistent with the theoretical analysis, the energy supply sector demonstrates the most rebound (80%) of any sector that does not backfire. In this case, general equilibrium effects reduce energy resource savings, as a partial equilibrium analysis would estimate rebound of only 42%. The recently proposed Affordable Clean Energy rule aims to reduce greenhouse gas emissions by increasing the efficiency of U.S. coal-fired power plants, but these results give reason to doubt that the emission reductions would be meaningful (see also Keyes et al., 2019).

I extend the setting to investigate the consequences of endogenous innovation and of costly improvements to energy efficiency. First, I incorporate a modern model of directed technical change (Acemoglu, 2002, 2007) in order to assess whether profit-maximizing innovation is likely to focus on sectors that are especially vulnerable to rebound. I show that the marginal research firm typically reduces overall energy resource use if, as in standard calibrations, labor and energy resources are complements in production and the various consumption goods are complements in utility.

Second, I explore the general equilibrium consequences of improvements in efficiency that come at the cost of disrupting other factors' productivity. Such costs may be especially important when improved efficiency is mandated by policy.[4] I show that if technologies are costly and the value share of labor is much larger than that of energy resources (as is true of many sectors of the U.S. economy), then many of the general equilibrium channels switch sign. In the quantitative application, these channels could now combine to reduce energy resource use. Further, a new general equilibrium channel tends to reinforce partial equilibrium changes in energy resource use. Accounting for the cost of improved energy efficiency is therefore likely to increase energy resource savings beyond those estimated in the quantitative application, potentially even driving a number of sectors to negative rebound. Experiments with computable general equilibrium models have in fact demonstrated these effects (Allan et al., 2007, 2009; Broberg et al., 2015).

[4] A large literature models environmental regulations as reducing factor productivity (e.g., Marten et al., 2019). Intermediate results obtained for the analysis of costly efficiency policies are also informative about the general equilibrium consequences of these policies.

There are only a few other analytic general equilibrium studies of rebound effects.[5] Wei (2007) restricts attention to a single energy good and a single non-energy good, assumes Cobb-Douglas functional forms for all production functions, and analyzes a linear demand system. Wei (2010) considers a setting with only a single consumption good and does not model the production of energy. Concurrently with the present paper, Böhringer and Rivers (2018) and Fullerton and Ta (2019) use linearization techniques to study settings with one or two consumption goods. By explicitly solving for energy resource use in a dual setting that treats prices as independent variables, my analysis demonstrates precisely which price changes generate each general equilibrium channel and thus further develops intuition for general equilibrium consequences. Further, whereas Böhringer and Rivers (2018) allow only a single general equilibrium channel at a time, I capture interactions between multiple general equilibrium channels. And whereas Fullerton and Ta (2019) study improvements in the efficiency with which households use energy, I study improvements in firms' energy efficiency, which are important both because households represent only 36% of U.S. energy use and because the computable general equilibrium literature has almost exclusively studied improvements in firms' energy efficiency.[6] The present setting also differs from Fullerton and Ta (2019) in allowing for channels such as labor supply decisions and households' substitution among consumption goods.[7]

All of this other analytic work abstracts from the use of energy resources in the energy supply sector. However, some computable general equilibrium modelers have emphasized the importance of accounting for energy demand by the firms that produce the energy needed for consumption good production (e.g., Allan et al., 2007; Sorrell, 2007; Hanley et al., 2009; Turner, 2009).[8] I show that the energy supply sector is critical to the possibility of super-

[5]There are quite a few numerical studies with computable general equilibrium models (see Sorrell, 2007; Allan et al., 2009; Turner, 2013; Broberg et al., 2015), which often report quite large effects from general equilibrium channels. There is also a large partial equilibrium literature. See Greening et al. (2000), Sorrell and Dimitropoulos (2008), Sorrell et al. (2009), and van den Bergh (2011). Neoclassical growth settings have emphasized how analogues of partial equilibrium income and substitution effects arise after improving the productivity of energy in the broader economy's production function (Saunders, 1992, 2000). Finally, Hart (2018) and Rausch and Schwerin (2019) study the role of rebound in explaining the long-run dynamics of aggregate energy use, emphasizing expanding varieties of energy-using goods and putty-clay production of energy-using capital, respectively.

[6]In 2018, the residential and transportation sectors accounted for 21% and 28% of U.S. energy consumption, respectively, with light-duty vehicles constituting 55% of transportation sector energy use. See `https://www.eia.gov/energyexplained/use-of-energy/`.

[7]Fullerton and Ta (2019) emphasize the fixed costs that households must bear when purchasing more efficient appliances. I instead introduce costly efficiency improvements in the way that computable general equilibrium models have introduced them, as reductions in the productivity of other factors (see Allan et al., 2007, 2009; Broberg et al., 2015).

[8]In their reviews, Greening et al. (2000) emphasize the potential for large adjustments in energy supply and Turner (2013) laments the lack of attention given to energy supply in analyses of rebound effects. Based on numerical experiments, Saunders (2014) conjectures that greater efficiency in energy production will inevitably backfire. I formally demonstrate that backfire is indeed especially likely in this case, but I also

conservation and that improvements to the efficiency of energy supply are much more likely to backfire than are improvements to the efficiency of consumption good production. Quantitatively, I find that energy supply sector responses are the most important of the general equilibrium channels and even change the sign of the overall effect of general equilibrium considerations. Abstracting from these responses is not innocuous.

The next section describes the setting. Section 3 derives the equilibrium prices and allocation. Section 4 recounts the familiar partial equilibrium analysis. Section 5 analyzes general equilibrium rebound from improvements in the energy efficiency of consumption good producers and energy producers. It also quantitatively explores the effects of greater efficiency in the U.S. economy. Section 6 extends the setting to allow for directed technical change and analyzes whether profit-driven research firms target sectors in which improved efficiency will backfire. Section 7 considers improvements in efficiency that impose costs on firms. The final section concludes. The appendix reports additional numerical results; connects the analysis to income effects of interest when energy services are direct inputs to households' utility; and extends the main analysis to consider the implications for total energy use of improvements in labor productivity, total factor productivity, and economy-wide energy efficiency.

2 Setting

There are N consumption goods, produced in quantity c_i for $i \in \{1, ..., N\}$. Households obtain utility from consuming these goods and leisure ℓ:

$$u(\ell, C) = \left((1-\nu)C^{\frac{\Theta-1}{\Theta}} + \nu\ell^{\frac{\Theta-1}{\Theta}}\right)^{\frac{\Theta}{\Theta-1}}, \text{ where } C \triangleq \left(\sum_{i=1}^{N} c_i^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}.$$

$\Theta > 0$ is the elasticity of substitution between leisure and the consumption index C.[9] $\epsilon > 0, \neq 1$ is the elasticity of substitution between the different consumption goods. I will be especially interested in the case with $\epsilon < 1$ because it is consistent with standard calibrations.[10] The price of each consumption good is p_i. Households consume energy resources only indirectly, through their choices of c_i.

Each consumption good is produced competitively using quantity R_i of energy resources

show that it is not inevitable and the central estimates in my quantitative application indeed do not show backfire.

[9]The given expression for $u(\ell, C)$ assumes $\Theta \neq 1$. If $\Theta = 1$, then $u(\ell, C)$ has the familiar Cobb-Douglas form.

[10]We will see in Section 5.3 that standard calibrations use $\epsilon < 1$ when the consumption good sectors represent different types of goods and services. $\epsilon > 1$ would be more likely if the sectors instead represented different varieties of a single product.

and quantity L_i of labor:

$$c_i = \left((1-\kappa_i)[A_i R_i]^{\frac{\sigma_i - 1}{\sigma_i}} + \kappa_i [B_i L_i]^{\frac{\sigma_i - 1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i - 1}}.$$

The production function has a constant elasticity of substitution $\sigma_i > 0$, with $\sigma_i \neq 1$. I will be especially interested in the case where $\sigma_i < \epsilon$ because it is consistent with standard calibrations for most sectors. I drop the subscript when considering special cases with identical σ_i for all consumption good firms. $\kappa_i \in (0,1)$ is the distribution parameter. The productivity of energy resources and of labor are determined by $A_i > 0$ and $B_i > 0$, respectively. We can interpret $A_i R_i$ as energy services such as heating, lighting, or mechanical motion, with A_i controlling the conversion from R_i into energy services.

The same energy resources are used in each sector. In equilibrium, each sector pays price p_R for each unit of energy resource. Energy resources are produced competitively via a CES function of energy resources and labor:

$$R = \left((1-\kappa_{N+1})[A_{N+1} R_{N+1}]^{\frac{\sigma_{N+1} - 1}{\sigma_{N+1}}} + \kappa_{N+1} [B_{N+1} L_{N+1}]^{\frac{\sigma_{N+1} - 1}{\sigma_{N+1}}}\right)^{\frac{\sigma_{N+1}}{\sigma_{N+1} - 1}}.$$

R is the total quantity of energy resources produced for the economy, with $\sigma_{N+1} > 0, \neq 1$, $\kappa_{N+1} \in (0,1)$, and $A_{N+1}, B_{N+1} > 0$. Assume that $(1-\kappa) A_{N+1}^{\frac{\sigma_{N+1}-1}{\sigma_{N+1}}} < 1$ so that energy producers' profit function is concave in R_{N+1}.

Each household is endowed with $H > 0$ units of time that can be allocated to either labor or leisure, yielding a household time constraint of $L + \ell \leq H$. Each household sells its labor hours to some sector i. In equilibrium, each sector pays the same wage w for a unit of labor. The household budget constraint is then $\sum_{i=1}^{N} p_i c_i \leq wL$.[11] There are a continuum of households, of measure 1.

3 Equilibrium Prices and Allocations

I study market equilibria.

Definition 1. *An* equilibrium *is given by consumption good prices ($\{p_i\}_{i=1}^N$), a price for labor (w), a price for energy resources (p_R), demands for inputs ($\{L_i, R_i\}_{i=1}^{N+1}$), demands for consumption goods ($\{c_i\}_{i=1}^N$), and a supply of labor (L) such that: (i) (L_i, R_i) maximizes profits of producers of consumption good i for $i \in \{1, ..., N\}$, (ii) (L_{N+1}, R_{N+1}) maximizes*

[11] I interpret the household factor endowment as labor, but it can also be interpreted as a capital endowment. The household is then choosing how much capital to use in household production and how much to rent to industrial sectors. We will see that the household's supply becomes fixed as $\Theta \to 1$, which corresponds to a short-run capital allocation problem. And we will see that the household's supply becomes perfectly elastic as $\Theta \to \infty$, which corresponds to a small economy open to world capital markets.

profits of energy resource producers, (iii) ($\{c_i\}_{i=1}^N, \ell, L$) maximizes household utility while satisfying the budget and time constraints, (iv) firms make zero profits, and (v) the prices w, p_R, and $\{p_i\}_{i=1}^N$ clear the markets for labor, energy resources, and consumption goods.

The equilibrium prices clear all factor markets, all firms maximize profits within competitive markets, and households maximize utility subject to their budget and time constraints.

Households do not leave hours unallocated, so that $\ell = H - L$ in equilibrium. Households solve the following maximization problem:

$$\max_{\{c_i\}_{i=1}^N, L} \left((1-\nu) \left(\left(\sum_{i=1}^N c_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \right)^{\frac{\Theta-1}{\Theta}} + \nu (H-L)^{\frac{\Theta-1}{\Theta}} \right)^{\frac{\Theta}{\Theta-1}} \quad \text{subject to} \quad \sum_{i=1}^N p_i c_i \leq wL.$$

Letting λ be the shadow value of the budget constraint and using subscripts to indicate partial derivatives, the first-order condition for c_i is

$$\frac{\lambda p_i}{u_C(H-L, C)} = \left(\frac{c_i}{C} \right)^{-\frac{1}{\epsilon}}.$$

Let P be the ideal price index, so that $\sum_{i=1}^N p_i c_i = P\,C$. Households' first-order condition for C implies that $P = u_C(H-L, C)/\lambda$. I choose the price index as the numeraire: $P = 1$. The household budget constraint then implies that $C = wL$ in equilibrium. Aggregate household demand for good i becomes

$$c_i = \left(\frac{p_i}{P} \right)^{-\epsilon} C = L\, p_i^{-\epsilon} w. \tag{1}$$

Households' first-order condition for L is

$$\lambda w = \nu \left(\frac{H-L}{u(H-L, C)} \right)^{-\frac{1}{\Theta}}.$$

Substituting for $u_C(H-L, C)$ in $\lambda = u_C(H-L, C)$, using $C = wL$, and taking the ratio with the previous expression, we find

$$w = \left(\frac{1-\nu}{\nu} \right)^{\frac{\Theta}{1-\Theta}} \left(\frac{H-L}{L} \right)^{\frac{1}{1-\Theta}}. \tag{2}$$

Labor supply is:

$$L = \frac{\left(\frac{1-\nu}{\nu}\right)^{\Theta}}{\left(\frac{1-\nu}{\nu}\right)^{\Theta} + w^{1-\Theta}} H \in (0, H).$$

Labor supplied increases in the wage if and only if $\Theta > 1$. A substitution effect drives workers to offer more labor when the wage increases but an income effect drives workers to offer less labor when the wage increases. The substitution effect dominates when $\Theta > 1$, and the income effect dominates when $\Theta < 1$. The two effects exactly offset each other when $\Theta = 1$, making this case equivalent to the case of fixed labor supply that has been one focal point in computable general equilibrium models of rebound effects (e.g., Allan et al., 2006, 2007; Broberg et al., 2015). As $\Theta \to \infty$, labor supply becomes perfectly elastic, a case that has been a second focal point in the same models.

Now consider the input mix chosen by firms in sector $i \in \{1, ..., N+1\}$. Firms solve:

$$\max_{L_i, R_i} \left\{ p_i \left((1-\kappa_i)[A_i R_i]^{\frac{\sigma_i - 1}{\sigma_i}} + \kappa_i [B_i L_i]^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}} - p_R R_i - w L_i \right\},$$

where $p_{N+1} \triangleq p_R$. The first-order conditions are:

$$w = p_i \kappa_i B_i^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{L_i}{c_i} \right)^{-\frac{1}{\sigma_i}}, \tag{3}$$

$$p_R = p_i (1-\kappa_i) A_i^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{R_i}{c_i} \right)^{-\frac{1}{\sigma_i}}, \tag{4}$$

where $c_{N+1} \triangleq R$. Rearranging the first-order conditions to solve for L_i and R_i and substituting into the zero-profit condition required by competitive markets, we obtain:

$$p_i = \left(w^{1-\sigma_i} B_i^{\sigma_i - 1} \kappa_i^{\sigma_i} + p_R^{1-\sigma_i} A_i^{\sigma_i - 1} (1-\kappa_i)^{\sigma_i} \right)^{\frac{1}{1-\sigma_i}}. \tag{5}$$

In the energy-producing sector, substituting p_R for p_i in equation (5) and rearranging yields:

$$p_R = \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1}{1-\sigma_{N+1}}} w, \tag{6}$$

where our assumption that $(1-\kappa_{N+1}) A_{N+1}^{\frac{\sigma_{N+1}-1}{\sigma_{N+1}}} < 1$ ensures that $p_R > 0$. Rearranging equations (3) and (4) and then substituting for p_R from equation (6), we have factor demand in the energy-producing sector:

$$L_{N+1} = \kappa_{N+1}^{\sigma_{N+1}} B_{N+1}^{\sigma_{N+1}-1} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} R, \tag{7}$$

$$R_{N+1} = (1-\kappa_{N+1})^{\sigma_{N+1}} A_{N+1}^{\sigma_{N+1}-1} R. \tag{8}$$

Now consider factor demand in the consumption good sectors. Rearrange equations (3) and (4) and substitute for c_i from equation (1) to obtain, for $i \in \{1, ..., N\}$:

$$L_i = L B_i^{\sigma_i - 1} \left(\frac{\kappa_i}{w}\right)^{\sigma_i} p_i^{\sigma_i - \epsilon} w, \tag{9}$$

$$R_i = L A_i^{\sigma_i - 1} \left(\frac{1-\kappa_i}{p_R}\right)^{\sigma_i} p_i^{\sigma_i - \epsilon} w. \tag{10}$$

Substituting for p_R from equation (6), the latter equation becomes:

$$R_i = L\, A_i^{\sigma_i - 1} (1-\kappa_i)^{\sigma_i} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{-\sigma_i}{1-\sigma_{N+1}}} p_i^{\sigma_i - \epsilon} w^{1-\sigma_i}. \tag{11}$$

Using equation (6) to substitute for p_R in equation (5), we have:

$$p_i = \left(B_i^{\sigma_i - 1}\kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{1}{1-\sigma_i}} w. \tag{12}$$

Substituting from equation (12), equations (9) and (11) become:

$$L_i = L\, B_i^{\sigma_i - 1}\kappa_i^{\sigma_i} \left(B_i^{\sigma_i - 1}\kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{\sigma_i - \epsilon}{1-\sigma_i}} w^{1-\epsilon}, \tag{13}$$

$$R_i = L\, A_i^{\sigma_i - 1} (1-\kappa_i)^{\sigma_i} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{-\sigma_i}{1-\sigma_{N+1}}}$$
$$\left(B_i^{\sigma_i - 1}\kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{\sigma_i - \epsilon}{1-\sigma_i}} w^{1-\epsilon}. \tag{14}$$

Market-clearing for labor implies that

$$L = \sum_{i=1}^{N+1} L_i$$
$$= \sum_{i=1}^{N} L\, B_i^{\sigma_i - 1}\kappa_i^{\sigma_i} \left(B_i^{\sigma_i - 1}\kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{\sigma_i - \epsilon}{1-\sigma_i}} w^{1-\epsilon}$$
$$+ B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} R.$$

Rearranging, we have:

$$w^{1-\epsilon} = \frac{L - B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}\left[\frac{B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} R}{L\sum_{i=1}^{N} B_i^{\sigma_i-1}\kappa_i^{\sigma_i}\left(B_i^{\sigma_i-1}\kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i}\left[\frac{B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}\right)^{\frac{\sigma_i-\epsilon}{1-\sigma_i}}}. \tag{15}$$

Substituting for w from equation (2), we find:

$$\left(\frac{\nu}{1-\nu}\right)^{\Theta\frac{1-\epsilon}{\Theta-1}}\left(\frac{L}{H-L}\right)^{\frac{1-\epsilon}{\Theta-1}}$$
$$= \frac{1 - B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}\left[\frac{B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} \frac{R}{L}}{\sum_{i=1}^{N} B_i^{\sigma_i-1}\kappa_i^{\sigma_i}\left(B_i^{\sigma_i-1}\kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i}\left[\frac{B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}\right)^{\frac{\sigma_i-\epsilon}{1-\sigma_i}}}. \tag{16}$$

Market-clearing for energy resources implies that

$$R = \sum_{i=1}^{N+1} R_i.$$

The equilibrium exists and is unique.[12]

Finally, define the value share (or cost share) of energy resources and labor in sector i as α_{Ri} and α_{Li}, respectively. These are:

$$\alpha_{Ri} \triangleq \frac{p_R R_i}{p_i c_i} = \frac{A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i}\left[\frac{B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}}{B_i^{\sigma_i-1}\kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i}\left[\frac{B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}},$$

$$\alpha_{Li} \triangleq \frac{w L_i}{p_i c_i} = \frac{B_i^{\sigma_i-1}\kappa_i^{\sigma_i}}{B_i^{\sigma_i-1}\kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i}\left[\frac{B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}}.$$

[12] Substituting for R_i from equation (14) and then for w from equation (15) gives R/L equal to a constant. Equation (16) gives R/L as a function of L. Combining gives a single equation in L, from which it is easy to establish that a unique equilibrium L exists.

4 Partial Equilibrium Rebound

I begin by reviewing partial equilibrium rebound from a 1% improvement in the efficiency of energy service production in some sector $k \in \{1, ..., N+1\}$.

First, the simplest "engineering" calculation does not consider changes in prices and does not allow for factor substitution by firms: it fixes c_k, $B_k L_k$, and $A_k R_k$. Let E_k be the energy services used prior to the improvement in A_k: $E_k \triangleq A_k R_k$. Totally differentiate and set $\mathrm{d}E_k = 0$ to hold energy services fixed: $0 = R_k \mathrm{d}A_k + A_k \mathrm{d}R_k$. The energy resource savings from a 1% improvement in A_k become:

$$Savings^{eng} \triangleq -A_k \frac{\mathrm{d}R_k}{\mathrm{d}A_k} = R_k.$$

The engineering calculation predicts that a 1% improvement in the efficiency of energy conversion leads to a 1% reduction in energy resource use.

Economists have long noted that improving the efficiency of energy conversion lowers the relative price of energy resources, which leads profit- or utility-maximizing agents to increase their use of energy resources. This substitution towards energy resource inputs is called rebound and is often analyzed in a partial equilibrium setting in which the prices of energy resources, of non-energy inputs, and of consumption goods are held fixed.[13] Equation (10) gives resource demand $R_k(A_k, p_k, p_R, w, L)$ as a function of technology, prices, and labor supplied. The partial equilibrium calculation does not allow p_k, p_R, w, or L to change with A_k, so that

$$Savings^{PE} \triangleq -A_k \frac{\partial R_k(A_k, p_k, p_R, w, L)}{\partial A_k} = (1 - \sigma_k) R_k.$$

Partial equilibrium rebound, as a fraction of the no-rebound or "engineering" savings from an improvement in energy efficiency, is then

$$Rebound^{PE} \triangleq \frac{Savings^{eng} - Savings^{PE}}{Savings^{eng}} = \sigma_k. \tag{17}$$

Partial equilibrium rebound is equal to σ_k, a result familiar from much previous work (e.g., Saunders, 1992; Sorrell and Dimitropoulos, 2008). This analysis suggests that a 1% improvement in energy efficiency most strongly reduces energy resource use when it occurs in a sector with high energy resource use (large R_k) and a small elasticity of substitution between energy and non-energy inputs (small σ_k). Partial equilibrium rebound goes to zero as $\sigma_k \to 0$, in which case the firm has a Leontief production function and so has no scope to adjust its input mix.

[13] In some settings, energy services are modeled as a direct input to utility. The appendix derives a partial equilibrium direct income effect that depends on the budget share of energy resources.

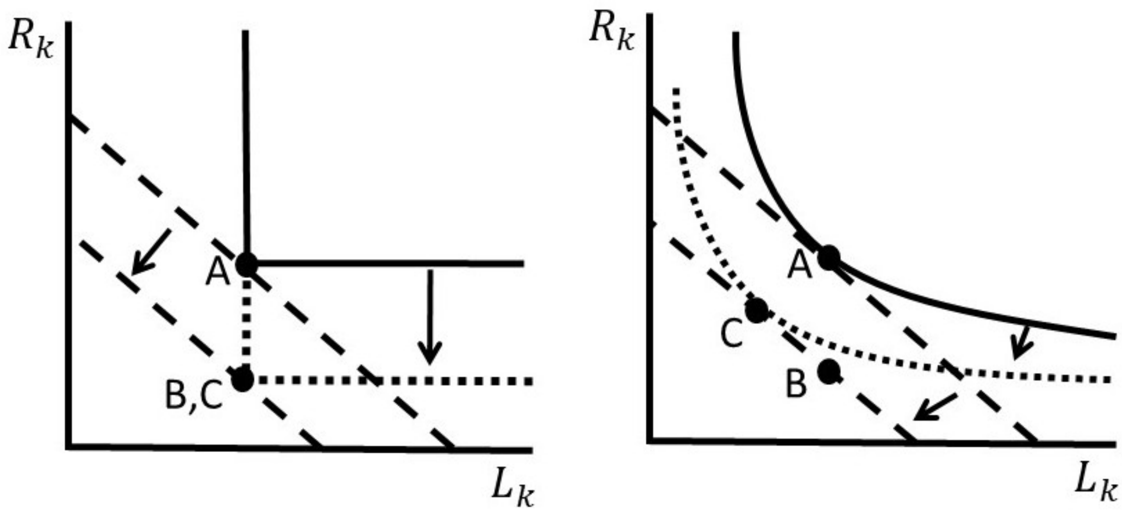
Figure 1 graphically describes the partial equilibrium effect. It plots the combinations of R_k and L_k that generate a given quantity of output c_k, and it also plots the isocost line (dashed). Prior to the improvement in energy efficiency, the firm's profit-maximizing point is at point A, where the solid isoquant is tangent to the isocost line. Improving the efficiency of energy conversion technology changes the isoquant to the dotted line. The improvement in efficiency shifts the frontier by more in regions of heavy energy resource use. The engineering calculation of the change in energy resource use holds L_k fixed, so it finds the point B on the altered isoquant that is directly below point A. Point B is on a lower isocost line. The vertical distance between points A and B defines the energy resource savings. The partial equilibrium calculation recognizes that the firm will reoptimize its input mix to return to a point of tangency with the isocost line. As $\sigma_k \to 0$ (left), point B is also the point of tangency with the altered isoquant. For larger σ_k (right), the new point of tangency (labeled C) is to the left and above point B. The vertical distance between points B and C determines partial equilibrium rebound. As σ_k becomes larger, the isoquant becomes flatter and the vertical distance between points B and C grows. For $\sigma_k > 1$ (not pictured), point C is above point A, in which case rebound is greater than 100% (a case of "backfire"). The general equilibrium analysis will account for movement to a different isocost line and will account for changes in factor prices that rotate the isocost line.

5 General Equilibrium Rebound

The previous, partial equilibrium analysis held factor and output prices fixed and asked how energy resource use changed in the sector with improved energy efficiency. However, pollution and other externalities are often related to the total change in energy resource use, including changes in other sectors induced by changes in factor and output prices.[14] I now consider this total, general equilibrium change in energy resource use from an improvement in some A_k. The appendix considers the general equilibrium consequences of an improvement in some B_k and of an improvement in sector k's total factor productivity.

Market-clearing in the energy supply sector required that $R = \sum_{i=1}^{N+1} R_i$. Equation (10) gives R_i as a function of A_i, p_i, p_R, w, and L for $i \in \{1, ..., N\}$, and equation (8) gives R_{N+1} as a function of R and A_{N+1}. Equation (5) gives p_i as a function of B_i, A_i, p_R, and w for $i \in \{1, ..., N\}$. Equation (6) gives p_R as a function of B_{N+1}, A_{N+1}, and w. Equation (2) gives w as a function of L. And equation (16) implicitly defines L as a function of each A_i, of each B_i, and of R. Now let variable y_k indicate some A_k or some B_k, for $k \in \{1, ..., N+1\}$.

[14] Many authors have observed that changes in energy use do not map into changes in welfare: rebound effects are not necessarily bad from a welfare perspective. However, those same authors nonetheless often focus on changes in energy use because that question has been of interest to historians, to policymakers, and to environmental economists concerned with externalities.



(a) $\sigma_k = 0$ (b) $\sigma_k \in (0, 1)$

Figure 1: Improving the quality of energy conversion technology A_k changes the isoquants of sector k's production technology from the solid line to the dotted line, with the dashed lines indicating the isocost lines. Point A indicates the initial equilibrium. The gap between point A and point B along the y-axis is the no-rebound calculation of energy resource savings, and the gap between point B and point C along the y-axis defines partial equilibrium rebound.

We have:

$$\begin{aligned}\frac{\mathrm{d}R}{\mathrm{d}y_k} =& \sum_{i=1}^{N}\Bigg[\frac{\partial R_i}{\partial y_k} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial y_k} + \left(\frac{\partial R_i}{\partial p_R} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R}\right)\frac{\partial p_R}{\partial y_k} + \frac{\partial R_i}{\partial L}\left(\frac{\partial L}{\partial y_k} + \frac{\partial L}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y_k}\right) \\ &\quad + \left(\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial w} + \left(\frac{\partial R_i}{\partial p_R} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R}\right)\frac{\partial p_R}{\partial w} + \frac{\partial R_i}{\partial w}\right)\frac{\partial w}{\partial L}\left(\frac{\partial L}{\partial y_k} + \frac{\partial L}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y_k}\right)\Bigg] \\ &+ \frac{\partial R_{N+1}}{\partial y_k} + \frac{\partial R_{N+1}}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y_k}.\end{aligned}$$

Let $\theta_{a,b}$ represent the elasticity of a with respect to b: $\theta_{a,b} \triangleq [b/a][\partial a/\partial b]$. Solving for $\mathrm{d}R/\mathrm{d}y_k$ and re-expressing in terms of elasticities, we have:[15]

$$\begin{aligned}\theta_{R,y_k} =& \overbrace{\left(1 + \frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\right)}^{\text{Energy supply amplifier}} \overbrace{\left(1 - \frac{L_{N+1}}{L} - \frac{\Theta - 1}{1-\epsilon}\frac{H-L}{H}\frac{L_{N+1}}{L}\right)}^{\text{Energy supply dampener}} \\ &\Bigg\{\overbrace{\sum_{i=1}^{N+1}\theta_{R_i,y_k}\frac{R_i}{R}}^{\text{PE effect}} + \overbrace{\sum_{i=1}^{N}\theta_{R_i,p_i}\theta_{p_i,y_k}\frac{R_i}{R}}^{\text{Output price effect}} + \overbrace{\sum_{i=1}^{N}\left(\theta_{R_i,p_R} + \theta_{R_i,p_i}\theta_{p_i,p_R}\right)\theta_{p_R,y_k}\frac{R_i}{R}}^{\text{Resource supply effect}} + \overbrace{\sum_{i=1}^{N}\theta_{R_i,L}\theta_{L,y_k}\frac{R_i}{R}}^{\text{Labor supply effect}} \\ &+ \overbrace{\sum_{i=1}^{N}\theta_{R_i,w}\theta_{w,L}\theta_{L,y_k}\frac{R_i}{R}}^{\text{Income effect}} + \overbrace{\sum_{i=1}^{N}\left(\theta_{R_i,p_i}\theta_{p_i,w} + \left(\theta_{R_i,p_R} + \theta_{R_i,p_i}\theta_{p_i,p_R}\right)\theta_{p_R,w}\right)\theta_{w,L}\theta_{L,y_k}\frac{R_i}{R}}^{\text{Input cost effect}}\Bigg\}.\end{aligned} \tag{18}$$

The second and third lines determine the sign of θ_{R,y_k}. I analyze them below. The first line contains two general equilibrium multipliers. The first multiplier is an energy supply amplifier that reflects how changes in consumer good sectors' use of energy resources affect the energy supply sector's demand for energy resources. This multiplier is large when the energy supply sector itself uses a larger share of available resources. It increases the magnitude of θ_{R,y_k} because expansions and contractions of the energy supply sector amplify changes in energy resource demand from other sectors.[16] If energy resources were not consumed in the

[15] The terms on the first line follow from the terms with $\mathrm{d}R/\mathrm{d}y_k$, which generate $R - \sum_{i=1}^{N}\theta_{R_i,L}\theta_{L,R}R_i - \sum_{i=1}^{N}\left(\theta_{R_i,p_i}\theta_{p_i,w} + \left(\theta_{R_i,p_R} + \theta_{R_i,p_i}\theta_{p_i,p_R}\right)\theta_{p_R,w} + \theta_{R_i,w}\right)\theta_{w,L}\theta_{L,R}R_i - \theta_{R_{N+1},R}R_{N+1}$ in the denominator. Substituting for those elasticities, the denominator becomes $R - \sum_{i=1}^{N}\theta_{L,R}R_i - \sum_{i=1}^{N}\left((\sigma_i - \epsilon)\alpha_{Li} + [-\sigma_i + (\sigma_i - \epsilon)\alpha_{Ri}] + 1\right)\theta_{w,L}\theta_{L,R}R_i - R_{N+1}$. Using $\alpha_{Li} + \alpha_{Ri} = 1$, this simplifies to $R - \sum_{i=1}^{N}\theta_{L,R}R_i + \sum_{i=1}^{N}(\epsilon - 1)\theta_{w,L}\theta_{L,R}R_i - R_{N+1}$. Using that $R - R_{N+1} = \sum_{i=1}^{N} R_i$ and $\theta_{w,L} = H[(\Theta - 1)(H-L)]^{-1}$, the denominator becomes $\sum_{i=1}^{N}\left[1 - \theta_{L,R} + \theta_{L,R}\frac{\epsilon-1}{\Theta-1}\frac{H}{H-L}\right]R_i$. The given expression follows from using the Implicit Function Theorem to obtain $\theta_{L,R}$ and using $L = \sum_{i=1}^{N} L_i + L_{N+1}$.

[16] As we will explore in the analysis of "super-conservation", this amplification is the multiplier effect discussed in Turner (2009).

energy supply sector ($R_{N+1} = 0$), then the energy supply amplifier would reduce to unity.

The second multiplier is an energy supply dampener. It is driven by the terms with $\theta_{L,R}$, reflecting that any change in the size of the energy supply sector changes aggregate demand for labor. Standard calibrations assume either that labor supply is fixed ($\Theta = 1$) or that labor supplied increases in the wage ($\Theta > 1$) with consumption goods complementary to each other ($\epsilon < 1$). In these cases, this second multiplier unambiguously shrinks the magnitude of θ_{R,y_k}, which is why I label it a "dampener". If energy resources were produced without labor ($L_{N+1} = 0$), then the energy supply dampener would reduce to unity.

Substituting the elasticities that do not depend on the choice of y_k, we find:

$$\theta_{R,y_k} = \overbrace{\left(1 + \frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\right)}^{\text{Energy supply amplifier}} \overbrace{\left(1 - \frac{L_{N+1}}{L} - \frac{\Theta - 1}{1 - \epsilon} \frac{H - L}{H} \frac{L_{N+1}}{L}\right)}^{\text{Energy supply dampener}}$$
$$\left\{ \overbrace{\theta_{R_k,y_k} \frac{R_k}{R}}^{\text{PE effect}} \overbrace{- (\epsilon - \sigma_k)\theta_{p_k,y_k} \frac{R_k}{R}}^{\text{Output price effect}} \overbrace{- \sum_{i=1}^{N} \left(\sigma_i + (\epsilon - \sigma_i)\alpha_{Ri}\right) \theta_{p_R,y_k} \frac{R_i}{R}}^{\text{Resource supply effect}} \right.$$
$$\left. + \overbrace{\theta_{L,y_k} \frac{\sum_{i=1}^{N} R_i}{R}}^{\text{Labor supply effect}} \overbrace{+ \underbrace{\theta_{w,L}\theta_{L,y_k} \frac{\sum_{i=1}^{N} R_i}{R}}_{\text{Income effect}} \underbrace{- \epsilon\, \theta_{w,L}\theta_{L,y_k} \frac{\sum_{i=1}^{N} R_i}{R}}_{\text{Input cost effect}}}^{\text{Wage effect}} \right\}.$$

The five terms in curly braces determine total rebound. The first two terms scale with sector k's share of total energy resource use. The first term captures the partial equilibrium channels: it reflects how an improvement in some sector's technology y_k affects that sector's factor use at constant prices. The second term captures how changes in energy conversion technology affect factor demand via output prices. Improving technology in some sector k reduces the cost of producing that sector's consumption good and thus reduces the price of its consumption good ($\theta_{p_i,y_k} \leq 0$). As a result, households substitute towards that consumption good. This substitution increases demand for energy resource and labor inputs in sector k. However, the lower output price also reduces demand for inputs in sector k. The net effect on sector k's demand for energy resources depends on (i) households' elasticity of substitution across consumption goods (ϵ) and (ii) sector k firms' elasticity of substitution across inputs (σ_k): from equation (10), $-\theta_{R_k,p_k} = \epsilon - \sigma_k$ for $k \in \{1, ..., N\}$. This general equilibrium channel increases total energy use if and only if (i) is larger than (ii), which is a common case in numerical models and will be true of most sectors in the quantitative application below.[17]

[17] Jevons' original story (see footnote 2) described this output price effect. θ_{p_k,y_k} derives from the zero-profit condition (5): the fact that θ_{p_k,y_k} is negative reflects that an increase in y_k produces positive profits ("the profits of the trade will increase") that increase entry ("new capital will be attracted"), which eventually

The previous channel vanishes if the improvement in technology occurs in the energy supply sector $N+1$, for there is no consumer good price p_k. Instead, the output price is p_R. As a result, improvements in the technology used to produce energy resources generate the third term instead of the second term. These improvements reduce the cost of energy resources to consumption good producers ($\theta_{p_R,y_k} \leq 0$). The reduction in the cost of energy resources directly works to increase their use ($-\theta_{R_i,p_R} = \sigma_i$ for $i \in \{1,...,N\}$), and by reducing the output price ($-\theta_{p_i,p_R} < 0$), the reduction in the cost of energy resources also indirectly increases their use if and only if $\epsilon > \sigma_i$.

The first term on the final line arises only when labor supply is endogenous. θ_{L,y_k} reflects how the change in y_k affects demand for labor in each sector. From equation (1), any resulting increase in labor supplied increases demand for each consumption good by increasing total income, and from equation (4), increased demand for a consumption good increases demand for the resources used to produce that consumption good. Because these two relationships are each linear conditional on prices, we have $\theta_{R_i,L} = 1$ (see equation (10)). Therefore, this labor supply channel works to increase total energy resource use if and only if increasing y_k increases aggregate demand for labor ($\theta_{L,y_k} > 0$).[18]

The remaining terms on the final line capture wage effects. A first term is an income effect: a 1% increase in the wage increases demand for energy resources by 1% because preferences are homothetic ($\theta_{R_i,w} = 1$, see equation (10)). A second term reflects how a higher wage reduces demand for energy resources by raising the cost of producing each consumption good i, both directly and by raising the cost of producing its energy resource inputs. On net, the income effect dominates the input cost effect if and only if $\epsilon < 1$, in which case a higher wage increases energy resource use (see equation (14)). A change in sector k's technology increases the wage if $\theta_{w,L}\theta_{L,y_k} > 0$ and reduces the wage otherwise.[19]

Define

$$Savings^{GE} \triangleq -A_k \frac{\mathrm{d}R}{\mathrm{d}A_k}.$$

Following Saunders (2008), I say that "backfire" occurs when $\theta_{R,A_k} > 0$ and that "super-conservation" occurs when $Savings^{GE} > Savings^{eng}$.

restores the zero-profit condition by reducing the output price ("the price of pig-iron will fall"). The reduction in p_k increases demand for c_k ("but the demand for it [pig-iron] increase") and, if $\epsilon > \sigma_k$, for R_k. If this were the only effect of increasing y_k, then we would have backfire ($\theta_{R,y_k} > 0$) if $-(\epsilon - \sigma_k)\theta_{p_k,y_k} > 1$, in which case "the greater number of furnaces will more than make up for the diminished consumption of each". The analysis below will show that this condition is equivalent to $(\epsilon - \sigma_k)\alpha_{Rk} > 1$. Jevons' story therefore requires a value of ϵ that is much larger than used in recent economic models.

[18]If labor supply were fixed exogenously, then aggregate demand for labor would affect demand for each consumption good in equation (1) only through the wage effect described next.

[19]The wage effect does not vanish if labor supply is exogenous: θ_{L,y_k} reflects changes in aggregate demand for labor, and the wage must adjust even if the aggregate supply of labor is fixed. The wage effect vanishes as the supply of labor becomes perfectly elastic (i.e., as $\Theta \to \infty$).

5.1 Improved energy efficiency in a consumption good sector

Now consider the consequences of improving A_k in some sector $k \in \{1, ..., N\}$:

$$\theta_{R,A_k} = \overbrace{\left(1 + \frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\right)}^{\text{Energy supply amplifier}} \overbrace{\left(1 - \frac{L_{N+1}}{L} - \frac{\Theta - 1}{1-\epsilon}\frac{H-L}{H}\frac{L_{N+1}}{L}\right)}^{\text{Energy supply dampener}}$$
$$\Bigg\{ \overbrace{(\sigma_k - 1)\frac{R_k}{R}}^{\text{PE effect}} + \overbrace{(\epsilon - \sigma_k)\alpha_{Rk}\frac{R_k}{R}}^{\text{Output price effect}} \overbrace{- (\epsilon - \sigma_k)\alpha_{Rk}\frac{L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i - L_{N+1}}\frac{\sum_{i=1}^{N} R_i}{R}}^{\text{Labor supply effect}}$$
$$\overbrace{-(\epsilon - \sigma_k)\alpha_{Rk}\frac{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i - L_{N+1}}\frac{\sum_{i=1}^{N} R_i}{R}}^{\text{Wage effect}}\Bigg\}. \tag{19}$$

The partial equilibrium effect increases energy resource use if and only if $\sigma_k > 1$, in accord with the analysis in Section 4. The sum of the labor supply and wage effects is

$$-(\epsilon - \sigma_k)\alpha_{Rk}\frac{\left(1 + \frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\right)L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i - L_{N+1}}\frac{\sum_{i=1}^{N} R_i}{R}.$$

To interpret this, apply the following reasonable restrictions: let the energy supply sector contribute a small share of overall labor demand (let L_{N+1} be small relative to $\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i$), let labor supply increase with the wage (let $\Theta \geq 1$), and let the consumption sectors' goods be complements (let $\epsilon < 1$). The combined labor supply and wage effects then have the opposite sign as $\epsilon - \sigma_k$, opposing the output price effect. If $\epsilon > \sigma_k$, then sector k's demand for labor increases through the exact same channels described for the output price effect: improved A_k induces household substitution towards the newly cheap consumption good k. Under the foregoing restrictions, increasing the wage (and thus the hours of labor supplied) would further increase demand for all consumption goods and thus for the labor needed to make them (see equation (13)—income effects are strong when $\epsilon < 1$). To clear the labor market, the wage must fall.[20] By the logic of the previous section, this change in the labor market reduces demand for energy resources, thereby opposing the output price effect. The output price effect dominates when sector k constitutes a large share of energy resource use but only a small share of the labor market. In this case, the net effect of the non-multiplier general equilibrium channels is to increase energy resource use if and only if $\epsilon > \sigma_k$.

[20]We will see immediately below that allowing L_{N+1} be large relative to $\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i$ (e.g., because of large Θ) changes this logic, because now a higher wage can induce households to supply sufficiently more labor to satisfy the additional demand for labor.

Two special cases are of interest because of their prominence in computable general equilibrium models. First, if labor supply is fixed exogenously (i.e., if $\Theta = 1$), then the sum of the labor supply and wage effects is[21]

$$-(\epsilon - \sigma_k)\alpha_{Rk}\frac{L_k}{\sum_{i=1}^N L_i}\frac{\sum_{i=1}^N R_i}{R},$$

which works to reduce energy use in the common case that $\epsilon > \sigma_k$. The labor market can adjust only through changes in the wage, and we just saw that increasing demand for labor in sector k must reduce that wage in order to bring demand back down to match the fixed supply. Second, if labor supply is perfectly elastic (i.e., as $\Theta \to \infty$), then the sum of the labor supply and wage effects is[22]

$$(\epsilon - \sigma_k)\alpha_{Rk}\frac{L_k}{L_{N+1}}\frac{\sum_{i=1}^N R_i}{R},$$

which works to increase energy resource use in the common case that $\epsilon > \sigma_k$. Now the wage is fixed. As improving A_k increases demand for labor in sector k (assuming $\epsilon > \sigma_k$), the labor market clears with more hours of labor. The additional hours worked increase demand for energy resources. Consistent with this analysis, computable general equilibrium models report greater rebound when modeling perfectly elastic labor supply as opposed to a fixed labor supply (Allan et al., 2007, 2009; Broberg et al., 2015).

The following proposition considers a case in which each consumption good sector has the same production function.

Proposition 1. *Assume that $\Theta = 1$ and either $N = 1$ or κ_i, σ_i, B_i, and A_i do not vary with i for $i \in \{1, ..., N\}$. Consider an improvement in A_k for some $k \in \{1, ..., N\}$.*

1. *The only general equilibrium channels are the multipliers, leaving $\theta_{R,A_k} = \frac{L_k}{L}(\sigma - 1)$.*

2. *Backfire occurs if and only if $\sigma > 1$.*

3. *If $R_{N+1}/L_{N+1} > R_k/L_k$, then there exists $\hat{\sigma} \in (0, 1)$ such that super-conservation occurs if and only if $\sigma < \hat{\sigma}$. If $R_{N+1}/L_{N+1} < R_k/L_k$, then super-conservation does not occur for any $\sigma > 0$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j/\sum_{i=1}^N R_i = L_j/\sum_{i=1}^N L_i = 1/N$ for all $j \in \{1, ..., N\}$. Substituting into θ_{R,A_k}, fixing $\Theta = 1$, and using $\sum_{i=1}^N L_i = NL_k$ and $\sum_{i=1}^N R_i = NR_k$ yields the first part of the proposition.

[21] This expression also holds as $\epsilon \to \infty$, for $\Theta \neq 1$. In this case of highly substitutable consumption goods, the labor market effects are driven by the input cost effect, irrespective of the the labor supply effect.

[22] This expression also holds as $\epsilon \to 1$, for $\Theta \neq 1$. In this case, the income and input cost effects cancel, leaving only the labor supply effect.

The second part of the proposition follows from that result and the definition of backfire as occurring if and only if $\theta_{R,A_k} > 0$. To establish the third part of the proposition, note that

$$Savings^{GE} > Savings^{eng} \Leftrightarrow -\theta_{R,A_k} R > R_k \Leftrightarrow 0 > \frac{R_k}{R} + \frac{L_k}{L}(\sigma - 1) \Leftrightarrow \sigma < \frac{\frac{L_k}{L} - \frac{R_k}{R}}{\frac{L_k}{L}}.$$

The result follows straightforwardly from this expression, the assumption that $\sigma > 0$, and the fact that, under the conditions of the proposition, $L_k/L < R_k/R$ if and only if $R_{N+1}/L_{N+1} < R_k/L_k$. □

If $N = 1$, then there is no scope for substitution when input costs change, and even if $N > 1$, substitution does not affect energy resource use if consumption good sectors are symmetric. In either case, the terms in braces in equation (19) reduce to only the partial equilibrium effect if labor supply is fixed ($\Theta = 1$). Backfire arises if and only if $\sigma > 1$, as in Section 4.

However, even though only the partial equilibrium effect remains from the terms in braces, θ_{R,A_k} is not identical to the pure partial equilibrium result, which would be $(\sigma - 1)R_k/R$. The difference arises because the general equilibrium analysis accounts for effects on energy supply, generating the multipliers in equation (18). Whereas the partial equilibrium analysis of changes in R_k in Section 4 showed that super-conservation was impossible, we now find that super-conservation is possible if σ is small and energy production is especially energy-intensive. When σ is small, the partial equilibrium effect on sector k reduces sector k's demand for resources. That reduction in resource demand leads the energy supply sector to contract, which in turn reduces demand for energy resources in that sector. This multiplier effect amplifies the energy resource savings from any given reduction in energy resource demand in the consumption good sectors. When σ is sufficiently small, the total energy resource savings can be greater even than predicted by an engineering calculation.[23]

Now imagine that consumption good sectors differ only in their energy efficiency A. Let $\bar{A} \triangleq \sum_{i=1}^{N} A_i/N$ indicate the average A across the consumption good sectors.

Proposition 2. *Assume that $\Theta = 1$, that κ_i, σ_i, and B_i do not vary with i for $i \in \{1, ..., N\}$, and that $Var(A)$ is small relative to $\bar{A}$. Consider an improvement in A_k for some $k \in \{1, ..., N\}$.*

1. *If $\sigma < 1$ and either $\sigma \leq \epsilon < 1$ or $(A_k - \bar{A})(\epsilon - 1) > 0$, then $\theta_{R,A_k} < 0$.*
2. *If $\sigma > 1$ and either $\sigma \geq \epsilon > 1$ or $(A_k - \bar{A})(\epsilon - 1) > 0$, then $\theta_{R,A_k} > 0$.*

[23] This analysis clarifies the conditions under which the changes in energy suppliers' demand for energy inputs considered in Turner (2009, 664) can lead to super-conservation. Of course, this multiplier effect works the other way as well, as it also amplifies backfire. Turner (2009) also discusses disinvestment effects, which can arise only in a dynamic model with imperfectly adjustable capital stocks, and Borenstein (2015) discusses other sources of negative rebound in a partial equilibrium setting of household energy use.

Proof. If $\Theta = 1$, then

$$\theta_{R,A_k} = \frac{\sum_{i=1}^N L_i}{L} \left\{ (\sigma_k - 1) \frac{R_k}{\sum_{i=1}^N R_i} + (\epsilon - \sigma_k)\alpha_{Rk} \left(\frac{R_k}{\sum_{i=1}^N R_i} - \frac{L_k}{\sum_{i=1}^N L_i} \right) \right\}. \tag{20}$$

From equations (13) and (14), we have:

$$\begin{aligned}
\frac{R_k}{\sum_{i=1}^N R_i} - \frac{L_k}{\sum_{i=1}^N L_i} &= \left\{ \frac{A_k^{\sigma-1} Z_k}{\sum_{i=1}^N A_i^{\sigma-1} Z_i} - \frac{Z_k}{\sum_{i=1}^N Z_i} \right\} \\
&= Z_k \left\{ \frac{A_k^{\sigma-1} \sum_{i=1}^N Z_i - \sum_{i=1}^N A_i^{\sigma-1} Z_i}{\left[\sum_{i=1}^N A_i^{\sigma-1} Z_i\right] \left[\sum_{i=1}^N Z_i\right]} \right\} \\
&= \frac{N Z_k}{\left[\sum_{i=1}^N A_i^{\sigma-1} Z_i\right] \left[\sum_{i=1}^N Z_i\right]} \left\{ \left[A_k^{\sigma-1} - \frac{1}{N} \sum_{i=1}^N A_i^{\sigma-1} \right] \frac{1}{N} \sum_{i=1}^N Z_i - Cov\left(A_i^{\sigma-1}, Z_i\right) \right\},
\end{aligned}$$

where

$$Z_i \triangleq \left(B_i^{\sigma_i - 1} \kappa_i^{\sigma_i} + A_i^{\sigma_i - 1} (1 - \kappa_i)^{\sigma_i} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1 - \kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{\epsilon - \sigma_i}{\sigma_i - 1}}.$$

Using a second-order Taylor series expansion around $A_i = \bar{A}$, we have:

$$\frac{1}{N} \sum_{i=1}^N A_i^{\sigma-1} \approx \bar{A}^{\sigma-1} + \frac{1}{2}(\sigma - 1)(\sigma - 2)\bar{A}^{\sigma-3} Var(A).$$

Using a second-order Taylor series expansion of Z_i around $A_i = \bar{A}$, we have:

$$\frac{1}{N} \sum_{i=1}^N A_i^{\sigma-1} \approx \bar{Z} + \frac{1}{2} \bar{Z} \bar{\alpha}_R \bar{A}^{-2} Var(A) \Big[(\epsilon - 2\sigma + 1)\bar{Z}\bar{\alpha}_R + (\sigma - 2)(\epsilon - \sigma) \Big],$$

where $\bar{Z}$ indicates Z_i evaluated at $A_i = \bar{A}$ and $\bar{\alpha}_R$ indicates the value share of energy resources in consumption good production evaluated at $\bar{A}$. And using first-order Taylor expansions of $A_i^{\sigma-1}$ and Z_i around $A_i = \bar{A}$, we have:

$$Cov\left(A_i^{\sigma-1}, Z_i\right) \approx (\epsilon - \sigma)(\sigma - 1)\bar{Z}\bar{A}^{\sigma-3}\bar{\alpha}_R Var(A).$$

Substituting and using the assumption that $Var(A)/\bar{A}$ is small, we find:

$$\frac{R_k}{\sum_{i=1}^N R_i} - \frac{L_k}{\sum_{i=1}^N L_i} \approx \frac{N Z_k \bar{Z}}{\left[\sum_{i=1}^N A_i^{\sigma-1} Z_i\right] \left[\sum_{i=1}^N Z_i\right]} \left\{ A_k^{\sigma-1} - \bar{A}^{\sigma-1} \right\}. \tag{21}$$

If $\sigma < 1$, then (21) is strictly negative if $A_k > \bar{A}$. In that case, all terms in (20) are negative if $\sigma \leq \epsilon$. If $\sigma < 1$ and $A_k < \bar{A}$, then (21) is strictly positive, but if in addition, $\epsilon \in [\sigma, 1)$, then (20) is strictly negative because $1-\sigma > (\epsilon-\sigma)\alpha_{Rk}$ and $\frac{R_k}{\sum_{i=1}^N R_i} > \frac{R_k}{\sum_{i=1}^N R_i} - \frac{L_k}{\sum_{i=1}^N L_i}$. We have established that θ_{R,A_k} is strictly negative if $\sigma \leq \epsilon < 1$ and also if $A_k > \bar{A}$ with $\sigma < 1 < \epsilon$.

If $\sigma \in (\epsilon, 1)$, then (21) is strictly positive if $A_k < \bar{A}$. All terms in (20) are strictly negative. Combining with the previous result, we have established that θ_{R,A_k} is strictly negative if $(A_k - \bar{A})(\epsilon - 1) > 0$.

If $\sigma > 1$, then (21) is strictly negative if $A_k < \bar{A}$. In that case, all terms in (20) are positive if $\sigma \geq \epsilon$. If $\sigma > 1$ and $A_k > \bar{A}$, then (21) is strictly positive, and if, in addition, $\epsilon \geq \sigma_k$, then all terms in (20) are positive. If $\sigma \geq \epsilon > 1$ and $A_k > \bar{A}$, then (21) is strictly positive but (20) is strictly negative because $\sigma - 1 > (\sigma - \epsilon)\alpha_{Rk}$ and $\frac{R_k}{\sum_{i=1}^N R_i} > \frac{R_k}{\sum_{i=1}^N R_i} - \frac{L_k}{\sum_{i=1}^N L_i}$. We have established that θ_{R,A_k} is strictly positive if $\sigma \geq \epsilon > 1$, if $A_k < \bar{A}$ with $\epsilon < 1 < \sigma$, and if $A_k > \bar{A}$ with $\sigma, \epsilon > 1$.

□

The proposition identifies sufficient conditions under which greater efficiency either avoids or generates backfire. Improved efficiency is likely to reduce energy resource use when σ is small and to backfire when σ is large. The partial equilibrium effect is strong for values of σ far from 1, and is strong enough to dominate any conflicting general equilibrium effects if ϵ is between σ and 1. However, matters are more complicated when σ is closer to 1 than is ϵ. We can clearly sign the effects when $(A_k - \bar{A})(\epsilon - 1) > 0$ because the general equilibrium effects then reinforce the partial equilibrium effect. The relationship between σ and 1 controls whether sectors with higher-than-average A_k are especially energy-intensive, the relation between ϵ and 1 controls whether households substitute towards or away from the sector with improved efficiency, and the relationship between σ and ϵ determines how changes in consumption good prices affect factor demand. As an example, consider the case with $A_k > \bar{A}$ and $\sigma \in (1, \epsilon)$. Households substitute towards sector k, which has above-average efficiency. When $\sigma \in (1, \epsilon)$, sector k's factor demand increases as its output price falls. The output price falls when the effect of improved A_k outweighs the effect of a higher wage, which occurs when sector k is especially energy-intensive (as implied by $A_k > \bar{A}$). In this case, the general equilibrium channels reinforce the partial equilibrium channel and backfire is unambiguous. If, instead, $A_k < \bar{A}$ then the general equilibrium channels reduce energy resource use and the net effect of improved efficiency is ambiguous.

Now let consumption good sectors differ only in their efficiency B of non-energy inputs, with $\bar{B} \triangleq \sum_{i=1}^N B_i/N$ indicating the average B across the consumption good sectors. We have the following corollary:

Corollary 3. *Assume that $\Theta = 1$, that κ_i, σ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$, and that $Var(B)$ is small relative to $\bar{B}$. Consider an improvement in A_k for some $k \in \{1, ..., N\}$.*

1. *If $\sigma < 1$ and either $\sigma \leq \epsilon < 1$ or $(B_k - \bar{B})(\epsilon - 1) < 0$, then $\theta_{R,A_k} < 0$.*

2. *If $\sigma > 1$ and either $\sigma \geq \epsilon > 1$ or $(B_k - \bar{B})(\epsilon - 1) < 0$, then $\theta_{R,A_k} > 0$.*

Proof. Follows from the proof of Proposition 2, noting that now

$$\frac{R_k}{\sum_{i=1}^N R_i} - \frac{L_k}{\sum_{i=1}^N L_i} = \frac{-NZ_k}{\left[\sum_{i=1}^N B_i^{\sigma-1} Z_i\right]\left[\sum_{i=1}^N Z_i\right]} \left\{\left[B_k^{\sigma-1} - \frac{1}{N}\sum_{i=1}^N B_i^{\sigma-1}\right]\frac{1}{N}\sum_{i=1}^N Z_i - Cov\left(B_i^{\sigma-1}, Z_i\right)\right\}$$

and

$$Cov\left(B_i^{\sigma-1}, Z_i\right) \approx (\epsilon - \sigma)(\sigma - 1)\bar{Z}\bar{B}^{\sigma-3}\bar{\alpha}_L Var(B).$$

□

Two limiting cases are of special interest: one in which sector k does not use any energy resources ($\kappa_k \to 1$), and one in which sector k uses only energy resources ($\kappa_k \to 0$). The latter case is closely related to settings in which households use energy resources directly.

Proposition 4. *Consider an improvement in A_k for some $k \in \{1, ..., N\}$.*

1. *$\theta_{R,A_k} \to 0$ as $\kappa_k \to 1$.*

2. *Assume $\Theta = 1$ and let $\kappa_k \to 0$. If $N = 1$, then $\theta_{R,A_k} \to 0$; if $N > 1$, then $\theta_{R,A_k} > 0$ if and only if $\epsilon > 1$.*

Proof. From equation (14), $R_k \to 0$ as $\kappa_k \to 1$. We also then have $\alpha_{Rk} \to 0$ as $\kappa_k \to 1$. The first result follows.

From equation (13), $L_k \to 0$ as $\kappa_k \to 0$. We also then have $\alpha_{Rk} \to 1$ as $\kappa_k \to 0$. Therefore $\theta_{R,A_k} \to \left((\epsilon - 1)\frac{\sum_{i=1}^N L_i}{L} + (\Theta - 1)(H - L)\frac{L_{N+1}}{L}\right)\frac{R_k}{\sum_{i=1}^N R_i}$ as $\kappa_k \to 0$. If $N = 1$, then $\sum_{i=1}^N L_i = 0$, but if $N > 1$, then $\sum_{i=1}^N L_i > 0$. The second result follows. □

If sector k does not use any energy resources (because κ_k is large), then an improvement in A_k is of limited importance and we have no change in use of energy resources. If, instead, sector k is highly energy-intensive (because κ_k is small), then the wage and labor supply effects become small because sector k does not use much labor. Further, the output price p_k declines by an especially large amount because the value share of energy resources goes to 1. The resulting reduction in producers' demand for R_k exactly offsets their partial equilibrium substitution towards R_k. We are left with the engineering savings, the effect of household substitution towards consumption good k, and the energy supply multipliers. If $N > 1$ and $\epsilon > 1$, then the effect of household substitution dominates the engineering savings, generating backfire. If $N > 1$ and $\epsilon < 1$, then the engineering savings dominate. If $N = 1$, then households must spend all of their income on the lone consumption good. In that case, a competitive sector expands its use of resources to the point where it fully destroys the profits generated by the engineering savings and we have no net change in energy resource use due to the improved technology.

5.2 Improved energy efficiency in the energy supply sector

Next consider the consequences of improving A_{N+1}:

$$\theta_{R,A_{N+1}} = \overbrace{\left(1+\frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\right)}^{\text{Energy supply amplifier}} \overbrace{\left(1-\frac{L_{N+1}}{L}-\frac{\Theta-1}{1-\epsilon}\frac{H-L}{H}\frac{L_{N+1}}{L}\right)}^{\text{Energy supply dampener}}$$
$$\left\{ \overbrace{(\sigma_{N+1}-1)\frac{R_{N+1}}{R}}^{\text{PE effect}} + \overbrace{\frac{R_{N+1}}{R}\sum_{i=1}^{N}\left(\sigma_i+(\epsilon-\sigma_i)\alpha_{Ri}\right)\frac{R_i}{\sum_{j=1}^{N} R_j}}^{\text{Resource supply effect}} \right.$$
$$+ \overbrace{\frac{\sigma_{N+1}L_{N+1}-\sum_{i=1}^{N}(\epsilon-\sigma_i)\alpha_{Ri}L_i}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i - L_{N+1}}\frac{R_{N+1}}{R}}^{\text{Labor supply effect}}$$
$$\left. + \overbrace{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\frac{\sigma_{N+1}L_{N+1}-\sum_{i=1}^{N}(\epsilon-\sigma_i)\alpha_{Ri}L_i}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i - L_{N+1}}\frac{R_{N+1}}{R}}^{\text{Wage effect}} \right\}. \tag{22}$$

The partial equilibrium effect is the same as in Section 5.1. However, the general equilibrium effects are quite different. First, we now have a resource supply effect arising because the improvement in efficiency reduces the price of energy resources. This price reduction increases use of energy resources because consumption good producers substitute towards the newly cheap energy resources (controlled by σ_i). In addition, this price reduction leads the price of each consumption good to fall (with implications determined by $\epsilon - \sigma_i$, as described earlier). The resource supply effect unambiguously increases energy resource use if $\epsilon > \sigma_i$ for all $i \in \{1, ..., N\}$.

Second, the labor supply and wage effects now each have two components. A first term ($\sigma_{N+1}L_{N+1}$) captures how improving A_{N+1} reduces p_R (see equation (6)) and thereby reduces demand for labor in the energy supply sector (controlled by σ_{N+1}). A second term captures how improving energy supply technology makes energy resources more abundant in all consumption good sectors, with effects on all consumption good sectors' output prices and demand for labor as described in the previous section. The first term opposes the second when $\epsilon > \sigma_i$ for each $i \in \{1, ..., N\}$.

These new general equilibrium effects can be large even when the value share of energy resources is small, as it often is in the U.S. economy. In contrast, the non-multiplier general equilibrium channels analyzed in Section 5.1 all scale with the value share of energy resources. In practice, we should therefore expect general equilibrium effects to be especially important when energy efficiency improvements arise in the energy supply sector.

The following propositions analyze especially tractable special cases:

Proposition 5. *Assume that $\Theta = 1$ and either $N = 1$ or κ_i, σ_i, B_i, and A_i do not vary with i for $i \in \{1, ..., N\}$. Consider an improvement in A_{N+1}.*

1. *The non-multiplier general equilibrium channels are strictly positive.*
2. *There exists $\hat{\sigma} \in (0, 1)$ such that backfire occurs if $\sigma + \sigma_{N+1} > \hat{\sigma}$.*
3. *If $R_{N+1}/L_{N+1} > R_i/L_i$ for $i \in \{1, ..., N\}$, then super-conservation can occur only if $\sigma + \sigma_{N+1} < 1$. If $R_{N+1}/L_{N+1} < R_i/L_i$ for $i \in \{1, ..., N\}$, then super-conservation does not occur for any $\sigma, \sigma_{N+1} > 0$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j / \sum_{i=1}^N R_i = L_j / \sum_{i=1}^N L_i = 1/N$ for all $j \in \{1, ..., N\}$. Label the σ_i simply σ. Using $\Theta = 1$, we find:

$$\theta_{R,A_{N+1}} = \frac{\sum_{i=1}^N L_i}{L} \frac{R_{N+1}}{\sum_{i=1}^N R_i} \left\{ (\sigma_{N+1} - 1) + \sigma + \sigma_{N+1} \frac{L_{N+1}}{\sum_{i=1}^N L_i} \right\}.$$

The first two parts of the proposition follow (recognizing that the partial equilibrium channel is the $\sigma_{N+1} - 1$). To prove the third part, note that

$$Savings^{GE} > Savings^{eng} \Leftrightarrow -\theta_{R,A_{N+1}} R > R_{N+1} \Leftrightarrow 0 > \frac{R_{N+1}}{R} + \theta_{R,A_{N+1}}.$$

As $\sigma, \sigma_{N+1} \to 0$, $\theta_{R,A_{N+1}} \to -\frac{\sum_{i=1}^N L_i}{L} \frac{R_{N+1}}{\sum_{i=1}^N R_i}$ from above and $Savings^{GE} > Savings^{eng}$ if and only if $0 > 1 - \frac{\sum_{i=1}^N L_i}{L} \frac{R}{\sum_{i=1}^N R_i}$. The third part of the proposition follows from noting that $\sum_{i=1}^N L_i = N L_i$ and $\sum_{i=1}^N R_i = N R_i$ and from the fact that, under the conditions of the proposition, $L_i/L < R_i/R$ if and only if $R_{N+1}/L_{N+1} < R_i/L_i$. .

□

Proposition 6. *Assume that either $\Theta = 1$ or L_{N+1}/L is small and that, for $i \in \{1, ..., N\}$, each $\sigma_i \approx 1$ and κ_i does not vary with i. Consider an improvement in A_{N+1}.*

1. *The general equilibrium channels are strictly positive.*
2. *Backfire occurs.*

Proof. The energy supply multipliers are strictly positive if either $\Theta = 1$ or L_{N+1}/L is small. The value share of energy resources in each consumption good sector is approximately κ_i when $\sigma_i \approx 1$ and is then independent of i when, in addition, κ_i is independent of i. From $\theta_{R,A_{N+1}}$, the general equilibrium channels are proportional to $\sum_{j=1}^N L_j + \sigma_{N+1} L_{N+1}$ if $\Theta = 1$ and proportional to

$$1 + \frac{(\epsilon - 1)\kappa L - \left(1 - \frac{\epsilon-1}{\Theta-1} \frac{H}{H-L}\right) \sigma_{N+1} L_{N+1}}{L_{N+1} + \frac{\epsilon-1}{\Theta-1} \frac{H}{H-L} \sum_{i=1}^N L_i}$$

otherwise. The denominator is strictly positive if $\Theta > 1$ and the numerator is strictly positive if $\Theta \in (1, \epsilon]$. The first part of the proposition follows. Under the same conditions, we also have that $\theta_{R,A_{N+1}}$ is proportional to σ_{N+1} if $\Theta = 1$ and otherwise proportional to

$$\sigma_{N+1} + \frac{(\epsilon - 1)\kappa L + \left(\frac{\epsilon-1}{\Theta-1}\frac{H}{H-L} - 1\right)\sigma_{N+1} L_{N+1}}{L_{N+1} + \frac{\epsilon-1}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i}.$$

The second part of the proposition follows. □

In contrast to Proposition 1, the non-multiplier general equilibrium channels in Proposition 5 are strictly positive even when $N = 1$ and even when consumption good sectors are symmetric. The reason is that general equilibrium channels now account for the effect of an outward shift in energy supply on every consumption good producer's input cost. As the cost of energy resources falls, consumption good firms substitute towards newly cheap energy resources. The sufficient condition for backfire becomes less demanding than in Proposition 1 and the necessary condition for super-conservation becomes even more demanding than in Proposition 1. As before, backfire occurs when the partial equilibrium effect favors backfire, but now backfire can also occur when the partial equilibrium effect is as negative as can be (i.e., even as $\sigma_{N+1} \to 0$). And Proposition 6 shows that backfire can now occur even when consumption good production is Cobb-Douglas. This result formalizes and confirms the results of numerical models that have emphasized rebound from the energy supply sector (see Allan et al., 2007; Sorrell, 2007; Hanley et al., 2009; Turner, 2009).

5.3 Evaluating the magnitude of general equilibrium rebound

The primary tools for evaluating general equilibrium rebound are computable general equilibrium models, which can be black boxes, can be difficult to calibrate convincingly, and are often not accessible to the broader research community. On the other hand, reduced-form, program evaluation methods have been restricted to evaluating partial equilibrium rebound in particular applications. I here develop a third approach, using the theoretical results in equations (19) and (22) to estimate the general equilibrium implications of improved energy efficiency. This approach is more structural than reduced-form methods in that the key equations derive from a fully specified economic model, but it is less structural than computable general equilibrium models in that it does not require knowledge of all structural parameters in order to calculate the effects of interest. This new approach is in the spirit of recommendations in Heckman (2010), of sufficient statistics methods as described in Chetty (2009), and of price theory as described in Weyl (2019).

Rather than detailed knowledge of sectoral production functions, equations (19) and (22) only require knowledge of sectoral input use and the several elasticities of substitution. I calculate the value share of energy in each 3-digit NAICS industrial sector from Bureau of

Economic Analysis data on the composition of U.S. output by industry.[24] I approximate each industry's share of total energy resource and labor use by its share of total spending on energy and labor. I aggregate the various mining sectors, the utility sector, the petroleum and coal products sector, and the pipeline transportation sector into a single energy supply sector. I calibrate each sector's σ_k to estimates from Koesler and Schymura (2015) and Marten and Garbaccio (2018).[25] I follow Herrendorf et al. (2013). Atalay (2017), and Baqaee and Farhi (2019) in setting $\epsilon = 0.9$.[26] Finally, I here fix $\Theta = 1$, which is equivalent to a short-run analysis that holds labor supply fixed. The appendix shows that calibrating Θ does not appreciably change the results.

The left panel of Figure 2 plots the calculated θ_{R,A_k} for all consumption good sectors against their estimated σ_k, with the markers scaled by R_k/R. Some computable general equilibrium analyses of rebound have set $\sigma_k = 0.3$ in all sectors (e.g., Allan et al., 2007; Hanley et al., 2009; Turner, 2009), but we here see substantial variation across sectors. Sectors with a small elasticity of substitution between energy resources and labor tend to show larger reductions in energy resource use, and for a given elasticity of substitution, sectors with a larger share of total energy resource use also tend to show larger effects. Both of these results follow directly from the partial equilibrium channel. Only five sectors show backfire, and these five are the only sectors with σ_k greater than 1.[27] Further, backfire is extremely slight in four of these five sectors, reflecting that their estimated σ_k is fairly close to 1. The right panel of Figure 2 includes 95% confidence intervals based on standard errors for σ_k reported in Koesler and Schymura (2015).[28] Several additional transportation sectors (i.e., truck, rail, and transit) now plausibly backfire, but efficiency improvements still clearly reduce total energy resource use in nearly all of the plotted sectors.

Table 1 decomposes the percentage change in economy-wide resource use resulting from a 1% improvement in a given sector's efficiency. The top panel reports the energy supply multipliers. The combined multiplier is greater than 1, indicating that expansion and contraction of the energy supply sector amplifies changes in other sectors' energy resource use. The lower panel decomposes θ_{R,A_k} for the sectors with the greatest reductions in resource use and for combinations of sectors.[29] Consider the non-energy supply sectors. The partial

[24] Available at `https://apps.bea.gov/iTable/index_industry_gdpIndy.cfm`. I use data for 2017 released on November 1, 2018. In some cases (such as "Other retail"), the data combine 3-digit sectors into a single sector. NAICS codes 531 (discussed below) and 541 are divided into multiple sectors.

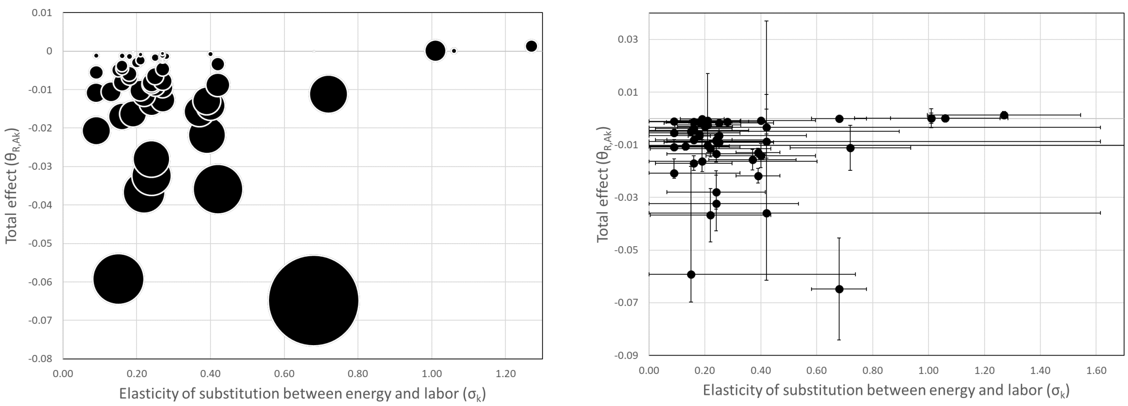
[25] I calibrate a single σ_{N+1} for the energy supply sector by taking an average of the underlying sectors' values, weighted by gross output. The underlying sectors' values are rather similar to each other.

[26] The value of ϵ used in computable general equilibrium models of rebound is not always reported, but it does appear that the analogous parameter in Broberg et al. (2015) is set to 0.9 (parameter "sovr" reported in Östblom and Berg, 2006). See also footnote 10.

[27] The sectors that backfire involve electronics, metals, and water transportation. The appendix reports their full labels and parameter values.

[28] The calculated intervals enforce a lower bound of zero on σ_k.

[29] The appendix reports the full set of sector-level results.



(a) Markers scaled by R_k/R (b) With confidence intervals for σ_k

Figure 2: The percentage change in energy resource use (θ_{R,A_k}) against the elasticity of substitution between energy resources and labor (σ_k). Left: Plots all non-energy supply sectors and scales markers by R_k/R. Right: Includes the 95% confidence interval for σ_k from Koesler and Schymura (2015). Plots only those sectors that can be clearly mapped to Koesler and Schymura (2015).

equilibrium effects are negative for all of these sectors, with many of them having quite small values of σ_k. The output price effect works to increase energy resource use but the wage effect works to reduce energy resource use. The output price usually dominates the wage effect in these sectors, so that the non-multiplier general equilibrium channels usually work to increase energy resource use on net.[30] However, this net effect is usually small relative to the partial equilibrium effect, in part because the value share of energy resources tends to be small. As a result, the most important general equilibrium channels are the energy supply multipliers. The total reduction in energy resource use is therefore even greater than suggested by partial equilibrium estimates despite the non-multiplier general equilibrium channels working to increase energy resource use.

Simultaneously improving energy efficiency in all non-energy supply sectors reduces energy resource use by 0.58%, with rebound of 28%.[31] The greatest total reduction in energy

[30] The appendix shows that the non-multiplier general equilibrium channels do work to reduce energy resource use on net for sectors that provide smaller energy savings. In sectors that backfire, the output price effect works to reduce energy resource use and the wage effect works to increase energy resource use. These changes occur because $\sigma_k > \epsilon$ in the sectors that backfire. The sectors with greater backfire see the output price effect dominate, but the sectors with less backfire see the wage effect dominate.

[31] This experiment matches a benchmark experiment undertaken by computable general equilibrium modelers in recent studies of European economies. They report rebound on the order of 30–70% (e.g., Allan et al., 2006, 2007; Turner, 2009; Broberg et al., 2015). There are many differences in model structure and calibration that could explain the higher values.

resource use comes from the "Other real estate" sector, which includes all components of NAICS code 531 ("Real estate") that are not categorized as housing. This sector has by far the largest share of total energy resource use among the non-energy supply sectors (over twice that of any other). This large share of total energy resource use overcomes its σ_k not being terribly small.[32] Even here, however, rebound reduces energy resource savings by 64% relative to an engineering calculation. Several of the next most promising sectors (such as construction and truck transportation) also use energy resources at a substantial scale. This scale makes the conservation benefits of efficiency especially large, but a full evaluation would have to consider that obtaining a 1% improvement in efficiency may be more difficult to achieve in sectors with a large stock of energy-using capital. Rebound is nontrivial in most sectors, but three of these sectors (construction, hospitals, and a financial sector) do show negative rebound, or "super-conservation". In all of these cases, especially small σ_k limits partial equilibrium rebound and the non-multiplier general equilibrium channels either roughly cancel each other or combine to reduce energy resource use.

Consistent with the theoretical analysis, the energy supply sector shows substantial rebound. General equilibrium channels here play an important role, increasing rebound to 80% from the 42% produced by partial equilibrium effects.[33] The most important general equilibrium channel is the resource supply effect, which almost offsets the partial equilibrium effect. The resource supply effect is so much larger than the output price effects seen in downstream sectors because it includes terms that do not scale with the value share of resources. We saw that general equilibrium channels enhance the energy resource savings from improving efficiency in all non-energy supply sectors (raising them from 0.49% to 0.58%), but the final row of Table 1 shows that rebound in the energy supply sector nearly cancels that effect when improving efficiency throughout the economy (general equilibrium channels now raise energy resource savings from 0.60% to only 0.62%).

6 Extension 1: Endogenous Innovation

Thus far I have considered the consequences of more efficient technology. I now consider which sectors tend to attract research effort and whether additional innovation will tend to

[32]This finding relates to a broader debate about the merits of energy efficiency in building codes. This debate has primarily focused on residential housing, with some studies reporting little to no gain from greater efficiency in that sector (Levinson, 2016). The "Housing" sector here has nearly no change in energy resource use from greater efficiency, and in fact sees the smallest reduction of any sector that does not backfire (see appendix). However, this sector here likely includes real estate lessors and agents rather than housing as thought of in the literature: its small energy savings arise because its R_k/R is tiny. The "Other real estate" sector can be interpreted as commercial real estate, for which the reduced-form literature has indeed detected sizable reductions in energy resource use from greater efficiency (Papineau, 2017).

[33]Note, however, that the energy supply sector's standard error for σ_k from Koesler and Schymura (2015) is enormous, above 10 even though the central estimate of σ_k is only around 0.4. It is entirely plausible that improved efficiency would backfire in this sector, even to a rather large degree.

Table 1: Decomposing the percentage change in energy resource use (θ_{R,A_k}) from a 1% increase in sector k's energy efficiency. Also reports rebound, as a percentage of engineering savings. The listed non-energy supply sectors are the ten that yield the greatest reduction in energy resource use.

Energy supply multipliers (same for all sectors)									
Amplifier	1.23								
Dampener	0.98								
Combined	1.20								
	Parameters				Channels				
Sector	σ_k	α_{Rk}	R_k/R	L_k/L	PE	Outp.*	Wage	Total	Reb.**
Other real estate	0.68	0.081	0.178	0.011	-0.057	0.0032	-0.0002	-0.065	64
Construction	0.15	0.022	0.058	0.061	-0.049	0.0010	-0.0008	-0.059	-2
Food services and drinking places	0.22	0.029	0.039	0.034	-0.031	0.0008	-0.0006	-0.037	7
Truck transportation	0.42	0.094	0.055	0.011	-0.032	0.0025	-0.0004	-0.036	35
Wholesale trade	0.24	0.011	0.035	0.063	-0.027	0.0002	-0.0004	-0.032	8
Other retail	0.24	0.018	0.031	0.035	-0.023	0.0004	-0.0003	-0.028	9
Other transportation and support activities	0.39	0.078	0.031	0.010	-0.019	0.0012	-0.0003	-0.022	30
Federal Reserve banks, credit intermediation, and related activities	0.09	0.012	0.019	0.032	-0.017	0.0002	-0.0003	-0.021	-10
Hospitals	0.16	0.011	0.017	0.047	-0.014	0.0001	-0.0003	-0.017	-2
Food and beverage and tobacco products	0.19	0.010	0.017	0.013	-0.014	0.0001	-0.0001	-0.016	3
All non-energy supply sectors					-0.49	0.014	-0.0070	-0.58	28
Energy supply	0.42	0.071	0.19	0.022	-0.11	0.077	0.00015	-0.038	80
All sectors					-0.60	0.091	-0.0069	-0.62	38

* Output price effect. For the energy supply sector, reports the resource supply effect.
** Rebound, as a percentage. Follows definition (17) except using the general equilibrium savings in place of the partial equilibrium savings and multiplying by 100 to obtain a percentage.
All columns: The energy supply multipliers are included only in the "Total" and "Rebound" calculations.

increase or decrease total resource use. I endogenize innovation by extending the setting to allow for directed technical change in the fashion of Acemoglu (2002, 2007): innovations will be driven by the market value of patents to improved technologies. I consider which sector a marginal research firm would target, with the predetermined technology parameters A_i reflecting both the incoming quality of technology and any pre-existing allocation of research effort.

Modify the production function for consumption goods to include an inner nest in which energy resources are combined with machines, which are produced according to the Dixit-Stiglitz model of monopolistic competition. The number of machines of variety j produced in sector i is z_{ij}, with the continuum of varieties indexed on the unit interval. Production of sector i's consumption good becomes:

$$c_i = \left((1-\kappa_i) \left[R_i^{1-\gamma} \int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j \right]^{\frac{\sigma_i - 1}{\sigma_i}} + \kappa_i [B_i L_i]^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}}$$

for $\gamma \in (0,1)$. We recover the previous setting as $\gamma \to 0$. Machine producers have a monopoly on their variety j, sell their machines at price p_{ij}, and have marginal cost of ζ units of energy resources. Market-clearing for energy resources now accounts for the resources used in machine production. Each household owns a share in each machine producer. Research firms choose which sector to target and are randomly allocated to a variety within that sector, as in Acemoglu et al. (2012). They succeed in innovating with probability η, in which case they improve the quality of their machine variety to $(1+\rho)A_{ij}$ and receive a patent for its production.

The appendix solves for the equilibrium. Household demand for consumption good i now increases in resource use because household income from sector i's machines increases in R_i. The appendix shows that equilibrium resource use in the machine sectors is

$$R_z = \frac{\gamma^2}{1-\gamma} \sum_{i=1}^{N} R_i,$$

so market-clearing for energy resources becomes

$$R = \frac{\gamma^2 - \gamma + 1}{1-\gamma} \sum_{i=1}^{N} R_i + R_{N+1}.$$

6.1 The effect of improved efficiency in some consumption good sector

Accounting for the dependence of R_i on R, we have:

$$\frac{\mathrm{d}R}{\mathrm{d}y_k} = \frac{\gamma^2-\gamma+1}{1-\gamma}\sum_{i=1}^{N}\Bigg[\frac{\partial R_i}{\partial y_k}+\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial y_k}+\left(\frac{\partial R_i}{\partial p_R}+\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R}\right)\frac{\partial p_R}{\partial y_k}+\frac{\partial R_i}{\partial L}\left(\frac{\partial L}{\partial y_k}+\frac{\partial L}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y_k}\right)$$
$$+\left(\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial w}+\left(\frac{\partial R_i}{\partial p_R}+\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R}\right)\frac{\partial p_R}{\partial w}+\frac{\partial R_i}{\partial w}\right)\frac{\partial w}{\partial L}\left(\frac{\partial L}{\partial y_k}+\frac{\partial L}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y_k}\right)+\frac{\partial R_i}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y_k}\Bigg]$$
$$+\frac{\partial R_{N+1}}{\partial y_k}+\frac{\partial R_{N+1}}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y_k}.$$

Consider an improvement in A_k, for some $k \in \{1, ..., N\}$. Following the earlier analysis but now using equilibrium relations given in the appendix, we obtain:

$$\theta_{R,A_k} = (1-\gamma)\overbrace{\left(1+\frac{R_{N+1}}{\sum_{i=1}^N R_i}\right)}^{\text{Energy supply amplifier}}\overbrace{\left(1-\frac{L_{N+1}}{L}-\frac{\Theta-1}{1-\epsilon}\frac{H-L}{H}\left[\frac{L_{N+1}}{L}+\frac{\frac{\gamma(1-\gamma)}{1-\gamma+\gamma^2}L_{N+1}}{L+\frac{\gamma(1-\gamma)}{1-\gamma+\gamma^2}L_{N+1}}\frac{\sum_{i=1}^N L_i}{L}\right]\right)}^{\text{Energy supply dampener}}$$
$$\Bigg\{\overbrace{(\sigma_k-1)\frac{R_k}{R}}^{\text{PE effect}}+\overbrace{(\epsilon-\sigma_k)(\alpha_{Rk}+\alpha_{zk})\frac{R_k}{R}}^{\text{Output price effect}}$$
$$\overbrace{-(\epsilon-\sigma_k)(\alpha_{Rk}+\alpha_{zk})\frac{L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^N L_i-L_{N+1}-\frac{\frac{\gamma(1-\gamma)}{1-\gamma+\gamma^2}L_{N+1}}{L+\frac{\gamma(1-\gamma)}{1-\gamma+\gamma^2}L_{N+1}}}\frac{\sum_{i=1}^N R_i}{R}}^{\text{Labor supply effect}}$$
$$\overbrace{-(\epsilon-\sigma_k)(\alpha_{Rk}+\alpha_{zk})\frac{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^N L_i-L_{N+1}-\frac{\frac{\gamma(1-\gamma)}{1-\gamma+\gamma^2}L_{N+1}}{L+\frac{\gamma(1-\gamma)}{1-\gamma+\gamma^2}L_{N+1}}}\frac{\sum_{i=1}^N R_i}{R}}^{\text{Wage effect}}\Bigg\}.$$

There are only a few differences with respect to the expression in Section 5.1. Most obvious are the leading $1-\gamma$ (reflecting that the technology becomes irrelevant as γ approaches 1) and the replacement of α_{Rk} with $\alpha_{Rk}+\alpha_{zk}$ (reflecting that the value share of machines is now also critical to the effect of A_k on the price of consumption goods).[34] It is easy to see

[34] The other differences are the inclusion of a term in the energy supply dampener that further reduces it when $\Theta > 1$ and $\epsilon < 1$ and the inclusion of a term in the denominator of the labor supply and wage effects that shrinks their magnitudes. Both terms derive from the new dependence of R_i on R through c_i: income from the machine sector increases in the number of machines produced and thus in total resource use, so factor demand now increases in total resource use.

that Propositions 1 and 2 apply here.[35] When consumption good sectors differ only in their average energy efficiency and $\Theta = 1$, additional research effort increases energy resource use if researchers target sectors that are already relatively (in)efficient with $\sigma > 1$ and $\epsilon > (<) 1$, but additional research effort reduces energy resource use if researchers target sectors that are already relatively (in)efficient with $\sigma < 1$ and $\epsilon > (<) 1$.

6.2 Researchers' incentives

I now consider which sectors researchers target. A research firm's expected profit from targeting sector i is:

$$\Pi_i \triangleq \eta \int_0^1 \pi_{ij}((1+\rho)A_{ij})\,\mathrm{d}j,$$

where $\pi_{ij}(\cdot)$ gives the profits obtained by producers of machine variety j in sector i, as a function of their machines' efficiency (see appendix). Researchers target sector i instead of sector j if and only if the expected relative profit Π_i/Π_j is at least 1. Using equations (A-9) and (A-8), we find:

$$\frac{\Pi_i}{\Pi_j} = \frac{R_i}{R_j}.$$

Now assume that consumption good sectors differ only in the average quality of their energy conversion technology. Substituting from equations (A-7) and (A-11), we have:

$$\frac{\Pi_i}{\Pi_j} = \left(\frac{p_i}{p_j}\right)^{\sigma-\epsilon} \left(\frac{A_i}{A_j}\right)^{(1-\gamma)(\sigma-1)}.$$

Defining p_i from equation (A-10) and holding w constant (because we are interested in the difference in A_i across sectors rather than in the effect of changing some sector's A_i), we have:

$$\begin{aligned}\frac{\mathrm{d}[\Pi_i/\Pi_j]}{\mathrm{d}A_i} &= (\sigma-1)(1-\gamma)A_i^{-1}\frac{\Pi_i}{\Pi_j} + (\sigma-\epsilon)p_i^{-1}\frac{\partial p_i}{\partial A_i}\frac{\Pi_i}{\Pi_j} \\ &= A_i^{-1}\frac{\Pi_i}{\Pi_j}(1-\gamma)\Big[\sigma - 1 + (\epsilon-\sigma)(\alpha_{Ri}+\alpha_{zi})\Big]. \end{aligned} \tag{23}$$

We now have the following proposition:

Proposition 7. *Assume that $\Theta = 1$, that κ_i, σ_i, and B_i do not vary with i for $i \in \{1, ..., N\}$, and that $Var(A)$ is small relative to $\bar{A}$. Without loss of generality, let $A_i \geq A_{i-1}$ for all $i \in \{2, ..., N\}$. Consider a marginal increase in the number of research firms.*

[35] The only difference, which is irrelevant to the analysis in this section, is that the condition for super-conservation to be possible must be modified to reflect R_z.

1. *If $\sigma, \epsilon < 1$, then the additional innovation occurs in sector* 1 *and total energy resource use decreases.*

2. *If $\sigma, \epsilon > 1$, then the additional innovation occurs in sector N and total energy resource use increases.*

Proof. If equation (23) is negative (positive) for all A_i with $i \in \{1, ..., N\}$, then the marginal research firm targets the sector with the smallest (largest) A_i. Equation (23) is negative (positive) for all A_i with $i \in \{1, ..., N\}$ if $\sigma, \epsilon < (>)$ 1. Section 6.1 established that Proposition 2 still holds in this setting with monopolistically competitive machine production. The proposition follows. □

Proposition 2 described how the effect of improved efficiency can depend on whether the sector that improves its efficiency was already more or less efficient than average, and we saw in Section 6.1 that this proposition still applies in our extension to monopolistically competitive machine production. We now see that profit-driven innovation directs itself towards the least efficient sector if $\sigma, \epsilon < 1$ and directs itself towards the most efficient sector if $\sigma, \epsilon > 1$. As a result, profit-driven innovation directs itself towards sectors that reduce total energy resource use if $\sigma, \epsilon < 1$ and directs itself towards sectors that increase total energy resource use if $\sigma, \epsilon > 1$.

7 Extension 2: Costly Improvements in Energy Efficiency

Most quantitative analyses of general equilibrium rebound consider a free improvement in energy efficiency, as did the analysis in Section 5. Even in Section 6, machine producers' equilibrium mark-ups end up being independent of their machines' efficiency. I now consider improvements to energy efficiency that impose costs on firms that adopt the improved technology, as when firms improve efficiency in order to satisfy a policy mandate.

In sensitivity analyses with computable general equilibrium models, Allan et al. (2007), Allan et al. (2009), and Broberg et al. (2015) numerically assess the implications of improvements in energy efficiency that come at the cost of reducing the productivity of either labor or a value-added aggregate.[36] These experiments set the penalty on other factors' productivity to neutralize the benefits of efficiency at initial prices. I here study an improvement in efficiency that reduces the productivity of labor by δ percent: the percentage change in energy

[36]There is also a long tradition of modeling pollution regulations as reducing factor productivity (e.g., Marten et al., 2019). The analysis of θ_{R,B_k} in the appendix is informative about the general equilibrium consequences of regulations that reduce the productivity of non-energy factors. The foregoing analysis of θ_{R,A_k} is informative about the general equilibrium consequences of regulations that reduce energy efficiency in the service of pollution abatement, as with requiring scrubbers on coal-fired power plants.

resource use from a 1% increase in efficiency is no longer θ_{R,A_k} but is instead $\theta_{R,A_k} - \delta\theta_{R,B_k}$.[37] Using equation (19) and the expression in the appendix for θ_{R,B_k}, we find:

$$\theta_{R,A_k} - \delta\theta_{R,B_k} = \overbrace{\left(1 + \frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\right)}^{\text{Energy supply amplifier}} \overbrace{\left(1 - \frac{L_{N+1}}{L} - \frac{\Theta - 1}{1-\epsilon}\frac{H-L}{H}\frac{L_{N+1}}{L}\right)}^{\text{Energy supply dampener}}$$
$$\Bigg\{ \overbrace{(\sigma_k - 1)\frac{R_k}{R}}^{\text{PE effect}} + \overbrace{(\epsilon - \sigma_k)(\alpha_{Rk} - \delta\alpha_{Lk})\frac{R_k}{R}}^{\text{Output price effect}}$$
$$\overbrace{-(\epsilon - \sigma_k)(\alpha_{Rk} - \delta\alpha_{Lk})\frac{L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i - L_{N+1}}\frac{\sum_{i=1}^{N} R_i}{R}}^{\text{Labor supply effect}}$$
$$\overbrace{-(\epsilon - \sigma_k)(\alpha_{Rk} - \delta\alpha_{Lk})\frac{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i - L_{N+1}}\frac{\sum_{i=1}^{N} R_i}{R}}^{\text{Wage effect}}$$
$$\overbrace{+\,\delta(\sigma_k - 1)\frac{\left(1 + \frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\right)L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N} L_i - L_{N+1}}\frac{\sum_{i=1}^{N} R_i}{R}}^{\text{GE effect of PE change in } B_k}\Bigg\}. \tag{24}$$

The partial equilibrium effect and energy supply multipliers are as before. However, the other general equilibrium effects now scale with $\alpha_{Rk} - \delta\alpha_{Lk}$ instead of α_{Rk}. In the numerical application in Section 5.3, α_{Rk} and α_{Lk} are of similar magnitude in a few sectors but α_{Rk} is substantially smaller than α_{Lk} in most sectors. The general equilibrium channels will therefore tend to switch sign if improved energy efficiency penalizes labor productivity to an appreciable degree. For the sectors listed in Table 1, the non-multiplier general equilibrium channels will then work to reduce energy resource use on net.

The final line in equation (24) is a new general equilibrium effect. Changes in B_k affect L_k through the same partial equilibrium channels already described for the effect of A_k on R_k. These changes in L_k in turn affect all sectors' resource use by changing the wage and labor supplied. This channel reinforces the familiar partial equilibrium effect if Θ is close to 1.

Combining these observations, we have seen that costly improvements to energy efficiency make the familiar non-multiplier general equilibrium channels in Table 1 smaller (or even negative) and introduce a new channel that often reinforces the negative partial equilibrium effects in Table 1. Therefore, we can expect costly improvements in energy efficiency to

[37] In a setting focused on household energy use, Fullerton and Ta (2019) consider the purchase of more efficient appliances. They assume a particular production function for appliance efficiency.

reduce energy resource use to a greater degree than reported in Table 1. Further, superconservation may arise for sectors with small σ_k and/or large $\delta\alpha_{Lk}$. These conclusions are consistent with the numerical results in Allan et al. (2007), Allan et al. (2009), and Broberg et al. (2015).

8 Conclusions

We have decomposed the general equilibrium consequences of improvements in energy efficiency. We have seen that these consequences are likely to be especially important when improvements occur in sectors with a large value share of energy resources and an elasticity of substitution among inputs that is very different from the elasticity of substitution among consumption goods. General equilibrium consequences are also likely to be especially important when improvements occur in sectors that produce the energy resources used as inputs to consumption good production. Quantitatively, the most important general equilibrium consequences arise from expansions and contractions in the size of the energy supply sector and from the effect of the energy supply sector's technology on other sectors' energy costs.

I have emphasized how household substitution among consumption goods, labor markets, the energy supply sector, and innovation incentives affect the implications of improved efficiency. Other work has emphasized trade (e.g., Allan et al., 2007; Hanley et al., 2009; Turner, 2009; Broberg et al., 2015), distortions such as non-marginal cost pricing (Borenstein, 2015), household energy use (Lecca et al., 2014; Fullerton and Ta, 2019), the introduction of new varieties of energy-using goods (Hart, 2018), the dynamics of capital allocation (Turner, 2009), and the dynamics of capital accumulation (Wei and Liu, 2017). Future work should integrate these and other features into the present setting and should adapt recent techniques (e.g., Baqaee and Farhi, 2019) to model the network structure of the economy in more detail.

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Appendix

The first section derives direct income effects discussed in previous literature by connecting the analysis to a case in which households purchase energy resources directly. The second section analyzes the implications for total energy resource use of improvements in the energy efficiency of every sector, in the productivity of labor, and in total factor productivity. The third section solves for equilibrium in the setting with endogenous innovation. The fourth section reports additional estimates of rebound in the U.S. economy.

A Direct income effects

I here provide a partial equilibrium analysis of a representative setting in which energy services are a direct input to utility (e.g., Lecca et al., 2014; Borenstein, 2015; Chan and Gillingham, 2015), as with gasoline purchases or household appliances. Define an additional consumption good as A_0R_0, where R_0 is energy resources purchased by the household and A_0 is the household's efficiency of energy conversion. There are still N standard consumption goods, produced in quantity c_i for $i \in \{1, ..., N\}$. Hold the hours of labor supply fixed, normalized to 1. The representative household's utility is now:

$$u(R_0, C) = \left(\nu(A_0R_0)^{\frac{\chi-1}{\chi}} + (1-\nu)C^{\frac{\chi-1}{\chi}}\right)^{\frac{\chi}{\chi-1}}$$

for $\chi > 0, \neq 1$. The price of energy resources is still p_R and the price of each consumption good is still p_i.

The representative household solves the following maximization problem:

$$\max_{\{c_i\}_{i=1}^N, R_0} \left(\nu(A_0R_0)^{\frac{\chi-1}{\chi}} + (1-\nu)\left(\left(\sum_{i=1}^N c_i^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}\right)^{\frac{\chi-1}{\chi}}\right)^{\frac{\chi}{\chi-1}} \text{ subject to } p_R R_0 + \sum_{i=1}^N p_i c_i \leq w.$$

Households will choose to sell all of their endowment. As before, define P such that $\sum_{i=1}^N p_i c_i = P\,C$. Again let λ be the shadow value of the budget constraint. Households' first-order condition for C implies that $P = u_C(R_0, C)/\lambda$. Again choose the price index as the numeraire: $P = 1$. The household budget constraint then implies that $C = w - p_R R_0$ in equilibrium. The first-order condition for R_0 is

$$\lambda p_R = \nu A_0^{\frac{\chi-1}{\chi}} \left(\frac{R_0}{u(R_0, C)}\right)^{-\frac{1}{\chi}},$$

and $\lambda = u_C(R_0, C)$ is equivalent to

$$\lambda = (1-\nu)\left(\frac{C}{u(R_0, C)}\right)^{-\frac{1}{\chi}}.$$

Taking the ratio of these expressions, household demand for R_0 is

$$R_0 = \left(\frac{\nu}{1-\nu}\right)^{\chi} p_R^{-\chi} A_0^{\chi-1} C. \tag{A-1}$$

Using $C = w - p_R R_0$ and solving for R_0, we find:

$$R_0 = \frac{\left(\frac{\nu}{1-\nu}\right)^{\chi} p_R^{-\chi} A_0^{\chi-1} w}{1 + \left(\frac{\nu}{1-\nu}\right)^{\chi} p_R^{-\chi} A_0^{\chi-1} p_R}. \tag{A-2}$$

Now consider how household resource use responds to an improvement in A_0, holding p_R, each p_i, and, at first, C fixed. Differentiating equation (A-1), we have:

$$\theta_{R_0,A_0} = \chi - 1.$$

This is the standard partial equilibrium result that resource use increases if and only if resources and other inputs are substitutes. However, by holding the consumption index C fixed, this expression does not respect the budget constraint. In fact, altering purchases of R_0 allows the household to alter purchases of C, even before accounting for general equilibrium changes in prices and wages. In order to see what has been called the “direct income effect” of improved A_0, instead differentiate equation (A-2) to obtain:

$$\theta_{R_0,A_0} = (\chi - 1)(1 - \alpha_{R_0}),$$

where the budget share of resources is

$$\alpha_{R_0} \triangleq \frac{p_R R_0}{w} = \frac{\left(\frac{\nu}{1-\nu}\right)^{\chi} p_R^{1-\chi} A_0^{\chi-1}}{1 + \left(\frac{\nu}{1-\nu}\right)^{\chi} p_R^{1-\chi} A_0^{\chi-1}} \in (0,1).$$

The new term $-(\chi-1)\alpha_{R0}$ is a direct income effect. Whether households end up using more ($\chi > 1$) or less ($\chi < 1$) R_0, the magnitude of any changes is limited by the household's desire to rebalance expenditures. Further, altered purchases of other consumption goods affect demand for their resource inputs in ways analyzed in the main text. These effects are typically described as indirect income effects. They do not arise in θ_{R_0,A_0} but would appear in a derivation of θ_{R,A_0}.

Let Υ represent the price elasticity of demand for R_0. Following Lecca et al. (2014), we have:

$$\Upsilon = \chi - (\chi - 1)\alpha_{R0}.$$

See also Gørtz (1977). Substituting, we find

$$\theta_{R_0,A_0} = \Upsilon - 1.$$

We have backfire if household demand for R_0 is elastic and have energy resource savings if household demand for R_0 is inelastic.

B Additional theoretical results

B.1 Improved energy efficiency in every sector

I here consider the consequences of improving the energy efficiency of a process or engine that is used in all sectors. Formally, consider improving every A_i by 1%, for $i \in \{1, ..., N+1\}$:

$$\begin{aligned}\sum_{i=1}^{N+1} \theta_{R,A_i} =& \left(1 + \frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\right)\left(1 - \frac{L_{N+1}}{L} - \frac{\Theta - 1}{1-\epsilon}\frac{H-L}{H}\frac{L_{N+1}}{L}\right) \\ & \left\{ \sum_{i=1}^{N+1}(\sigma_i - 1)\frac{R_i}{R} + \frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\left(\sum_{i=1}^{N}\sigma_i\frac{R_i}{R} + \sigma_{N+1}\frac{L_{N+1}\left(\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}+1\right)}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{j=1}^{N} L_j - L_{N+1}}\frac{\sum_{i=1}^{N} R_i}{R}\right)\right. \\ & \left. + \frac{R}{\sum_{i=1}^{N} R_i}\sum_{i=1}^{N}(\epsilon - \sigma_i)\alpha_{Ri}\left(\frac{R_i}{R} - \frac{L_i\left(\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}+1\right)}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{j=1}^{N} L_j - L_{N+1}}\frac{\sum_{j=1}^{N} R_j}{R}\right)\right\}.\end{aligned}$$

We now have the following proposition:

Proposition 8. *Assume that $\Theta = 1$ and that κ_i, σ_i, B_i, and A_i do not vary with i for $i \in \{1, ..., N\}$. Consider an improvement in every sector's technology A_i.*

1. $\sum_{i=1}^{N+1} \theta_{R,A_i} > 0$ *if* $\sigma > 1$.
2. $\sum_{i=1}^{N+1} \theta_{R,A_i} > 0$ *if* $\sigma_{N+1} > \frac{\sum_{i=1}^{N} L_i}{L}\frac{R}{R_{N+1}}$.
3. *As $\sigma_{N+1} \to 0$, $\sum_{i=1}^{N+1} \theta_{R,A_i} > 0$ if and only if $\sigma > 1$ for all $i \in \{1, ..., N\}$.*
4. *If $R_{N+1}/L_{N+1} > \sum_{i=1}^{N} R_i / \sum_{i=1}^{N} L_i$, then super-conservation can occur only if $\sigma < 1$ and σ_{N+1} is sufficiently small. If $R_{N+1}/L_{N+1} < \sum_{i=1}^{N} R_i / \sum_{i=1}^{N} L_i$, then super-conservation does not occur for any $\sigma, \sigma_{N+1} > 0$.*

Proof. Because each consumption good sector has identical parameters, we have $R_j / \sum_{i=1}^{N} R_i = L_j / \sum_{i=1}^{N} L_i = 1/N$ for all $j \in \{1, ..., N\}$. Using $\Theta = 1$, we then have:

$$\begin{aligned}\sum_{i=1}^{N+1} \theta_{R,A_i} =& \frac{\sum_{i=1}^{N} L_i}{L}\left\{(\sigma - 1) + (\sigma_{N+1} - 1)\frac{R_{N+1}}{\sum_{j=1}^{N} R_j} + \sigma\frac{R_{N+1}}{\sum_{j=1}^{N} R_j} + \sigma_{N+1}\frac{L_{N+1}}{\sum_{i=1}^{N} L_i}\frac{R_{N+1}}{\sum_{j=1}^{N} R_j}\right\} \\ =& \frac{\sum_{i=1}^{N} L_i}{L}\left\{(\sigma - 1)\frac{R}{\sum_{j=1}^{N} R_j} + \sigma_{N+1}\frac{L}{\sum_{i=1}^{N} L_i}\frac{R_{N+1}}{\sum_{j=1}^{N} R_j}\right\}.\end{aligned}$$

The first three parts of the proposition follow.

To prove the final part of the proposition, note that

$$Savings^{GE} > Savings^{eng} \Leftrightarrow -\sum_{i=1}^{N+1} \theta_{R,A_i} R > \sum_{i=1}^{N+1} R_i \Leftrightarrow 0 > \sum_{i=1}^{N+1} \theta_{R,A_i} + 1.$$

For given R_i and L_i, this must hold at very small σ and σ_{N+1} if it holds anywhere. As $\sigma, \sigma_{N+1} \to 0$, this condition becomes

$$0 > 1 - \frac{\sum_{i=1}^{N} L_i}{L} \frac{R}{\sum_{j=1}^{N} R_j}.$$

The result follows straightforwardly from this expression and the fact that, under the conditions of the proposition, $\sum_{i=1}^{N} L_i/L > \sum_{i=1}^{N} R_i/R$ if and only if $R_{N+1}/L_{N+1} > \sum_{i=1}^{N} R_i/\sum_{i=1}^{N} L_i$.
□

B.2 Improved efficiency of the non-energy input in a consumption good sector

Now consider the consequences of improving B_k in some sector $k \in \{1, ..., N\}$:

$$\begin{aligned}\theta_{R,B_k} = & \overbrace{\left(1 + \frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\right)}^{\text{Energy supply amplifier}} \overbrace{\left(1 - \frac{L_{N+1}}{L} - \frac{\Theta - 1}{1 - \epsilon} \frac{H - L}{H} \frac{L_{N+1}}{L}\right)}^{\text{Energy supply dampener}} \\ & \Bigg\{ \overbrace{(\epsilon - \sigma_k)\alpha_{Lk} \frac{R_k}{R}}^{\text{Output price effect}} \overbrace{- [(\epsilon - \sigma_k)\alpha_{Lk} - (1 - \sigma_k)] \frac{L_k}{\frac{1-\epsilon}{\Theta-1} \frac{H}{H-L} \sum_{i=1}^{N} L_i - L_{N+1}} \frac{\sum_{i=1}^{N} R_i}{R}}^{\text{Labor supply effect}} \\ & \overbrace{- [(\epsilon - \sigma_k)\alpha_{Lk} - (1 - \sigma_k)] \frac{\frac{1-\epsilon}{\Theta-1} \frac{H}{H-L} L_k}{\frac{1-\epsilon}{\Theta-1} \frac{H}{H-L} \sum_{i=1}^{N} L_i - L_{N+1}} \frac{\sum_{i=1}^{N} R_i}{R}}^{\text{Wage effect}} \Bigg\}.\end{aligned}$$

This is nearly identical to θ_{R,A_k}, but for two differences. First, we now have α_{Lk} in place of α_{Rk}, reflecting that the response of output prices to B_k depends on the value share of L rather than the value share of R. Second, we now have $1 - \sigma_k$ appearing in the labor supply and wage effects in place of $\sigma_k - 1$ appearing in the partial equilibrium effect. The improvement in B_k does indeed affect L_k through partial equilibrium channels that are proportional to $\sigma_k - 1$, as was true for the response of R_k to A_k. However, that partial equilibrium effect maps into energy resource use only through general equilibrium channels. An increase in L_k on net reduces energy resource use if $\Theta = 1$, as the wage effect dominates the labor supply effect. Finally, note that $\theta_{R,B_k} \to \theta_{R,A_k}$ as $\kappa \to 0.5$ and $\sigma_k \to 1$, reflecting that the two types of factor-augmenting technical change are equivalent in a Cobb-Douglas specification.

The following proposition is the analogue of Proposition 1:

Proposition 9. *Assume that $\Theta = 1$ and either $N = 1$ or κ_i, σ_i, B_i, and A_i do not vary with i for $i \in \{1, ..., N\}$. Consider an improvement in B_k for some $k \in \{1, ..., N\}$. Then $\theta_{R,B_k} > 0$ if and only if $\sigma < 1$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j / \sum_{i=1}^N R_i = L_j / \sum_{i=1}^N L_i = 1/N$ for all $j \in \{1, ..., N\}$. The proposition follows. □

The condition for backfire is now reversed from the case of an improvement in A_k. The usual partial equilibrium analysis suggests that demand for L_k increases if and only if $\sigma > 1$. When the supply of labor is fixed ($\Theta = 1$), increasing demand for L_k reduces energy resource use through the wage effect, as described in the main text. Because $\sigma < 1$ may be the more common case empirically, we may be especially concerned about the potential for backfire when improving the productivity of the non-energy input. This result formalizes some previous conjectures (e.g., Saunders, 1992; Sorrell, 2007; Saunders, 2013).

We now have the analogues of Proposition 2, Corollary 3, and Proposition 12:

Proposition 10. *Assume that $\Theta = 1$, that κ_i, σ_i, and B_i do not vary with i for $i \in \{1, ..., N\}$, and that $Var(A)$ is small relative to $\bar{A}$. Consider an improvement in B_k for some $k \in \{1, ..., N\}$.*

1. *If $\sigma < 1$ and either $\sigma \leq \epsilon < 1$ or $(A_k - \bar{A})(\sigma - \epsilon) > 0$, then $\theta_{R,B_k} > 0$.*
2. *If $\sigma > 1$ and either $\sigma \geq \epsilon > 1$ or $(A_k - \bar{A})(\sigma - \epsilon) > 0$, then $\theta_{R,B_k} < 0$.*

Proof. If $\Theta = 1$, then

$$\theta_{R,B_k} = \frac{\sum_{i=1}^N L_i}{L} \left\{ (1 - \sigma_k) \frac{L_k}{\sum_{i=1}^N L_i} + (\sigma_k - \epsilon)\alpha_{Lk} \left(\frac{L_k}{\sum_{i=1}^N L_i} - \frac{R_k}{\sum_{i=1}^N R_i} \right) \right\}. \tag{A-3}$$

Following the proof of Proposition 2,

$$\frac{L_k}{\sum_{i=1}^N L_i} - \frac{R_k}{\sum_{i=1}^N R_i} \approx \frac{N Z_k \bar{Z}}{\left[\sum_{i=1}^N A_i^{\sigma-1} Z_i\right] \left[\sum_{i=1}^N Z_i\right]} \left\{ \bar{A}^{\sigma-1} - A_k^{\sigma-1} \right\}. \tag{A-4}$$

If $\sigma < 1$, then (A-4) is strictly positive if $A_k > \bar{A}$. In that case, all terms in (A-3) are positive if $\epsilon \leq \sigma$. If, instead, $\epsilon \in (\sigma, 1)$ with $A_k > \bar{A}$, then (A-3) is strictly positive because $1 - \sigma > (\epsilon - \sigma)\alpha_{Lk}$ and $\frac{L_k}{\sum_{i=1}^N L_i} > \frac{L_k}{\sum_{i=1}^N L_i} - \frac{R_k}{\sum_{i=1}^N R_i}$. If $\sigma < 1$ and $A_k < \bar{A}$, then (A-4) is strictly negative, but if in addition, $\epsilon \geq \sigma$, then all terms in (A-3) are positive. We have established that θ_{R,B_k} is strictly positive if $\sigma \leq \epsilon < 1$, if $A_k > \bar{A}$ with $\epsilon \leq \sigma$, and if $A_k < \bar{A}$ with $\epsilon > 1$.

If $\sigma > 1$, then (A-4) is strictly negative if $A_k > \bar{A}$. In that case, all terms in (A-3) are negative if $\epsilon \leq \sigma$. If $\sigma > 1$ and $A_k < \bar{A}$, then (A-4) is strictly positive, but if in addition, $\epsilon \geq \sigma$, then all terms in (A-3) are negative. Finally, if $\sigma > 1$, $A_k < \bar{A}$, and $\epsilon \in (1, \sigma)$, then (A-3) is strictly negative because $\sigma - 1 > (\sigma - \epsilon)\alpha_{Lk}$ and $\frac{L_k}{\sum_{i=1}^N L_i} > \frac{L_k}{\sum_{i=1}^N L_i} - \frac{R_k}{\sum_{i=1}^N R_i}$. We have established that θ_{R,B_k} is strictly negative if $1 < \epsilon \leq \sigma$, if $A_k > \bar{A}$ with $\epsilon < 1$, and if $A_k < \bar{A}$ with $\epsilon > \sigma$.

□

Corollary 11. *Assume that $\Theta = 1$, that κ_i, σ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$, and that $Var(B)$ is small relative to $\bar{B}$. Consider an improvement in B_k for some $k \in \{1, ..., N\}$.*

1. *If $\sigma < 1$ and either $\sigma \leq \epsilon < 1$ or $(B_k - \bar{B})(\sigma - \epsilon) < 0$, then $\theta_{R,B_k} > 0$.*
2. *If $\sigma > 1$ and either $\sigma \geq \epsilon > 1$ or $(B_k - \bar{B})(\sigma - \epsilon) < 0$, then $\theta_{R,B_k} < 0$.*

Proof. Follows from the proofs of Corollary 3 and Proposition 10. □

Proposition 12. *Consider an improvement in B_k for some $k \in \{1, ..., N\}$.*

1. *$\theta_{R,B_k} \to 0$ as $\kappa_k \to 0$.*
2. *If $\Theta = 1$ and $\kappa_k \to 1$, then $\theta_{R,B_k} > 0$ if and only if $\epsilon < 1$.*

Proof. From equation (13), $L_k \to 0$ as $\kappa_k \to 0$. We also then have $\alpha_{Lk} \to 0$ as $\kappa_k \to 0$. The first result follows.

From equation (14), $R_k \to 0$ as $\kappa_k \to 1$. We also then have $\alpha_{Lk} \to 1$ as $\kappa_k \to 1$. Then $1 - \sigma_k - (\epsilon - \sigma_k)\alpha_{Lk} = -(\epsilon - 1)$. The second result follows from noting that the general equilibrium multiplier is positive for $\Theta = 1$ and that the labor supply effect vanishes for $\Theta = 1$.

□

The logic is as in the main text, noting that an increase in L_k corresponds to a decrease in R_k when wage effects dominate labor supply effects.

B.3 Improved efficiency of the non-energy input in the energy supply sector

Now consider the consequences of improving B_{N+1}:

$$\theta_{R,B_{N+1}} = \overbrace{\left(1+\frac{R_{N+1}}{\sum_{i=1}^{N} R_i}\right)}^{\text{Energy supply amplifier}} \overbrace{\left(1-\frac{L_{N+1}}{L}-\frac{\Theta-1}{1-\epsilon}\frac{H-L}{H}\frac{L_{N+1}}{L}\right)}^{\text{Energy supply dampener}}$$
$$\left\{ \overbrace{\sum_{i=1}^{N}(\sigma_i+(\epsilon-\sigma_i)\alpha_{Ri})\frac{R_i}{R}}^{\text{Resource supply effect}} + \overbrace{\frac{L_{N+1}-\sum_{i=1}^{N}(\epsilon-\sigma_i)\alpha_{Ri}L_i}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N}L_i-L_{N+1}}\frac{\sum_{i=1}^{N}R_i}{R}}^{\text{Labor supply effect}} \right.$$
$$\left. + \overbrace{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\frac{L_{N+1}-\sum_{i=1}^{N}(\epsilon-\sigma_i)\alpha_{Ri}L_i}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^{N}L_i-L_{N+1}}\frac{\sum_{i=1}^{N}R_i}{R}}^{\text{Wage effect}} \right\}.$$

We then have:

Proposition 13. *Consider an improvement in B_{N+1}.*

1. *If $\Theta = 1$ and either $N = 1$ or κ_i, σ_i, B_i, and A_i do not vary with i for $i \in \{1, ..., N\}$, then $\theta_{R,B_{N+1}} > 0$ and there exists $\hat{\sigma} \in (0, 1)$ such that $\theta_{R,B_{N+1}} > 1$ if and only if $\sigma > \hat{\sigma}$.*
2. *If $\Theta = 1$, each $\sigma_i \approx 1$, and κ_i does not vary with i for $i \in \{1, ..., N\}$, then $\theta_{R,B_{N+1}} \approx 1$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j / \sum_{i=1}^{N} R_i = L_j / \sum_{i=1}^{N} L_i = 1/N$ for all $j \in \{1, ..., N\}$. Using $\Theta = 1$, we then have:

$$\theta_{R,B_{N+1}} = \frac{\sum_{i=1}^{N} L_i}{L}\left\{\sigma + \frac{L_{N+1}}{\sum_{i=1}^{N} L_i}\right\}.$$

This increases in σ and is equal to 1 if $\sigma = 1$. The first part of the proposition follows.

The value share of energy resources in each consumption good sector is approximately κ_i when $\sigma_i \approx 1$ and is then independent of i when, in addition, κ_i is independent of i. Substituting into $\theta_{R,B_{N+1}}$ and using $\Theta = 1$, we have:

$$\theta_{R,B_{N+1}} \approx \frac{\sum_{i=1}^{N} L_i}{L}\left\{1 + \frac{L_{N+1}}{\sum_{i=1}^{N} L_i}\right\} = 1.$$

The second part of the proposition follows.

□

The primary difference with respect to the analysis of Section 5.2 is that now there are no partial equilibrium effects. Instead, the partial equilibrium change in L affects R through the wage and labor supply effects. Partial equilibrium substitution towards L in energy production is offset by the decline in the price of the energy output, so that we are left with the "engineering" savings in L_{N+1} increasing energy resource use through the wage effect (if $\Theta = 1$). As a result, backfire must arise when the output price channels in the resource supply and wage effects offset each other (whether because $N = \Theta = 1$ or because sectors are symmetric with $\Theta = 1$). That backfire is driven both by the adjustment in the wage and also by consumption good producers' substitution towards newly cheap energy resources. In fact, not only does backfire occur, but a 1% improvement in L_{N+1} can increase total energy resource use by more than 1%. Finally, if consumption good sectors are Cobb-Douglas with similar factor shares, then energy resource use increases by the same percentage that B_{N+1} improved.

B.4 Improved total factor productivity in a consumption good sector

Now consider improving total factor productivity in some consumption good sector. Note that $\theta_{R,TFP_k} = \theta_{R,A_k} + \theta_{R,B_k}$.[38] We then have:

$$\begin{aligned}\theta_{R,TFP_k} =&\left(1 + \frac{R_{N+1}}{\sum_{i=1}^N R_i}\right)\left(1 - \frac{L_{N+1}}{L} - \frac{\Theta - 1}{1-\epsilon}\frac{H-L}{H}\frac{L_{N+1}}{L}\right)\\ &[\epsilon - 1]\left[\frac{R_k}{R} - \frac{\left(\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}+1\right)L_k}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^N L_i - L_{N+1}}\frac{\sum_{i=1}^N R_i}{R}\right].\end{aligned}$$

Proposition 14. *Consider an improvement in TFP_k for some $k \in \{1, ..., N\}$.*

1. *If $\Theta = 1$ and κ_i, σ_i, B_i, and A_i do not vary with i for $i \in \{1, ..., N\}$, then $\theta_{R,TFP_k} = 0$.*
2. *Assume that $\Theta = 1$, that κ_i, σ_i, and B_i do not vary with i for $i \in \{1, ..., N\}$, and that $Var(A)$ is small relative to $\bar{A}$. Then:*
   (a) If $\sigma < 1$, then $\theta_{R,TFP_k} < 0$ if and only if $(A_k - \bar{A})(\epsilon - 1) > 0$.
   (b) If $\sigma > 1$, then $\theta_{R,TFP_k} > 0$ if and only if $(A_k - \bar{A})(\epsilon - 1) > 0$.

Proof. If $\Theta = 1$, then

$$\theta_{R,TFP_k} = \frac{\sum_{i=1}^N L_i}{L}[\epsilon - 1]\left[\frac{R_k}{\sum_{i=1}^N R_i} - \frac{L_k}{\sum_{i=1}^N L_i}\right].$$

[38] We can write $B_k \triangleq TFP_k \tilde{B}_k$ and $A_k \triangleq TFP_k \tilde{A}_k$ in sector k firms' production functions. The claim follows from totally differentiating $R(A_k, B_k)$ with respect to TFP_k.

Because each consumption good sector has identical parameters, we have $R_j/\sum_{i=1}^N R_i = L_j/\sum_{i=1}^N L_i = 1/N$ for all $j \in \{1, ..., N\}$. The first part of the proposition follows. The second part follows from θ_{R,TFP_k} and the proof of Proposition 2. □

Now consider improving total factor productivity in all consumption good sectors at once.

Proposition 15. *If $\Theta = 1$, then $\sum_{i=1}^N \theta_{R,TFP_i} = 0$.*

Proof. Follows directly from the given expression for θ_{R,TFP_k}. □

B.5 Improved total factor productivity of energy resource production

Following previous analysis, an improvement in total factor productivity in the energy supply sector yields the following change in total energy resource use:

$$\begin{aligned}\theta_{R,TFP_{N+1}} =& \left(1 + \frac{R_{N+1}}{\sum_{i=1}^N R_i}\right)\left(1 - \frac{L_{N+1}}{L} - \frac{\Theta - 1}{1-\epsilon}\frac{H-L}{H}\frac{L_{N+1}}{L}\right) \\ &\Bigg\{(\sigma_{N+1} - 1)\frac{R_{N+1}}{R} + \frac{R}{\sum_{i=1}^N R_i}\sum_{i=1}^N \sigma_i \frac{R_i}{R} \\ &+ \left(1 + \frac{R_{N+1}}{\sum_{i=1}^N R_i}\sigma_{N+1}\right)\frac{\left(\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L} + 1\right)L_{N+1}}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^N L_i - L_{N+1}}\frac{\sum_{i=1}^N R_i}{R} \\ &+ \frac{R}{\sum_{i=1}^N R_i}\sum_{i=1}^N (\epsilon - \sigma_i)\alpha_{Ri}\left(\frac{R_i}{R} - \frac{\left(\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L} + 1\right)L_i}{\frac{1-\epsilon}{\Theta-1}\frac{H}{H-L}\sum_{i=1}^N L_i - L_{N+1}}\frac{\sum_{j=1}^N R_j}{R}\right)\Bigg\}.\end{aligned}$$

We then have:

Proposition 16. *If $\Theta = 1$ and either $N = 1$ or κ_i, σ_i, B_i, and A_i do not vary with i for $i \in \{1, ..., N\}$, then there exists $\hat{\sigma} < 1$ such that $\theta_{R,TFP_{N+1}} > 0$ if $\sigma_{N+1} + \sigma > \hat{\sigma}$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j/\sum_{i=1}^N R_i = L_j/\sum_{i=1}^N L_i = 1/N$ for all $j \in \{1, ..., N\}$. Using $\Theta = 1$, we then have:

$$\theta_{R,TFP_{N+1}} = \frac{\sum_{i=1}^N L_i}{L}\left\{\left(\sigma_{N+1}\frac{L}{\sum_{i=1}^N L_i} + \sigma - 1\right)\frac{R_{N+1}}{\sum_{i=1}^N R_i} + \sigma + \frac{L_{N+1}}{\sum_{i=1}^N L_i}\right\}.$$

The proposition follows. □

Proposition 17. *Assume that $\Theta = 1$ and that, for $i \in \{1, ..., N\}$, each $\sigma_i \approx 1$ and κ_i does not vary with i. Then $\theta_{R,TFP_{N+1}} > 1$.*

Proof. The value share of energy resources in each consumption good sector is approximately κ_i when $\sigma_i \approx 1$ and is then independent of i when, in addition, κ_i is independent of i. Substituting into $\theta_{R,B_{N+1}}$ and using $\Theta = 1$, we have:

$$\theta_{R,TFP_{N+1}} \approx \frac{\sum_{i=1}^{N} L_i}{L} \left\{ \sigma_{N+1} \frac{L}{\sum_{i=1}^{N} L_i} \frac{R_{N+1}}{\sum_{i=1}^{N} R_i} + 1 + \frac{L_{N+1}}{\sum_{i=1}^{N} L_i} \right\}.$$

The proposition follows.

□

C Solving for equilibrium in the setting with endogenous innovation

Consumption good firms now solve:

$$\max_{L_i, R_i, z_{ij}} \left\{ p_i \left(\kappa_i [B_i L_i]^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \kappa_i) \left[R_i^{1-\gamma} \int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j \right]^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}} - w L_i - p_R R_i - \int_0^1 p_{ij} z_{ij} \, \mathrm{d}j \right\}.$$

The first-order conditions become:

$$\begin{aligned}
w =& p_i \kappa_i B_i^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{L_i}{c_i} \right)^{-\frac{1}{\sigma_i}}, \\
p_R =& p_i (1 - \kappa_i)(1 - \gamma) \left[\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j \right]^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{R_i}{c_i} \right)^{-\frac{1}{\sigma_i}} R_i^{-\gamma \frac{\sigma_i - 1}{\sigma_i}}, \qquad \text{(A-5)} \\
p_{ij} =& p_i (1 - \kappa_i) \gamma R_i^{(1-\gamma)\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j}{c_i} \right)^{-\frac{1}{\sigma_i}} A_{ij}^{1-\gamma} z_{ij}^{\gamma - 1}.
\end{aligned}$$

Rearranging the latter condition, we find

$$z_{ij} = \left[\frac{p_i}{p_{ij}} (1 - \kappa_i) \gamma \left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j}{c_i} \right)^{-\frac{1}{\sigma_i}} \right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i - 1}{\sigma_i}} A_{ij}.$$

Demand is isoelastic, so the monopolist chooses a constant mark-up over marginal cost: $p_{ij} = \zeta p_R / \gamma$.[39] Substituting into z_{ij}, profit-maximizing machine production is:

$$z_{ij} = \left[p_i \frac{\gamma^2}{\zeta p_R} (1 - \kappa_i) \left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j}{c_i} \right)^{-\frac{1}{\sigma_i}} \right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i - 1}{\sigma_i}} A_{ij}.$$

[39] In line with much literature, the monopolist does not account for its effect on $\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j$.

Note that:

$$\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma}\,\mathrm{d}j = \int_0^1 A_{ij}^{1-\gamma}\left(\left[p_i\frac{\gamma^2}{\zeta p_R}(1-\kappa_i)\left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma}\,\mathrm{d}j}{c_i}\right)^{-\frac{1}{\sigma_i}}\right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i-1}{\sigma_i}} A_{ij}\right)^{\gamma}\mathrm{d}j.$$

Rearranging, we find:

$$\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma}\,\mathrm{d}j = \left(\left[p_i\frac{\gamma^2}{\zeta p_R}(1-\kappa_i)\left(\frac{1}{c_i}\right)^{-\frac{1}{\sigma_i}}\right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i-1}{\sigma_i}}\right)^{\gamma\frac{\sigma_i(1-\gamma)}{\gamma+\sigma_i(1-\gamma)}} A_i^{\frac{\sigma_i(1-\gamma)}{\gamma+\sigma_i(1-\gamma)}}, \tag{A-6}$$

where A_i is the average technology in sector i. Equation (A-5) then implies:

$$R_i = p_R^{-\sigma_i}(1-\kappa_i)^{\sigma_i}(1-\gamma)^{\gamma+\sigma_i(1-\gamma)}\left(\frac{\gamma^2}{\zeta}\right)^{(\sigma_i-1)\gamma} A_i^{(\sigma_i-1)(1-\gamma)} p_i^{\sigma_i} c_i. \tag{A-7}$$

Demand for R_{N+1} is unchanged from equation (8), and equation (6) still defines p_R as a function of w. Machine production in sector i is:

$$\begin{aligned}
\int_0^1 z_{ij}\mathrm{d}j &= \left[p_i\frac{\gamma^2}{\zeta p_R}(1-\kappa_i)\left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma}\,\mathrm{d}j}{c_i}\right)^{-\frac{1}{\sigma_i}}\right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i-1}{\sigma_i}} A_i \\
&= \left[p_i\frac{\gamma^2}{\zeta p_R}(1-\kappa_i)\right]^{\frac{\sigma_i}{\gamma+\sigma_i(1-\gamma)}} c_i^{\frac{1}{\gamma+\sigma_i(1-\gamma)}} R_i^{\frac{\sigma_i-1}{\sigma_i}\frac{\sigma_i(1-\gamma)}{\gamma+\sigma_i(1-\gamma)}} A_i^{\frac{(1-\gamma)(\sigma_i-1)}{\gamma+\sigma_i(1-\gamma)}} \\
&= \frac{\gamma^2}{\zeta}\frac{1}{1-\gamma}R_i,
\end{aligned} \tag{A-8}$$

where I substitute from equations (A-6) and (A-7) and simplify. Finally, producers of machine variety j in sector i obtain profits of

$$\pi_{ij}(A_{ij}) \triangleq \frac{\zeta p_R}{\gamma} z_{ij} - \zeta p_R z_{ij} = \frac{1-\gamma}{\gamma}\zeta p_R z_{ij}.$$

Substituting for p_R from equation (6), we have:

$$\pi_{ij}(A_{ij}) = \frac{1-\gamma}{\gamma}\zeta z_{ij}\left[\frac{B_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1}{1-\sigma_{N+1}}} w. \tag{A-9}$$

Consumption good firms' zero-profit condition becomes

$$p_i c_i = wL_i + p_R R_i + \int_0^1 p_{ij} z_{ij}\,\mathrm{d}j,$$

which yields

$$p_i = \left\{ w^{1-\sigma_i} \kappa_i^{\sigma_i} B_i^{\sigma_i - 1} + \frac{1}{1-\gamma} p_R^{1-\sigma_i} (1-\kappa_i)^{\sigma_i} (1-\gamma)^{\gamma + \sigma_i(1-\gamma)} \left(\frac{\gamma^2}{\zeta}\right)^{(\sigma_i - 1)\gamma} A_i^{(\sigma_i-1)(1-\gamma)} \right\}^{\frac{1}{1-\sigma_i}}.$$

Substituting for p_R from equation (6), we obtain:

$$\begin{aligned} p_i =& w \Bigg\{ \kappa_i^{\sigma_i} B_i^{\sigma_i - 1} + \frac{1}{1-\gamma} (1-\kappa_i)^{\sigma_i} (1-\gamma)^{\gamma + \sigma_i(1-\gamma)} \left(\frac{\gamma^2}{\zeta}\right)^{(\sigma_i - 1)\gamma} \\ & \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} A_i^{(\sigma_i-1)(1-\gamma)} \Bigg\}^{\frac{1}{1-\sigma_i}}. \end{aligned} \tag{A-10}$$

The representative household's budget constraint is now $\sum_{i=1}^N p_i c_i \leq wL + \sum_{i=1}^N \int_0^1 \pi_{ij}(A_{ij}) \, \mathrm{d}j$. The household budget constraint implies that $C = wL + \sum_{i=1}^N \int_0^1 \pi_{ij}(A_{ij}) \, \mathrm{d}j$ in equilibrium. Aggregate household demand for good i becomes

$$\begin{aligned} c_i =& p_i^{-\epsilon} \left(wL + \sum_{i=1}^N \int_0^1 \pi_{ij}(A_{ij}) \, \mathrm{d}j \right) \\ =& p_i^{-\epsilon} w \left(L + \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1}{1-\sigma_{N+1}}} \sum_{i=1}^N \gamma R_i \right), \end{aligned} \tag{A-11}$$

where we substitute from equation (A-9) and then from equation (A-8).

Following the analysis in Section 3 but using equations (A-10) and (A-11), we obtain equilibrium demand for labor, for $i \in \{1, ..., N\}$:

$$\begin{aligned} L_i =& B_i^{\sigma_i - 1} \kappa_i^{\sigma_i} w^{1-\epsilon} \left(L + \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1}{1-\sigma_{N+1}}} \sum_{i=1}^N \gamma R_i \right) \\ & \Bigg\{ \kappa_i^{\sigma_i} B_i^{\sigma_i - 1} + \frac{1}{1-\gamma} (1-\kappa_i)^{\sigma_i} (1-\gamma)^{\gamma + \sigma_i(1-\gamma)} \left(\frac{\gamma^2}{\zeta}\right)^{(\sigma_i - 1)\gamma} \\ & \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} A_i^{(\sigma_i-1)(1-\gamma)} \Bigg\}^{\frac{\sigma_i - \epsilon}{1-\sigma_i}}. \end{aligned} \tag{A-12}$$

Demand for L_{N+1} is unchanged from equation (7). Market-clearing for labor implies that

$$\begin{aligned} L &= \sum_{i=1}^{N+1} L_i \\ &= \sum_{i=1}^{N} L_i + B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}} \left[\frac{B_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} R. \end{aligned} \tag{A-13}$$

The analogue of equation (16) follows. Market-clearing for energy resources implies that

$$R = \sum_{i=1}^{N+1} R_i + R_z,$$

where R_z is energy resource demand by machines. Using equation (A-8), we have:

$$R_z \triangleq \sum_{i=1}^{N} \zeta \int_0^1 z_{ij} \mathrm{d}j = \frac{\gamma^2}{1-\gamma} \sum_{i=1}^{N} R_i.$$

Market-clearing, R_z, and R_{N+1} from equation (8) imply:

$$\sum_{i=1}^{N} R_i = \frac{1-\gamma}{1-\gamma+\gamma^2} (R - R_{N+1}) = \frac{1-\gamma}{1-\gamma+\gamma^2} \left(1 - (1-\kappa_{N+1})^{\sigma_{N+1}} A_{N+1}^{\sigma_{N+1}-1} \right) R. \tag{A-14}$$

Substituting into equation (A-11) and then into equation (A-7) yields an expression for R_i analogous to equation (10) in the main analysis. From here, we can follow steps similar to those in the main analysis to solve for and analyze equilibrium.

D Additional numerical results

D.1 Allowing $\Theta \neq 1$

I here generalize the quantitative analysis to allow for $\Theta \neq 1$. The value of Θ in Marten and Garbaccio (2018) varies by household, but aggregating yields a value of around 2.5 for the representative household (personal communication). I now require an estimate of $(H-L)/H$. As a rough proxy, I use the U-6 measure of unemployment, which accounts for marginally attached and part-time workers. This measure was around 8.5% in 2017.[40] Table A-1 is the analogue of Table 1. The results are only slightly impacted by the change in Θ. Endogenizing labor supply reduces the energy supply dampener, which mitigates

[40] `https://fred.stlouisfed.org/series/U6RATE`

any energy resource savings. (However, the combined multiplier is still greater than 1.) The wage effect is roughly unchanged for the ten sectors with the greatest energy resource savings, but the labor supply effect now works to further reduce energy resource use. On net, the change in the energy supply dampener dominates in these ten sectors, so that improving their efficiency now generates slightly smaller energy resource savings (i.e., slightly greater rebound) than reported in the main text. Aggregating over all consumption good sectors, the combined wage and labor supply effects produce smaller reductions in energy resource use than did the wage effect reported in the main text, which reinforces the change in the energy supply dampener and increases rebound slightly. Finally, in the energy supply sector, the combined wage and labor supply effects work to increase energy resource use to an even greater degree than did the wage effect reported in the main text, which again reinforces the change in the energy supply dampener and mitigates energy resource savings.

D.2 Full results for $\Theta = 1$

Table A-2 reports the results omitted from the main text, reporting θ_{R,A_k} for each non-energy supply sector. It aggregates the output price and wage effects into a single "GE" effect. Sectors are sorted from most negative to most positive total effect. The energy supply multipliers are as reported in the main text.

Table A-1: Decomposing the percentage change in energy resource use (θ_{R,A_k}) from a 1% increase in sector k's energy efficiency, with $\Theta = 2.5$. Also reports rebound, as a percentage of engineering savings. The listed non-energy supply sectors are the ten that yield the greatest reduction in energy resource use.

Energy supply multipliers (same for all sectors)									
Amplifier	1.23								
Dampener	0.95								
Combined	1.17								
	Parameters				Channels				
Sector	σ_k	α_{Rk}	R_k/R	L_k/L	PE	Outp.*	LS+W**	Total	Reb.***
Other real estate	0.68	0.081	0.178	0.011	-0.057	0.0032	-0.0004	-0.063	64
Construction	0.15	0.022	0.058	0.061	-0.049	0.0010	-0.0020	-0.059	-1
Food services and drinking places	0.22	0.029	0.039	0.034	-0.031	0.0008	-0.0013	-0.037	7
Truck transportation	0.42	0.094	0.055	0.011	-0.032	0.0025	-0.0010	-0.036	35
Wholesale trade	0.24	0.011	0.035	0.063	-0.027	0.0002	-0.0009	-0.032	9
Other retail	0.24	0.018	0.031	0.035	-0.023	0.0004	-0.0008	-0.028	9
Other transportation and support activities	0.39	0.078	0.031	0.010	-0.019	0.0012	-0.0008	-0.022	30
Federal Reserve banks, credit intermediation, and related activities	0.09	0.012	0.019	0.032	-0.017	0.0002	-0.0006	-0.021	-9
Hospitals	0.16	0.011	0.017	0.047	-0.014	0.0001	-0.0007	-0.017	-2
Food and beverage and tobacco products	0.19	0.010	0.017	0.013	-0.014	0.0001	-0.0002	-0.016	5
All non-energy supply sectors					-0.49	0.014	-0.00034	-0.58	29
Energy supply	0.42	0.071	0.19	0.022	-0.11	0.077	0.00036	-0.037	80
All sectors					-0.60	0.091	0.00002	-0.62	38

* Output price effect. For the energy supply sector, reports the resource supply effect.
** Sum of the labor supply and wage effects.
*** Rebound, as a percentage. Follows definition (17) except using the general equilibrium savings in place of the partial equilibrium savings and multiplying by 100 to obtain a percentage.
All columns: The energy supply multipliers are included only in the "Total" and "Rebound" calculations.

Table A-2: The percentage change in energy resource use (θ_{R,A_k}) from a 1% improvement in sector k's energy efficiency. Sectors are ordered by total effect. See the main text for the energy supply sector.

	Parameters				Channels			
Sector	σ_k	α_{Rk}	R_k/R	L_k/L	PE	GE*	Total	Reb.**
Other real estate	0.68	0.081	0.178	0.011	-0.05688	0.003009	-0.064812	64
Construction	0.15	0.022	0.058	0.061	-0.04934	0.000122	-0.059218	-2
Food services and drinking places	0.22	0.029	0.039	0.034	-0.03072	0.000213	-0.036704	7
Truck transportation	0.42	0.094	0.055	0.011	-0.03188	0.002053	-0.035887	35
Wholesale trade	0.24	0.011	0.035	0.063	-0.02681	-0.00012	-0.032399	8
Other retail	0.24	0.018	0.031	0.035	-0.02329	0.000015	-0.028009	9
Other transportation and support activities	0.39	0.078	0.031	0.01	-0.01901	0.000905	-0.021783	30
Federal Reserve banks, credit intermediation, and related activities	0.09	0.012	0.019	0.032	-0.01714	-0.000073	-0.02071	-10
Hospitals	0.16	0.011	0.017	0.047	-0.01395	-0.000177	-0.017	-2
Food and beverage and tobacco products	0.19	0.01	0.017	0.013	-0.01359	0.000041	-0.016305	3
Air transportation	0.37	0.063	0.022	0.007	-0.01359	0.000532	-0.015713	27
Farms	0.4	0.031	0.02	0.004	-0.01202	0.000262	-0.014147	29
Food and beverage stores	0.24	0.036	0.015	0.011	-0.01132	0.000132	-0.013464	10
Warehousing and storage	0.39	0.081	0.018	0.006	-0.01128	0.000548	-0.012914	30
Management of companies and enterprises	0.27	0.015	0.014	0.038	-0.01038	-0.000165	-0.012681	11
Accommodation	0.22	0.026	0.012	0.01	-0.00962	0.000076	-0.011479	7
Chemical products	0.72	0.024	0.034	0.012	-0.0094	0.000102	-0.011184	67
Securities, commodity contracts, and investments	0.09	0.01	0.01	0.03	-0.00888	-0.000116	-0.010826	-11
Educational services	0.13	0.016	0.01	0.022	-0.00864	-0.000109	-0.010527	-6
Other services, except government	0.21	0.009	0.011	0.036	-0.00839	-0.000118	-0.010232	4

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	Parameters				Channels			
Sector	σ_k	α_{Rk}	R_k/R	L_k/L	PE	GE*	Total	Reb.**
Miscellaneous professional, scientific, and technical services	0.27	0.004	0.011	0.07	-0.00775	-0.00013	-0.009481	11
Paper products	0.25	0.033	0.01	0.004	-0.00771	0.000154	-0.009085	12
Rail transportation	0.42	0.101	0.014	0.003	-0.00785	0.000547	-0.008782	35
General merchandise stores	0.24	0.023	0.009	0.011	-0.00716	0.000004	-0.008606	9
Motor vehicle and parts dealers	0.24	0.016	0.009	0.014	-0.00677	-0.000029	-0.008177	8
Ambulatory health care services	0.16	0.004	0.008	0.064	-0.00662	-0.000145	-0.008135	-3
Administrative and support services	0.27	0.006	0.009	0.048	-0.00638	-0.00011	-0.007803	11
Plastics and rubber products	0.18	0.017	0.007	0.006	-0.00562	0.000027	-0.006725	2
Nonmetallic mineral products	0.25	0.033	0.007	0.003	-0.00552	0.000097	-0.006528	11
Rental and leasing services and lessors of intangible assets	0.18	0.01	0.006	0.005	-0.00491	0.000015	-0.005895	2
Funds, trusts, and other financial vehicles	0.09	0.018	0.005	0.0002	-0.00467	0.000072	-0.005538	-8
Broadcasting and telecommunications	0.15	0.003	0.005	0.013	-0.00408	-0.000015	-0.004921	-3
Amusements, gambling, and recreation industries	0.27	0.021	0.005	0.006	-0.00388	0.000001	-0.004661	12
Motor vehicles, bodies and trailers, and parts	0.16	0.004	0.004	0.009	-0.00374	-0.000008	-0.004509	-1
Nursing and residential care facilities	0.16	0.009	0.004	0.016	-0.00316	-0.00006	-0.003879	-3
Transit and ground passenger transportation	0.42	0.04	0.005	0.003	-0.00288	0.000054	-0.003401	32
Machinery	0.2	0.005	0.003	0.011	-0.00247	-0.000021	-0.002992	3
Wood products	0.21	0.013	0.003	0.003	-0.00203	0.000004	-0.002437	5

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	Parameters				Channels			
Sector	σ_k	α_{Rk}	R_k/R	L_k/L	PE	GE*	Total	Reb.**
Data processing, internet publishing, and other information services	0.27	0.004	0.002	0.008	-0.00163	-0.000011	-0.001968	12
Social assistance	0.27	0.006	0.002	0.013	-0.0015	-0.000033	-0.001844	10
Printing and related support activities	0.25	0.014	0.002	0.003	-0.00141	-0.000006	-0.001707	9
Computer systems design and related services	0.27	0.002	0.002	0.031	-0.00125	-0.000034	-0.001545	10
Performing arts, spectator sports, museums, and related activities	0.27	0.005	0.002	0.007	-0.00113	-0.000013	-0.001369	11
Miscellaneous manufacturing	0.18	0.005	0.001	0.006	-0.00112	-0.000013	-0.001367	0.2
Waste management and remediation services	0.27	0.009	0.002	0.004	-0.00113	-0.000008	-0.001363	12
Textile mills and textile product mills	0.28	0.017	0.002	0.001	-0.00111	0.000003	-0.001331	14
Other transportation equipment	0.16	0.002	0.001	0.009	-0.00101	-0.000011	-0.001224	-2
Insurance carriers and related activities	0.09	0.001	0.001	0.032	-0.00093	-0.000011	-0.001138	-11
Publishing industries, except internet (includes software)	0.27	0.002	0.001	0.016	-0.00088	-0.000014	-0.00107	11
Furniture and related products	0.21	0.007	0.001	0.003	-0.00068	-0.000006	-0.000821	4
Forestry, fishing, and related activities	0.4	0.011	0.001	0.003	-0.00062	-0.000008	-0.000751	27
Legal services	0.27	0.001	0.001	0.015	-0.0005	-0.000008	-0.000612	11
Motion picture and sound recording industries	0.27	0.003	0.001	0.004	-0.0005	-0.000005	-0.000607	11
Apparel and leather and allied products	0.19	0.004	0.0002	0.001	-0.00014	-0.000002	-0.000169	1

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	Parameters				Channels			
Sector	σ_k	α_{Rk}	R_k/R	L_k/L	PE	GE*	Total	Reb.**
Housing	0.68	0.00005	0.0002	0.002	-0.00005	0	-0.000066	61
Electrical equipment, appliances, and components	1.06	0.005	0.001	0.004	0.00006	0.000002	0.000077	107
Fabricated metal products	1.01	0.01	0.006	0.012	0.00006	0.000004	0.000077	101
Computer and electronic products	1.06	0.002	0.001	0.018	0.00006	0.000004	0.000079	108
Primary metals	1.01	0.028	0.011	0.004	0.00011	-0.000025	0.000104	101
Water transportation	1.27	0.052	0.004	0.001	0.00116	-0.000068	0.001308	131

* The "GE" channel sums the output price and wage effects.

** Rebound, as a percentage. Follows definition (17) except using the general equilibrium savings in place of the partial equilibrium savings and multiplying by 100 to obtain a percentage.

All columns: The energy supply multipliers are included only in the "Total" calculations.